PI Sliding Mode Control for Active Magnetic Bearings in Flywheel

Van Van Huynh[1(✉)] and Yao-Wen Tsai[2]

[1] Modeling Evolutionary Algorithms Simulation and Artificial Intelligence Faculty of Electrical and Electronics Engineering, Ton Duc Thang University, Ho Chi Minh City, Vietnam
huynhvanvan@tdt.edu.vn

[2] Department of Mechanical and Automation Engineering, Da-Yeh University, No. 168, University Rd., Changhua 51591, Taiwan R.O.C.

Abstract. This paper proposes the PI sliding mode control approach in order to control the nonlinear multiple-input-multiple-output active magnetic bearing system in flywheel. A nonlinear model of a one degree of freedom (DOF) active magnetic bearing system in flywheel obtained using Lagrange's equation is proposed. In this model, the current in each coil is treated as a state variable and the control input is the voltage applied to each coil, this approach offers more advantages than current control input approach. The proportional and integral switching surface is constructed for active magnetic bearing system to improve system dynamic performance in reaching intervals. The robust controller is proposed by the reaching law method to assure that the rotor stays close at the desired displacement even when disturbance and dynamic effect of rotating are taken into considering.

Keywords: Flywheel energy storage system (FESS) · Active manegtic bearing (AMB) · Model predictive control (MPC) · Nonlinear system

1 Introduction

The alternative solution of the clean energy storage system are flywheels [1, 2]. The traditional (low speed) Flywheel Energy Storage System has a steel wheel supported by the mechanical contact bearings and coupled with motor/generator, such that they increase moment of inertia and limit rotational speed. The traditional Flywheel Energy Storage System is capable of delivering approximately 70% of the flywheel's energy as usable. Thus, they have many disadvantages such as low power density, high mechanical friction and aerodynamic losses and noise.

Magnetic bearings are electromechanical devices that use magnetic forces to completely levitate a rotor or suspend it in an air gap without physical contact. Based on the noncontact and frictionless characteristics, active magnetic bearing (AMB) offers many practical and promising advantages over conventional bearings such as longer life, lower rotating frictional losses, higher rotational speed, and elimination of the lubrication. The active magnetic bearing system in flywheel energy storage system has emerged as a viable option, as it encompasses these aforementioned properties. [3–7].

V.H. Duy et al. (eds.), *AETA 2017 - Recent Advances in Electrical Engineering and Related Sciences: Theory and Application*, Lecture Notes in Electrical Engineering 465,
https://doi.org/10.1007/978-3-319-69814-4_46

Sliding mode control (SMC) is known to be an effective robust control technique, and has been successfully and widely applied for both linear and nonlinear systems such as robot manipulators, aircrafts, underwater vehicles, spacecraft, flexible space structures, electrical motors, power systems, and automotive engines [1]. The main advantages of SMC are fast response and strong robustness with respect to uncertainties and external disturbances [2–4]. Also, as SMC was developed in the process industry where slower processes with larger time constants are usually encountered, another challenge will be to implement real-time control on the active magnetic bearing system in flywheel where fast time-constants are required [8–12].

In this paper, a nonlinear model of a one degree of freedom (DOF) active magnetic bearing system in flywheel obtained using Lagrange's equation is proposed. In this model, the current in each coil is treated as a state variable and the control input is the voltage applied to each coil, this approach offers more advantages than current control input approach. The proportional and integral switching surface is constructed for active magnetic bearing system to improve system dynamic performance in reaching intervals. The robust controller is proposed by the reaching law method to assure that the rotor stays close at the desired displacement even when disturbance and dynamic effect of rotating are taken into considering.

2 Active Magnetic Bearing System in Flywheel Modeling

In this section, a model of AMB in flywheel with single mechanical degree of freedom is introduced in Fig. 1.

