Under-Actuated Systems: Nonlinear Control Showcase

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Abstract. Most of controllers for nonlinear systems are designed by using linearly approximated models and by applying linear control theory. In most of such cases, nonlinear control theory cannot improve control performance as long as we are controlling the systems in the vicinity of the equilibrium point. However, there are many under-actuated systems which are not stabilized with this linearization strategy: some sorts of singularity at the equilibrium point cause uncontrollability of the approximated linear model even though the system is controllable in nonlinear control theory. This note will present such nature of under-actuated systems and their control strategies.

Keywords: Under-actuated systems · Nonlinear control · Non-holonomic systems · Motion control · Bilinear systems · Unstable zero-dynamics · Periodic motion

1 Nonlinear Control Theory

Why should we research Nonlinear Control Theory? Some say, "Every real systems have nonlinearity, thus, we need nonlinear control theory." In my opinion, this answer is partially correct, but not perfectly.

The commonly used controller design strategy for nonlinear systems is as follows, first deriving approximate linear model of the system, and then designing controller using linear control theory. For example, stabilizing controllers for famous inverted pendulum [1, 2] are designed with this strategy.

It should also be noted that "Even though we design nonlinear optimal controllers for nonlinear systems by using nonlinear optimal control theory, linear approximations of those nonlinear controllers are identical to the linear optimal controllers for approximately linearized systems designed by using linear optimal control theory, where their performance indices are quadratic approximation of those of the original nonlinear optimal control." Thus, approximate linearization is a powerful tool as long as we are controlling the systems in the vicinity of the equilibrium point.

This is not a bad news for nonlinear control researchers, however, since this also implies that <u>NONLINEAR CONTROL THEORY is NEEDED in the following cases:</u>

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- we should control the system away from the equilibrium point.
- the linearized system is uncontrollable even though the original system is controllable in nonlinear control theory, i.e., the equilibrium point is a singular point of controllability.

The former case includes swing-up control of the inverted pendulum and other motion controls, i.e., not stabilization problem. The latter case includes theoretically interesting nonlinear systems.

Under-Actuated Systems have many of those control difficulties. This note will introduce our control challenges to those problems.

2 Under-Actuated Systems

Under-Actuated Systems are mechanical systems whose number of inputs (actuators) is strictly less than the degrees of freedom (DOF). A commonly used 3-link manipulator has 3 DOF and 3 motors (inputs), therefor this system is not an under-actuated system but is a full-actuated system.

Examples of under-actuated systems include the followings:

- The cart-pendulum (inverted pendulum) system [1, 2] is 2 DOF (cart position and pendulum orientation) and should be controlled with 1 input (cart acceleration).
- The Quad-Rotor Drone has 6 DOF (3D position and 3 orientation) and 4 inputs (rotors).
- The Car has 3 DOF (2D position and 1 orientation) and 2 inputs (forward/backward velocity and steering).

In the following sections, we will show several under-actuated systems and our control approaches to them.

3 Velocity Constrained Systems (Driftless System)

Brockett's Theorem [3] shows a necessary condition for the systems to be stabilized with static continuous feedbacks, such as linear feedbacks $u = Fx$.

If the system does not satisfy Brockett's condition, it cannot be stabilized with any static continuous feedbacks, i.e., at least we should design time-varying or discontinuous controllers.

It is known that the wheeled vehicle is modeled as driftless system and does not satisfy Brackett's condition [4]. Thus many researchers have worked on its control and have proposed time-varying controllers and discontinuous controllers [for example, 4, 5, 6, 7]. Our approach is time-state control form [8, 9] based on time scale transformation [10], a hybrid type controller.

