Geometry Description of Local Scouring Process in Various Laboratory Water Structure Models

Marta Kiraga and Zbigniew Popek

Abstract Water structures are usually used to stabilise river bed, although they also are a cause of degradation processes like erosion and scouring. The aim of this study was to correlate scour geometry, characterized by the scour medium and maximal depth and its length with hydraulic and granulometric properties. The study presents results obtained for three laboratory models (with and without weir flow) with partially or totally erodible bed, with median grain size $d_{50} = 0.91$ mm in clear-water and live-bed conditions. The experiment duration was sufficient to obtain equilibrium, stable scour shape. Analysed relationships were parameterized using linear and exponential functions. The intensity of the sediment transport was investigated using the modified principle of fluvial hydraulics—the Lane's relation, originally derived from basic rules of the sediment transport as a qualitative expression.

1 Introduction

Water structures are usually used to stabilise river bed, although they also are a cause of degradation processes like erosion and scouring. The downstream erosion not only causes morphological changes, but also could be dangerous for structure safety because of the erosion of foundations and loss of structure stability (Petts 1977).

Many studies have been devoted to the problem of local scouring near different hydraulic structures. Laboratory experiments allowed to develop many formulas and more complex numerical models of this process. However, all the methods used in practical applications are still very inaccurate, because of the complexity of the

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transport processes and the fact that there is no general equation, so far, for describing a local scour geometry. Some attempts using real *in situ* data (Lenzi et al. 2003; Pagliara et al. 2016) revealed the problem of unknown initial conditions, such as the shape of a river bed before the scour was formed.

A comprehensive analysis was performed for mountain rivers—the study dealing with scour process analysis occurring downstream of block ramps located in river bends was presented by Pagliara et al. (2016). Studies proved that the river curvature is the most important parameter affecting the maximum scour depth occurring downstream the structure. Mountain river experiments indicated differences even up to 300% between laboratory measurements and results of calculations with scour forecasting formulas (Ślizowski and Radecki-Pawlik 2003).

The case of local scouring in a bed made of sequences of sills is well recognized and described in the literature. Gaudio et al. (2000) used dimensional analysis to obtain predictive formulae for clear-water conditions of scour hole dimensions for rivers with a moderate slope. It was emphasized that maximum scour depth and the scour hole length could be described as functions of 'morphological jump' (defined as the multiplication outcome of difference between initial and equilibrium bed slopes and the distance between subsequent sills), critical specific energy, relative submerged grain density and median grain size with r = 0.98 correlation coefficient. Gaudio et al. (2000) attempt was confirmed by others (Lenzi et al. 2002) for mountain high-gradient streams (for non-uniform sediments with presence of protruding cobbles), however the best fit to experimental data was obtained for the non-linear equation, both for scour length and maximum scour depth, with the 0.99 and 0.95 correlation coefficients. Other experiment results (Gaudio and Marion 2003) confirmed the validity of Gaudio's method for maximum depth of scour assessment for mild bed slopes, with correlation coefficient 0.94-the outcome of the study was a linear formula describing maximal scour depth geometry using the hydraulic, morphological and granulometric variables proposed by Gaudio et al. (2000). Although Gaudio's experiment gave good results, the accuracy of the proposed methods is still inadequate in other experimental flume model variants (Ben Meftah and Mossa 2006). The general conclusion for bed sills sequence experiments was that the maximal scour depth and length strongly depend on the distance between sills and that maximal scour depth and length are not correlated (Ben Meftah and Mossa 2006; Gaudio et al. 2000; Gaudio and Marion 2003; Lenzi et al. 2002).

It was shown that for mild energy grade line slopes the maximal scour depth could be described as a function of dimensionless shear stress with correlation coefficient of 0.67 for laboratory flumes with a rectangular cross-section, in which the solid bottom transforms into sandy bottom in the intake part. Due to the increase of flow resistance on the whole length of the bed, resulting from varied roughness of the solid and sandy bottom, the hydraulic gradient increases causing an increase of shear stress on the bottom (Kiraga and Popek 2016). The present paper comprises an extension of this experimental case.

