Finite-Time Adaptive Attitude Stabilization for Spacecraft Based on Modified Power Reaching Law

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Abstract. In this paper, a finite-time adaptive sliding mode control scheme is proposed for the attitude stabilization of spacecrafts with lumped uncertainties. By introducing an exponential function in the reaching law design, an improved reaching law is developed such that the faster convergence of sliding mainfold can be achieved. Then, an adaptive controller is proposed based on the modified reaching law to guarantee the finite time attitude stabilization of spacecrafts by adaptive estimating the bounds of uncertainties. Besides, the chattering problem is reduced by using a power rate term in the controller design. Simulations are given to illustrate the effectiveness and superior performance of the proposed method.

Keywords: Finite-time adaptive stabilization · Spacecraft system · Sliding mode control · Power reaching law

1 Introduction

Attitude stabilization for spacecrafts has gained extensive interest in recent years, however, it is still a challenge to achieve the attitude stabilization with rapid convergence and high accuracy. Recently, there have been numerous researches in the literature on spacecraft attitute control (see, for instance, [1–4]).

Due to the excellent properties such as robustness to uncertainties and faster convergence, sliding mode control has been widely used in spacecraft attitude control. In [2], two sliding mode controllers are proposed to drive system states to the origin with the finite-time convergence for spacecraft attitude stabilization. In [3], an adaptive sliding mode control (SMC) is proposed for spacecrafts to ensure that the attitude control can be achieved with actuator saturation. In [4], an adaptive finite-time fault-tolerant controller has been proposed for rigid spacecrafts with external disturbances subject to four types of actuator faults. In [5], a power rate reaching strategy based on the conventional reaching law is applied to reduce chattering, but it increased the reaching time. Recently, an exponential reaching law (ERL) was proposed in [6]. By introducing an exponential function in the reaching law design, the faster convergence of sliding

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mode variable can be achieved. Considering the convergence rate and the chattering problem, the choice of coefficient of sign function becomes sensitive. Based on ideas in [6], this paper proposes an adaptive controller based on the modified reaching law, and the finite time attitude stabilization is guaranteed for spacecrafts with system uncertainties and disturbances. Besides, the chattering problem is reduced by using a power rate term in the controller design.

The rest of this paper is organized as follows. In Sect. 2, a spacecraft attitude model is constructed based on unit quaternion and the transformed attitude dynamics is developed in a more convenient way. In Sect. 3, an adaptive controller with modified power reaching law (MPRL) is designed to ensure that the sliding states can converge in finite time rapidly, then the system states can converge into a small region through Lyapunov stability analysis. Simulation results are presented in Sect. 4. Finally, this paper is concluded in Sect. 5.

2 Preliminaries

2.1 Spacecraft Dynamics and Kinematics Equations

Consider the following attitude kinematics and dynamics equations of the spacecraft in terms of quaternion [7]:

$$J\dot{\omega} = -\omega^{\times} J\omega + u + d\left(t\right) \tag{1}$$

$$\dot{q}_v = \frac{1}{2}\left(q_v^{\times} + q_0 I_3\right)\omega \tag{2}$$

$$\dot{q}_0 = -\frac{1}{2} q_v^T \omega \tag{3}$$

where $\omega \in \mathbb{R}^3$ is the angular velocity of the spacecraft; $I \in \mathbb{R}^{3\times 3}$ is the identity matrix; $J \in \mathbb{R}^{3\times 3}$ is the innertia matrix of the spacecraft, $u \in \mathbb{R}^3$ and $d(t) \in \mathbb{R}^3$ are the control torque and the external unknow disturbances including environmental disturbances, respectively. The unit quaternion $Q = \left[q_0, q_1, q_2, q_3\right]^T = \left[q_0, q_v^T\right]^T \in \mathbb{R} \times \mathbb{R}^3$ describes the attitude orientation and satisfies the constraint $q_0^2 + q_v^T q_v = 1$. The notation $a^{\times}$ for a vector $a = [a_1, a_2, a_3]^T$ is used to denote the skew-symmetric matrix $a^{\times} = [0, -a_3, a_2; a_3, 0, -a_1; -a_2, a_1, 0]$.