One of the canonical forms of driftless system is the following chained form.

$$\begin{aligned} \dot{x}_1 &= u_1, \\ \dot{x}_2 &= u_2, \\ \dot{x}_3 &= x_2 u_1. \end{aligned}$$

Since all terms in the right hand side are activated by the input u, i.e., there are no drift term, it is called “driftless system.” Its linear approximation is uncontrollable as follows.

$$\dot{x}_1 = u_1,$$
$$\dot{x}_2 = u_2,$$
$$\dot{x}_3 = O^2(x, u).$$

If we ignore $O^2(x, u)$, $\dot{x}_3 = 0$ and x_3 is uncontrollable. But if we set $u_1 = 1$,

$$\dot{x}_2 = u_2,$$
$$\dot{x}_3 = x_2,$$

and if we set $u_1 = -1$,

$$\dot{x}_2 = u_2,$$
$$\dot{x}_3 = -x_2.$$

Since those systems are linear controllable systems, we can stabilize x_2 and x_3 while $u_1 = 1$ or $u_1 = -1$. x_1 can be controlled by changing $u_1 = 1$ and $u_1 = -1$. x_1 is called “generator.” Our time-state control approach is more sophisticated way to use non-constant u_1 to control the system. In the case of wheeled car system, u_1 corresponds to the forward or the backward movement, and u_2 corresponds to the rotation of the vehicle (Fig. 1).

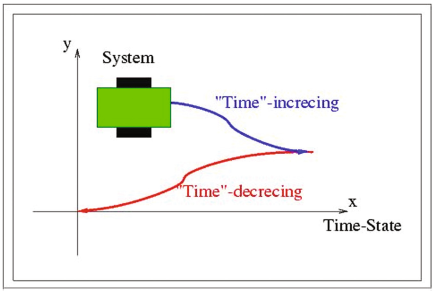

Fig. 1. Control strategy with time-state control form

Fig. 2. Double trailer system

We showed that our controller can control many driftless systems such as a wheeled vehicle, a trailer [11], and a double-trailer [12] (Fig. 2). We also showed that an under-actuated space robot [13, 14] (Fig. 3) and a dexterous manipulation of the ball/plate position (a ball sandwiched by two plates: the plate's movement is the input) [15] (Fig. 4) can be modeled as chained forms, and controlled with our strategy. We showed that the dexterous manipulation of the ball/plate position together with the ball orientation is more difficult problem, i.e., it cannot be approximated to chained form since one generator is not enough for the controllability of the rest of the system. We proposed using two generators to control this system [16] (Fig. 5).

Recently, we proposed a discontinuous controller, whose trajectory is similar to time varying controllers, using Semiconcave Control Lyapunov Function [17].

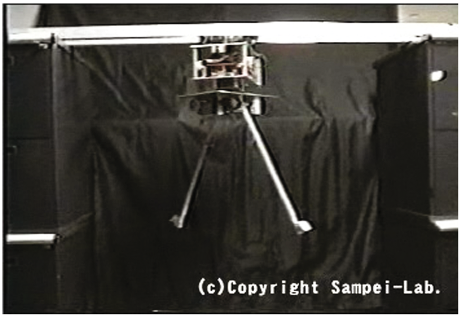

Fig. 3. Space robot simulator

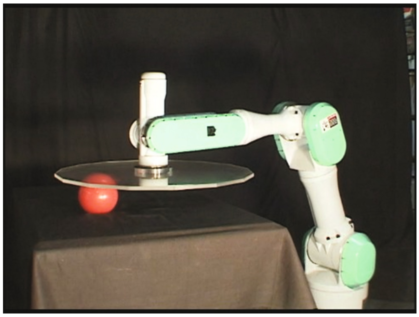

Fig. 4. Dexterous manipulation (ball/plate position)

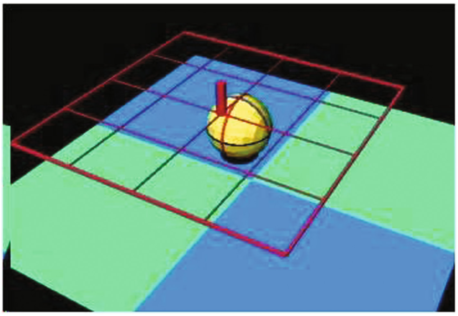

Fig. 5. Dexterous manipulation (position and orientation)

4 Velocity Constrained Systems (Constant Drift)

If the velocity constrained system has some constant drift, how the control strategy should be changed. When we model the space robot as the driftless system, we assume that the initial momentum of the space robot is zero. In such a case, any positions or orientations of the space robot are equilibria. On the other hand, if it has non-zero initial momentum, no configurations are equilibria. In this case, we should change the control objective. In [18], we proposed controllers for the spacecraft with only two reaction wheels with non-zero initial momentum (Fig. 6). Our control objective was to make the antenna of the spacecraft face to the earth. By choosing the appropriate output, we succeeded in controlling the direction of the antenna, and left the angular momentum as the rotating motion of the craft (zero-dynamics).

