State Estimation of a Yoyo Based on a Model with Elasticity of a String

Takuma Nemoto[1(✉)], Sho Komagata[2], Koichi Asada[2], and Masami Iwase[2]

[1] Singapore University of Technology and Design, 8 Somapah Road, Singapore 487372, Singapore
takuma_nemoto@sutd.edu.sg

[2] Tokyo Denki University, 5 Senju Asahi-cho, Adachi-ku, Tokyo 120-8551, Japan

Abstract. This paper presents an approach to estimate a state of a yoyo, as one of tools including a deformable component like a string, based on its detailed model. In the proposed approach, we model a yoyo describing the elasticity of its string with a tension provided by a spring-damper model and employ the unscented Kalman filter on the estimation. We design an estimator based on the approach and evaluate its performance through numerical simulations. The simulations results demonstrate the effectiveness of the proposed approach in conclusion.

Keywords: State estimation · UKF · Yoyo · Deformable object

1 Introduction

Observation of states of deformable objects which include tools with deformable components, such as strings, cables, fishing rods, and yoyos, is an element required for robotic dexterous manipulation of them. Such manipulation of the deformable objects is one of skilled tasks which are easy for humans but complex for robots due to difference of motility and sensors. Progress in the skilled tasks by robots allows them to work in human living environments, where robotic manipulation of the deformable objects can especially offer application to household services and high-place works with safety tethers. As robotic manipulation of the deformable objects, this paper focuses on robotic yoyo.

Most of the conventional researches on robotic yoyo have discussed the fundamental up-and-down motion of a yoyo with simple observation [3–6,8,11,12]. Jin et al. have modeled complex yoyo dynamics by separating the up-and-down motion of a yoyo into four phases and achieved the robotic motion by designing a controller for a robot arm based on a simplified yoyo model [5,6]. Hashimoto et al. [3,4] and Žlajpah et al. [11,12] have also achieved it with a simplified model and a visual feedback controller. Mombaur has reported the up-and-down motion with the biped robot HRP-2 [8]. While the aforementioned researches

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have observed the yoyo height and utilized it to control the robotic yoyo, more complex tasks, like controlling trajectories of a thrown yoyo and high-speed yoyo motion, require the accurate observation of a state of a yoyo.

In this paper, we propose an approach for model-based state estimation of a yoyo to improve accuracy of its observation. For the accurate model-based state estimation of a yoyo, we require an adequate model of a yoyo. On the issue, we have developed a yoyo model and identified its parameters [9,10]. The yoyo model describes the influence of the elasticity of a string with a tension provided by a spring-damper model. In the yoyo model, we have identified some of its parameters from experiments and demonstrated its adequacy. We hence apply the yoyo model as a model for the unscented Kalman filter (UKF) [7]. The estimation results with the UKF are verified through numerical simulations.

2 Problem Establishment

In this paper, we address the state estimation of a yoyo in free fall in numerical simulations. In this problem, we take the use of a commercial yoyo which is painted white for trajectory measurement (Fig. 1) into account. The yoyo starts free-falling with its whole string wound, whose end is tied to a rigid bar. The yoyo repeats the up-and-down motion decreasing the top height of bounce and finally stops at the bottom. In this motion, the yoyo involves the rotation around the string and the lean. We provisionally neglect these actions because they don't occur when the yoyo rotates at high speed. However, we would consider these motions for more accurate estimation in future work.

Fig. 1. White painted commercial yoyo

In this problem, we can observe a position of the yoyo based on the end of its string tied to a rigid bar at a constant frequency. The position of the yoyo is acquired from images captured with a camera by image processing and subject to noise. Utilizing the position data of the yoyo, we estimate its state, such as the rotational angle of the yoyo, the swing angle of the string and these angular velocities.

