Using Cases and Events in Teacher Education: Prospective Teachers' Preferences

Pessia Tsamir, Dina Tirosh, Esther Levenson, and Ruthi Barkai

Abstract The use of cases as a pedagogical tool in teacher education is seen as one way of bringing practice closer to theory. This study describes the use of cases in a university course for secondary school prospective mathematics teachers. The study investigates participants' views of cases taken from different sources and presented in different situations. In general, participants felt that the use of cases had an impact on their understanding of common mathematical errors but that cases based on mistakes they had themselves made during homework assignments were most meaningful.

Keywords Cases • Prospective secondary teachers • Preferences • Mathematical errors • Views

Introduction

Preparing future mathematics teachers is a complex process involving both academic and practical elements. Academic elements often include university or college courses aimed at promoting prospective teachers' mathematics knowledge needed for teaching (Ball, Thames, & Phelps, 2008). Practical elements often include fieldwork such as classroom observations and student teaching where the aim is to practice and apply what was learned in theory. Yet, bridging out-of-university practice and within-university academic courses can be a challenge (Zeichner, 2010). Some educators argue that clinical experiences should be central to teacher education and that all teacher preparation should stem from those experiences (Ball & Forzani, 2009). Others point out that prospective teachers' lack of experience may limit their observational skills and consequently limit what may be learned from fieldwork (Masinglia & Doerr, 2002). While we agree that clinical experiences are essential to teacher preparation, additional tools, such as analyzing classroom videos and cases, may also assist in bringing the classroom practice

P. Tsamir (✉) • D. Tirosh • E. Levenson • R. Barkai
Tel Aviv University, Tel Aviv, Israel
e-mail: pessia@tauex.tau.ac.il

H. Palmér, J. Skott (eds.), *Students' and Teachers' Values, Attitudes, Feelings and Beliefs in Mathematics Classrooms*,
https://doi.org/10.1007/978-3-319-70244-5_7

closer to future teachers, while enhancing future teachers' mathematics knowledge and pedagogical content knowledge (e.g., Markovits & Smith, 2008; Santagata & Guarino, 2011).

In this paper we describe a university course which, among other tools, used cases as a tool for promoting participants' mathematics knowledge, as well as their knowledge of common mathematical errors made by secondary school students. We use the term case in a broad sense to mean an actual event that occurred at some time in some learning situation and which may be generalized beyond the specific event to a larger set of mathematics education ideas. The cases used in this study all included an instance or several instances of students making mathematical errors. However, the cases were taken from different sources, such as research articles, the participants' own homework assignments, and classroom observations. In addition, the cases were introduced to participants in different situations such as homework assignments and classroom activities. Studies have shown that one's beliefs and affect may impact on the way that individual engages in professional development (Roesken-Winter, 2013). Thus, if we want to improve prospective teachers' engagement with the cases, it is relevant to investigate their views on the ways in which cases are used in the course. In addition, Ball (1990) claimed that the experiences of prospective teachers during their university methods course may impact on the future teachers' ideas, ways of seeing, and ways of acting. Thus, the way prospective teachers view their experiences with students' errors may impact on the way they see those errors when they become teachers and the way they act on those errors in their future classrooms. Our main research questions are: How do prospective teachers view the impact of cases taken from different sources on their understanding of mathematical errors? How do prospective teachers view the impact of cases presented in different situations on their understanding of mathematical errors?

Using Cases, Events, and Situations in Teacher Education

The idea of using cases in teacher education is not new. Shulman (1986), in his seminal work on teachers' knowledge, argued that although the case method was historically used in teaching law and medicine, it could also be used for teaching future teachers. He used the term "case knowledge" to describe "knowledge of specific, well documented, and richly described events" (p. 11). He warned, however, that a case "is not simply the report of an event or incident. To call something a case is to make a theoretical claim – to argue that it is a 'case of something,' or to argue that it is an instance of a larger class" (p. 11). Furthermore, Shulman claimed that the use of the case method in teacher education can illuminate both the practical and theoretical sides of teaching. In other words, it can help bridge the gap between fieldwork and course work.