Another example is landing control [19]. Our problem was to control a flying under-actuated mechanism to land at the specified posture, i.e., we should control its posture at a certain time. We assumed that it had non-zero initial angular momentum. Since we could not control the angular momentum, we decided to use the change of the moment of inertia as an input, i.e., by moving its joints, we could change the moment of inertia. The change of the moment of inertia with the constant angular momentum caused a change of the angular velocity. In this case, we could design a conventional trajectory tracking controller for its posture.

Another example is surface vessel with unknown disturbances [20]. We assumed that the dynamics of surface vessel was same as wheeled vehicle, except it suffered from constant tidal stream (Fig. 7).

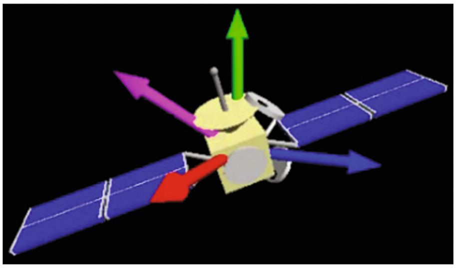

Fig. 6. Spacecraft with initial angular momentum

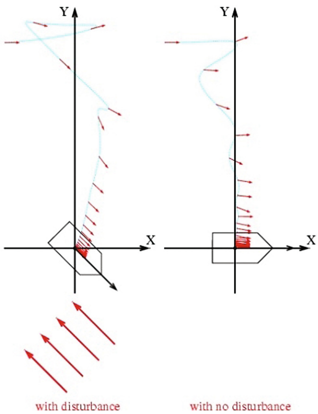

Fig. 7. Surface vessel with tidal stream

5 Under-Actuated Systems Without Gravity

What would happen if we try to control manipulators with passive joints in a horizontal plane (without gravity)? Such systems do not satisfy Brockett's condition and cannot be stabilized with static continuous feedbacks. Arai et al. [21] proposes such a problem, and solves it using the concept of “the center of percussion.” Our group mathematically formulated this system as a second order chained system as follows, and proposed a discontinuous controller [22].

$$\begin{aligned}\ddot{x}_1 &= u_1,\\ \ddot{x}_2 &= u_2,\\ \ddot{x}_3 &= x_2 u_1.\end{aligned}$$

6 Under-Actuated Systems with Gravity

Manipulators with passive joints without gravity are quite hard to control. If there is a gravity, however, they are quite easy to be controlled. The inverted pendulum can be modeled as a manipulator with a passive joint. Since its approximate linearization is controllable, it is easy to be controlled. This is because of the structure of equilibria. If there are no gravity, any angles of the pendulum are equilibria. However, if there is a gravity, the equilibrium point of the pendulum is the upright position. This implies that the dimension of the equilibrium manifold is changed. It is known that if the dimension of the equilibrium manifold is strictly greater than the number of inputs, the system cannot be stabilized with any static continuous feedback [23].

Similarly, the quadrotor system is 6 DOF with 4 inputs. Since the dimension of the equilibrium manifold of the quadrotor is 4 (pitch and roll should be zero), the quadrotor can be controlled with linear controllers.

7 Bilinear Systems

The conventional inverted pendulums are controlled by using the horizontal movement of the cart (actuated part) (Fig. 8). In contrast, human uses not only the horizontal movement but also the vertical movement to control the pendulum (Fig. 9). How can we design controllers for the inverted pendulum using both horizontal and vertical movement? This problems is not easy because the controllability of this system has a singularity at the origin.

You can easily imagine that, if the pendulum is at the upright position, the vertical movement will not affect the pendulum’s angle. On the other hand, if the pendulum is declined, then the vertical movement affects the pendulum’s angle. This implies that the controllability from the vertical input to the pendulum has a singularity at the upright position, i.e., it is controllable almost everywhere except the upright position.

We found that this system can be modeled (or approximated) as a bilinear system. We proposed an inverse optimal type controller for bilinear systems and showed that our controller efficiently used the vertical movement to control the pendulum [24, 25] (Fig. 10).

