Concept Cartoons as a Representation of Practice

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Abstract The chapter focuses on using Concept Cartoons as a representation of practice in pre-service primary school teachers' education, especially on the possibility to employ them as a tool for investigating informal foundations of pedagogical content knowledge. The chapter introduces Concept Cartoons, and reports qualitative empirical research with a preparatory study. The preparatory study suggests the form of the Concept Cartoons environment suitable for investigating pedagogical content knowledge, and the main study analyzes displays of pedagogical content knowledge revealed in data collected from pre-service primary school teachers before their entering the course on didactics of mathematics. The results confirmed that Concept Cartoons were suitable for the studied purpose.

Keywords Concept Cartoons · Pedagogical content knowledge
Pre-service primary school teachers · Representation of practice
Teacher education

Introduction

In this chapter, I will present an educational tool called Concept Cartoons in a novel role—as a representation of practice and a diagnostic tool in pre-service primary school teachers' education. I will show how this new approach to Concept Cartoons can lead to successful investigation of informal foundations of pedagogical content knowledge of pre-service primary school teachers. From a general perspective, the chapter intends to contribute to the discussion about how representations of practice can help to investigate aspects of teacher expertise.

As an educator of pre-service primary school teachers I appreciate that data collected with the help of Concept Cartoons during mathematics content courses

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can give me valuable information on pre-service teachers' knowledge: data related to content knowledge provide me with continuous feedback on my own teaching, and data related to pedagogical content knowledge provide me with an introductory overview that allows to prepare didactics courses tailored to a particular sample of pre-service teachers.

The reported research focuses on two questions: "What form of the Concept Cartoons environment is suitable for investigating pedagogical content knowledge of pre-service teachers?", and "What are the informal foundations of pedagogical content knowledge with which pre-service primary school teachers enter the course on didactics of mathematics?" The research on the first question serves as a preparatory study for the research on the second question. During the preparatory study, more than 20 various Concept Cartoons with a set of questions were assigned to more than 100 pre-service teachers, in order to investigate how compositions of particular Concept Cartoons relate to quality and amount of displays of pedagogical content knowledge found in the collected data. A set of eight Concept Cartoons was selected on the basis of this preparatory study. During the main study, the selected Concept Cartoons were assigned to a group of 29 pre-service primary school teachers in the time before entering the course on didactics of mathematics, in order to analyze which particular displays of informal foundations of pedagogical content knowledge can be found in the collected data.

Background of the Research

Teachers and Their Knowledge

Teachers and their knowledge that influences the course of teaching are the focus of many educational frameworks. This contribution deals with *pedagogical content knowledge* in the sense of Shulman (1986) and Grossman (1990), i.e. with the construct that includes four central components: knowledge of teaching purposes, curricular knowledge, knowledge of pupils, and instructional knowledge.

As pedagogical content knowledge is a combination of miscellaneous components, also the range of the methods used to investigate pedagogical content knowledge is vast: tests, questionnaires, lesson observations, etc. (Depaepe et al. 2013). In mathematics education, an extensive study on pedagogical content knowledge was conducted under the research project COACTIV (Krauss et al. 2008). One of the studies that built on the COACTIV project investigated pedagogical content knowledge of lower-secondary mathematics teachers at different points in their teaching careers (Kleickmann et al. 2013). Its tests with open questions assessed three facets of pedagogical content knowledge: *knowledge of pupils* (of their strategies, conceptions and misconceptions, possible difficulties, sources of pupils' misunderstanding, etc.), *knowledge of tasks* (of multiple ways of solving, potential for pupils' learning), and *knowledge of instruction* (of different representations, models, modes of explanation, etc.). For instance, the following assignment belonged to a question on knowledge of tasks: “How does the surface area of a square change when the side length is tripled? Show your reasoning. Please note down as many different ways of solving this problem (and different reasoning) as possible.” (ibid., p. 102), and the other to a question on knowledge of pupils: “The area of a parallelogram can be calculated by multiplying the length of its base by its altitude… Please sketch an example of a parallelogram to which students might fail to apply this formula.” (ibid., p. 102).

Depaepe et al. (2015) conducted a research focusing on pedagogical content knowledge of pre-service primary and lower-secondary teachers. They also employed tests with open questions in their study but distinguished only two components of pedagogical content knowledge: *knowledge of pupils' misconceptions*, and *knowledge of instructional strategies and representations*. For instance, the following assignment belonged to a test question on knowledge of pupils' misconceptions: “Below are illustrations of elementary students' answers to the problem … For each student's answer write down the presumable student's reasoning and evaluate whether the answer is correct.” (ibid., p. 87).

Based on my previous experience that knowledge related to tasks might originate from different sources than knowledge related to pupils, I prefer to distinguish between knowledge of tasks and knowledge of pupils. Thus the reported study employs the classification of pedagogical content knowledge provided by Kleickmann et al. (2013).

Concept Cartoons

The name *Concept Cartoons* belongs to an educational tool that was developed in the 1990s by Keogh and Naylor (1993). They introduced Concept Cartoons as an instrument that might help support teaching and learning in primary school science classrooms by generating discussion, stimulating investigations, and promoting learners' involvement and motivation.

Each Concept Cartoon is a simple independent picture that shows a situation well known to pupils from school or everyday reality, and a group of several children discussing the situation through a bubble-dialog. The texts in bubbles are short and employ simple language. The discussion is composed in such a way that each of the children presents an alternative viewpoint on the situation or an alternative solution to a problem arising from the situation. Some alternatives may be correct, some incorrect, the correctness may also be unclear or conditional, depending on additional conditions not explicitly mentioned in the picture. One of the bubbles is blank, in order to indicate that there might exist other alternatives that have not been included in the dialog yet.

For a sample of one of the first Concept Cartoons created by Keogh and Naylor (1993) see Fig. 1. In that picture, the correctness of alternatives depends on two main factors: on the material of the coat, and on actual weather conditions