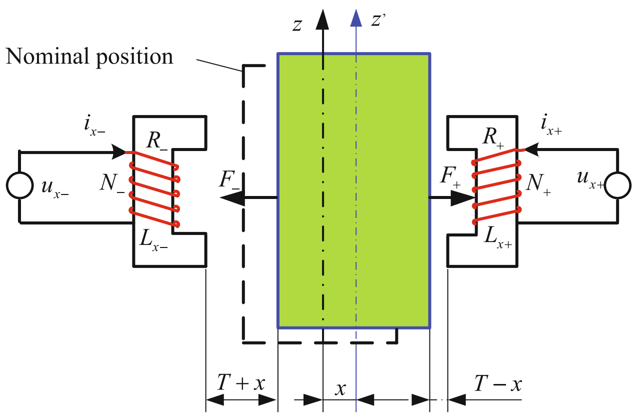

Fig. 1. Single DOF AMB in flywheel model

The energy contributions entering the Lagrangian function that characterizes an electromechanical system are as follows

and
$$L = E_c + E_m - V_m - V_e \tag{1}$$

$$\begin{cases} E_m = \frac{1}{2} m\dot{x}^2 \\ E_e = \frac{1}{2} L_{x+}\dot{q}_{x+}^2 + \frac{1}{2} L_{x-}\dot{q}_{x-}^2, \\ V_M = mgx \\ V_e = 0. \end{cases} \tag{2}$$

where: E_m, V_M are the kinetic and potential energy of mechanical part; E_e and V_e are the kinetic and potential energy of electrical part; The electrical charge in each coil, q_{x+}, q_{x-} is generalized coordinates of electrical part; x is the displacement of the rotor; L_{x+}, L_{x-} are coil inductances.

The dynamic equation of single DOF AMB model can be derived from Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial \mathsf{L}}{\partial \dot{\mathbf{s}}}\right) - \frac{\partial \mathsf{L}}{\partial \mathbf{s}} + \frac{\partial P}{\partial \dot{\mathbf{s}}} = \mathbf{Q} \tag{3}$$

where $\mathbf{s}$ is the generalized coordinate vector

$$\mathbf{s} = [q_{x+}, q_{x-}, \dot{x}, x]^T, \tag{4}$$

$\mathbf{Q}$ is a vector of generalized external forces (control input voltage and mechanical force)

$$\mathbf{Q} = [u_{x+}, u_{x-}, 0, F_x]^T, \tag{5}$$

and P is the dissipation of copper losses in the coils as follow

$$P = \frac{1}{2} R_R \dot{q}_{x+}^2 + \frac{1}{2} R_R \dot{q}_{x-}^2 \tag{6}$$

The equations of motion of the system can be derived in a standard form of differential equation

$$N\dot{\mathbf{s}}_{state} = h, \tag{7}$$

where $N \in R^{nxn}$ is the inertial matrix and $h \in R^{n \times m}$ is the vector of nonlinear function. The Eq. (7) have nonlinear relationships with the control currents and displacements of rotor. Using Maple and from (7) we obtain

$$\begin{bmatrix} \frac{-\Gamma}{-T+x} & 0 & 0 & \frac{\Gamma i_1}{(-T+x)^2} \\ 0 & \frac{\Gamma}{-T+x} & 0 & \frac{\Gamma i_2}{(T+x)^2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -Ri_1 + u_1 \\ -Ri_2 + u_2 \\ -mg + \frac{1}{2}\frac{\Gamma i_1^2}{(T-x)^2} - \frac{1}{2}\frac{\Gamma i_2^2}{(T+x)^2} + F_x \\ \dot{x} \end{bmatrix} \tag{8}$$

Substitute Eq. (8) into (7), where $N \in R^{4x4}$ and $h \in R^{4x1}$

$$\dot{s}_{state} = N^{-1}h = \begin{bmatrix} -\frac{(-T+x)(-Ri_1+u_1)}{\Gamma} + \frac{i_1\dot{x}}{-T+x} \\ \frac{(T+x)(-Ri_2+u_2)}{\Gamma} + \frac{i_2\dot{x}}{T+x} \\ \frac{-mg + \frac{1}{2}\frac{\Gamma i_1^2}{(T-x)^2} - \frac{1}{2}\frac{\Gamma i_2^2}{(T+x)^2} + F_x}{m} \\ \dot{x} \end{bmatrix} \tag{9}$$

