Mining Schedule Optimisation for Conditionally Simulated Orebodies

M. Menabde, G. Froyland, P. Stone and G.A. Yeates

Abstract Traditionally the process of mine development, pit design and long-term scheduling is based on a single deterministic orebody model built by the interpolation of drill hole data using some form of spatial interpolation procedure, e.g. kriging. Typical steps in mine design would include the determination of the ultimate pit, the development of a number of mining phases (pushbacks) and then the development of a life-of-mine schedule. All of these steps would have the aim of maximising the mine's net present value (NPV), along with meeting numerous other business and physical constraints. There are a number of software packages commercially available and widely used in the mining industry that deal with some or all of these issues. The methods employed by all of these packages treat the process described above in a strictly deterministic way. In reality, given the sparse drill hole data, there is usually significant and variable uncertainty associated with a single or unique deterministic block model. This uncertainty is not captured or used in the planning process. This paper describes work undertaken by the Exploration and Mining Technology Group within BHP Billiton to develop a new mathematical algorithm for mine optimisation under orebody uncertainty. This uncertainty is expressed as a number of conditionally simulated orebody models. This optimisation algorithm is implemented in a new software package. The software uses a number of proprietary algorithms along with the commercially available mixed integer-programming package ILOG CPLEX. The development targets all phases of mine optimisation, including the NPV optimal block extraction sequence, pushback design, and simultaneous cut-off grade and mining schedule optimisation. AQ1

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R. Dimitrakopoulos (ed.), *Advances in Applied Strategic Mine Planning*,
https://doi.org/10.1007/978-3-319-69320-0_8

Introduction

This paper describes the development and implementation of a new software package for open pit mine development and scheduling optimisation under conditions of orebody uncertainty and is based on the mixed integer programming method. The approach uses multiple conditionally simulated realisations of the orebody as input to characterise the orebody along with the uncertainty in the estimate.

Traditionally open pit mine planning, pit design and long-term scheduling is based on a block model of the orebody built by interpolation techniques such as kriging from the drill hole sample data. This single model is assumed to be a fair representation of reality and is used for mine design and optimisation. The design process consists of four main steps:

1. Determining the ultimate pit shell to define the scheduling universe.
2. Finding the block extraction sequence which produces the best net present value (NPV) whilst satisfying the geotechnical slope constraints.
3. Designing the practically minable mine phases (pushbacks) which are roughly based on the optimal block sequence.
4. Optimising the mining schedule and cut-off grades (COG) within a set of business and operational constraints. The NPV of this 'optimal' schedule is considered as a main criterion of the economical viability of the project.

In reality, there are many uncertainties in the models and parameters used in optimisation. Thus, the adoption of a single economic criterion for a project can be very questionable. One of the most important sources of uncertainty is the block model itself. The drill hole data for a mining project is typically sparse, particularly at the scale of the selective mining unit and could support a range of possible outcomes for the orebody. A unique deterministic block model will often be a good representation of the global resource, but will not be representative of the potential local variability or the uncertainty in the estimate. An approach that quantifies both the local variability and the potential uncertainty is to use multiple conditional simulation realisation to represent the orebody (see Dimitrakopoulos 1998). This approach allows the generation of a number of equally probable realisations of the block model, at the selective mining unit (SMU) scale, with all of them honouring the drill hole data along with the first and second order statistics of the orebody represented, respectively, by the probability distribution and variogram (e.g. Isaaks and Srivastava 1989).

The simplest and most straightforward use of this set of orebody realisations is to estimate the variability in the project NPV associated with the orebody uncertainty by valuing the 'optimal' schedule obtained from the kriged deterministic model through each of the conditionally simulated realisations.

The more interesting question is whether it is possible to use the set of conditionally simulated realisations to produce a better mine design and production schedule. By 'better' we mean here a higher expected NPV (which becomes a

Layout: T1_Standard Book ID: 457301_3_En Book ISBN: 978-3-319-69319-4
Chapter No.: 8 Date: 14-10-2017 Time: 2:29 pm Page: 3/10

random variable in case of multiple realisations of the orebody model) and/or less variability from one realisation to other (i.e. lower variance of NPV). A new promising approach to this problem is presented in Ramazan and Dimitrakopoulos (2007, this volume); Jewbali (2006).

In this paper we address one particular aspect of the optimisation under uncertainty, namely the simultaneous optimisation of the extraction sequence and COG. The use and importance of optimal (variable) COG to mining projects has been known for a long time (e.g. Lane 1988). It will be demonstrated here that the use of variable COG optimised under uncertainty, using the set of equi-probable realisations of the orebody can provide a substantial improvement in terms of expected NPV. The approach based on mixed integer programming techniques can provide a truly optimal schedule, as opposed to various heuristic methods used in most of the commercially available mining optimisation software packages.

Mining Schedule Optimisation as a Mixed Integer Programming Model

Typically, the orebody block model contains between 50,000 and 5,000,000 blocks, which must be scheduled over a period of say 5–25 years. The objective of any scheduling procedure is to find the block extraction sequence, which produces the maximum possible net present value (NPV) and obeys a number of constraints. The latter include:

1. geotechnical slope constraints, which are modelled by a set of precedence arcs between individual blocks;
2. mining constraints, i.e. total maximum amount of rock which can be mined in one time period (usually one year);
3. processing constraints, i.e. maximum amount of ore which can be processed through a given processing plant in one time period; and
4. the market constraints, i.e. the maximum amount of metal that can be sold in one time period.

