Photoelasticity-Based Study of Stress-Strain State in the Area of the Plain Domain Boundary Cut-Out Area Vertex

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Abstract. In the article, the local stress-strain state of structures and constructions is investigated, various variants for the design of the boundary are taken into account: special lines, points. The acting forced deformations don't satisfy the compatibility conditions. They have a finite discontinuity along the contact line (surface) of the domains, including the irregular point of the boundary, causing stresses. The subject of article is stress concentrators - the singularity of the stress-strain state of structures and constructions exhibiting "constructive heterogeneity" and discontinuous forced deformations determined on polymer models of photoelasticity and defrosting of deformations. A complex theoretical-numerical-experimental approach, for obtaining and analyzing the stress state in the neighborhood of the irregular point of the plane domain boundary, is proposed to extrapolate reliable experimental data to a domain where the fringe contour is not readable.

Keywords: Stress concentrators · Stress-Strain state
Transport Buildings and Structures

1 Introduction

The stress-strain state (SSS) of structures and constructions in the domain of significant constructive heterogeneity (incoming angles, special lines, points) is characterized by the emergence of stress concentration. Acting forced deformations, that don't satisfy the compatibility conditions, have a finite discontinuity along the contact line (surface) of the domains, entering the irregular point of the boundary, and cause stresses. The shape of the boundary and the final discontinuity of the given forced deformations determine the appearance of a singularity of the stress-strain state of the structures.

The relevance of the of investigation of stress from the action of incompatible deformations arises when investigating the SSS of structural elements under the action of temperature gradients, the temperature changes in joints of dissimilar materials with different coefficients of thermal expansion, abrupt changes in distortions having a finite discontinuity in joints of domains with different mechanical properties, as well as the stresses from the installation and the sequence of fabrication of structures, force fit, etc.

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The main objective of the study - to obtain with the strict certainty a local stress-strain state in the vertex domain of the angular cut-out of the domain boundary into which the forced deformations gap leaves. The singularities of the SSS of structures and constructions with "constructive heterogeneity" and discontinuous forced deformations are determined as stress concentrators on polymer models of photoelasticity and defrosting of free temperature deformations [1–7]. Fundamentals of the method of photoelasticity are given in fundamental works [1–5].

2 Materials and Methods

We consider a plane problem of the theory of elasticity in the neighbourhood of an irregular boundary point [8–19], into which the contact line of the domains with the jump in the values of the forced deformations comes out. A homogeneous or piecewise-homogeneous body, being in a plane stress state, has an angular point at the boundary. Along the contact boundary $\gamma = \gamma_1 \cup \gamma_2$ of Ω_1 and Ω_2 domains, components of the elastic body, forced deformation and volume forces have a jump (finite discontinuity) as follows:

$$\Delta\varepsilon_{ij}^b = \varepsilon_{ij}^b\Big|_{\gamma_2} - \varepsilon_{ij}^b\Big|_{\gamma_1}; \quad \Delta F_i = F_i\big|_{\gamma_2} - F_i\big|_{\gamma_1}; \quad i,j = x,y$$

Young's modulus, Poisson's coefficients, coefficients of linear expansion of Ω_1 and Ω_2 domains are constant and different: $E_1, v_1, \alpha_1, i,j \in \Omega_1$ and $E_2, v_2, \alpha_2, i,j \in \Omega_2$, respectively.

Boundary conditions in the neighborhood of an irregular point $\mathrm{O}(0,0)$ on boundary L_0 are homogeneous. We consider a small neighborhood of an irregular point $\mathrm{O}(0,0)$ of the part B of an elastic body: $x^2 + y^2 < \varepsilon_1^2$; $z < \varepsilon_0$, $\varepsilon_1, \varepsilon_0$ – are positive small numbers. We use the following similarity group:

$$x_1 = tx; \quad y_1 = ty; \quad z_1 = z$$

$$\sigma_{ij} = t\bar{\sigma}_{ij}; \quad \varepsilon_{ij} = t\bar{\varepsilon}_{ij}; \quad U_i = \bar{U}_i$$