Assume that the inertia matrix J is a form of $J = J_0 + \Delta J$, where J_0 and ΔJ denote the nominal part and the uncertain part of J, respectively. Then, (1) can be rewritten as

$$J_0\dot{\omega} = -\omega^{\times} J_0\omega + u + d\left(t\right) - \Delta J\dot{\omega} - \omega^{\times}\Delta J\omega \tag{4}$$

Property 1. The nominal part J_0 is a symmetric and positive definite matrix and satisfies:

$$J_1\|x\|^2 \le x^T J_0 x \le J_2\|x\|^2, \forall x \in \mathbb{R}^3, \tag{5}$$

where J_1 and J_2 are positive constants, denoting the lower and upper bounds of J_0, respectively.

2.2 Transformed Spacecraft Attitude Dynamics

For a more convenient way to express the attitude dynamics controller design, the Lagrange-like equation in [4] is utilized to describe the spacecraft attitude dynamic (1). Let $T = \frac{1}{2}\left(q_v^{\times} + q_0 I_3\right) \in \mathbb{R}^{3\times 3}$, and (2) can be rewritten as

$$\omega = P\dot{q}_v \tag{6}$$

with

$$P = T^{-1} = \left[\frac{1}{2}\left(q_v^{\times} + q_0 I_3\right)\right]^{-1} \tag{7}$$

Then, differentiating (6) yields:

$$\dot{\omega} = \dot{P}\dot{q}_v + P\ddot{q}_v \tag{8}$$

Substituting (6) and (8) into (4) and premultiplying both sides of the resulting expression by P^T leads to

$$J^*\ddot{q}_v = -\Xi\dot{q}_v + P^T u + T_d \tag{9}$$

where $J^* = P^T J_0 P$, $\Xi = P^T J_0 \dot{P} - P^T (J_0 P\dot{q}_v)^{\times} P$, and $T_d = P^T d\left(t\right) - P^T \Delta J\dot{\omega} - P^T \omega^{\times}\Delta J\omega$. Here, T_d is considered as the lumped disturbances and uncertainties. Regarding the dynamic model given in (9) and Property 1, some more properties and assumptions are given as follows.

Property 2. [8] The inertia matrix J^* is symmetric and positively definite, and the matrix $\dot{J}^* - 2\Xi$ satisfies the following skew-symmetric relationship:

$$x^T\left(\dot{J}^* - 2\Xi\right)x = 0, \quad \forall x \in \mathbb{R}^3, \tag{10}$$

Property 3. [8] The inertia matrix J^* satisfies the following bounded condition:

$$J_{\min}\|x\|^2 \le x^T J^* x \le J_{\max}\|x\|^2, \forall x \in \mathbb{R}^3, \tag{11}$$

where J_{min} and J_{max} are positive constants, denoting the lower and upper bounds of J^*, respectively.

Assumption 1. [8] *To guarantee the existence of P defined in (7), the following condition should be satisfied:*

$$\det\left(T\right) = \frac{1}{2}q_0 \ne 0 \quad \forall t \in [0, \infty) \tag{12}$$

Assumption 2. [9] *The lumped term T_d of the disturbances and uncertainties satisfies the following relationship:*

$$\|T_d\| \le \gamma_0 \Phi \tag{13}$$

where $\Phi = 1 + \|\omega\| + \|\omega\|^2$ and γ_0 is a positive constant.

3 Finite Time Adaptive Control

3.1 Modified Power Reaching Law

In this subsection, the sliding mainfold $s \in \mathbb{R}^3$ is selected as

$$s = \dot{q}_v + \alpha q_v + \beta sig(q_v)^r \tag{14}$$

where α and β are positive constants; $r = \frac{r_1}{r_2}$, r_1 and r_2 are positive odd integers and $0 < r_1 < r_2$; the function $sig(q_v)^r$ is defined as

$$sig(q_v)^r = \left[|q_{v1}|^r sign\left(q_{v1}\right), |q_{v2}|^r sign\left(q_{v2}\right), |q_{v3}|^r sign\left(q_{v3}\right)\right]^T$$

Differentiating (14) with respect to time yields

$$\dot{s} = \ddot{q}_v + \alpha \dot{q}_v + \beta \cdot r \cdot diag\left(|q_v|^{r-1}\right) \dot{q}_v \tag{15}$$

where $diag\left(|q_v|^{r-1}\right) = diag\left(\left[|q_{v1}|^{r-1}, |q_{v2}|^{r-1}, |q_{v3}|^{r-1}\right]\right) \in \mathbb{R}^{3\times3}$.