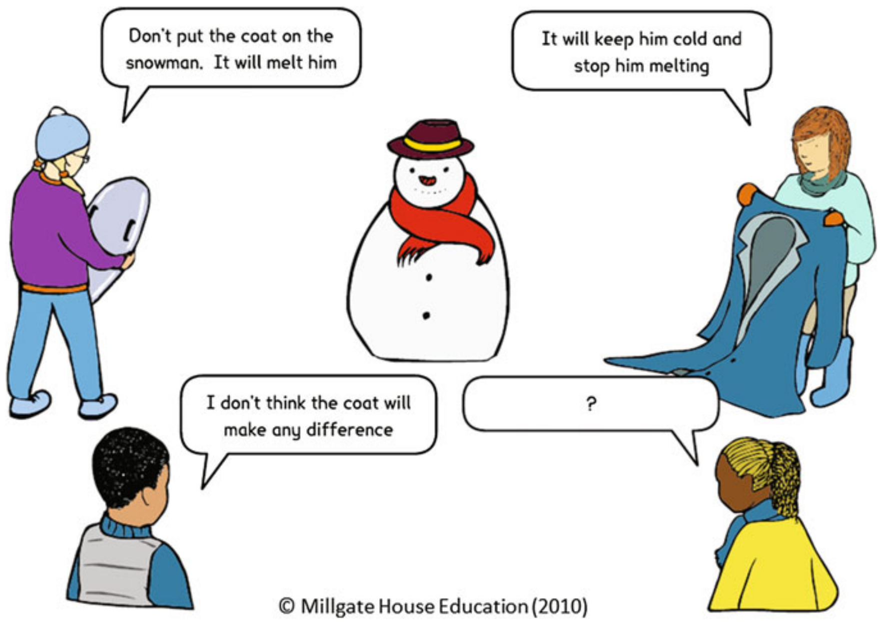

Fig. 1 Original Concept Cartoon; picture taken from (Naylor and Keogh 2010, no. 3.2)

(sun, temperature, etc.). This Concept Cartoon was composed in such a way that each of the alternatives can be true if you choose a suitable combination of the factors.

When using Concept Cartoons as an educational tool in the classroom, the teacher usually presents the picture to pupils with questions "What do you think about it?", "Which of the children are right?", "Why?", "What can we write into the blank bubble?", and the pupils discuss the answers.

Authors of Concept Cartoons performed several studies on the use of the tool in science classroom. The large-scale research investigated how pupils responded to the use of Concept Cartoons at primary and secondary school levels (Keogh and Naylor 1999). Among other results, the research confirmed that Concept Cartoons are able to support teaching and learning, and promote learners' motivation and engagement. The latter was confirmed even in case of usually less confident pupils—having pictured children speaking for them gives the pupils the confidence to discuss their ideas—from their point of view, the blame for a potential incorrect idea is not on the pupil but on the pictured children. As Keogh and Naylor pointed out in the study, also the evaluation looked differently with Concept Cartoons—pupils' ideas were not evaluated directly by the teacher as usual, instead the pupils themselves evaluated ideas of the pictured children, and this fact might have positively affected willingness to participate in the discussion as well. These attributes of Concept Cartoons relate to learners' motivation as a consequence of cognitive incongruity, a matter that was discussed by Hatano (1988). Hatano distinguishes three types of cognitive incongruity, and suitably composed Concept Cartoons may meet some or all of them: *surprise* (which is induced when a person encounters information that disconfirms a prediction based on prior knowledge), *perplexity* (which is induced when a person is aware of equally plausible but competing ideas), and *discoordination* (which is induced when one recognizes a lack of coordination among some of the pieces of knowledge involved).

Another research performed by authors of Concept Cartoons focused in detail on the form of the discussion and on the quality of arguments that appeared there (Naylor et al. 2007). Its results show that the lack of agreement amongst the pictured children encourages pupils to join the discourse with their own opinions, explanations and justifications, and that such discourse can take a form of sustainable and purposeful argumentation.

Since Concept Cartoons proved to be useful in science education, they naturally expanded to education of other school subjects, including mathematics (Dabell et al. 2008). In that case no large-scale research was conducted by the authors; the authors suppose that the results related to motivation, engagement and argumentation are of general character, and that they do not depend on the subject.

For a sample of one of the mathematical Concept Cartoons see Fig. 2. In that picture, one alternative is correct, and the others are incorrect. No conditionality appears there.

Fig. 2 Original Concept Cartoon; picture taken from (Dabell et al. 2008, no. 2.16)

Concept Cartoons as a Representation of Practice and a Diagnostic Tool

When I first encountered Concept Cartoons, I was attracted by the fact that each of the pictures shows various children's opinions in a certain situation related to topics that are taught in the classroom, so that the pictures can be considered as models of classroom discussions of pupils—as specific representations of practice.

The role of the teacher is not included in these representations, thus there is a big space for its integration and elaboration. For instance, I may assign the picture to a person, and ask the person to play the role of the teacher, i.e. to moderate the discussion, judge and evaluate the opinions, provoke further contributions to the debate, provide hints, explanation or advice that would be comprehensible for pupils, seek possible sources of misunderstanding or misconceptions, provoke or plan other activities that would clarify the situation, encourage looking for other alternatives that could be filled in the blank bubble.

Such use of Concept Cartoons resembles some of the test items intended for assessing teachers' pedagogical content knowledge presented in Section "Teachers and Their Knowledge":

- the setting consisting of various opinions (answers) of pupils and the requirement to judge and evaluate them, to provide advice or to present other possible answers appears in test items that focus on knowledge of pupils (e.g. on the ability to recognize pupils' misconceptions, difficulties and solving strategies—Kleickmann et al. 2013; Depaepe et al. 2015);
- the requirement to present other possible answers also appears in test items that focus on knowledge of tasks (e.g. on the knowledge of multiple ways to solve a problem—Kleickmann et al. 2013);
- the requirement to provide explanation also appears in some test items that focus on knowledge of instruction (e.g. on knowledge of different representations and explanations to standard problems—Kleickmann et al. 2013).

Following this resemblance, I decided to probe Concept Cartoons as a diagnostic tool for assessing teachers' pedagogical content knowledge. As the question of using Concept Cartoons for such a purpose is new, and the terrain is unknown, I narrowed the range of the question to an environment that is rather informal (similarly as Concept Cartoons are). So that I decided to address only pre-service primary school teachers and the informal foundations of pedagogical content knowledge that they might have gained from their own learning experiences (K–12, non-didactical teacher training courses). Thus, the research survey presented in this chapter focuses on pedagogical content knowledge of pre-service primary school teachers in the time before their entering the course on didactics of mathematics.

In comparison with the original use of Concept Cartoons, some modifications came about during the survey:

- instead of primary school pupils, pre-service primary school teachers worked with Concept Cartoons;
- instead of joint discussion in the classroom, the assessment was conducted individually and in written form;
- the original set of questions assigned with Concept Cartoons is not sufficient for such diagnostic purposes, so that questions on possible pupil's considerations and possible sources of pupil's misunderstanding or misconceptions, as well as requirements to provide explanations comprehensible to the pupil were added to the set;
- the original purpose of Concept Cartoons was educational, so that some particular pictures may not be suitable as diagnostic—the suitability of particular pictures had to be tested, some new pictures created.

To resolve and clarify all the above-mentioned differences, a preparatory study took place ahead of the main survey.

Preparatory Study

As indicated above, the main research required a preparatory study, and the research question for this preparatory study was "What form of the Concept Cartoons environment is suitable for investigating pedagogical content knowledge of pre-service teachers?" The preparatory study was conducted in two separate stages: the first stage explored Concept Cartoons from the original educational set, while the second stage dealt with Concept Cartoons that were newly created specifically for the purpose of this study. Participants of the preparatory study were 127 pre-service teachers, full time or distance university students from various years of the teacher training program.