Taking into account that not every event may be considered a case, it becomes relevant to discuss how and why certain cases are chosen specifically for the

education of future mathematics teachers. Markovits and Smith (2008) describe two kinds of cases used in mathematics teacher education – exemplars and problem situations. Exemplars consist of lengthy descriptions of an entire instructional episode that highlight key ideas about mathematics teaching and learning. Key ideas include not only pedagogical moves but also key mathematical ideas in a specific mathematical domain. For example, cases may be used to demonstrate the crucial role of teachers' actions and their interactions with students during classroom instruction that includes cognitively challenging mathematical tasks (Henningsen & Stein, 1997). Exemplars illustrate authentic practice. They do not necessarily exemplify best practice but may be used in teacher education to study factors which inhibit students' learning (Markovits & Smith, 2008).

The second type of case mentioned by Markowits and Smith (2008) is problem situations. As opposed to the first type described above, problem situations are usually relatively short and may convey real events that took place in a classroom or a hypothetical situation based on research related to students' ways of thinking. In general, they describe classroom events involving mathematics, which raise a problem or dilemma inviting readers to analyze the situation and to suggest ways of responding to the problem (Markovits & Even, 1999).

Different studies described different ways in which cases or situations were used in teacher education. Conner, Wilson, and Kim (2011) developed a tool they called "Situations" which consisted of three parts. The first part contained a prompt which included a brief description of a mathematical event along with students' and teachers' questions and insights. The second part was a description of various aspects of mathematical proficiency relevant to the event. The last part included a commentary which discussed a summary of key ideas to be found in the first two parts. In their study, participants discussed the Situation along with the facilitator, after being given time to individually reflect on the Situation. Pang (2011) used video cases accompanied by comprehensive narratives which included the background of the recorded lesson as well as related theory, when working with prospective teachers. The videos were taken from both planned and unplanned lessons given by experienced teachers and publically available video libraries, as well as recording of the prospective teachers' teaching during their fieldwork. Prospective teachers were required to view recording and read the accompanying text before coming to class and then discuss key elements during the class.

While most studies which investigated the use of cases in teacher education focused on promoting teachers' mathematical and pedagogical content knowledge, some studies also noted affective issues. Conner et al. (2011) reported that prospective teachers said that engagement with Situations was one of the most helpful aspects of their methods course. Furthermore, the instructor and prospective teachers found the Situations to be very relevant. While discussing video cases, Santagata and Guarino (2011) mentioned that when participants view exemplar cases, there can be a problem with the "distance PSTs [preservice teachers] might feel between their teaching abilities and the ability of the teachers portrayed in the videos" (p. 143). Lin (2005) remarked that the "video-cases motivated preservice teachers to rethink the importance of a student-oriented approach and to emphasize the need

for engaging students with challenging mathematical tasks" (p. 372). Working with practicing teachers, Walen and Williams (2000) suggested that discussions and reflections of cases elicited powerful reactions among the teachers and that the cases played a surprisingly powerful role in helping the teachers acknowledge and deal with their classroom concerns. In other words, using cases in teacher education has the potential to affect participants' engagement in their learning as well as their beliefs and practice regarding teaching mathematics.

Methodology

The Teacher Preparation Program and Course

Participants in this study were 31 students enrolled in a university program for preparing secondary school mathematics teachers. All students had a first degree in mathematics or a mathematically rich field such as engineering and after successfully completing the program would receive a teaching license. The program included attending university courses as well as doing 130 h of fieldwork in secondary school mathematics classes under the guidance of a mathematics teacher. In addition, all participants attended a workshop at the university, run by an expert mathematics teacher, who discussed with the participants their fieldwork experience. The expert teacher at the university was in contact with other university lecturers who taught these participants. In general, the aims of the university courses were to promote participants' mathematics content knowledge and pedagogical knowledge for teaching (Ball et al., 2008).