3 Yoyo Model with the Elasticity of the String

This section describes a yoyo model with the elasticity of the string. The model diagram of the yoyo is shown in Fig. 2. The yoyo model describes its vertical and horizontal motion, rotation and swinging based on an assumption that it has 4 degrees of freedom (DOF) and defines three mass points: the end of the string, the connection point, and the yoyo discs. The end of the string is constrained at the origin. The connection point is the virtual mass point defined for considering the elasticity of the string. With respect to the yoyo discs, θ_y describes the accumulative rotational angle based on the string. The absolute value of the accumulative rotational angle is 0 when the yoyo unwinds the whole string and its center of gravity (COG) is the extended line of the string and the maximum when the yoyo winds the whole string.

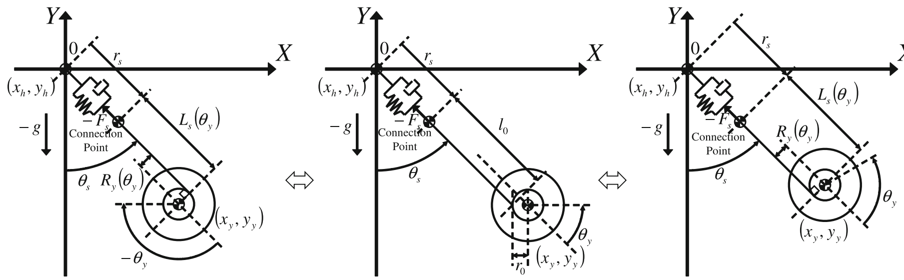

Fig. 2. Model diagram of the yoyo

In the yoyo model, the yoyo has two phases: the winding-and-unwinding phase with $|\theta_y| \geq \pi/2$ (Figs. 2(a), (c)) and the transition phase with $|\theta_y| < \pi/2$ (Fig. 2(b)). It is based on the assumption that the radius of the axle is always perpendicular to the string while the string is wound and unwound and the axle rotates by π radians into the other side of the string only when the whole string is unwound.

In the yoyo model, the radius of the axle $R_y(\theta_y)$ and the length of the string $L_s(\theta_y)$ are respectively given by

$$R_y(\theta_y) = \begin{cases} r_0 + k_r\left(|\theta_y| - \frac{\pi}{2}\right) & \left(|\theta_y| \geq \frac{\pi}{2}\right) \\ r_0 & \left(|\theta_y| < \frac{\pi}{2}\right) \end{cases}, \tag{1}$$

$$L_s(\theta_y) = \begin{cases} l_0 - \left(r_0\left(|\theta_y| - \frac{\pi}{2}\right) + \frac{k_r}{2}\left(|\theta_y| - \frac{\pi}{2}\right)^2\right) & \left(|\theta_y| \geq \frac{\pi}{2}\right) \\ l_0 & \left(|\theta_y| < \frac{\pi}{2}\right) \end{cases}, \tag{2}$$

where k_r is a constant relating the change in the radius of the axle to the diameter of the string, r_0 is the minimum radius of the axle, and l_0 is the maximum length of the string. The force for the tension of the string $F_s(r_s, \dot{r}_s)$ is also given by

$$F_s(r_s, \dot{r}_s) = \begin{cases} k_s r_s + d_s \dot{r}_s & (r_s \geq 0, \dot{r}_s \geq 0) \\ k_s r_s & (r_s \geq 0, \dot{r}_s < 0) \\ 0 & (r_s < 0) \end{cases}, \tag{3}$$

where k_s and d_s are the spring coefficient and the damper coefficient of the string.

The equation of motion of the yoyo model is represented by the following formula which has been derived by applying the projection method [1,2,10]:

$$\boldsymbol{D}_y^T \boldsymbol{M}_y \boldsymbol{D}_y \ddot{\boldsymbol{q}}_y + \boldsymbol{D}_y^T \boldsymbol{M}_y \dot{\boldsymbol{D}}_y \dot{\boldsymbol{q}}_y = \boldsymbol{D}_y^T \boldsymbol{h}_y,$$
$$\dot{\boldsymbol{q}}_y := \left[\dot{\theta}_y \ \dot{\theta}_s \ \dot{r}_s \right]^T, \tag{4}$$

where θ_s is the swing angle of the string, and r_s is the translational distance of the connection point. Each coefficient in (4) is given as follows:

$$\boldsymbol{M}_y := \begin{bmatrix} I_y & I_y & 0 & 0 & 0 & 0 & 0 \\ I_y & m_s r_s^2 + I_y + I_s & 0 & m_s r_s \cos(\theta_s) & m_s r_s \sin(\theta_s) & 0 & 0 \\ 0 & 0 & m_s & m_s \sin(\theta_s) & -m_s \cos(\theta_s) & 0 & 0 \\ 0 & m_s r_s \cos(\theta_s) & m_s \sin(\theta_s) & m_s + m_h & 0 & 0 & 0 \\ 0 & m_s r_s \sin(\theta_s) & -m_s \cos(\theta_s) & 0 & m_s + m_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_y & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_y \end{bmatrix}, \tag{5}$$

where I_y and I_s are the inertia moment of the yoyo and the string, m_h, m_s, and m_y are the mass of the end of the string, the connection point, and the yoyo. While the inertia moment of the string actually varies in proportion to the length of the string, it is approximated by the constant I_s since the mass of the string is much less than the mass of the yoyo body.

$$\begin{aligned} \boldsymbol{h}_y := \Big[& -R_y(\theta_y) c_y \dot{\theta}_y, \ -(2 m_s r_s \dot{r}_s + c_s)\dot{\theta}_s - m_s r_s g \sin(\theta_s), \\ & -F_s(r_s, \dot{r}_s) + m_s r_s \dot{\theta}_s^2 + m_s g \cos(\theta_s), \ m_s r_s \dot{\theta}_s^2 \sin(\theta_s) - 2 m_s \dot{r}_s \dot{\theta}_s \cos(\theta_s), \\ & -m_s r_s \dot{\theta}_s^2 \cos(\theta_s) - 2 m_s \dot{r}_s \dot{\theta}_s \sin(\theta_s) - (m_s + m_h) g, \ 0, \ -m_y g \Big]^T, \end{aligned} \tag{6}$$

where c_y is the viscous friction constant of the yoyo, c_s is the viscous friction of the string, and $g = 9.81$ m/s^2 is the gravity acceleration. In (6), the first term including c_y describes that the friction for rotation of the yoyo increases depending on a contact area between the yoyo discs and the wound string. The orthogonal complement matrix $\boldsymbol{D}_y \in \mathbb{R}^{7\times 3}$ is defined as follows according to the phases:

$$\boldsymbol{D}_y := \left[\boldsymbol{I}_3 \ \boldsymbol{O}_{3\times 2} \ \boldsymbol{d}_6^T \ \boldsymbol{d}_7^T \right]^T, \tag{7}$$

$$\boldsymbol{d}_6 := \begin{cases} [k_r \cos(\theta_s) - R_y \sin(\theta_s)\mathrm{sgn}(\theta_y), \\ \quad (L_s + r_s)\cos(\theta_s) - R_y \sin(\theta_s)\mathrm{sgn}(\theta_y), \ \sin(\theta_s)] & \left(|\theta_y| \geq \frac{\pi}{2}\right) \\ [r_0 \cos(\theta_s + \theta_y), \ (l_0 + r_s)\cos(\theta_s) + r_0 \cos(\theta_s + \theta_y), \ \sin(\theta_s)] & \left(|\theta_y| < \frac{\pi}{2}\right) \end{cases},$$

$$\boldsymbol{d}_7 := \begin{cases} [k_r \sin(\theta_s) + R_y \cos(\theta_s)\mathrm{sgn}(\theta_y), \\ \quad (L_s + r_s)\sin(\theta_s) + R_y \cos(\theta_s)\mathrm{sgn}(\theta_y), -\cos(\theta_s)] & \left(|\theta_y| \geq \frac{\pi}{2}\right) \\ [r_0 \sin(\theta_s + \theta_y), \ (l_0 + r_s)\sin(\theta_s) + r_0 \sin(\theta_s + \theta_y), \ -\cos(\theta_s)] & \left(|\theta_y| < \frac{\pi}{2}\right) \end{cases},$$

where $\boldsymbol{I}$ and $\boldsymbol{O}$ denotes the identity matrix and the zero matrix respectively and these indexes denotes the dimension.