The First Stage

For the first stage, four Concept Cartoons were selected from the original educational set created by Dabell et al. (2008); one of them is shown in Fig. 3.

The selected pictures differed in several composition factors: type of the pictured situation (classroom vs. everyday event), type of the text in bubbles (a proposal of a result vs. a proposal of a procedure and a result vs. an advice to a pupil who made a mistake), and/or number of alternatives that could be declared as correct (one vs. three). These Concept Cartoons were assigned to students on a worksheet with six common questions:

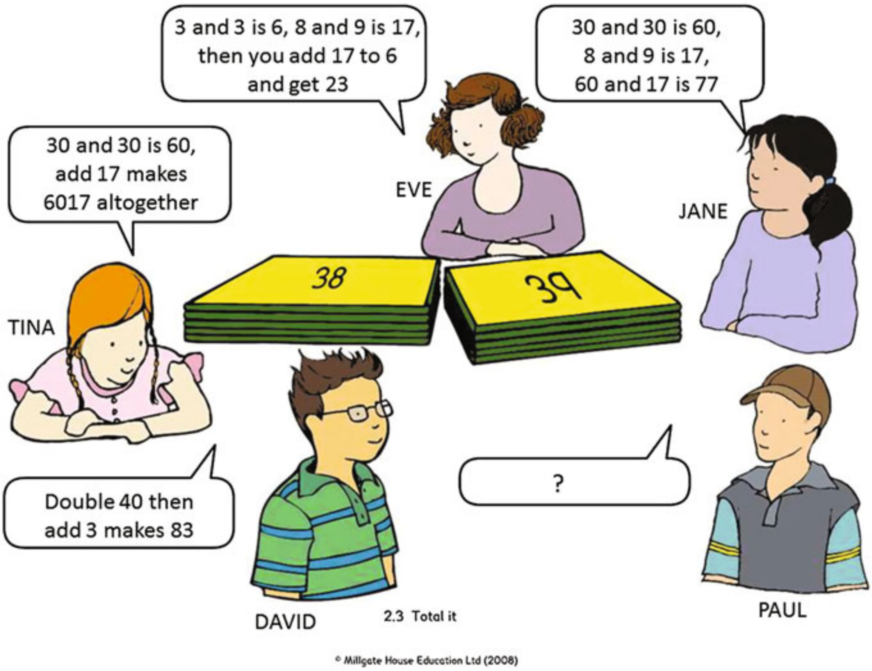

Fig. 3 Original Concept Cartoon; picture taken from (Dabell et al. 2008, no. 2.3), names added

(1) Which child do you strongly agree with?
(2) Which child do you strongly disagree with?
(3) Decide which ideas are right and which are wrong. Give reasons for your decision.
(4) Try to discover the cause of the mistakes.
(5) Advise the children who made the mistakes how to correct them.
(6) Propose a text that could be filled in the blank bubble—does not matter whether correct or incorrect. It might relate to another correct way of solving, or to another misconception.

Students worked on worksheets individually, during a lesson. The work took them approximately 80 min.

Original Concept Cartoons do not have the children in the picture named (see Figs. 1 and 2) which appeared uncomfortable for the respondents—many of them announced during the work that they did not know how to refer to particular pictured children. So that I let the respondents add letters (A, B, C, D …) to the children, and from that time I always label the pictured children. I prefer labeling by names (as in Fig. 3), to make the Concept Cartoons authentic—pupils in the classroom are also called by names, not by letters.

Data from worksheets were processed qualitatively, using the method of substantive coding and constant comparison (Bryant and Charmaz 2007). I focused on displays of pedagogical content knowledge, e.g. displays related to provision and recognition of right and wrong answers, to recognition of procedures used by pictured children, to identification of the causes of mistakes. Detailed description of analysis of data connected with two of the Concept Cartoons, and partial results belonging to 64 pre-service primary school teachers we already reported in Samková and Hošpesová (2015). We presented there two of the Concept Cartoons in detail, and showed how they allowed us to distinguish between subject matter knowledge and pedagogical content knowledge in the sense of Shulman (1986), as well as between procedural knowledge and conceptual knowledge in the sense of Baroody et al. (2007).

The other two Concept Cartoons that were not reported in Samková and Hošpesová (2015) appeared to be problematic from the perspective of the diagnostic purpose, because data collected with them offered almost no information on pedagogical content knowledge. Respondents' responses to indicative questions were very short, often giving only the opinion on the correctness, without attempts to comment on reasons or search for possible sources of misconceptions. Since the four tasks from the four Concept Cartoons were of similar difficulty, the obvious inequality in the responses turned my attention to the possible unsuitability of some Concept Cartoons for my purpose, and to the need to observe closely particular composition factors related to particular Concept Cartoons.

As turned out, both the pictures with enough collected data displayed an everyday event, the first of them used proposals of procedures and results in its bubbles, the second one used just results. Both the pictures with lack of collected data displayed a classroom event, the first of them used advices to a pupil in its bubbles, the second one used just results. From the perspective of alternatives in bubbles, the picture with advices to a pupil offered three alternatives that could be declared as correct, all other pictures just one. For further study I decided to focus closely on diverse combinations of the three composition factors, and explore the further potential of the factors. Since the original Concept Cartoons do not offer enough diversity for such a study (e.g. most of the pictures offer just one correct alternative, and bubbles within the pictures usually have the same type of the text), I had to prepare some new pictures.

The Second Stage

With focus on variability of combinations of the three composition factors from the first stage of the preparatory study, 22 new Concept Cartoons were created and used in the second stage. Among them, 11 Concept Cartoons were slight modifications of original Concept Cartoons. The modifications consisted for instance in changing the number of correct alternatives (e.g. by modifying numbers in the assignment of the task in such a way that the task gained more than one solution, or by replacing one of the incorrect alternatives by a correct one), in adjusting some incorrect alternatives to look more plausible (e.g. to take a form of a typical pupil's