This study focuses on a semester-long university course, given by a senior university lecturer, which emphasized the analysis of students' mistakes as a way for promoting prospective teachers' knowledge of common student errors and, in addition, as a way for promoting participants' mathematics knowledge. During the course, which met once a week for 2 h, students were introduced to theories for analyzing the reasons behind students' common mistakes such as the conflict between concept images and concept definitions (Tall & Vinner, 1981), intuitive rules (Stavy & Tirosh, 2000), and the interaction between formal, algorithmic, and intuitive elements of mathematics (Fischbein, 1993).

The mathematical errors analyzed during the course came from different sources and were presented and discussed among participants in different ways. In total, there were six different error analysis situations. The sources of the first two situations were errors made by the participants of the course. Every week, students were given a series of four to five mathematics problems to solve at home, and then during the class, the lecturer went over the participants' correct and incorrect solutions. Thus the source of the first situation was the participants' homework errors. At times, a new problem was presented in class, and students solved the problem in class, with some participants solving the problem on the whiteboard up front. Thus,

the second source was participants' errors made during the class. The source of the third and fourth error situations came from research papers. From the fourth lesson on, two students presented to the class their summary of common mistakes taken from a research paper assigned to them by the lecturer. The third source came from research papers describing quantitative studies of students' common errors. The fourth source came from papers describing classroom interactions and a qualitative analysis of a case which involved students' mathematical errors. The fifth and sixth sources came from the participants' current experiences with secondary school students. During the semester, participants took part in classroom observations and were required twice during the semester to report and analyze cases taken from those observations. The fifth error situation was a case chosen by the lecturer to be used in a homework assignment. All participants were required to analyze errors which arose during the case, according to theories learned during the course. The sixth source came from the participants' experience with students. Participants were given two mathematics problems known to cause student errors. Participants were required to ask two high school students to solve those problems, interview the students, and then analyze the students' solutions, including errors that arose during the solution process.

Research Tool

At the end of the course, students were requested to fill out a questionnaire that had the following instructions:

> During the course, you had the opportunity to analyse incorrect solutions which arose in different situations. For each situation, rate the extent to which the activity helped you to understand error analysis: greatly, to some extent, a little bit, not at all.

Following this instruction was a list of the six different situations, as described above. The lecturer also clarified each situation orally ensuring that all of the participants recognized the different situations. At the end of the six situations, participants were asked to answer the following question: If you had to choose only two situations (from the above six) which would you choose?

Results

Recall that participants were requested to rate the extent of each of the six situations' impact on their understanding of common mathematical errors. Each rating was assigned a numerical value: 1, not at all; 2, a little bit; 3, to some extent; and 4, greatly. To begin with, we note that only one participant rated one situation (Situation 6) as not having any impact at all. In other words, results indicated that prospective teachers perceive the use of cases, regardless of the source of the case or the

Table 1 Mean ratings of each situation's impact ($N = 31$)

Source of mistakes	Mistakes made by participants		Mistakes reported in research papers		Eyewitness to others' mistakes	
Situation	1 H.W.	2 Classwork	3 Quantitative study	4 Qualitative study	5 Classroom observation	6 Student interview
Mean	3.84	3.52	3.39	3.13	3.39	2.97
SD	0.37	0.57	0.72	0.72	0.72	0.84

situation in which it is presented, as having an impact on their understanding of mathematical errors. Furthermore, the mean ratings (see Table 1) indicate that overall, the use of cases was perceived as being more than just a little bit meaningful.

Taking a closer look at the results, we note slight differences. Situation 1, where mistakes made by participants in their homework assignments were discussed in class, was perceived as having the most impact on participants' understanding of the errors, followed by mistakes made by participants while engaged in classwork. In other words, participants viewed that analyzing their own mathematical mistakes was more meaningful than analyzing mistakes made by others. Participants viewed Situations 3 and 5 as having less of an impact than Situations 1 and 2 but more than Situations 4 and 6.

That participants viewed Situations 3 and 5 as having the same impact on their learning is surprising. First, the errors presented in the two situations come from very different sources. Situation 3 consisted of cases taken from research papers that reported on quantitative studies of students' common mistakes, while Situation 5 was one case taken from a participant's classroom observation. Second, the way each situation was used was also different. Situation 3 was discussed in class, and Situation 5 was a homework assignment. While the same surprise might be felt for the similar preference for Situations 4 and 6, those situations are at least similar in that both deal in depth with one or two students and the mathematical errors those few students made. In essence, it may be said that Situation 4 prepared them to deal with Situation 6 although Situation 6 was more personal in that the participant actually conducted firsthand an informal qualitative study.