From the equation of motion of the yoyo model (4), the state equation and the observation equation of the system in this paper are represented as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \begin{bmatrix} \left(\boldsymbol{D}_y^T \boldsymbol{M}_y \boldsymbol{D}_y\right)^{-1} \left(\boldsymbol{D}_y^T \boldsymbol{h}_y - \boldsymbol{D}_y^T \boldsymbol{M}_y \dot{\boldsymbol{D}}_y \dot{\boldsymbol{q}}_y\right) \\ \dot{\boldsymbol{q}}_y \end{bmatrix}, \tag{8}$$

$$\boldsymbol{y} = \begin{cases} \begin{bmatrix} (L_s + r_s)\sin(\theta_s) + R_y \cos(\theta_s)\mathrm{sgn}(\theta_y) \\ -(L_s + r_s)\cos(\theta_s) + R_y \sin(\theta_s)\mathrm{sgn}(\theta_y) \end{bmatrix} & (|\theta_y| \geq \frac{\pi}{2}) \\ \begin{bmatrix} (l_0 + r_s)\sin(\theta_s) + r_0 \sin(\theta_s + \theta_y) \\ -(l_0 + r_s)\cos(\theta_s) + r_0 \cos(\theta_s + \theta_y) \end{bmatrix} & (|\theta_y| < \frac{\pi}{2}) \end{cases}, \tag{9}$$

$$\boldsymbol{x} := \left[\dot{\boldsymbol{q}}_y^T,\ \boldsymbol{q}_y^T\right]^T.$$

The observation equation (9) represents the position of the yoyo and also switches according to the phases.

4 State Estimation Based on the Yoyo Model

This section presents the state estimation based on the yoyo model in numerical simulations. The simulations for the state estimation are performed in MATLAB/SIMULINK environment. The simulations scheme is presented in Fig. 3. In this simulations model, the plant block contains the yoyo model and outputs the signal of the state of the yoyo in free fall, which includes the state of (8) and the position and the velocity of the yoyo. We treat the output signal of the plant block as the true values of the state of the yoyo on the simulations. The signal of the state of the yoyo is input into the sensor block. The sensor block generates the positional data of the yoyo at a constant period adding the observation noise with the mean value of 0 and the variance of $\sigma_w^2 = 1 \times 10^{-4}$ to the input signal. Using the positional data, we compute the estimate values of the state of the yoyo in the estimator block. The estimator block employs the UKF [7] with the Eqs. (8) and (9) which are discretized by the Runge-Kutta method and works with the same period as the sensor block.

In this paper, we obtain the estimate values with the simulation model for 10.0 seconds and repeatedly perform the simulations changing the sampling time of the sensor and estimator blocks. We set the parameters of the yoyo model in the simulations are as follows: $m_y = 5.46 \times 10^{-2}$ kg, $m_h = 1.00 \times 10^{-3}$ kg, $m_s = 1.00 \times 10^{-6}$ kg, $I_y = 1.56 \times 10^{-5}$ kgm^2, $I_s = 3.44 \times 10^{-4}$ kgm^2, $d_s = 6.08$ Ns/m, $c_y = 1.91 \times 10^{-4}$ Ns/rad, $c_s = 7.59 \times 10^{-3}$ Nms/rad, $k_s = 1.00 \times 10^3$ N/m, $r_0 = 4.00 \times 10^{-3}$ m, $l_0 = 1.05$ m, and $k_r = 5.85 \times 10^{-5}$ m. Here I_y, I_s, d_s, c_y, c_s, and k_s have been identified in the previous researches [9,10], and m_s is a virtual minute value. The other parameters have been determined from actual measurement of the yoyo. We also set the initial state as $\boldsymbol{x}_0 = \left[0.00,\ 0.00,\ 0.00,\ -1.34 \times 10^2,\ -1.13 \times 10^{-1},\ 0.00\right]$, where $|\theta_y| = 1.34 \times 10^2$ rad is the maximum rotational angle of the yoyo. As the parameters of the