All the above-mentioned experiments (Gaudio et al. 2000; Lenzi et al. 2002; Gaudio and Marion 2003; Ben Meftah and Mossa 2006; Kiraga and Popek 2016)

were conducted with no sediment feeding system adopted, namely, in clear water conditions. Live-bed conditions case is still an undeveloped branch of experimental research, the present paper attempts to fulfil.

The aim of this paper is to complement Gaudio et al. (2000) and Kiraga and Popek (2016) attempts, in a scope of three experimental test stands with or without hydraulic structure (stone weir) in the laboratory flume for moderate energy grade lines, including live-bed conditions. Besides the maximal scour depth and length, the medium scour depth was also included. It was an attempt made to find functional correlations between hydrodynamic parameters of water and sediment discharge and geometrical parameters of local scour holes in various physical models. Inter alia, modified cause-and-effect Lane's relation was included, commonly used to describe river and valley morphology formation.

2 Methods

2.1 Initial Assumptions

For clear water conditions, Gaudio et al. (2000) proposed the following formulas:

$$\left(\frac{z_{max}}{H_s}\right) = c \frac{a_1}{sd_{50}} + d,\tag{1}$$

$$\left(\frac{l_s}{H_s}\right) = f \frac{a_1}{sd_{50}} + j,\tag{2}$$

where c, d, f, j stand for linear coefficients, z_{max} the maximal scour depth (in equilibrium conditions) (m), l_s for scour hole length (m), s for relative particle density (–) and d_{50} for median grain size (m). $H_s = 1.5 \sqrt[3]{q^2/g}$ stands for the critical specific energy in proximity of the water structure (m) with q for unit discharge (m³ s⁻¹ m⁻¹) and g for gravity acceleration (m s⁻²). $a_1 = (S_0 - S_{eq})L$ stands for the morphological jump, S_0 and S_{eq} denote, respectively, the initial and the final longitudinal bed slope (–) and L is the distance between two successive sills (m). Equations (1) and (2) are recognized to be valid in the range $1.3 \le a_1/(s d_{50}) \le 9.1$ with the correlation coefficient r = 0.98. For the stone weir investigated in the present paper, the above equations were modified—'morphological jump' was substituted by 'energy grade line jump', using total working section length instead of the distance between the sills, and the initial and equilibrium longitudinal bed slopes.

The original Lane's relation (Lane 1955), $Q_s d \sim Q_w S$, involves bedload transport discharge Q_s , particle diameter d, water discharge Q_w and energy grade line slope S and comprises a qualitative expression because of incompatibility of

units. Despite its simplified character, original Lane's principle serves well to describe the process of forming alluvial stream channels. Additionally, it was demonstrated on physical model in which the solid bottom transforms into sandy bed in the intake part in clear-water conditions, that it is possible to correlate multiplication outcome of bedload transport discharge, grain parameter $D_* = d_{50}[(s-1)g/\nu^2]^{1/3}$ (–) (Van Rijn 1993), considering kinematic viscosity parameter ν (m² s⁻¹) and width-to-depth (*W*/*H*) ratio with multiplication outcome of water discharge and equilibrium longitudinal energy grade line slope *S* with r = 0.95 correlation coefficient (Kiraga and Popek 2016):

$$Q_s D_* (W/H)^{-1} = 0.158 (Q_w S) - 1.459.$$
(3)

Relationship (3) was examined in praxis for the water maximum discharge of $5.06 \text{ m}^3 \text{ s}^{-1}$ during a flood event in Zagożdżonka River (Poland) on the grounds of in situ measurements. The river channel properties were similar to laboratory model —the solid bottom precedes sandy bed, and the formation of local scour takes place in clear-water conditions. The results are satisfactory because of the same order of magnitude (Kiraga and Popek 2016). It was established that miscalculation could result from the lack of knowledge of exact bottom shape before the flood occurrence, the variability of water surface level during flood conditions and variable energy grade line slope in natural flow conditions. The newly introduced form of Eq. (3), including local scour geometry properties (medium scour depth z_m and l_s scour length) eventually takes the form of linear empirical equation with parameters *a* and *b*:

$$Q_{s}D_{*}(W/H)^{-1}\left(\frac{z_{m}}{l_{s}}\right) = aQ_{w}(S-S_{0}) + b.$$
(4)