Remark 1. If $q_{vj} = 0$ and $\dot{q}_{vj} \neq 0$, the singularity occurs because of a negative fractional power $r - 1$. To avoid singularity, the first-order derivative of s is modified as [10]

$$\dot{s} = \ddot{q}_v + \alpha \dot{q}_v + \beta q_{vr} \tag{16}$$

with $q_{vr} \in \mathbb{R}^3$ defined as

$$q_{vr,j} = \begin{cases} r|q_{vj}|^{r-1}\dot{q}_{vj}\ , \text{if}\, |q_{vj}| \geq \epsilon \text{ and } \dot{q}_{vj} \neq 0 \\ r|\epsilon|^{r-1}\dot{q}_{vj}\ \ , \ \text{if}\, |q_{vj}| < \epsilon \text{ and } \dot{q}_{vj} \neq 0 \\ \quad 0, \qquad \dot{q}_{vj} = 0 \end{cases} \tag{17}$$

where ϵ is a small constant. Then, considering (9), (14), and (16), it can be shown that

$$J^* \dot{s} = -\Xi s + P^T u + F + T_d \tag{18}$$

where $F = \Xi \alpha q_v + \Xi \beta sig(q_v)^r + J^* \alpha \dot{q}_v + J^* \beta q_{vr}$.

In this paper, a modified reaching law is proposed and expressed as

$$\dot{s} = -\frac{K}{D\left(s\right)} |s_j|^\theta sign\left(s\right) \tag{19}$$

$$D\left(s\right) = \mu + \left(\varphi - \mu\right) e^{-\vartheta \|s\|} \tag{20}$$

where $0 < \theta < 1$, $K > 0$, $0 < \mu < 1$, $\varphi = 1$ and $\vartheta > 0$.

In the proposed approach, as pointed out in [6], the $D(s)$ is strictly positive, so it does not affect the stability of SMC. If $\|s\|$ grows, $D(s)$ goes towards μ and $K|s_j|^\theta/D(s)$ would be $K|s_j|^\theta/\mu$, which is greater than K. In contrast, when $\|s\|$ decreases, it tends to $K|s_j|^\theta/\varphi$. This phenomenon makes the controller gain to be modified between $K|s_j|^\theta/\mu$ and $K|s_j|^\theta/\varphi$. Therefore, the MPRL specifies faster reaching speed compared with the conventional reaching law in [5] considering similar gain K. In addition, a term $|s_j|^\theta$ is employed to reduce the chattering problem which compares with the ERL in [6].

3.2 Controller Design and Stability Analysis

The finite-time adaptive control law is designed as

$$u = -P\left[u_{nom} + \frac{K}{D(s)\left\|P\right\|^2}\left|s_j\right|^\theta sign(s)\right] \tag{21}$$

with

$$u_{nom} = \frac{(\|F\| + \hat{\gamma}_0 \varPhi)\|s\| s}{\|Ps\|^2} \tag{22}$$

where $\hat{\gamma}_0$ is the estimated values of γ_0, and the adaptive law is chosen as

$$\dot{\hat{\gamma}}_0 = c_0\left(\varPhi\|s\| - \varepsilon_0\hat{\gamma}_0\right) \tag{23}$$

where c_0 and $\varepsilon_0 > 0$ are the designed parameters then the selection of c_0 is according to (29) and the initial estimated values satisfy $\hat{\gamma}_0(0) > 0$.

Lemma 1. [4] *Suppose $a_1, a_2, \ldots, a_n$ are positive numbers and $0 < p < 2$. Then, the following relationship exists:*

$$\left(a_1^2 + a_2^2 + \cdots + a_n^2\right)^p \le \left(a_1^p + a_2^p + \cdots + a_n^p\right)^2 \tag{24}$$

Lemma 2. [2] *Consider the nolinear system $\dot{x} = f(x,u)$. Suppose that there exist continuous function $V(x)$, scalars $\lambda > 0$, $0 < \alpha < 1$ and $0 < \eta < \infty$ such that*

$$\dot{V}(x) \le -\lambda V^\alpha(x) + \eta \tag{25}$$

Then, the trajectory of system $\dot{x} = f(x,u)$ is pratical finite-time stable (PFS).