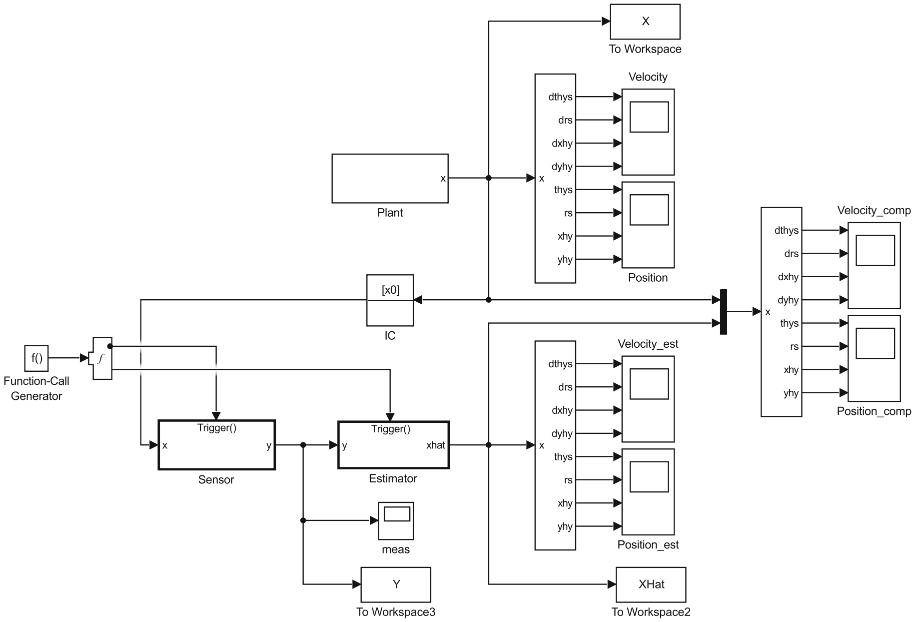

Fig. 3. Block scheme of the simulations in Simulink

UKF, we give the initial error covariance $\boldsymbol{P}_0 = \gamma \boldsymbol{I}_6$, $\gamma = 5.7955 \times 10^{-1}$, the variance of system noise $\sigma_v^2 = 1 \times 10^{-12}$, and the scaling parameter for the unscented transform $\kappa = 5$. These values are determined by a heuristic method and applied to every simulation with the different sampling time.

We show the estimation results when the sampling time is 1.00×10^{-3} seconds and 2.00×10^{-2} seconds for examples. Figures 4, 5, and 6 show the positions of the yoyo, the angles of the yoyo and the string, and the angular velocities of the yoyo and the string in the sampling time of 1.00×10^{-3} seconds, respectively. Figures 7, 8, and 9 show the positions of the yoyo, the angles of the yoyo and the string, and the angular velocities of the yoyo and the string in the sampling time of 2.00×10^{-2} seconds, respectively. Figures 4 and 7 also include the measured values generated by the sensor block. We leave out the estimation results with respect to the connection point because the connection point is a virtual mass and doesn't describe actual behavior of the yoyo. These results show that the proposed estimator can estimate the state of the yoyo accurately in the sampling time of 1.00×10^{-3} seconds and also estimate it in the sampling time of 2.00×10^{-2} seconds while the estimated values have errors. However, the estimator causes errors in the x direction position of the yoyo and the swing velocity of the string instantaneously when the yoyo bounds at the bottom as shown in Figs. 4, 6, 7, and 9. These errors are caused by the rotation by π radians of the yoyo into the other side of its string. When this rotation is involved, the dynamics of the yoyo changes. That change makes the transition of the models and comparatively