It was also demonstrated that maximal scour depth could be expressed as a function of dimensionless shear stress in bed region, $z_{max} = f(\theta)$ (with r = 0.67 correlation coefficient, as a linear function) (Kiraga and Popek 2016); however, re-examination of data suggested better correlation if function would be expressed as exponential (correlation coefficient r = 0.82). To determine the hydraulic radius of the sandy bottom part in the chosen cross-section, necessary for bed shear stress calculations, the Einstein's division of the velocity field was used (for detailed description, see Kiraga and Popek 2016).

It was investigated if it is possible to delineate the non-linear formula including medium scour depth-to-length ratio. The function would eventually take the following form, with exponential function parameters k and m:

$$\left(\frac{z_m}{l_s}\right) = k e^{m\theta},\tag{5}$$

Equations (1), (2), (4) and (5) were analysed using three models of the flume structure.

2.2 Experimental Set-up

The research of local scours properties was conducted in the hydraulic laboratory of Warsaw University of Life Sciences (Poland). The experiment was performed on three test stands with and without water structure and it included 13 runs for each stand. The flume of the rectangular cross-section was 0.58 m wide and 0.60 m high. For the full description of the laboratory flume and hydraulic system, see Kiraga and Popek (2016). The sediment used in all tests was sand with the median diameter $d_{50} = 0.91$ mm and relative submerged density s = 2.61.

The first model (I) had the following bottom construction: about 4-meter long solid bottom that transforms in the intake part into the sandy bottom, long for L = 2.18 m (Fig. 1a). The experimental conditions in this study may be compared to a case of the transport continuity being disrupted by the accumulation of the bedload material in the retention reservoir located in the upstream (Dust and Wohl 2012; Huang et al. 2014). Due to the increase of flow resistance along the whole flume, resulting from varied roughness of solid and sandy bottom, the hydraulic gradient increases, also causing the increment of shear stress at the bottom. After exceeding the critical shear stress, the motion of sediment grains starts, followed by gradual scouring of the bed. The experiment was carried out until the scour hole shape stabilized. Then, the local scour hole obtained its equilibrium length l_s , medium depth in clear-water scour conditions z_m and maximal depth z_{max} (Kiraga and Popek 2016).

In the second (II) and third model (III), water structure was introduced in the flume with a totally or partially sandy bed (Fig. 1b, c); the weir was made of stone with four slots (summary area of slots $A_{sII} = 0.018 \text{ m}^2$ and $A_{sIII} = 0.035 \text{ m}^2$). The weir was situated downstream the 4.0 m section of the sandy bed. In the third model, a 0.35 m long section of the bottom, immediately downstream the weir was reinforced. Upstream the weir, the thickness of the sand layer was 5 cm, while downstream it was 25 cm. Sandy bed above the structure was washed out by the approaching flow and the sediment load was moved out from the upper part towards the lower part of the water structure, ensuring live-bed conditions. Velocity of the flowing water in the structure's slots is increased because of flow area reduction. Therefore, in this case, the shear stress reinforcement is a result not only of roughness variability of material, but also of the velocity increment. Local scour was allowed to form gradually, until live-bed equilibrium conditions were obtained, just to achieve a stable shape, characterized by geometrical parameters such as length l_s , medium and maximal depth z_m and z_{max} (Fig. 4).

During an experimental research, there were few cases in which the scour length l_s had seemed to be slightly longer than the total length of erodible part of the bed



Fig. 1 Experimental channel schemes: model I (a); model II (b), and model III (c) (all dimensions in meters), where: A—solid bottom; B—working section with washout bed (sandy); C—stone weir (with or without solid bottom); a—pin water gauge; b—disc probe; c—collection chamber; d —glass panels, e—the regulatory gate

L (Fig. 2)—i.e. the scour hole would have been longer if erodible bed had been longer too. It such cases, its value was extrapolated.