The resolving system of equations of the plane problem of the theory of elasticity in new variables in this neighborhood will be rewritten as follows:

$$\sum_{j1} \frac{\partial \bar{\sigma}_{ij}}{\partial j1} + \frac{1}{t^2} F_j = 0 \quad \bar{\varepsilon}_{ij} = \frac{\partial \bar{U}_i}{\partial j1} + \frac{\partial \bar{U}_j}{\partial i1}, \quad i,j \in \Omega_1, \Omega_2 \tag{1a}$$

$$\sum_{j1} \bar{\sigma}_{ij} n_{j1}\big|_{L_0} = 0; \qquad \sum_{j1} \bar{\sigma}_{ij} n_{j1}\big|_{\gamma\mathrm{B}} = \frac{1}{t}\sigma_{in}^b\big|_{\gamma\mathrm{B}} \tag{1b}$$

$$\bar{\sigma}_{in}\big|_{\gamma_1} = \bar{\sigma}_{in}\big|_{\gamma_2}; \quad \bar{U}_i\big|_{\gamma_1} = \bar{U}_i\big|_{\gamma_2} \tag{1c}$$

$$\bar{\varepsilon}_{ij} = \frac{1}{E_k}\left[(1+\nu_k)\bar{\sigma}_{ij} - \nu_k \bar{S}\delta_{ij}\right] + \frac{1}{t}\varepsilon^b_{ij} + \frac{1}{t}\alpha_k T\delta_{ij} \tag{1d}$$

when $k = 1, (x_1, y_1) \in \Omega_1; \quad k = 2, (x_1, y_1) \in \Omega_2, \; j1, i1 = x_1, y_1, \quad n_{j1} = n_j$ - is a normal to the contact line of the domains $\gamma = \gamma_1 \cup \gamma_2$, γB - is a domain boundary, containing a neighbourhood $O_\delta(0)$ of an irregular boundary point.

Depending on move away or moving closer to the irregular boundary point, which is determined by the change in the geometric parameter t, the form of the resolving system of Eqs. (1) in $O_\delta(0)$ varies as follows:

(a) With an unlimited increase in the geometric parameter t = $\frac{\varepsilon_1}{\varepsilon_2} \to \infty$, $\varepsilon_1 << \varepsilon_2$ the plane problem of the theory of elasticity of a piecewise homogeneous body $(E_{\mathrm{k}}, \nu_k, \alpha_k, k = 1, 2)$ with given loads (forced deformations, temperature deformations, volumetric forces, in the neighborhood of an irregular point $O(0, 0)$ of a boundary) is reduced to a homogeneous boundary value problem $\bar{\xi}^{\mathrm{E}} = (\bar{\sigma}^E_{ij}, \quad \bar{\varepsilon}^E_{ij}, \quad \overline{U}^E_i)$ for a piecewise-homogeneous body with homogeneous boundary conditions (canonical, singular).

The solution of the obtained homogeneous boundary-value problem with homogeneous boundary conditions characterizes the singularity of the SSS at an irregular point $O(0, 0)$ and its neighborhood, depends on the given form of the boundary, type of homogeneous boundary conditions and the values of the mechanical characteristics of the material $(E_{\mathrm{k}}, \nu_k, k = 1, 2)$. A nontrivial solution of the obtained homogeneous problem is defined as a eigensolution.

(b) When $t \to 1$, the system of Eqs. (1) coincides with the initial one taken under the action of given loads. When $t \to 1$, SSS $\xi^S = (\sigma^S_{ij}, \quad \varepsilon^S_{ij}, \quad U^S_i)$ is conditioned by given loads and boundary conditions.

(c) In the intermediate range of variation of the parameter values $t \in (1 + \alpha, N)$, where $N > 0$ - is sufficiently large, $\alpha > 0$ - is quite small, it operates both like own SSS $\bar{\xi}^E$ and SSS given from the loads $\bar{\xi}^S$ of the system (1).

According to the considered cases of change in the parameter t, the solution of the plane problem of a piecewise homogeneous body in a neighborhood of an irregular boundary point can be represented in the following form:

$$\bar{\sigma}_{ij} = \bar{\sigma}^E_{ij} + \bar{\sigma}^S_{ij}, \bar{\varepsilon}_{ij} = \bar{\varepsilon}^E_{ij} + \bar{\varepsilon}^S_{ij}, U_i = \bar{U}_i = \bar{U}^E_i + \bar{U}^S_i, \text{ or } \bar{\xi} = \bar{\xi}^E + \bar{\xi}^S$$

when $\bar{\xi}^E = (\bar{\sigma}^E_{ij}, \bar{\varepsilon}^E_{ij}, \bar{U}^E_i)$ - is a singular (eigen) solution of a homogeneous boundary value problem that characterizes the singularity of the SSS in the neighbourhood of an irregular boundary point; $\bar{\xi}^S = (\bar{\sigma}^S_{ij}, \bar{\varepsilon}^S_{ij}, \bar{U}^S_i)$ - is solution of system (1), caused by the effect of the action of given loads of the following form:

$$F_i^a = \frac{1}{t^2} F_i; \quad \sigma_{in}^b \Big|_{\gamma B} = \frac{1}{t} \sigma_{in}^b \Big|_{\gamma B}, \varepsilon_{ij}^a = \frac{1}{t} \varepsilon_{ij}^b + \frac{1}{t} \alpha_k T \delta_{ij}$$

and also the influence of the action of the jump of the forced deformations and the volume forces along the line of contact $\gamma = \gamma_1 \cup \gamma_2$ of Ω_1 and Ω_2 domains:

$$\Delta F_i^a = \frac{1}{t^2} \left(F_i|_{\gamma_2} - F_i|_{\gamma_1} \right) = \frac{1}{t^2} \Delta F_i,$$

$$\Delta \varepsilon_{ij}^a = \frac{1}{t} \left(\varepsilon_{ij}^b \Big|_{\gamma_2} - \varepsilon_{ij}^b \Big|_{\gamma_1} \right) = \frac{1}{t} \Delta \varepsilon_{ij}^b,$$

$$\Delta \varepsilon_{ij}^{a1} = \frac{1}{t} \left(\alpha_2 T|_{\gamma_2} - \alpha_1 T|_{\gamma_1} \right) \delta_{ij} = \frac{1}{t} \Delta\alpha \, \Delta T \, \delta_{ij}.$$

Considering the relationship between the terms in the SSS representation: $\xi = \xi^E + \xi^S$ in the neighbourhood of the irregular point of the boundary, we can distinguish the following specific areas of SSS action:

(a) There exists a neighborhood of the irregular point of the boundary of a plane domain in which the singular solution of the homogeneous boundary value problem is true, characterizes that $\sigma_{ij} \to \sigma_{ij}^E, \sigma_{ij}^S \to 0$. The singularity of inner stresses σ_{ij}^E (deformation ε_{ij}^E) has a power-law form $r^{\mathrm{Re}\,\lambda - 1}$, $\lambda \in [0, 0.5]$. The fringe order in the domain of the stress concentrator on the model (the domain of the singular solution) are not readable for any increase in the neighbourhood of the irregular point.

(b) There exists a neighborhood of an irregular point of the boundary of the domain in which $\sigma_{ij} \approx \sigma_{ij}^E$, $\sigma_{ij}^S \approx 0$ and the nonsingular homogeneous elastic problem with the same "eigenvalue" value $\min \mathrm{Re}\,\lambda$ is valid as is in the singular problem. The domain of a nonsingular solution does not contain a neighbourhood of the singular solution and the irregular point itself, but adjoins it. When approaching from the outside to the boundary of the domain of a singular solution, the stress and the deformations change continuously, their values are large, but finite. The fringe order on the model corresponding to the nonsingular domain of the solution are readable with some possible exceptions.

(c) If there is a sufficient distance from the irregular point of the boundary, there exists a domain in which $\sigma_{ij} = \sigma_{ij}^S, \sigma_{ij}^E = 0$ and stresses are caused by the given loads (common stress field).

It is possible to give estimates in the domain of a nonsingular solution of a homogeneous planar elastic problem. While using them, it is possible to extrapolate the data to sections close to the irregular point of the boundary, taking into account the experimental data and the practical accuracy of the data measurement by the photoelasticity.

Analyzing the stress state at the vertex of a rectangular wedge by the example of the known [2] experimental solution of Frocht (Fig. 1), choosing the domain of the nonsingular solution (m = 7), which adjoins the domain of the singular solution, the load value is restored. On the basis of this, it is concluded that it is possible to restore the fringe order in a small neighborhood of the wedge vertex according to the practical possibility of the experimental solution and the accuracy of photoelasticity within the limits of linear elastic statement.

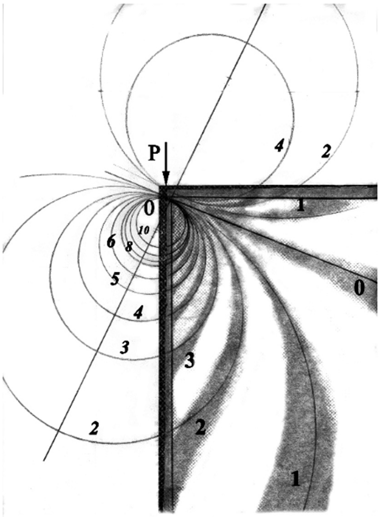

Fig. 1. Diagrams of the fringe orders according to the experimental data. The fringe orders for the right angle was obtained in [2].