Jacobian (9) follow states, we consider a system described by the state-space form as

$$\dot{s}(t) = As(t) + Bu(t) + f \tag{10}$$

where $A = \begin{bmatrix} -\frac{TR}{\Gamma} & 0 & -\frac{i_1}{T} & \frac{Ri_1}{\Gamma} \\ 0 & -\frac{TR}{\Gamma} & \frac{i_2}{T} & -\frac{Ri_2}{\Gamma} \\ \frac{\Gamma i_1}{mT^2} & -\frac{\Gamma i_2}{mT^2} & 0 & \frac{\Gamma i_1^2}{mT^3} + \frac{\Gamma i_2^2}{mT^3} \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} \frac{T}{\Gamma} - \frac{x}{\Gamma} & 0 \\ 0 & \frac{T}{\Gamma} + \frac{x}{\Gamma} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$,

$f = \begin{bmatrix} \sin(10\pi t) + \cos t \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $s = [i_1 \quad i_2 \quad \dot{x} \quad x]^T$ and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $\Gamma = \mu_0 \frac{N^2 A}{2}$, T is nominal air gap; R is coil resistance; Γ is summarizes the coil characterizing parameters, m is the body mass, x is the displacement with respect to the nominal air gap Γ; F_x is mechanical force i_1 and i_2 are AMB currents. We consider disturbance effect direct on the coils.

3 PI Sliding Mode Control Design

3.1 PI Sliding Surface

To improve the dynamic performance and robustness during the reaching phase against the matched and unmatched perturbations, the PI switching surface is selected as

$$\sigma(t) = Gs(t) - G(A - BK)\int_0^t s(\tau)d\tau \tag{11}$$

where G and K are constant matrices. Matrix G is selected to assure that matrix GB is nonsingular. Matrix K is designed through pole assignment such that the eigenvalues of matrix are less than zero.

By using the equivalent control method and setting $\sigma(t) = \dot{\sigma}(t) = 0$, we can see that the equivalent control is given by

$$u_{eq}(t) = -(GB)^{-1}\{GBKs(t) + Gf\} \tag{12}$$

By setting $\sigma(t) = \dot{\sigma}(t) = 0$ and substituting $u(t)$ with u_{eq}, we can show that the sliding mode dynamics restricted to the switching surface (11) is given by

$$\dot{s}(t) = (A - BK)s(t) + (I - B(GB)^{-1}G)f \tag{13}$$

Because there is a gain matrix K for stable $(A - BK)$, a symmetric positive definite matrix exists for the following Lyapunov equation:

$$(A - BK)P + P(A - BK)^T = -Q \tag{14}$$

where P is the solution of (14) for a given positive definite symmetric matrix Q.

Theorem 1: For $s(t) \in B_c(\kappa)$ system dynamic performance in sliding mode is stable at any time, where $B_c(\kappa)$ is the complement of the closed ball centered at $s(t) = 0$ with radius.

$$\kappa = \frac{2\left\|(I - B(GB)^{-1}G)f\right\|\|P\|}{\lambda_{\min}(Q)} \tag{15}$$

Proof: Let us consider the following positive definition function.

$$V(t) = s^T(t)Ps(t) \tag{16}$$

where $P \in R^{n\times n} > 0$. Then, the time derivative of V along the state trajectories of system (10) is given by

$$\begin{aligned}\dot{V}(t) =& s^T(t)(A - BK)^T Ps(t) + s^T(t)P(A - BK)s(t)) \\ & + [(I - B(GB)^{-1}G)f]^T Ps(t) + s^T(t)P[(I - B(GB)^{-1}G)f] \\ & = -s^T(t)Qs(t) + [(I - B(GB)^{-1}G)f]^T Ps(t) \\ & + s^T(t)P[(I - B(GB)^{-1}G)f] \\ & \le -\lambda_{\min}(Q)\|s(t)\|^2 + 2\left\|(I - B(GB)^{-1}G)f\right\|\|P\|\|s(t)\|\end{aligned} \tag{17}$$

when the state trajectory enter into the closed ball $B_c(\kappa)$ and the eigenvalue $\lambda_{\min}(Q)$, the Lyapunov function satisfy $\dot{V}(t) < 0$. Therefore, the system is stable in the sliding mode.