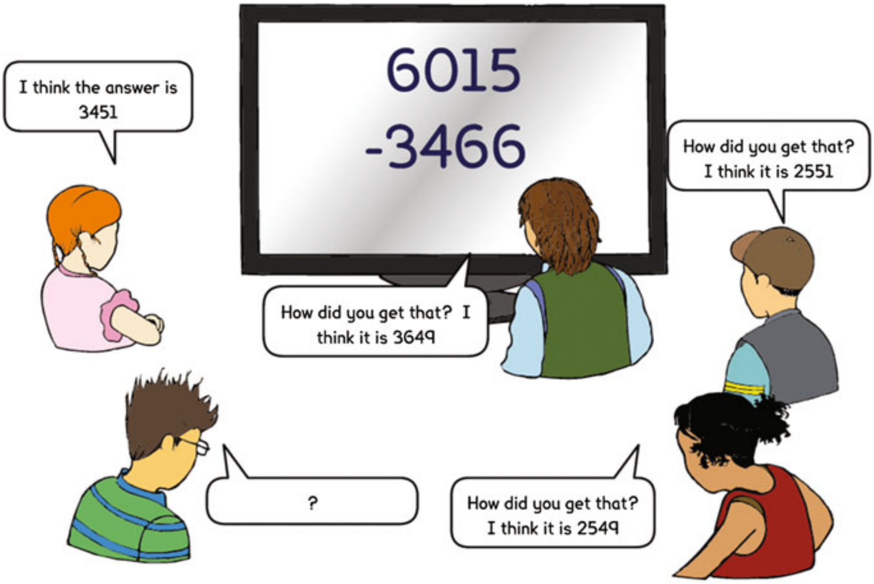

Fig. 4 Slightly modified Concept Cartoon (just decimal marks after the second digit deleted from all numbers, i.e. the range of the task changed from decimal to natural numbers); picture taken from (Dabell et al. 2008, no. 2.13)

misconception known from educational research), or in making small shifts in mathematical content (for such a shift see Fig. 4).

The other 11 Concept Cartoons were brand new. They presented a new pictured situation, a new perspective on the situation, and/or a quite new mathematical content (compare Figs. 2 and 5). I searched for inspiration in my own teaching experience and in the teaching experience of my colleagues (e.g. Tichá and Hošpesová 2010), in results of educational research (e.g. Ryan and Williams 2011; Bana et al. 1995), in books and textbooks (Ashlock 2002, 2010). Again, the newly created Concept Cartoons contained in their bubbles various more or less usual pupils' conceptions or misconceptions, descriptions of various correct ways of solving, or plausible incorrect ways of solving, and also some intentionally prepared unusual but authentically looking misconceptions (Samková and Tichá 2015). In comparison to original Concept Cartoons, some new types of the text in bubbles were established for the newly created pictures: a proposal of a procedure, a proposal of a statement, an opinion on the validity of a statement, an opinion on the number of solutions, and a reference to a drawing that was not displayed in the picture (i.e. a reference to a missing drawing). Some of the Concept Cartoons had the same mathematical content but different types of the text in bubbles (Samková et al. 2015), in order to allow monitoring the influence of the composition on collected data. All the 22 Concept Cartoons were assigned to various groups of participants, under the same conditions as in the first stage.

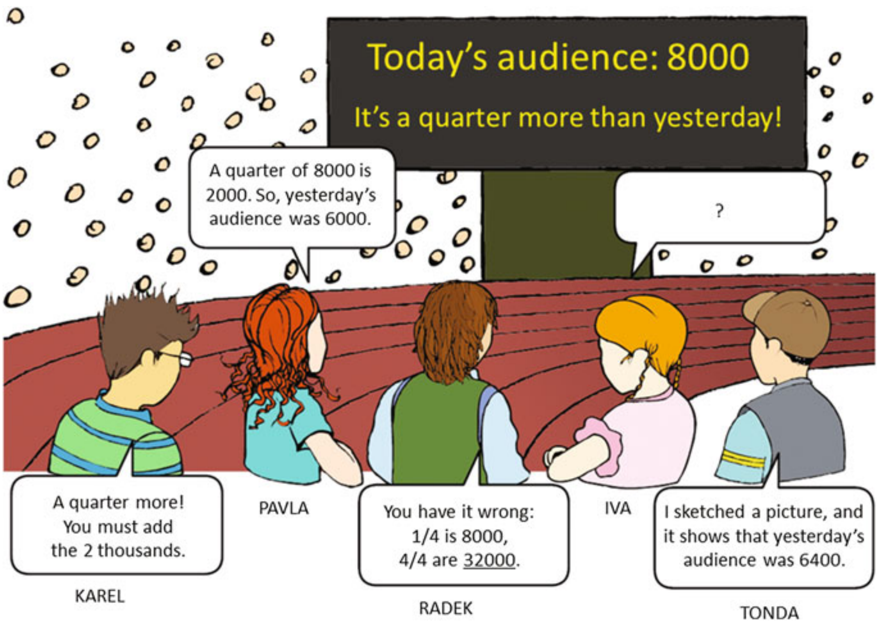

Fig. 5 Concept Cartoon re-designed for this study; template with empty bubbles and empty notice board taken from Fig. 2, new texts and names added

Data from the second stage were analyzed qualitatively. This time, the analysis focused on displays of pedagogical content knowledge in relation to composition factors of given Concept Cartoons. I also monitored the amount of relevant data obtained from participants in relation to various Concept Cartoons. According to quality and amount of relevant data collected with the Concept Cartoons, data analysis revealed three significant types of bubble content: bubbles with procedures and results, bubbles with references to a missing drawing, and bubbles with just results.

When using Concept Cartoons containing both procedures and results in their bubbles (as in Fig. 3), respondents could comment on described results and procedures, and look for errors in procedures leading to incorrect results as well as in procedures leading to correct results. This kind of Concept Cartoons offered the respondents a lot of concrete facts to judge and discuss, and the responses provided a lot of relevant data.

Another type of Concept Cartoons that appeared diagnostically valuable was the one that combined several bubbles containing procedures and results with a bubble introducing a result together with a reference to a missing drawing leading to this result (as in Fig. 5 where Tonda's bubble refers to such a drawing). These combinations of bubbles were often thought-provoking: respondents first commented on the procedures described in the bubbles, and then focused on the bubble without the procedure, attempting to find out what drawing the child was talking about. They often proposed their own drawings that could lead to the result. In this case, the Concept Cartoon played a similar diagnostic and thought-provoking role as problem posing, e.g. as posing problems corresponding to a given calculation (Tichá and Hošpesová 2010).

Some of the Concept Cartoons with just results proved to be problematic from the diagnostic perspective, especially when the task was in a form of a calculation (e.g. as in Fig. 4). The respondents often tended just to compare the correct result of the calculation with the numbers in bubbles, did not comment on reasons, and did not attempt to seek the procedures hidden behind the incorrect results or possible sources of misconceptions. This kind of Concept Cartoons offered the respondents few concrete facts to judge and discuss, and the responses provided few relevant data on pedagogical content knowledge. On the other hand, with an unusual composition, the Concept Cartoon with a calculation task and just results in bubbles attracted respondents' attention, and provoked them to respond widely. Such an unusual composition belonged to one of the original Concept Cartoons from the first stage of the study, which presented a task based on a calculation 5904 + 5106. What was unusual about the results proposed in bubbles is that all of them were composed only of digits 1 and 0: 1110, 11100, 11010, and 1010010. With this Concept Cartoon, I obtained a lot of relevant data; the respondents sought for the procedures hidden behind the proposed results, and suggested various sources of mistakes. Since some of the respondents' responses were not correct, this Concept Cartoon also helped to reveal weaknesses in pedagogical content knowledge: there were respondents who just compared the incorrect results in the bubbles with the correct result, and gave the children advices regardless of the possible source of the mistakes. For instance, one of the participants gave the children with the 11100 result the following advice: "Peter, recount it again, your result is 90 more than the correct result." (Samková and Hošpesová 2015).