Lemma 3. [4] *Consider the sliding mode mainfold s defined by (14). If the sliding mode mainfold satisfies $s = 0$, then the system states q_v and $\dot{q}_v$ can converge to $q_v = 0$ and $\dot{q}_v = 0$ in finite time, respectively.*

Theorem 1. *Considering the attitude control systems (1)–(3), the adaptive controllers in (21)–(22), and the update law in (23) under Assumptions 1-2, the sliding mode mainfold s, spacecraft attitude q_{vj} and angular velocity ω_j $(j = 1,2,3)$ are locally finite-time uniformly ultimately bounded.*

Proof. Consider the following Lyaounov function candidate V_1

$$V_1 = \frac{1}{2}s^T J^* s + \frac{1}{2c_0}\tilde{\gamma}_0^2$$

$$\begin{aligned}
\dot{V}_1 &= \tfrac{1}{2}s^T \dot{J}^* s + s^T J^* \dot{s} - \tfrac{1}{c_0}\tilde{\gamma}_0\dot{\hat{\gamma}}_0 \\
&= \tfrac{1}{2}s^T \dot{J}^* s + s^T\left(-\varXi s + P^T u + F + T_d\right) - \tfrac{1}{c_0}\tilde{\gamma}_0\dot{\hat{\gamma}}_0 \\
&\le -\|F\|\|s\| - \tfrac{K}{D(s)}\sum_{j=1}^{3}|s|^{\theta+1} + \|F\|\|s\| + (\|T_d\| - \gamma_0\varPhi)\|s\| + \varepsilon_0\tilde{\gamma}_0\hat{\gamma}_0 \\
&\le -\tfrac{K}{D(s)}\sum_{j=1}^{3}|s|^{\theta+1} + \varepsilon_0\tilde{\gamma}_0\hat{\gamma}_0
\end{aligned} \tag{26}$$

Note that for any positive scalar $\delta_0 > \frac{1}{2}$, the following inequlity exists:

$$\varepsilon_0 \tilde{\gamma}_0 \hat{\gamma}_0 = \varepsilon_0 \tilde{\gamma}_0 \left(-\tilde{\gamma}_0 + \gamma_0\right) \leq \tfrac{-\varepsilon_0(2\delta_0 - 1)}{2\delta_0} \tilde{\gamma}_0^2 + \tfrac{\varepsilon_0 \delta_0}{2} \gamma_0^2 \tag{27}$$

Thus inequality (26) can be expressed as

$$\begin{aligned} \dot{V}_1 &\leq -\tfrac{K}{D(s)} \sum_{j=1}^{3} |s|^{\theta+1} - \left(\tfrac{\varepsilon_0(2\delta_0-1)}{2\delta_0} \tilde{\gamma}_0^2\right)^{\frac{\theta+1}{2}} + \left(\tfrac{\varepsilon_0(2\delta_0-1)}{2\delta_0} \tilde{\gamma}_0^2\right)^{\frac{\theta+1}{2}} + \varepsilon_0 \tilde{\gamma}_0 \hat{\gamma}_0 \\ &\leq -\varsigma \left[\left(\tfrac{1}{2} s^T J^* s\right)^{\frac{\theta+1}{2}} + \left(\tfrac{1}{2c_0} \tilde{\gamma}_0^2\right)^{\frac{\theta+1}{2}} \right] + \left(\tfrac{\varepsilon_0(2\delta_0-1)}{2\delta_0} \tilde{\gamma}_0^2\right)^{\frac{\theta+1}{2}} + \varepsilon_0 \tilde{\gamma}_0 \hat{\gamma}_0 \end{aligned} \tag{28}$$

where

$$\varsigma = \frac{K}{D\left(s\right)\left(\frac{1}{2} J_{\max}\right)^{(\theta+1)/2}}, \; c_0 = \frac{\delta_0 \varsigma^{2/(\theta+1)}}{\varepsilon_0 \left(2\delta_0 - 1\right)} \tag{29}$$

Note that Lemma 1, $\delta_0 > \frac{1}{2}$ and $\frac{1}{2} < \frac{\theta+1}{2} < 1$

$$\dot{V}_1 \leq -\varsigma V_1^{\frac{\theta+1}{2}} + \left(\frac{\varepsilon_0 \left(2\delta_0 - 1\right)}{2\delta_0} \tilde{\gamma}_0^2\right)^{\frac{\theta+1}{2}} + \varepsilon_0 \tilde{\gamma}_0 \hat{\gamma}_0 \tag{30}$$