All four equations' parameters were identified for each experiment (Eqs. 1, 2, 4, and 5). Tables 1, 2 and 3 report conditions of each run: water discharge Q_w and water depth H in control profile, situated in the nearest region of the weir (in D-D cross section), observed directly after reaching equilibrium conditions (during total time of experiment t_i). After the flume was drainaged, the volume of sand captured in the collection chamber V_t was measured, providing information on the total volume of scour, that was transfromed into sediment transport discharge by dividing it by the total time. Besides the bed shape and water surface elevation, allowing initial and equilibrium energy grade slope line appointment S_0 and S, also temperature T was measured in each time step during each experiment and the kinematic viscosity parameter ν was assigned (Kiraga and Popek 2016). The S slope was specified while obtaining stable scour shape, i.e. when local scouring equilibrium was ensured.



Fig. 2 Scheme of local scour forming in washable area of sandy bed in model III, where: A—solid bottom; B—sandy washout bed; C—stone weir; H_1 , H_2 , H_3 —water depths; Q_w —water flow discharge; S_1 , S_2 —energy grade line slope; z_{max} —maximal depth of local scour while achieving stabilization in time t_t with s_t scour shape (live-bed equilibrium scour depth); s_1 , s_2 , s_r —bed elevation below the weir while duration of experiment in time, *D-D*—computational cross-section, *E-E*—cross section in water structure's region for water velocity calculations

3 Results and Discussion

Four empirical equations were verified in the scope of three test stands with 13 experimental runs for each. Two of them (Eqs. 1 and 2) were originally derived from the relations specified for bed sills sequences. Because the hydraulic structure inserted into the flume was a single stone weir, there was a necessity to modify original equations. The third of them (Eq. 4) is modified Lane's relation, applicable so far in dynamic balance conditions in the context of alluvial rivers, and the fourth (Eq. 5) is an extension for previously conducted experiment (Kiraga and Popek 2016). The obtained relations are shown in Figs. 3, 4, 5 and 6 with regression lines sketched and *r* correlation coefficient remarked. In few experimental series, performed on test stand I, it was noticed that the initial energy grade line slope exceeded the final slope. When a stable scour shape was obtained, the bump was formed in the bed downstream the scour hole, which led to water surface level lifting. Those cases interfered with the general conclusion about the regressions obtained for this test stand.

The following equations were obtained for each test stand:

(1) For test stand I

$$\left(\frac{z_m}{l_s}\right) = 0.003 e^{25.6\,\theta}; r = 0.82.$$
 (6)

(2) For test stand II

$$\left(\frac{z_{max}}{H_s}\right) = 0.145 \frac{a_1}{sd_{50}} + 0.256; r = 0.86;$$
(7)

Table 1	Main paramet	ers of the f	first test case	ſ							
No	Q_w	Н	t_t	V_t	Z_m	Zmax	l_s	S_0	S	Т	ν
	(m ³ s ⁻¹)	(m)	(h)	(m ³)	(m)	(m)	(m)	(-)	(-)	(°C)	$(10^{-6} \text{ m}^2 \text{ s}^{-1})$
-	0.020	0.10	7.25	0.00153	0.0016	0.0042	0.20	0.0005	0.0005	16.8	1.086
2	0.025	0.10	10.50	0.01841	0.0150	0.0341	2.20	0.0008	0.0007	16.5	1.094
3	0.025	0.12	6.50	0.00151	0.0016	0.0041	0.26	0.0004	0.0003	16.1	1.104
4	0.030	0.10	5.00	0.03701	0.0293	0.0474	2.35	0.0013	0.0014	16.5	1.094
5	0.030	0.15	6.00	0.00151	0.0016	0.0047	0.19	0.0004	0.0004	16.7	1.089
9	0.035	0.12	8.50	0.04681	0.0372	0.0564	2.20	0.0011	0.0010	16.3	1.099
7	0.035	0.15	7.50	0.00404	0.0032	0.0217	1.00	0.0005	0.0005	15.9	1.109
8	0.040	0.10	9.25	0.09746	0.0783	0.1182	2.35	0.0012	0.0013	16.0	1.107
6	0.040	0.12	10.50	0.05500	0.0443	0.0928	2.29	0.0010	0.0010	17.2	1.076
10	0.040	0.15	8.00	0.01900	0.0150	0.0423	0.55	0.0008	0.0009	17.0	1.081
11	0.040	0.20	6.00	0.00240	0.0016	0.0059	0.44	0.0002	0.0002	16.8	1.086
12	0.043	0.12	8.50	0.06800	0.0538	0.0764	2.20	0.0013	0.0012	16.6	1.091
13	0.045	0.15	8.50	0.04500	0.0356	0.0629	2.31	0.0008	0.0009	16.0	1.107