Unlike the first stage of the preparatory study, some Concept Cartoons based on classroom events appeared suitable for assessing pedagogical content knowledge. All of them had a common characteristic: at least two correct alternatives in the picture that could be declared as correct, and procedures or procedures with results in bubbles. The correctness of the alternatives was either general, or conditional. With these Concept Cartoons, the respondents seemed to be surprised by the existence of multiple correct alternatives, and thus paid more attention to the reasoning related to the alternatives that looked correct. As a positive consequence, some of them paid more attention also to alternatives that looked incorrect, and offered detailed justifications on the incorrectness. As a negative consequence, some of the participants paid too much attention to the alternatives that looked correct, and improperly found mistakes in some completely correct formulations.

Generally, the results of the preparatory study showed that various pictured situations, various numbers of alternatives that can be considered as correct, and various types of the text in bubbles allowed to reach diverse components of pedagogical content knowledge, so that a set of Concept Cartoons with various combinations of composition factors is needed to get a comprehensible overview of participants' pedagogical content knowledge (Samková 2017).

On the basis of the results of the preparatory study I selected a set of eight Concept Cartoons for the main survey. This set contained Concept Cartoons from the preparatory study, four of them original Concept Cartoons (e.g. the one in Fig. 3), and four newly created (e.g. the one in Fig. 5). Tasks on the pictures were of diverse focus and difficulty, number of alternatives in a picture that could be declared as correct varied from one to all, and texts in bubbles were of diverse types: just results, just procedures, procedures and results, references to a missing drawing, opinions on validity, recommendations, and proposals of general statements or rules. Four of the Concept Cartoons were based on an everyday event, and four on a classroom event.

Main Survey

As already mentioned above, the research question for the main survey was "What are the informal foundations of pedagogical content knowledge with which pre-service primary school teachers enter the course on didactics of mathematics?"

Participants and Data Collection

Respondents of the research were 29 full time university students of master degree training for pre-service primary school teachers. In our country, pre-service primary school teachers' training covers all the primary school curriculum subjects and lasts 5 years, i.e. it serves as an equivalent of a 3-year bachelor program followed by a 2-year master program. Students come to the university directly from the secondary school, with no experience in teaching. During the first and second years of the training program, the students attend mandatory courses on mathematics, and during the third year they attend mandatory courses on didactics of mathematics. The survey took place in the second year of the training program, i.e. in the time before the students entered the courses on didactics of mathematics. All of the second year students participated in the research, none of them participated in the preparatory study.

The eight Concept Cartoons selected on the basis of the preparatory study were assigned to the respondents in two separate stages (due to time constraint reasons), four Concept Cartoons per stage. I placed them on a worksheet with six common questions that were the same as in the preparatory study (Section "The First Stage"). Students worked on worksheets individually, during a lesson. The work took them approximately 80 min each stage.

Data Analysis

First, all the materials were open-coded, and the codes sorted to 12 categories in such a way that, if applicable, the categories included both strengths and weaknesses related to the category label. In the following list, the categories including both strengths and weaknesses are provided with two examples of codes in brackets, the first example is considered as referring to some strength, and the second one to some weakness:

A. strong (dis)agreement (e.g. "strongly disagrees with Radek", "strongly disagrees with Tonda");
B. recognition of a correct/incorrect statement (e.g. "found all incorrect statements", "thinks that a correct statement is incorrect");
C. recognition of a procedure and its particular steps (e.g. "reveals a step that is incorrect", "sees the procedure as one indivisible whole");
D. explanation (e.g. "illustrative explanation", "imprecise explanation");
E. advice (e.g. "helpful advice", "missing advice");
F. identification of the cause of a mistake (e.g. "plausible cause of a mistake", "just compares the result in a bubble with his own result");
G. differentiation between identification of a mistake, its cause, and its remedy (e.g. "successful differentiation", "one common answer for questions 3, 4, 5");
H. own respondent errors and mistakes that appeared as a part of explanation or advice (e.g. "confuses part and whole");
I. blank bubble (e.g. "alternative way of solving", "unrealistic alternative");
J. formal arrangement (e.g. "carefully follows the order of questions and answers", "does not number answers");
K. orientation in the picture (e.g. "did not link bubbles to names");
L. not specified (e.g. "interesting", "unclear", "read again").

Then the method of constant comparison was employed, data were repeatedly read over, labeled by new codes when needed, codes repeatedly compared with data and among themselves, rearranged, adjusted. Some codes were removed. For better clarity of the process, the codes were marked with plus or minus sign to denote aspects that were considered as positive or negative from the perspective of teachers' pedagogical content knowledge. During the process, the list of categories was re-organized:

- codes from J, K and L categories were adjusted and replaced or removed, the three categories were canceled;
- codes from categories C, D, E, F appeared to be too tight together, so that these categories were unified under one common category labeled CF;
- when comparing codes within particular respondents, the heterogeneity of data related to Concept Cartoons with diverse composition was disruptive, so that the analysis was additionally enriched with codes related to the composition of particular Concept Cartoons (type of the text in bubbles, number of bubbles with

correct statement, number of solutions, etc.), and a new category M was established for them.

Thus, categories A, B, CF, G, H, I, M remained for the final analysis. The final analysis identified B, CF and I as the categories with the biggest density of data.

General Findings

In this section, I present general findings of the research, accompanied in brackets by references to particular responses that appear as transcripts in Section "Illustrative Data Excerpts". The references are in the following form: (respondent number: name of the child or children to which the response relates).

In spite of the fact that respondents of the research were students who had not attended a course on didactic of mathematics yet, data revealed 15 of the 29 students with good informal foundations of pedagogical content knowledge. These students were able to

- recognize various pupils' misconceptions (S30: Pavla, S31: Pavla, Radek);
- find a mistake in a procedure, clearly describe its possible cause, and advice how to remedy it (S4: Pavla, Karel, Radek);
- check/verify results of a task that they themselves did not solve, and explain to children with wrong results why their results cannot be correct (S6: Tonda, Pavla, Radek, S10: Pavla, Karel, Radek);
- usefully employ visualization (S2: Tonda, S4: Tonda, S22: Tonda, S24: Tonda);
- present various alternative ways of solving (S5: Paul, S10: Paul), also the elegant ones that advantageously utilize certain specific relations (S3: Paul, S4: Paul, S18: Paul, S26: Paul);
- present plausible potential pupils' misconceptions (S23: Paul).