Although participants were not requested to explain their ratings, some participants did add clarifications. For example, regarding the high rating for the first two situations, one participant wrote for Situation 1, "the attention given to our solutions sharpened my understanding of mistakes and corresponding theories." Regarding Situation 2 that same participant wrote, "when we talk about 'our' solutions, I relate better to the material." One participant commented on his or her high rating for Situation 6, "when I interviewed the students, I went on to other topics which sharpened my understanding of the source of those mistakes and this will help me in my teaching." In other words, this participant used the interview situation as an opportunity to test out other theories and thereby strengthen knowledge gained regarding errors.

For the most part, as noted above, Situation 6 received relatively low ratings. This may be explained by one participant's comment: "We only had to interview

Table 2 Frequency (%) of participants' choices for most preferred situations ($N = 31$)

Source of mistakes	Mistakes made by participants		Mistakes reported in research papers		Eyewitness to others' mistakes	
Situation	1 H.W.	2 Classwork	3 Quantitative study	4 Qualitative study	5 Classroom observation	6 Student interview
Frequency	25 (81)	11 (35)	12 (39)	4 (13)	6 (19)	2 (6)

two people. Perhaps if we interviewed more people it would have helped more in our understanding of students' common mistakes (two is not a representative sample)." While this comment was given by only one participant, it hints at a possible reason for the relatively low ratings given to both Situations 4 and 6. In their attempt to understand why some errors are more common than others, perhaps participants may have felt the need to read about or to experience many students making the same error as opposed to hearing about or even personally interacting with one or two who made those errors. On the other hand, another participant wrote that the reports on quantitative studies were less effective than the reports from qualitative studies because, "in the case of the quantitative studies, we did not go into depth, and not enough time was given, but for the other papers we analysed concrete and clear mistakes and that helped our understanding." It is not clear from this comment if the participant means that the quantitative studies dealt with too many errors at once and thus it was impossible to analyze and understand all of them in depth or, for some reason, during the class, there was less time devoted to those papers. In any case, for this participant, it was important to understand each mistake in detail.

On the second part of the questionnaire, participants were asked to choose the two situations they prefer most. Two participants chose only one situation. Table 2 presents the number of teachers who chose each situation as one of their most preferred. As can be seen from Table 2, Situation 1 was chosen by over three-quarters of the teachers, Situations 2 and 3 by approximately a third of the participants, and the rest by even less participants. Combining situations from the same source, 27 participants (87%) would choose at least one case stemming from their own mistakes (Situations 1 or 2), 15 (48%) participants would choose at least one situation based on errors reported in a research paper (Situations 3 and 4), and 8 (26%) participants would choose cases based on others' mistakes that they or their peers had witnessed.

In light of the responses to the first part of the questionnaire, it is not surprising that so many participants chose the first situation as one of the two they most preferred. One participant who chose Situation 1 commented:

> Sometimes, when working on the assignments, we discussed problems that were more complex than the usual broad common mistakes and you could get lost. Still, the solutions and their analysis were a tool that allowed me to solidify my understanding of the concept and the mistake by going over it several times.

That same participant also chose Situation 3 and wrote:

> Relating to common mistakes found in research allowed me to focus on the specific mathematical mistake (and not on all kinds of different mistakes) that stem from a specific mathematical problem. For example, the intuitive rule 'More of A, More of B', when talking about a simple function such $f(x) = 0.5^x$ and the question of which is greater $f(2)$ or $f(1)$. Together with the detailed frequencies and the discussion of the students' explanations (along with the researcher's analysis), helped me to understand better this type of error.