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Table 2	Main parame	ters of the	second test	case							
No	ϱ_{w}	Н	t_t	V_t	Zm	Zmax	l_s	S_0	S	Т	v
	$(m^3 s^{-1})$	(m)	(h)	(m ³)	(m)	(m)	(m)	(-)	(-)	(°C)	$(10^{-6} \text{ m}^2 \text{ s}^{-1})$
1	0.020	0.10	8.00	0.00151	0.0016	0.0200	0.78	0.0010	0.0010	16.7	1.089
5	0.025	0.10	8.00	0.01560	0.0076	0.0323	1.60	0.0023	0.0032	16.3	1.099
3	0.025	0.12	8.00	0.00300	0.0025	0.0220	1.20	0.0012	0.0014	15.9	1.109
4	0.030	0.10	8.00	0.04800	0.0133	0.0400	1.80	0.0023	0.0026	16.8	1.086
5	0.030	0.15	8.00	0.00151	0.0124	0.0457	1.70	0.0010	0.0012	16.6	1.091
9	0.035	0.12	8.00	0.16000	0.0491	0.0520	1.80	0.0040	0.0067	16.0	1.107
7	0.035	0.15	8.00	0.00550	0.0040	0.0274	1.40	0.0012	0.0014	16.3	1.099
8	0.040	0.10	8.00	0.13450	0.0440	0.0850	3.10	0.0056	0.0085	15.9	1.109
6	0.040	0.12	8.00	0.20400	0.0644	0.0770	2.70	0.0060	0.0090	16.8	1.086
10	0.040	0.15	8.00	0.02050	0.0110	0.0304	2.00	0.0017	0.0019	16.5	1.094
11	0.040	0.20	8.00	0.00200	0.0042	0.0150	1.00	0.0013	0.0016	16.8	1.086
12	0.043	0.12	8.00	0.19000	0.0634	0.0832	3.00	0.0050	0.0077	16.8	1.086
13	0.045	0.15	8.00	0.05050	0.0063	0.0300	1.91	0.0020	0.0024	16.5	1.094

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No	Q_w	Н	t_t	V_t	Z_m	Zmax	l_s	S_0	S	T	ν
	$(m^3 s^{-1})$	(m)	(h)	(m ³)	(m)	(m)	(m)	(-)	(-)	(°C)	$(10^{-6} \text{ m}^2 \text{ s}^{-1})$
1	0.020	0.10	8.00	0.01000	0.0199	0.0613	1.35	0.0052	0.0063	16.0	1.107
2	0.025	0.10	8.00	0.04800	0.0397	0.0872	2.60	0.0066	0.0082	16.8	1.086
3	0.025	0.12	8.00	0.01400	0.0206	0.0592	1.12	0.0039	0.0047	16.0	1.107
4	0.030	0.10	8.00	0.07480	0.0479	0.0843	2.31	0.0059	0.0073	16.7	1.089
5	0.030	0.15	8.00	0.00040	0.0050	0.0270	0.53	0.0013	0.0015	16.8	1.086
6	0.035	0.12	8.00	0.07900	0.0457	0.0809	2.44	0.0061	0.0075	16.8	1.086
7	0.035	0.15	8.00	0.01340	0.0192	0.0472	1.68	0.0026	0.0027	16.6	1.091
8	0.040	0.10	8.00	0.11200	0.0562	0.0870	2.03	0.0081	0.0092	16.3	1.099
9	0.040	0.12	8.00	0.12400	0.0516	0.0826	2.29	0.0053	0.0066	17.0	1.081
10	0.040	0.15	8.00	0.03600	0.0281	0.0516	1.86	0.0027	0.0028	16.3	1.099
11	0.040	0.20	8.00	0.00065	0.0014	0.0142	0.28	0.0002	0.0003	16.8	1.086
12	0.043	0.12	8.00	0.14452	0.0596	0.0868	2.38	0.0085	0.0099	16.7	1.089
13	0.045	0.15	8.00	0.06000	0.0144	0.0235	1.80	0.0013	0.0015	16.1	1.104