According to [2], the following inequality can be obtained:

$$\left(\frac{\varepsilon_0 \left(2\delta_0 - 1\right)}{2\delta_0} \tilde{\gamma}_0^2\right)^{\frac{\theta+1}{2}} + \varepsilon_0 \tilde{\gamma}_0 \hat{\gamma}_0 \leq \frac{\varepsilon_0 \delta_0}{2} \gamma_0^2 \tag{31}$$

Thus, from (30) and (31), we can obtain

$$\dot{V}_1 \leq -\varsigma V_1^{\frac{\theta+1}{2}} + \phi \tag{32}$$

where $\phi = \frac{\varepsilon_0 \delta_0}{2} \gamma_0^2$.

From (32), the sliding mainfold is finite-time uniformly ultimately bounded by using Lemma 2. Hence, the bounded convergence region Δs is obtained as

$$|s_j| \leq \Delta s = \sqrt{\frac{2}{J_{\max}}} \left(\frac{\phi}{\varsigma}\right)^{\frac{1}{\theta+1}}, j = 1, 2, 3 \tag{33}$$

Then, the sliding mode mainfold defined in (14) can be expressed as follows:

$$\dot{q}_{vj} + \alpha q_{vj} + \beta sig(q_{vj})^r = \eta_j \;, \quad |\eta_j| \leq \Delta s \tag{34}$$

Then, (34) can be written in the following two forms:

$$\dot{q}_{vj} + \left(\alpha - \frac{\eta_j}{q_{vj}}\right) q_{vj} + \beta sig(q_{vj})^r = 0, \tag{35}$$

$$\dot{q}_{vj} + \alpha q_{vj} + \left(\beta - \frac{\eta_j}{sig(q_{vj})^r}\right) sig(q_{vj})^r = 0, \tag{36}$$

From (35) and (36), if $\alpha - \frac{\eta_j}{q_{vj}} > 0$ and $\beta - \frac{\eta_j}{sig(q_{vj})^r} > 0$, they have similar structures to the proposed sliding mode mainfold. Therefore, by using Lemma 3, the attitude q_{vj} converges to the regions

$$|q_{vj}| \leq \frac{|\eta_j|}{\alpha} \leq \frac{\Delta s}{\alpha} \tag{37}$$

$$|q_{vj}| \leq \left(\frac{|\eta_j|}{\beta}\right)^{\frac{1}{r}} \leq \left(\frac{\Delta s}{\beta}\right)^{\frac{1}{r}} \tag{38}$$

in finite time. Finally, the attitude q_{vj} converges to the region

$$|q_{vj}| \leq \min\left\{\frac{\Delta s}{\alpha}, \left(\frac{\Delta s}{\beta}\right)^{\frac{1}{r}}\right\} \tag{39}$$

in finite time. Moreover, from (34), $\dot{q}_{vj}$ converges to the region

$$|\dot{q}_{vj}| \leq |\eta_j| + \alpha\,|q_{vj}| + \beta|q_{vj}|^r \leq 3\Delta s \tag{40}$$

in finite time.

It should be noticed that $\|q_v^\times + q_0 I_3\| = 1$. From (2), $\|\omega\|_\infty \leq 2\sqrt{3}\|\dot{q}_v\|_\infty$ is obtained. However, because $|\dot{q}_{vj}| \leq 3\Delta s\ \ (j = 1, 2, 3)$ in finite time, $\|\dot{q}_v\|_\infty \leq 3\Delta s$ can be satisfied in finite time. Therefore, considering (6) and Assumption 1, $|\omega_j| \leq 6\sqrt{3}\Delta s$ can be concluded. Based on the above analysis, the sliding mode mainfold s, spacecraft attitude q_{vj} and angular velocity ω_j are locally finite-time uniformly ultimately bounded. This completes the proof.

Remark 2. From (33), it can be seen that the larger parameter ς or the smaller parameter ϕ will lead to the smaller Δs. Besides, as seen from (39), larger parameters α and β or smaller parameter r can result in the smaller accuracy of the attitude stabilization.