Similarly, the semi-active suspension (Fig. 11) of the vehicle is modeled as a bilinear system, and can be controlled with a similar strategy [26, 27]. This controller was employed by TOYOTA as H-infinity TEMS.

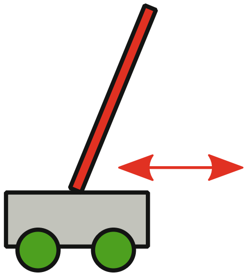

Fig. 8. Conventional inverted pendulum

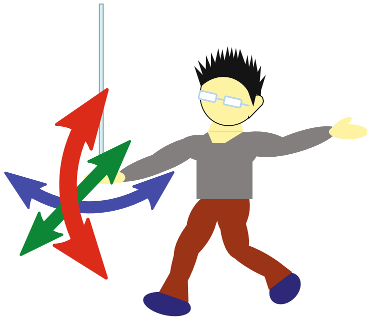

Fig. 9. Human control of inverted pendulum

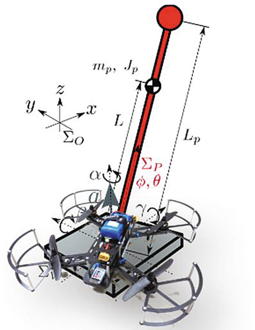

Fig. 10. Inverted pendulum on the drone

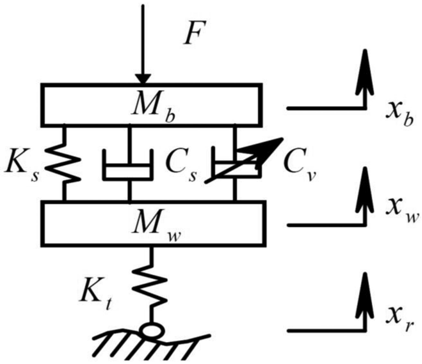

Fig. 11. Semi-active suspension

8 Motion Control (Unstable Zero-Dynamics)

Can we control under-actuated manipulators to throw a ball? When we use a fully actuated manipulator, we first design a time trajectory and control the manipulator to follow this trajectory. However, it is well known that, in the case of human throwing, torque of the elbow joint is small. That is why we made a throwing machine with passive elbow joint. Since the elbow joint did not have a heavy motor, the arm was light and moved quickly. However, since this was an under-actuated system, it could not exactly follow the designed time trajectory.

For this system, we designed a controller based on Un-Stable Zero-Dynamics. Since there was only one input (shoulder motor), we could control only 1 degree of freedom. Thus, we decided to control the end-effector to stay on the path designed for throwing motion. The controller was only to make the end-effector on the path, and the dynamics along the path was not controlled, i.e., zero-dynamics. We designed this path unstable, i.e., the end-effector was accelerated along the path so that it could throw the ball [28] (Figs. 12 and 13).

The key point of this control was the choice of the output function which made the zero-dynamics unstable. We proposed the Relative Degree Structure [29] which helped us to design such an output function.

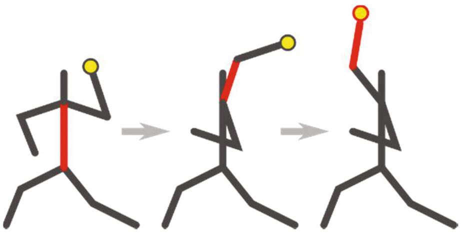

Fig. 12. Human throwing

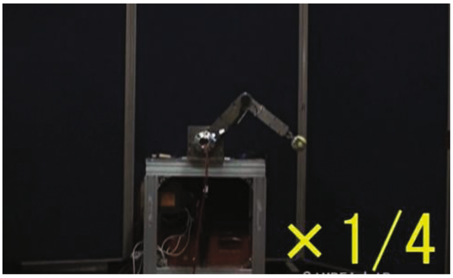

Fig. 13. Under-actuated throwing machine

9 Motion Control (Periodic Motion)

If the path for the unstable zero-dynamics is a closed path, we may activate a periodic motion of the system. Of course, if the path is merely unstable, the zero-dynamics in the path continuously accelerates the motion and the periodic motion becomes unstable. In order to stabilize the periodic motion (control the velocity of the zero-dynamics), we should find some factors to control the velocity of the zero- dynamics. We designed controller for Devil Stick (Juggling) [30] (Figs. 14 and 15), Biped Running [31] (Fig. 16), Denglibot (Rolling Acrobot: such motion is called “DENGURIGAESHI” in Japanese) [32, 33] (Figs. 17 and 18).