Table 3 Main parameters of the third test case



Fig. 3 Regression of equation $\left(\frac{z_{max}}{H_s}\right) = c_{\frac{a_1}{sd_{50}}} + d$ (Eq. 1)



Fig. 4 Regression of equation $\left(\frac{l_s}{H_s}\right) = f \frac{a_1}{sd_{50}} + j$ (Eq. 2)

$$\left(\frac{l_s}{H_s}\right) = 3.50 \frac{a_1}{sd_{50}} + 13.7; r = 0.76;$$
(8)

$$Q_s D_* \left(\frac{W}{H}\right)^{-1} \left(\frac{z_m}{l_s}\right) = 0.005 Q_w (S - S_0) - 0.033; r = 0.91;$$
(9)

$$\left(\frac{z_m}{l_s}\right) = 0.002 e^{6.4\,\theta}; r = 0.87.$$
 (10)



Fig. 5 Regression of equation $Q_s D_* (W/H)^{-1} \left(\frac{z_m}{l_s}\right) = a Q_w (S - S_0) + b$ (Eq. 4)



Fig. 6 Regression of equation $\left(\frac{z_m}{l_s}\right) = k e^{m\theta}$ (Eq. 5)

(3) For test stand III

$$\left(\frac{z_{max}}{H_s}\right) = 0.455 \frac{a_1}{sd_{50}} + 0.240; r = 0.91;$$
(11)

$$\left(\frac{l_s}{H_s}\right) = 10.4 \frac{a_1}{sd_{50}} + 8.54; r = 0.79;$$
(12)

$$Q_s D_* \left(\frac{W}{H}\right)^{-1} \left(\frac{z_m}{l_s}\right) = 0.007 Q_w (S - S_0) - 0.038; r = 0.84;$$
(13)

$$\left(\frac{z_m}{l_s}\right) = 0.007 e^{2.6\theta}; r = 0.91.$$
 (14)

4 Conclusions

The experiment including a total of thirty-nine test runs, performed on three test stands (13 test runs for each stand), was conducted in the hydraulic laboratory to investigate the local scours properties, aiming at relating scour geometry with hydrodynamic parameters of water and sediment discharge. Measurements were taken when the scour hole has obtained its stable shape in clear-water and live-bed conditions. One non-uniform sand was used as a bed material with $d_{50} = 0.91$ mm. Test stands included total and partially sandy bed with and without the water structure (stone weir).

According to initial assumptions, four empirical formulas—Eqs. (1), (2), (4) and (5)—were confirmed to predict the medium scour depth z_m , maximal scour depth z_{max} and the length of the scour hole l_s as far as correlation coefficient r > 0.70 signalizes satisfying match and r > 0.80 good match of data in test cases with the stone weir. It must be mentioned that the local scour geometrical properties are strongly responsive to the difference between the equilibrium and initial energy grade line slope. The exponential and linear functions were identified, hence there was no general equation for cases with water structure input into the flume. The obtained values of linear function coefficients c, d, f, j are similiar to those proposed by Gaudio et al. (2000) and Gaudio and Marion (2003). Further development could involve higher energy grade slopes (the high equilibrium energy slope line surplus to the initial slope) or different stone weir configurations, for example, similar to the water structure presented in this research, with diminished slots area to intensify scour forming processes in the proposed test run conditions.

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