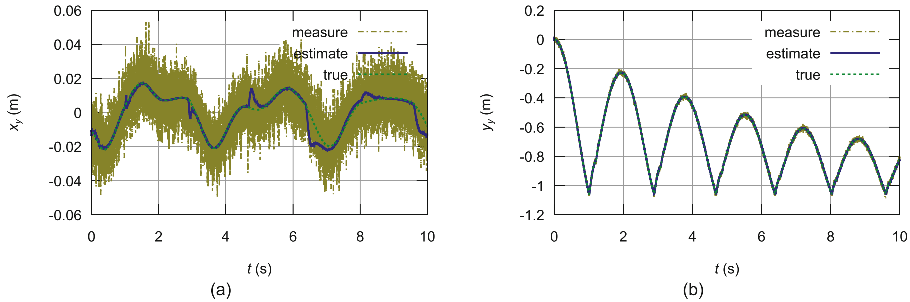

Fig. 4. Positions of the yoyo in the sampling time of 1.00×10^{-3} s: (a) X direction position of the yoyo (b) Y direction position of the yoyo

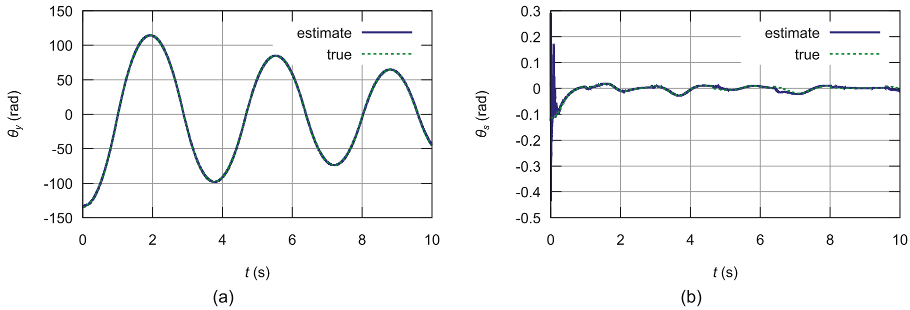

Fig. 5. Angles of the yoyo and the string in the sampling time of 1.00×10^{-3} s: (a) Rotational angle of the yoyo (b) Swing angle of the string

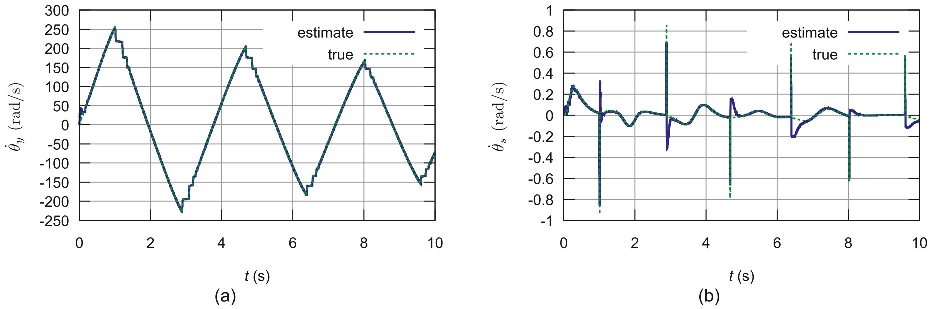

Fig. 6. Angular velocities of the yoyo and the string in the sampling time of 1.00×10^{-3} s: (a) Rotational velocity of the yoyo (b) Swing velocity of the string

affects the horizontal motion of the yoyo. Due to that high non-linearity, the estimate values of the x direction position of the yoyo and the swing velocity of the string have the errors when the yoyo bounds at the bottom.

The above estimation results demonstrate that the estimator is effective in accurate estimation of the state of the yoyo and can be implemented in a system