3.2 Control Law Design

In this section, the result of designing of reaching control law is given.

Theorem 2: Let us consider the system (10) with the sliding surface given by Eq. (11). The trajectory of the closed loop system (10) can be driven onto the sliding manifold $\sigma(t)$ in finite time by using the controller given by (18).

$$u(t) = -(GB)^{-1}\{GBKs(t) + Gf\} - (GB)^{-1}\alpha\frac{\sigma(t)}{\|\sigma(t)\|} \tag{18}$$

where $\alpha > 0$.

Proof: Let us define a Lyapunov function V_0 as follows.

$$V_0 = \sigma^T(t)\sigma(t) \tag{19}$$

The time derivative of $V_0(t)$ is obtained as

$$\begin{aligned}\dot{V}_0 =& 2\sigma^T(t)\dot{\sigma}(t) = 2\sigma^T(t)\{G\dot{s}(t) - G(A - BK)s(t)\}\\ =& 2\sigma^T(t)\{GBu(t) + GDf + GBKs(t)\}\end{aligned} \tag{20}$$

According to Eqs. (18) and (20), we achieve

$$V_0 \leq -\alpha\|\sigma(t)\| \tag{21}$$

Therefore the hitting condition (21) is assured by the designed controller (18) in Theorem 2.

4 Simulation

The physical parameters of this Flywheel Energy Storage System model for simulation are given follow Table 1

In this section, dynamic behaviors of the system and control performance are discussed in simulation results.

Table 1. Rotor parameters

Parameters	Symbol		Value		Unit
	Axial	Radial	Axial	Radial	
Air permeability	μ_0	μ_0	4πe−3	4πe−3	$[H/m]$
Nominal air gap	T_A	T_R	0.6e−3	0.5e−3	$[m]$
Cross section area	A_A	A_R	2.1e−3	0.6e−3	$[m^2]$
Number of coil	N_A	N_R	150	208	–
Coil resistance	R_A	R_R	0.2	0.515	$[\Omega]$
Sensor gain	K_{AV}	K_{RV}	1000	1000	$[V/m]$