A similar concept, Hybrid Zero-Dynamics, was proposed by Westervelt, Grizzle and Koditschek [34], which design a stable periodic motion path. Our control forces the periodic motion stable by modifying the path (or other parameter) as a control input.

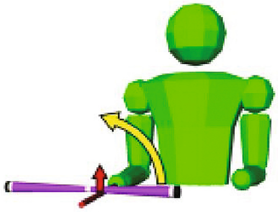

Fig. 14. Devil stick: propeller motion

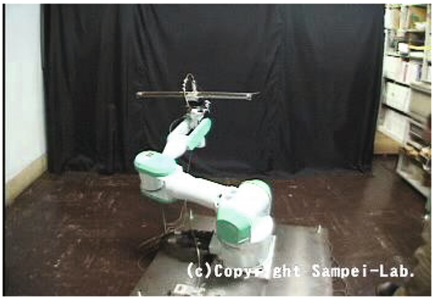

Fig. 15. Devil stick experiment

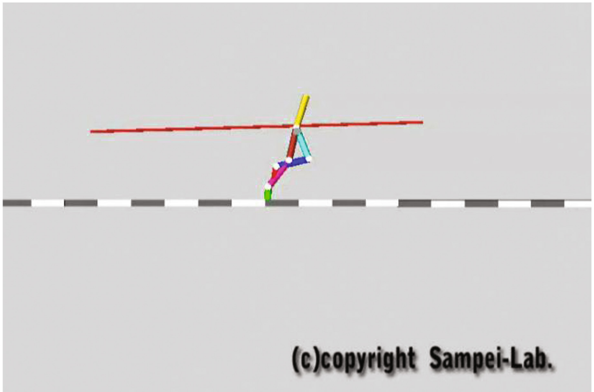

Fig. 16. Biped running

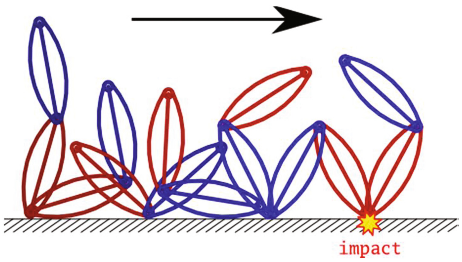

Fig. 17. Denguribot (simulation)

Fig. 18. Dengribot (experiment)

10 Motion Control (Snake)

Another example of the motion control is seen in the manipulation of the snake-like robot. It is known that the Serpentine Motion causes the forward movement of the snake-like robot [35]. We showed that the Serpentine-Like Motion was automatically generated if the robot tried to move the head forward while suppressing the side force arisen on the body (wheel) [36] (Figs. 19 and 20).

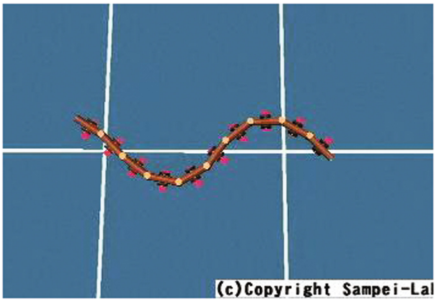

Fig. 19. Snake robot (simulation)

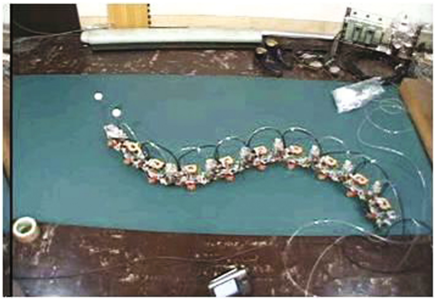

Fig. 20. Snake robot (experiment)

11 Concluding Comment

This note showed the results of our work regarding the control of under-actuated systems. As shown in this note, many under-actuated systems need advanced nonlinear control theory. Thus, under-actuated systems are the show case of advanced nonlinear control theory.

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