On the other side, data revealed also 8 students with low level of knowledge related to pedagogical content. These students

- proposed possible pupils' alternative solutions that were unrealistic (S29: Iva);
- proposed explanations of pupils' procedures that were unrealistic (S1: Tonda, S21: Tonda) or with no relation to the task (S15: Tonda);
- tended to reject procedures which they themselves did not grasp (S12: Tonda);
- did not know common misconceptions (S16: Pavla).

The remaining 6 students showed unbalanced knowledge since some of their responses could be considered as displays of good knowledge, and some could not. For instance, the respondent S11 presented a proper alternative way of solving as a response to Paul's bubble (S11: Paul) but an unrealistic incorrect alternative as a response to Iva's bubble (S11: Iva).

Further, 10 of the 29 respondents in their worksheets made an effort to differentiate between identification of a mistake, its cause, and its remedy. All of them were successful in this endeavor. Without exception, they were all students who were successful also in other aspects of pedagogical content knowledge, e.g. in presenting various alternative solutions of a task (S4: Paul).

The remaining 19 respondents offered answers that did not differentiate between identification, cause and remedy. They often gave one common answer for questions 3, 4 and 5 (S12: Tonda, S15: Tonda). This fact is not surprising since the respondents had not attended any didactical courses yet.

Illustrative Data Excerpts

The findings reported generally in the previous section will now be illustrated by particular data excerpts. The transcripts will be presented in the following form:

respondent no.	question no.	respondent's answer to the question or its part (name of the pictured child included)

In cases when respondents gave one common answer for questions 3, 4 and 5, the question number in the transcript will be denoted by 3/4/5.

As an illustrative example, I have selected the Concept Cartoon from Fig. 5. The scene described in this Concept Cartoon is located outside the classroom, is based on a word problem with fractions, and the word problem has a unique solution. A similar task appeared in 2015 in state matriculation exam, where only 33% of the students solved the task correctly (Řídká 2015). I changed the context of the task, and enlarged the quantity given in the task from 800 to 8000. When creating the bubbles, I based three of them on three most frequent incorrect solutions (Pavla, Karel, Radek), and the fourth on the correct solution (Tonda). The incorrect solutions are offered in the form of procedures with results, the correct solution is in the form of a result with a reference to a missing drawing.

Like in the state exam, the task appeared to be rather difficult to solve: only 41% of the respondents properly choose Tonda's bubble as the correct one, the other 59% improperly choose Pavla's bubble.

Agreement with Pavla was often supported by the justification that is known as a common misconception:

S16	2)	Pavla—right answer, 8000: 4 = 2000 → yesterday 6000.

The respondents who choose Pavla, often strongly disagreed with Tonda. Some of them admitted that it is because they did not understand Tonda at all, that they did not grasp how could have he come to the result 6400:

S12	2)	I do not agree with Radek and Tonda.
	3/4/5)	Tonda's opinion is not right. I entirely did not understand his thinking.

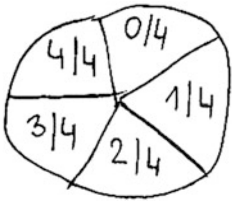

Fig. 6 Picture with a "pie" drawn by the respondent S21

Some respondents presented explanations of potential Tonda's procedure that were based on unrealistic misconceptions:

S1 3) Tonda—this opinion is the most wrong → first he subtracted one quarter (2000), and then added 100 from each quarter.

S21 4) Tonda—he probably drew a "pie", and included zero to his calculations (see Fig. 6).

Some of the explanations did not even relate to numbers from the assignment of the task:

S15 3/4/5) Tonda calculated $80 \cdot 80$.

Among the responses, I also found some explanations of Tonda's procedure that were plausible but incorrectly rejected by their authors as wrong:

S9 2) Tonda

4) Tonda—he probably drew a picture with four quarters, and added one quarter to it, so that he divided 8000 by 5.

5) Tonda—draw a picture, and the amount 8000 has to be divided to how many pieces if you want to find 1/4?

Respondents who agreed with Tonda supported their agreement diversely. Some of them offered a picture (see Fig. 7), others just solved the task without a picture, and compared their result with Tonda's.

These respondents were usually able to find mistakes in procedures presented by other children (Pavla, Karel and Radek), and gave reasons why the mistakes might occur. Some of them also advised a remedy:

S30 4) Pavla did not realize that the quarter must be calculated from the previous whole. That today's audience is

yesterday's whole + its quarter = 8000.

S31 4) Karel—he proceeded from 8000, i.e. from today's audience, not from the yesterday's

Pavla—the same as Karel

S4 4) Radek—he calculated that today is quarter of yesterday's, instead of quarter more than yesterday

5) Radek, read the task carefully: quarter of ≠ quarter more!

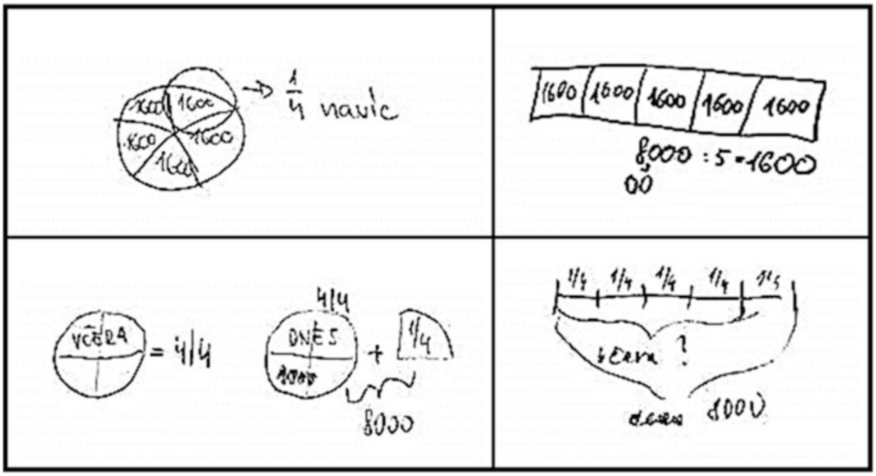

Fig. 7 Excerpts from four different worksheets where respondents agreed with Tonda (S24, S4, S22, S2). Translation: včera = yesterday, dnes = today, navíc = extra

Among the respondents there were also some that did not solve the task at all. They just verified all the results from the bubbles, one by one, and then gave explanations to children with wrong results which showed why these results could not be correct:

S10 3) Tonda—right, 6400 + quarter = 8000
Pavla—quarter of the yesterday's audience, not today's
Radek—wrong, when yesterday was some audience, and today is quarter more, then yesterday could not be more people than today
(bigger number—32,000—nonsense)

S6 3) Pavla—if yesterday came 6000 people, one quarter would be 1500, which does not result in 8000 but only in 7500.
Karel—if yesterday came 8000 people, one quarter would be 2000, and this number would be added to 8000 to get the people that have come today.
Radek—if his bubble were true, then the notice board would say:
Today's audience: 8000
It's a quarter of yesterday's.