Two differences between the results of the first and second parts of the questionnaires can be seen. First, on the second part of the questionnaire, only a third of the participants chose Situation 2 as one of the two most preferred situations, while almost all of the participants gave this situation a high impact rating on the first part of the questionnaire. It could be that participants viewed the first two situations as being very similar, and thus if they could only choose two situations and they already chose Situation 1, then there was no need to also choose Situation 2. The second difference between the two parts of the questionnaires was participants' responses to Situations 3 and 5. While on the first part of the questionnaire, participants gave these situations similar impact ratings, on the second part, twice as many chose Situation 3 as Situation 5.

Summary, Discussion, and Implications

The first question addressed by this paper was: How do prospective teachers view the impact of cases taken from different sources on their understanding of mathematical errors? The errors in the cases came from three sources: the participants' own errors (Situations 1 and 2), errors reported in research studies (Situations 3 and 4), and participants' observations of others' errors (Situations 5 and 6). Findings from both parts of the questionnaire indicated that participants' viewed learning from their own errors as most meaningful. When it came to choosing only two situations, most participants chose Situation 1 (learning from mistakes they made when solving homework assignments) over Situation 2 (learning from mistakes made during classwork). Although we cannot know for sure the reasons for this preference, it could be that there was more anonymity in discussing mistakes made in the privacy of one's home than mistakes made in class. It could also be that participants had more time at home to work on mathematics problems, and thus discussing those problems was more meaningful than discussing mistakes made on the spot during class. Finally, mistakes taken from the homework assignments were specifically chosen by the lecturer for discussion because at least several participants made the same mistake and thus prospective teachers could relate to those mistakes.

Regarding learning from mistakes made by others, on the one hand, participants found that learning about mistakes from research papers and learning from observations were both meaningful. On the other hand, if they had to choose one or the other, most would choose cases based on errors reported in research studies. This last result is surprising because learning from research studies was thought to be more connected with theory and rather removed from practice, while analyzing

errors made by students whom the participants directly interviewed was thought to bring practice closer to the theory discussed in the course. According to Zeichner (2010), bridging the gap between theory and practice is important and is thought to impact greatly on teacher preparation. In trying to understanding this result, we take a look at the differences between the ways these cases were presented and used in the course and now turn toward the second research question.

The second question addressed by this paper was: How do prospective teachers view the impact of cases presented in different situations on their understanding of mathematical errors? We begin with the problem of why participants preferred cases taken from research papers over cases based on their own observations. The cases based on research papers were presented by different participants to the whole class and then discussed and analyzed in the course along with the teacher educator and the other participants. The cases based on the participants' own observations were analyzed by individual participants as part of a homework assignment and as part of the final project. Although the participants received feedback on their work, these cases were not discussed by the whole group of participants along with the teacher educator. While researchers agree that it is important for prospective teachers to observe and reflect on students' mathematical thinking (e.g., Ball et al., 2008), one of the problems with fieldwork placements is that the prospective teachers lack a common experience to discuss with their peers (Masingila & Doerr, 2002). The same might be said for analyzing students' mathematical thinking through the analysis of cases. Students preferred to learn from situations that were discussed together in class (Situations 1, 2, 3, and 4) over situations that were analyzed alone (Situations 5 and 6).

There are several possible implications of this study for teacher education. First, the use of cases, in general, is seen as a positive learning activity for prospective teachers. However, not all cases have the same impact. Although participants agreed that it was fruitful to learn from their own mistakes, they preferred to learn from mistakes they made individually but then discussed collectively. Thus, teacher educators might take into consideration prospective teachers' comfort zone when discussing mistakes in class as well as their need to discuss mistakes with their peers. Second, although bringing practice closer to theory is important, it does not preclude learning from previous research reports. This is in line with Tsamir (2008) who showed the effectiveness of using theories as tools in teacher education. Finally, that participants felt it was less meaningful to analyze errors made by students they interviewed may inform teacher educators who work with prospective teachers that have limited access to field practice.