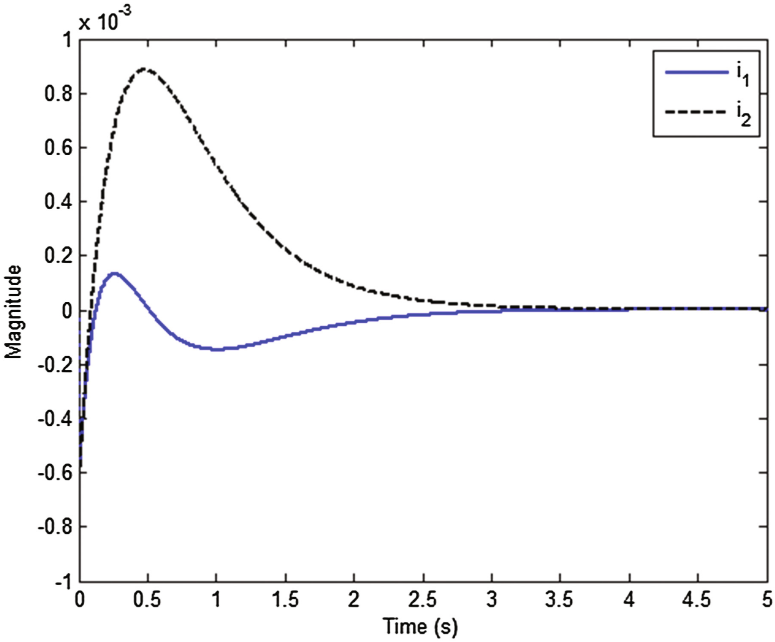

Fig. 2. Current inside each coil

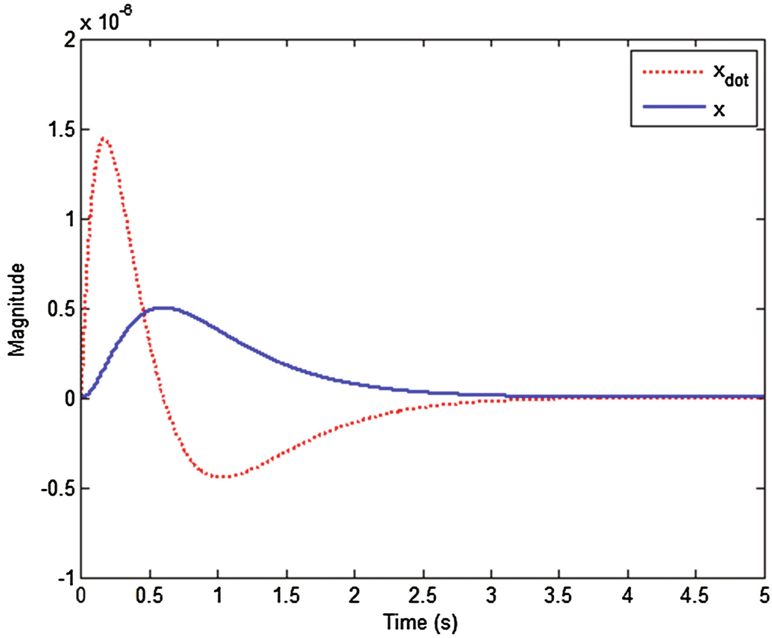

Fig. 3. Displacements of the rotor

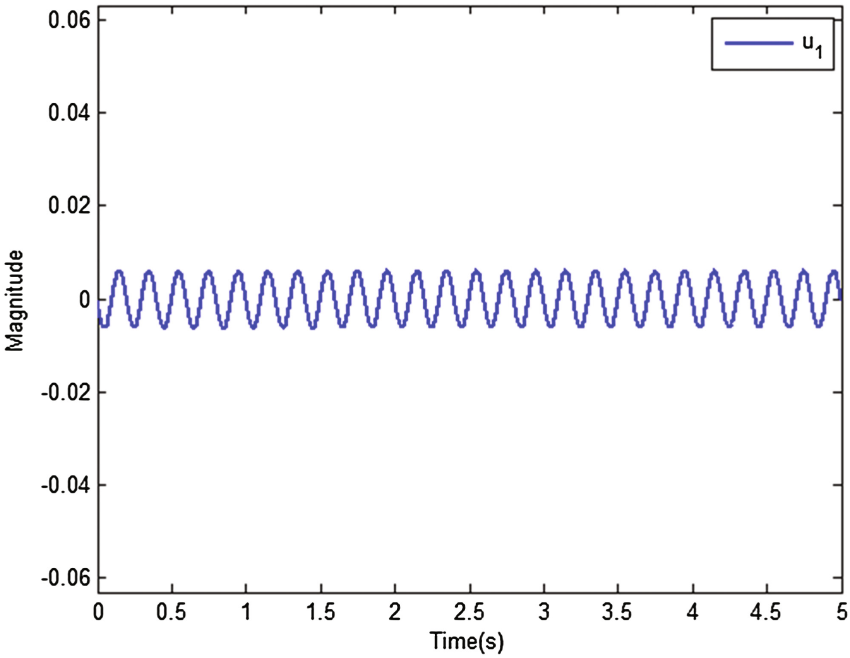

Fig. 4. Control input coil 1

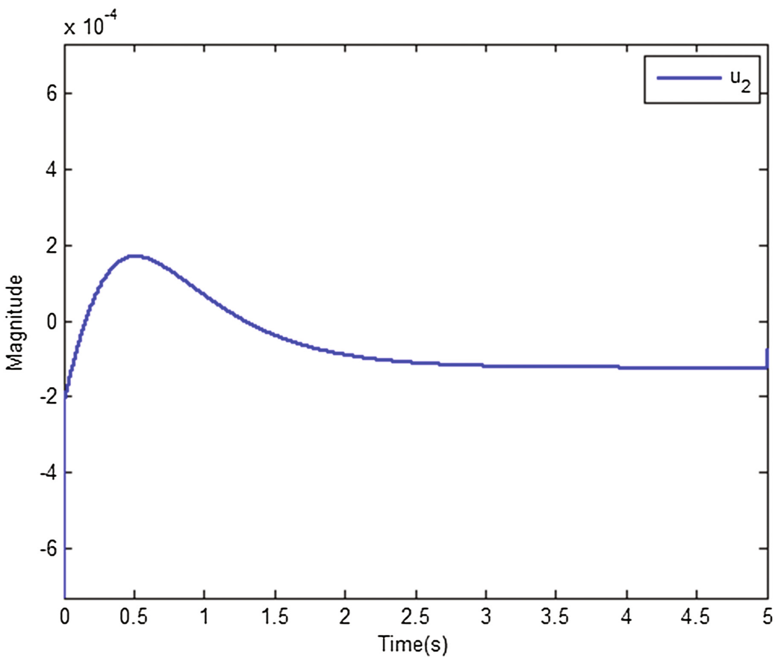

Fig. 5. Control input coil 2

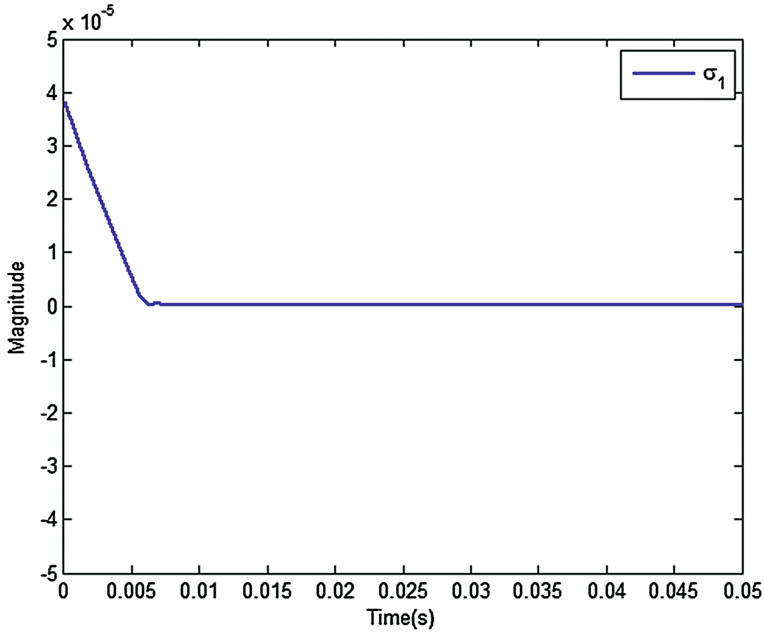

Fig. 6. Sliding surface

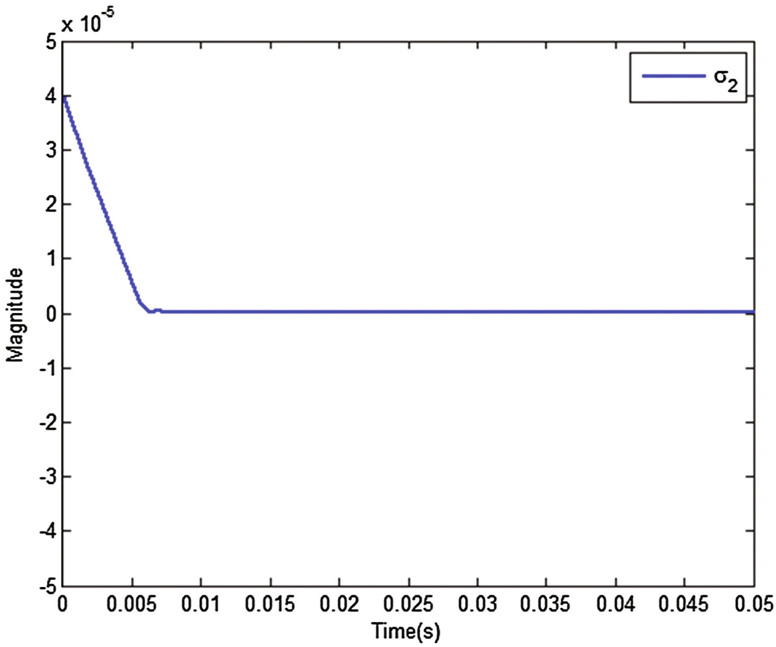

Fig. 7. Sliding surface

Figures 2, 3, 4, 5, 6 and 7 show the simulation results, i.e., Current inside each coil, displacements of rotor, control input and sliding surface. The simulation results indicate that the rotor can be stabilized to the bearing center with settling time around 3.5 s. The steady state errors are all less than 1 μm, approaching the resolution of the position sensors. The results show the good performance of the levitation controller, and clearly verify the accuracy of the magnetic force models.

5 Conclusion

PI sliding mode control for AMB rotor system of Flywheel Energy Storage System is presented in this paper. The results show that the proposed PI sliding mode control has better floating performance compared