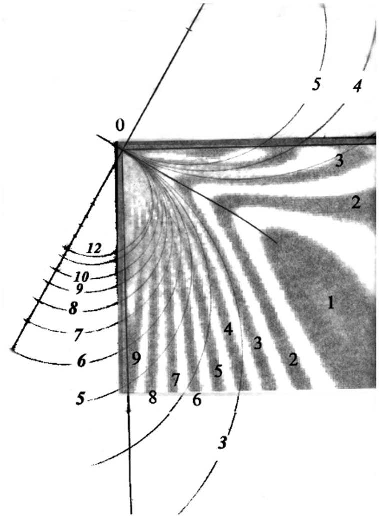

Fig. 2. A comparison of the theoretical and experimental fringe contour for the domain of the straight end of the beam

The experimental solution of the thermoelastic problem is considered for a beam model in one of thedomains with created free temperature deformations $\alpha T\delta_{ij}$, and the other domain is free of loads. The jump of forced deformations $\Delta\varepsilon_{ij} = \alpha T\delta_{ij}$ on the contact line of the domains making up the model, goes to an irregular point O $(0, 0)$ of the boundaries of the straight end of the beam. The fringe contour obtained by the "defrosting" method for one of the beam domains is shown in Fig. 2. The inner stresses in the neighborhood of the irregular point of the boundary of the straight end of the beam have the following form:

$$\sigma_r = \frac{\alpha ET}{r}(c_1 \cos\,\theta + c_2 \sin\,\theta); \quad \sigma_\theta = \tau_{r\theta} = 0$$

The singularity of the inner radial stresses in the domain of the straight end of the beam (Fig. 2) is the same as the singularity of radial stresses for a rectangular wedge (Fig. 1) under the action of force in problem of Frocht, when. Therefore, the isochrome contour of Fig. 1, corresponding to the radial stresses in the experimental solution of Frocht [2], is a picture of the inner radial stresses in the neighbourhood of the irregular boundary of the straight end of the beam of Fig. 2.

The diagrams of fringe order (isochrom) for several radial sections, plotted in the domain of the straight end of the beam, are shown in Fig. 3. The similarity is established for the fringe order [20, 21]. From the experimental data, the calculated cross section in the domain of the nonsingular solution of the homogeneous problem (d-d) is chosen. Taking into account the continuity and similarity of the changes in fringe order in the cross section (k-k), isochrome order diagrams of (Fig. 3) are constructed, as well as made on their basis radial stresses diagrams (Fig. 4). The section (k-k) is located in the domain of the singular solution, where fringe contour is not readable or is "badly" readable.

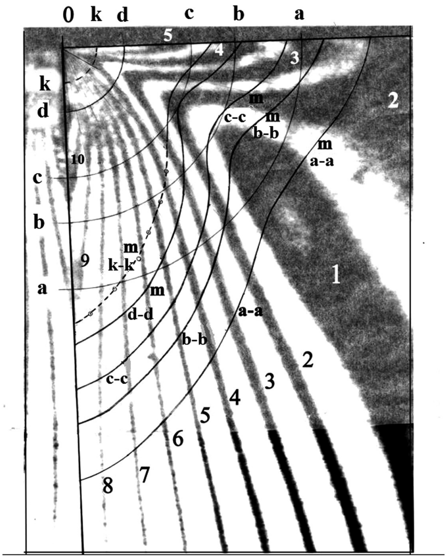

Fig. 3. Diagrams of fringe order according to the experimental data. The section d-d is the calculated section.

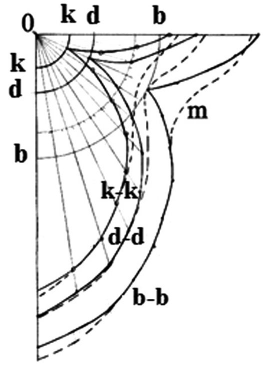

Fig. 4. Diagrams of fringe order and radial stresses (dashed lines) in the domain of the straight end of the beam.

3 Results

According to the theoretical-experimental analysis, the stress state in the neighborhood of the irregular point of the boundary of the plane domain into which the jump of the forced deformations comes out, the following formula is proposed for extrapolating the experimental data:

$$m_{i+1} = \left(\frac{r_i}{r_{i+1}}\right)^{1-\lambda_0} m_i \tag{2}$$

when m_i - is a fringe order from the experimental data in the calculated cross section r_i in a neighbourhood of a nonsingular solution of a homogeneous boundary value problem, m_{i+1} - is a fringe order in the cross section of smaller radius $r_{i+1} < r_i$, located in a domain with not readable or "badly" readable isochrome contour of the model, $\lambda_0 = \min \mathrm{Re}\,\lambda$ - the minimal value of the real part of the complex root of the characteristic equation of a homogeneous boundary value problem for a model wedge.