Remark 3. In order to avoid the chattering problems caused by the discontinous term $\frac{s}{\|Ps\|^2}$ in (22), we employ the continuous function $\frac{s}{\|Ps\|^2+\xi}$ to replace it in the following simulation section, where $\xi > 0$.

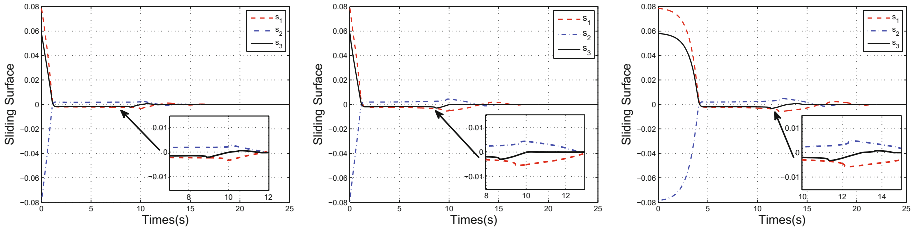
4 Simulation Results

In this section, some simulation results are provided to illustrate the effectiveness of the proposed controller. For comparison, the ERL in [5] and conventional reaching law in [6] are also simulated. The expressions can be writen as follows respectively

$$\dot{s} = -\frac{K}{D(s)} sign(s) \tag{41}$$

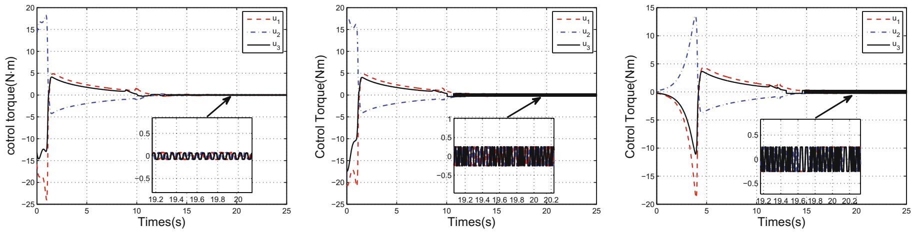
$$\dot{s} = -K \cdot sign(s) \tag{42}$$

where K and $D(s)$ is similar chosen as (19) and (20), respectively.



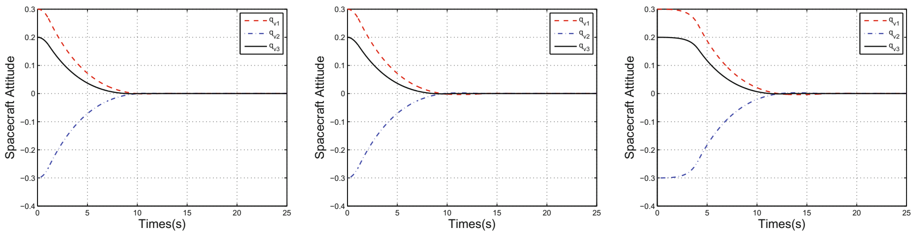
(a) (b) (c)

Fig. 1. Sliding surface with different reaching laws. (a)MPRL. (b)ERL. (c)Conventional reaching law.



(a) (b) (c)

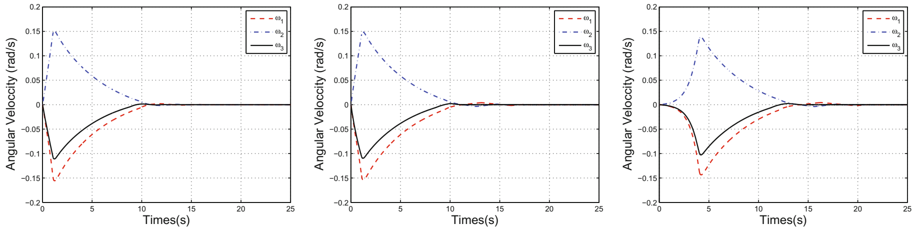
Fig. 2. Control torque with different reaching laws. (a)MPRL. (b)ERL. (c)Conventional reaching law.