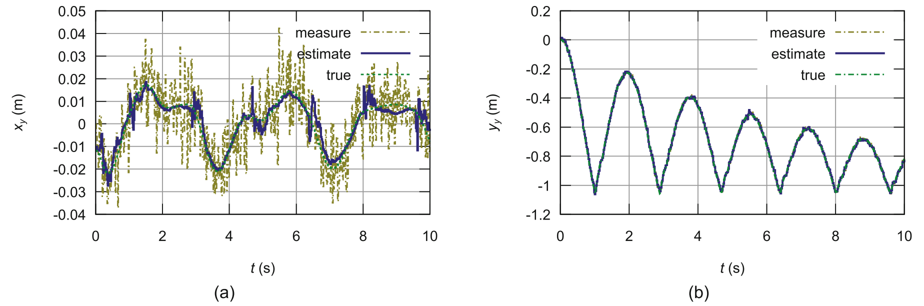

Fig. 7. Positions of the yoyo in the sampling time of 2.00×10^{-2} s: (a) X direction position of the yoyo (b) Y direction position of the yoyo

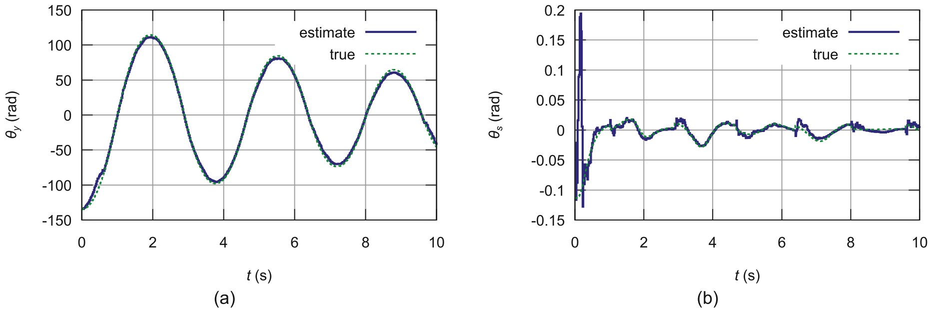

Fig. 8. Angles of the yoyo and the string in the sampling time of 2.00×10^{-2} s: (a) Rotational angle of the yoyo (b) Swing angle of the string

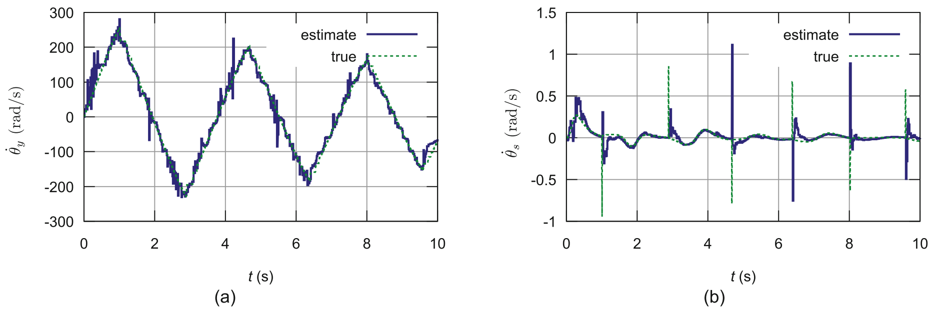

Fig. 9. Angular velocities of the yoyo and the string in the sampling time of 2.00×10^{-2} s: (a) Rotational velocity of the yoyo (b) Swing velocity of the string

including a camera with the sampling time of less than 2.00×10^{-2} seconds. They also imply that the proposed approach would be applied to state estimation of deformable objects.

In future work, we implement the estimator to an experimental system and address estimation experiments with the use of actual observed data. However, the estimation results in this paper imply that the estimator can calculate accurate values from observed data because it has been verified in the previous researches [9,10] that the simulation results with the yoyo parameters used in this paper track observed yoyo behavior.

5 Conclusion

In this paper, we have proposed an approach for state estimation of a yoyo as one of tools including a deformable component. The proposed approach employs the UKF and a detailed model of the yoyo which describes the influence of the elasticity of its string with a spring-damper model. An estimator based on the approach has been evaluated through numerical simulations. The simulations results have shown that the estimator can calculate the state of the yoyo accurately in the sampling time of less than 2.00×10^{-2} seconds and demonstrated the effectiveness of the proposed approach. Since we have not addressed estimation with actual observed data in this paper, we would experiment on it and implement the estimator to an experimental system in future work.

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