Alternatives proposed to Iva's bubble depended on what result the respondent considered as correct. Students who agreed with Tonda often proposed hints to Tonda's result:

S10 6) Iva: 8000 is 5/4
S22 6) Iva: I think that yesterday's audience was 1600 less.
S4 6) Iva: It is 6400, because the picture shows me that it is 4/5 of 8000.

Students who agreed with Pavla often proposed pictures illustrating her procedure—a pie divided into quarters (S16, S18) or a hint about 8000 being 4/4 (S12). Some of the worksheets contained unrealistic alternative solutions to the task:

S29 6) Iva: A quarter of 8000 is 6000, so that yesterday came only 2000 people.
S11 6) Iva: Yesterday came 4000 people. Because 1/4 is 4000.

The Concept Cartoon from Fig. 5 was unique because students did not offer any alternative ways of solving to the blank bubble. However, students offered many alternative ways of solving to the Concept Cartoon from Fig. 3. This Concept Cartoon focuses on a numerical task 38 + 39 from the perspective of mental calculation, and its bubbles offer various pupil's solutions in the form of a procedure with a result. The worksheets contained many diverse correct alternative procedures proposed to Paul's bubble:

S5 6) Paul: First I add tens, 30 + 30 = 60, then I add units, 8 + 9 = 17. Then I add these numbers, 60 + 17, and get the number 77.
S11 6) Paul: 38 + 30 = 68, 68 + 9 = 77
S10 6) Paul: $2 \cdot 30 = 60$, and 9 is 69, and 8 is 77.

Besides the usual standard alternatives mentioned above, some other alternatives advantageously utilized the fact that both addends are close to 40 which is a multiple of ten, or that they are close to each other:

S26 6) Paul: I borrow one from 38, and add it to 39 → 40 + 37 = 77. It is better to count, because you don't need to carry any figures.
S3 6) Paul: 2 are missing in 38 to get 40, I borrow them from 39. So that I will have 40 + 37 = 77.
S18 6) Paul: Two times 40, and subtract 3.
S4 6) Paul: $(38 \cdot 2) + 1 = 77$

Some of the worksheets also offered plausible potential pupils' misconceptions:

S23 6) Paul: (3 + 3) = 6
(8 + 9) = 17
38 + 39 = 617

Discussion

Results of the presented research are in accordance with findings of similar studies focusing on pedagogical content knowledge. Like in Kleickmann et al. (2013), Krauss et al. (2008), the research showed that some of the pre-service teachers were able to gain informal foundations of pedagogical content knowledge from their own learning experience prior to didactical courses and own teaching practice.

As an object in the first part of the research and a tool in the second part of the research I used the Concept Cartoons environment. Results of the research showed that Concept Cartoons provided with a set of indicative questions might become a suitable tool for exploring various aspects of pedagogical content knowledge in mathematics. In accordance with the earlier research on Concept Cartoons (Keogh and Naylor 1999; Naylor et al. 2007), Concept Cartoons appeared to be able to encourage presenting own opinions on texts in bubbles as well as answering the indicative questions about the picture. Although unlike the earlier research, I used Concept Cartoons in slightly different ways: with pre-service teachers instead of pupils, individually instead of collectively, in mathematics, and in written form.

As an important component of Concept Cartoons I see the blank bubble which allows to gain insight into knowledge of alternative ways of solving and knowledge of common pupils' misconceptions. The spectrum of responses to these bubbles that appeared in collected data is vast but the analysis of data shows that the wording of the sixth question which required to propose *one* alternative *or* misconception was unnecessary limiting the possible range of responses. For future use, it would be better to get an inspiration from the study of Kleickmann et al. (2013), and make some changes in the sixth question: divide the question into two parts, in the first part ask for *as many as possible* alternative ways of solving, and in the second part ask for *as many as possible* potential pupils' misconceptions.

The reported research was conducted at the Faculty of Education as a part of a three-year project focusing on opportunities to influence professional competences of pre-service primary school teachers. In some other studies under this project that I performed together with my colleagues, we investigated the possible use of Concept Cartoons in pre-service teachers' education from various perspectives and with various groups of pre-service primary school teachers. In mathematics courses, we used Concept Cartoons to support problem solving, reasoning and argumentation (Samková and Tichá 2016a, 2017b), and as a diagnostic tool for assessing pre-service teachers' mathematical content knowledge (Samková and Tichá 2015, 2016b, 2017a). In didactics courses, we used them in problem posing activities (Samková and Tichá 2016b, 2017b). Among others, Concept Cartoons helped us to show how diverse information could be provided by problem posing and problem solving, and thus confirmed the importance of linking problem solving and problem posing that was emphasized in a recently issued monograph on problem posing (Singer et al. 2015). We also used Concept Cartoons when preparing pre-service teachers for their own teaching practice: such a tool helped them to become aware of potential pupils' mistakes and misconceptions that might appear in the classroom. A similar but not the same tool called *discussion prompt sheets* was mentioned by Ryan and Williams (2011).

To place this current research on Concept Cartoons into the broader framework, it is useful to realize that I employed Concept Cartoons to ascertain responses of pre-service teachers to various alternative opinions of virtual (pictured) pupils. So, in a sense it can be said that I investigated informal foundations of pre-service teachers' ability to notice, namely of the aspects related to comments on pupils' talks (category *pupil commentary* in Vondrová and Žalská 2015).

Further, Concept Cartoons also address the concerns that were raised in the study of Depaepe et al. (2015) which focused on investigating mathematical content knowledge and pedagogical content knowledge of pre-service teachers. Depaepe et al. reproached some previous surveys on pedagogical content knowledge for not investigating pedagogical content knowledge independently of mathematical content knowledge, i.e. that respondents of those surveys first solved a task to demonstrate mathematical content knowledge, and then in the context of the *same* task commented on issues related to pedagogical content knowledge. By Depaepe et al., such circumstances negatively affect data related to pedagogical content knowledge. When using Concept Cartoons, no similar dependence occurs, because no solution of the task is explicitly required, and the respondents may demonstrate some aspects of their pedagogical content knowledge even in the case when they do not know how to solve the task or do not want to solve it. For instance, by proper checking and verifying all offered results, or by proper justification of why a result or a procedure offered in a bubble cannot be correct, the respondents can present their ability to evaluate pupils' answers—an ability that is an integral part of pedagogical content knowledge. In particular, the ability to evaluate pupils' answers without solving the task turns out to be important in teaching in the moment when, as a direct consequence of a discussion or as a prompt support to an explanation, a quite new task not previously prepared by the teacher appears in the classroom.