Studying cases can support prospective teachers' analysis and reflection of their own emerging practices (Masingila & Doerr, 2002). Thus, teachers' preferences for the different case sources and different ways of working with the cases may impact on the way participants will use students' mistakes during their future practice. The positive views prospective teachers had with regard to learning from their own mistakes may encourage these participants to build on their future students' mistakes as part of their future teaching practice, instead of trying to avoid or simply "fix" mistakes. Kaur (2009) suggested that an important element of good teaching prac-

tice is to encourage students to learn from their mistakes, not only by stressing the final answer, but by focusing on the kinds of mistakes made. Similarly, participants added comments on their questionnaires stressing the importance of analyzing the errors and not just fixing them. These positive experiences will hopefully impact on their future teaching.

Acknowledgment This research was supported by the Trump Foundation grant no. 145.

References

Ball, D. L. (1990). Breaking with experience in learning to teach mathematics: The role of a pre-service methods course. *For the learning of mathematics, 10*(2), 10–16.

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5), 497–511. https://doi.org/10.1177/0022487109348479

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Conner, A., Wilson, P. S., & Kim, H. J. (2011). Building on mathematical events in the classroom. *ZDM, 43*(6–7), 979–992. https://doi.org/10.1007/s11858-011-0362-1

Fischbein, E. (1993). The interaction between the formal, the algorithmic and the intuitive components in a mathematical activity. In R. Biehler, R. Scholz, R. Strässer, & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline* (pp. 231–245). Dordrecht, The Netherlands: Kluwer.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*, 524–549. https://doi.org/10.2307/749690

Kaur, B. (2009). Characteristics of good mathematics teaching in Singapore grade 8 classrooms: A juxtaposition of teachers' practice and students' perception. *ZDM, 41*(3), 333–347. https://doi.org/10.1007/s11858-009-0170-z

Lin, P. J. (2005). Using research-based video-cases to help pre-service primary teachers conceptualize a contemporary view of mathematics teaching. *International Journal of Science and Mathematics Education, 3*(3), 351–377. https://doi.org/10.1007/s10763-004-8369-5

Markovits, Z., & Even, R. (1999). The decimal point situation: A close look at the use of mathematics-classroom-situations in teacher education. *Teaching and Teacher Education, 15*(6), 653–665. doi.org/10.1016/S0742-051X(99)00020-7

Markovitz, Z., & Smith, M. (2008). Cases as tools in mathematics teacher education. In D. Tirosh & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 2, pp. 39–64). Rotterdam, The Netherlands: Sense Publishers.

Masingila, J. O., & Doerr, H. M. (2002). Understanding pre-service teachers' emerging practices through their analyses of a multimedia case study of practice. *Journal of Mathematics Teacher Education, 5*(3), 235–263. https://doi.org/10.1023/A:1019847825912

Pang, J. (2011). Case-based pedagogy for prospective teachers to learn how to teach elementary mathematics in Korea. *ZDM, 43*(6–7), 777–789. https://doi.org/10.1007/s11858-011-0352-3

Roesken-Winter, B. (2013). Capturing mathematics teachers' professional development in terms of beliefs. In *Proficiency and beliefs in learning and teaching mathematics* (pp. 157–178), Rotterdam, The Netherlands: Sense Publishers. https://doi.org/10.1007/978-94-6209-299-0_11

Santagata, R., & Guarino, J. (2011). Using video to teach future teachers to learn from teaching. *ZDM, 43*(1), 133–145. https://doi.org/10.1007/s11858-010-0292-3

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Stavy, R., & Tirosh, D. (2000). *How students (mis-) understand science and mathematics: Intuitive rules*. Teachers College Press, NY, USA.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*(2), 151–169. https://doi.org/10.1007/BF00305619

Tsamir, P. (2008). Using theories as tools in mathematics teacher education. In D. Tirosh & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 2, pp. 211–234). Rotterdam, The Netherlands: Sense Publishers.

Walen, S. B., & Williams, S. R. (2000). Validating classroom issues: Case method in support of teacher change. *Journal of Mathematics Teacher Education, 3*(1), 3–26. https://doi.org/10.1023/A:1009917510 31

Zeichner, K. (2010). Rethinking the connections between campus courses and field experiences in college-and university-based teacher education. *Journal of Teacher Education, 61*(1–2), 89–99. https://doi.org/10.1177/0022487109347671