(a) (b) (c)

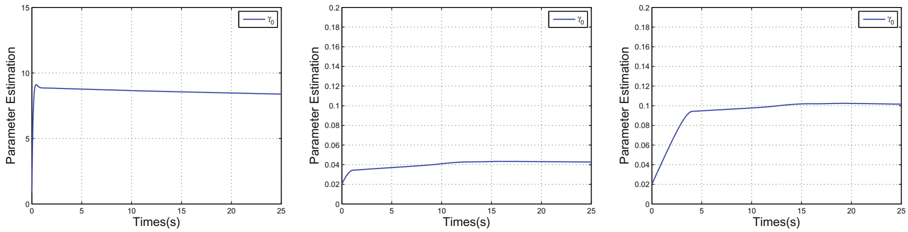
Fig. 3. Spacecraft attitude with different reaching laws. (a)MPRL. (b)ERL. (c)Conventional reaching law.

Considering the spacecraft model given in (1)–(3), the nominal inertia matrix of the spacecraft is $J_0 = diag\,([140, 120, 130])$ kg $\cdot$ m^2 and the uncertainty in the inertia matrix $\Delta J = diag[sin(0.1t), 2sin(0.2t), 3sin(0.3t)]$ kg $\cdot$ m^2. The initial attitude orientation is chosen as $q_v(0) = [0.3, -0.3, 0.2]^T$ and $q_0(0) = 0.8832$. The initial angualar velocity is $\omega(0) = [0, 0, 0]^T$ rad/s. The external disturbance model is $d(t) = 0.005 \times [sin(0.8t), cos(0.5t), cos(0.3t)]^T$ N $\cdot$ m. For the sake of fairness, the parameters given in (19)–(20), the ERL in (41) and conventional reaching law in (42) are identical. Those parameters are chosen as $K = 0.5$, $\mu = 0.01$, $\theta = 0.1$, $\vartheta = 50$ and $\varphi = 1$. The parameters defined in (14) are chosen



(a) (b) (c)

Fig. 4. Angular velocity response with different reaching laws. (a)MPRL. (b)ERL. (c)Conventional reaching law.



(a) (b) (c)

Fig. 5. Parameter estimation with different reaching laws. (a)MPRL. (b)ERL. (c)Conventional reaching law.

as $\alpha = 0.1$, $\beta = 0.1$, $r_1 = 3$, $r_2 = 5$. The parameters of adaptive law defined in (23) is chosen $\varepsilon_0 = 0.01$. The parameters in (29) is set as $J_{max} = 560$, $\delta_0 = 1$. The initial value of $\hat{\gamma_0}(0) = 0.02$. The parameter ξ is 0.0002.

Figures 1 and 2 show the sliding surface and control torque response, respectively. If $\|s\|$ grows, $D(s)$ goes towards 0.01 and $K/D(s)$ in (19) and ERL would be 50, which is greater than $K = 0.5$ in the conventional reaching law. In contrast, when $\|s\|$ decreases, it tends to 0.5. This phenomenon makes the controller gain to be modified between 50 and 0.5. As shown in Figs. 1 and 2, the convergence time of sliding surface using the MPRL and ERL are approximately 1.2 s, then the convergence time of sliding surface using the conventional reaching law is approximately 4.2 s. The MPRL and ERL outperforms the conventional reaching law, with higher steady performance and shorter reaching time. In addition, a term $|s_j|^{\theta}$ in (19) of MPRL reduces the chattering problem which compares with the ERL and conventional reaching law obviously.

The spacecraft attitude quaternion and angular velocity are shown in Figs. 3 and 4, respectively. The results show that both approaches can realize finite-time uniformly ultimately bounded. The convergence time of attitude quaternion using the MPRL and ERL are approximately 10 s, which are almost faster 2 s than the conventional reaching law. Moreover, the convergence time of angular velocity using the MPRL and ERL are approximately 11 s, which are nearly faster 3 s than the conventional reaching law. Based on similar analysis, the convergence speed of attitude quaternion and angular velocity using the MPRL and ERL are faster than that using the conventional reaching law. The corresponding estimated parameter is shown in Fig. 5. From Figs. 1, 2, 3, 4 and 5, it is clear that the proposed MPRL method can achieve superior control performance than the other two methods.

5 Conclusion

In this paper, the problem of attitude stabilization for spacecrafts with external disturbance and internal uncertainty has been considered. The main contribution of this paper is to propose an adaptive controller based on the modified reaching law, and the finite time attitude stabilization is guaranteed for spacecrafts with system uncertainties and disturbances. Besides, the chattering problem has been reduced by using a power rate term in the controller design. Simulation studies have been presented to verify that the proposed controller has stronger robustness and better control performance.

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