Conclusion

In this chapter I introduced a study focusing on the opportunity to use an educational tool called Concept Cartoons as a representation of practice in pre-service primary school teachers' education, especially on the possibility to employ Concept Cartoons as a tool for investigating informal foundations of pedagogical content knowledge of pre-service primary school teachers. The study suggests the form of the Concept Cartoons environment suitable for such a purpose, and analyzes displays of pedagogical content knowledge that were collected in the environment from pre-service primary school teachers before entering the course on didactics of mathematics. The results confirmed that Concept Cartoons were suitable for the studied purpose, namely for investigating informal foundations of knowledge of pupils (e.g. ability to recognize pupils' misconceptions, difficulties and solving strategies), knowledge of tasks (e.g. knowledge of multiple ways to solve a problem), and knowledge of instruction (e.g. knowledge of different representations and explanations to standard problems).

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References

Ashlock, R. B. (2002). *Error patterns in computation: Using error patterns to improve instruction*. Upper Saddle River: Merrill Prentice Hall.

Ashlock, R. B. (2010). *Error patterns in computation: Using error patterns to help each student learn*. Boston: Allyn & Bacon.

Bana, J., Farrell, B., & McIntosh, A. (1995). Error patterns in mental computation in years 3–9. In B. Atweh, & S. Flavel (Eds.), *Galtha: Conference Proceedings of the 18th Annual Conference of MERGA* (pp. 51–56). Darwin: MERGA.

Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education, 38,* 115–131.

Bryant, A., & Charmaz, K. (Eds.). (2007). *The SAGE handbook of grounded theory*. London: SAGE.

Dabell, J., Keogh, B., & Naylor, S. (2008). *Concept Cartoons in mathematics education (CD-ROM).* Sandbach: Millgate House.

Depaepe, F., Torbeyns, J., Vermeersch, N., Janssens, D., Janssen, R., Kelchtermans, G., et al. (2015). Teachers' content and pedagogical content knowledge on rational numbers: A comparison of prospective elementary and lower secondary school teachers. *Teaching and Teacher Education, 47,* 82–92.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34,* 12–25.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. New York: Teachers College Press.

Hatano, G. (1988). Social and motivational bases for mathematical understanding. In G. B. Saxe & M. Gearhart (Eds.), *Children's mathematics. New directions for child development* (pp. 55–70). San Francisco: Jossey-Bass.

Keogh, B., & Naylor, S. (1993). Learning in science: Another way in. *Primary Science Review, 26*, 22–23.

Keogh, B., & Naylor, S. (1999). Concept Cartoons, teaching and learning in science: An evaluation. *International Journal of Science Education, 21*(4), 431–446.

Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S., et al. (2013). Teachers' content and pedagogical content knowledge: The role of structural differences in teacher education. *Journal of Teacher Education, 64,* 90–106.

Krauss, S., Baumert, J., & Blum, W. (2008). Secondary mathematics teachers' pedagogical content knowledge and content knowledge: Validation of the COACTIV constructs. *ZDM Mathematics Education, 40,* 873–892.

Naylor, S., & Keogh, B. (2010). *Concept Cartoons in science education* (2nd ed.). Sandbach: Millgate House Education.

Naylor, S., Keogh, B., & Downing, B. (2007). Argumentation and primary science. *Research in Science Education, 37,* 17–39.

Řídká, E. (2015, September). *Současný stav maturit z matematiky*. [In Czech: *The current state of matriculation exam in mathematics*]. Paper presented at the 64th Academic Forum of the Expert Group of Czech Physical Society of the Union of Czech Mathematicians and Physicists, Praha.

Ryan, J., & Williams, J. (2011). *Children's mathematics 4-15. Learning from errors and misconceptions*. Berkshire: Open University Press.

Samková, L. (2017). Using Concept Cartoons to investigate future teachers' knowledge—new findings and results. In S. Zehetmeier, B. Rösken-Winter, D. Potari, & M. Ribeiro (Eds.), *Proceedings of the Third ERME Topic Conference on Mathematics Teaching, Resources and Teacher Professional Development (ETC3, October 5 to 7, 2016)* (pp. 207–216). Berlin, Germany: Humboldt-Universität zu Berlin.

Samková, L., & Hošpesová, A. (2015). Using Concept Cartoons to investigate future teachers' knowledge. In K. Krainer & N. Vondrová (Eds.), *Proceedings of CERME 9* (pp. 3241–3247). Praha: Univerzita Karlova, Pedagogická fakulta.

Samková, L., & Tichá, M. (2015) Investigating future primary teachers' grasping of situations related to unequal partition word problems. In C. Sabena & B. Di Paola (Eds.), *Quaderni di Ricerca in Didattica (Mathematics),* n. 25, Supplemento n. 2. *Proceedings CIEAEM 67, Teaching and learning mathematics: Resources and obstacles* (pp. 295–303), Palermo, Italy: G.R.I.M.

Samková, L., & Tichá, M. (2016a). Developing views of proof of future primary school teachers. In L'. Balko, D. Szarková, & D. Richtáriková (Eds.), *Proceedings, 15th Conference on Applied Mathematics Aplimat 2016* (pp. 987–998). Bratislava: STU.

Samková, L., & Tichá, M. (2016b). On the way to develop open approach to mathematics in future primary school teachers. *ERIES Journal, 9*(2), 37–44.

Samková, L., & Tichá, M. (2017a). Observing how future primary school teachers reason about fractions. In M. Houška et al. (Eds.), *Proceedings of the 14th International Conference Efficiency and Responsibility in Education 2017* (pp. 363–371). Prague: Czech University of Life Sciences.

Samková, L., & Tichá, M. (2017b). Observing how future primary school teachers reason and generalize: The case of number triangles and Concept Cartoons. In D. Szarková et al. (Eds.), *Proceedings, 16th Conference on Applied Mathematics Aplimat 2017* (pp. 1354–1368). Bratislava: STU.

Samková, L., Tichá, M., & Hošpesová, A. (2015). Error patterns in computation in Concept Cartoons. In J. Novotná, & H. Moraová (Eds.), *International Symposium Elementary Maths Teaching SEMT '15. Proceedings* (pp. 390–391). Prague: Charles University, Faculty of Education.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Singer, F. M., Ellerton, N. F., & Cai, J. (Eds.). (2015). *Mathematical problem posing. From research to effective practice*. New York: Springer.

Tichá, M., & Hošpesová, A. (2010). Problem posing and development of pedagogical content knowledge in pre-service teacher training. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of CERME 6* (pp. 1941–1950). Lyon: INRP.

Vondrová, N., & Žalská, J. (2015). Ability to notice mathematics specific phenomena: What exactly do student teachers attend to? *Orbis Scholae, 9*(2), 77–101.