4 Discussion

The article presents a solution to the problem of obtaining the local stress-strain state with the greatest certainty in the domain of the vertex of the corner cut-out of the boundary of the domain into which the rupture of forced deformations occurs.

The considered singularities of the stressed state are relevant for structural elements working with temperature changes in the joints of dissimilar materials with different coefficients of thermal expansion, during the installation, the sequence of fabrication of structures, force fit.

The presented complex investigations of the stress state in the neighborhood of the irregular point of the domain boundary allows us to compare the data of numerical, analytical and experimental methods and to reveal the advantages and disadvantages of each of them.

The singularity of stresses and deformations due to the corner cut-out of the boundary of the domain is determined by the idealization of the statement of the linear problem of the theory of elasticity in the neighborhood of the irregular point of the boundary.

5 Conclusions

The theoretical-numerical-experimental analysis of the stress state in the neighborhood of the irregular point of the boundary of a plane domain, in which a finite discontinuity (jump) of the forced deformations occurs, and the extrapolation formula of the experimental data (2) proposed on its basis, allow us to restore the fringe order in the domain of the singular solution of the elastic problem, in which isochromes on the model are not readable or "badly" readable. The nearness of the cross section to the irregular point of the boundary is due to the linear-elastic statement of the problem, the accuracy of measuring the experimental data and the practical accuracy of photoelasticity method.

References

1. Hesin, G.L., et al.: The photoelasticity method, vol. 3, p. 311. Stroyizdat, Moscow (1975)
2. Frocht, M.M.: Photoelasticity, edited by N.I. Prigorovskiy, vol. 2, p. 432. GITTL, Moscow – Leningrad (1948)
3. Durelli, A., Riley, W.: Introduction to photoelasticity, edited by N.I. Prigorovskiy. Mir, Moscow (1970)

4. Aleksandrov, A.Ja., Ahmetzjanov, M.H.: Polarization-optical methods for deformable solid mechanics, p. 576. Nauka, Moscow (1974)
5. Kobayashi, A.: Experimental Mexhanics, vol. 1, p. 615. Mir, Moscow; vol. 2, p. 551. Mir, Moscow (1990)
6. Koshelenko, A.S., Poznjak, G.G.: Theoretical foundations and practice of photomechanics in mechanical engineering, p. 296. Granica, Moscow (2004)
7. Razumovskij, I.A.: Interference-optical methods of deformable solid mechanics, p. 240. Publisher MGTU named after N Je Bauman, Moscow (2007)
8. Kuliev, V.D.: Singular boundary value problems, p. 719. Nauka, Moscow (2005)
9. Netrebko, V.P.: Photoelasticity method-based study of stress intensity factors near inclined cracks in orthotropic plates, pp. 69–77. Mechanical Engineering Research Institute of the Russian Academy of Sciences, Moscow (2003)
10. Ahmetzjanov, M.H., Tihomirov, V.M., Surovin, P.G.: News of higher educational institutions. Construction **1**, 19–25 (2003)
11. Zadojan, M.A.: Bulletin of the Russian Academy of Science. Mechanics of Rigid Body, vol. 1, pp. 111–122 (2003)
12. Xu, L.R., Kuai, H., Sengupta, S.: Exp. Mech. **44**, 608–615 (2004)
13. Xu, L.R., Kuai, H., Sengupta, S.: Exp. Mech. **44**, 616–621 (2004)
14. Yavari, A., Sarkani, S., Moyer, E.T.: J. Appl. Mech. **69**, 65–72 (2002)
15. Yao, X.F., Yeh, H.Y., Xu, W.: Int. J. Solid. Struct. **43**, 1189–1200 (2006)
16. Frishter, L.Ju., Vatanskij, V.A.: Vestnik MGSU **8**, 51–58 (2013)
17. Pestrenin, V.M., Pestrenina, I.V., Landik, L.V.: Vestnik TGU. Math. Mech. **4**, 80–87 (2013)
18. Matveenko, V.P., Nakarjakova, T.O., Sevodina, N.V., Shardakov, I.N.: Stress singularity at the top of homogeneous and composite cones with different boundary conditions. J. Math. Mech. **72**, 477–484 (2008)
19. Albaut, G.N., Kharinova, N.V., Sadovnichij, V.P., Semenova, J.I., Fedin, S.A.: Nonlinear problems of mechanics of destruction. Vestnik of Lobachevsky University of Nizhni Novgorod, vol. 4, pp. 1344–1348 (2011)
20. Vardanjan, G.S., Frishter, L.Ju.: Int. J. Comput. Civil struct. Eng. **3**, 75–81 (2007)
21. Frishter, L.Ju.: Vestnik MGSU **1**, 169–174 (2008)