with the conventional sliding mode control, and is insensitive to system disturbance. The simulation results show that the rotor can be stabilized to the bearing center, confirming the accuracy of the magnetic force model. The proposed PI sliding mode control method shows good potential for AMB rotor system in the Flywheel Energy Storage System.

References

1. Toh, C.S., Chen, S.L.: Design, modeling and control of magnetic bearings for a ring-type flywheel energy storage system. Energies **9**(1051), 1–12 (2016)
2. Fang, J., Wen, T.A.: A wide linear range eddy current displacement sensor equipped with dual-coil probe applied in the magnetic suspension flywheel. Sensors **12**, 10693–10706 (2012)
3. Mystkowski, A.: Energy saving robust control of active magnetic bearing in flywheel. ACTA Mech. Autom. **6**(3), 72–76 (2012)
4. Mystkowski, A., Rowinski, A.: Contruction and control of AMBs high speed flywheel. Arch. Mech. Eng. **8**(1), 79–89 (2011)
5. Zhu, K.Y., Xiao, Y., Rajendra, A.U.: Optimal control of the magnetic bearings for a flywheel energy storage system. Mechatronics **19**, 1221–1235 (2009)
6. Sivrioglu, S., Nonami, K.: Active Permanent Magnet Support for A Superconducting Magnetic-Bearing Flywheel Rotor. IEEE Trans. Appl. Supercond. **10**(4), 1673–1677 (2000)
7. Hajjaji, A.E., Ouladsine, M.: Modeling and nonlinear control of magnetic levitation systems. IEEE Trans. Industr. Electron. **48**(4), 831–838 (2001)
8. Chen, S.Y., Lin, F.J.: Robust nonsingular terminal sliding-mode control for nonlinear magnetic bearing system. IEEE Trans. Control Syst. Technol. **19**, 636–643 (2011)
9. Sig, K.M., Lyou, J., Lee, J.K.: Sliding mode control for an active magnetic bearing system subject to base motion. Mechatronics **20**, 171–178 (2010)
10. Yeh, T.J., Chung, Y.J., Wu, W.C.: Sliding control of magnetic bearing systems. J. Dyn. Syst. Meas. Control, Trans. ASM **123**, 353–362 (2001)
11. Li, Y.J., Li, R.H.: Sliding mode variable structure control of active magnetic bearings based on boundary layer approach. J. Syst. Simul. **17**, 1962–1965 (2005)
12. Rundell, A.E., Drakunov, S.V., DeCarlo, R.A.: A sliding mode observer and controller for stabilization of rotatinal motion of a vertical shaft magnetic bearing. IEEE Trans. Control Syst. Technol. **4**, 598–608 (1996)