The mathematical formulation of the scheduling procedure in terms of binary decision variables describing in which period the particular block is extracted and what its destination is (either processing plant, stockpile or waste dump), is quite straightforward. The size of the problem is, however, prohibitively large. Apart from the computational difficulties, the hypothetical optimal block extraction sequence may be completely impractical due to the requirements for the mining equipment access and relocation.

Because of these problems the mine scheduling is done using much bigger elementary units that are typically aggregations of hundreds or even thousands of blocks. The aggregation of blocks is a nontrivial problem. For example, simply combining rectangular blocks into a larger rectangular block with dimensions multiples of that of individual blocks can effectively reduce the size of the problem but will provide a very poor approximation for the geotechnical slopes. An interesting approach to block aggregation based on the concept of 'fundamental trees' has been recently developed by Ramazan (2007, this volume). In this method the aggregations of blocks—fundamental trees—obey the slope constraints and can substantially reduce the number of integer variables required for the scheduling model. However, the number of these aggregations is not user controllable and in many cases the problem can be still too big to be solved by a direct application of the mixed integer programming techniques.

We have recently developed a new algorithm for block aggregation, which preserves the slope constraints, and is very flexible allowing the user to fully control the size and shape of these aggregations. The details of this algorithm will not be discussed here. The optimisation procedure, however, can be applied to any aggregation of blocks with a set of precedence arcs, prescribing which blocks should be extracted before the given one. As an example we consider here the scheduling of mining phases.

In practice, the open pit mine is divided into a number of mining phases, which are mined bench by bench, each bench represented by a horizontal layer of blocks within the given mining phase and having the same elevation. A bench within a mining phase is sometimes referred to as a "panel". The mining phases can be mined one by one from top to bottom, however this kind of schedule is usually suboptimal. Mining several phases simultaneously and applying variable COG can produce much better results in terms of NPV. There are several commercially available packages, which use proprietary (and undisclosed) heuristics to optimise the schedule and COG. It is difficult to estimate their effectiveness, as the upper theoretical limit on NPV remains unknown. Moreover, these methods can only be used on a single orebody representation and cannot be directly used on a set of conditionally simulated orebody realisations.

The standard optimsation technique widely used in many industrial applications is the leanear and integer programming (e.g. industrial applications is the linear and integer programming (e.g. Padberg 1995). The main difficulty in its application to mining scheduling is that the optimsation with variable COG in its direct formulation leads to a non-linear problem, which is much harder to solve. Our approach provides an effective linearsation of this problem, making it possible to use a mixed integer programming (MIP) formulation for a simultaneous optimization of the extraction sequence and COG for a number of conditionally simulated orebody models. The MIP formulation we use here is similar to the one used by Caccetta and Hill (2003) but is generalised to include the multiple realisations of conditional simulations and variable cut-off grades. This approach also allows one to estimate the gap between the obtained solution and the upper theoretical limit.

We consider the simplest case when we have one rock type containing one metal type, which can be processed through one processing plant. Generalisation to the case of multiple rock types, metals and processing streams is cumbersome but straightforward. For simplicity we consider here only the case of a discrete set of COGs, though it is possible to generalise the results to the continuous COG case. We use the following notations:

Layout: T1_Standard | Book ID: 457301_3_En | Book ISBN: 978-3-319-69319-4
Chapter No.: 8 | Date: 14-10-2017 Time: 2:29 pm | Page: 5/10

T is the number of scheduling periods
N is the number of simulations
P is the total number of panels
G is the number of all possible cut-off grades
R_i^n is the total rock in the panel i in simulations n
Q_{ij}^n is the total ore in the panel i, simulation n, when mined with the COG j
V_{ij}^n is the value of the panel i, simulation n, when mined and processed with the COG j
R_t^0 is the maximum mining capacity in period t
Q_t^0 is the maximum processing rate in period t
Si is the set of panels that must be removed before starting the panel i
d^t is the time discount factor
$Xijt$ is the fraction of the panel i is extracted with the COG j in period t
Yit is a binary variable equal to 1 if the extraction of the panel I has started in periods 1 to t, and equal to 0 otherwise;
δjt is a binary variable controlling the selection of the COG applied in period t

The MIP formulation is:

$$Maximise\left(\frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{P}\sum_{j=1}^{G}\sum_{t=1}^{T}V_{ij}^{n}x_{ijt}d^{t}\right) \tag{1}$$

Subject to the following constraints:

$$\frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{P}\sum_{j-1}^{G}R_i^n x_{ijt}\le R_{t,}^0 \quad \mathit{for\ all\ t} \tag{2}$$

$$\frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{P}\sum_{j=1}^{G}Q_{ij}^n x_{ijt}\le Q_{t,}^0 \quad \mathit{for\ all\ t} \tag{3}$$

$$y_{i}, \quad t-1\le y_{it}, \quad \mathit{for\ all\ i\ and\ t} \tag{4}$$

$$\sum_{\tau=1}^{t}\sum_{j=1}^{G}x_{ij\tau}\le y_{it}, \quad \mathit{for\ all} \tag{5}$$

$$y_{it}\le\sum_{j=1}^{G}\sum_{\tau=1}^{T}x_{kj\tau}, \quad \mathit{for\ all\ i,} \quad t\ \mathit{and}\ k\subset S_i \tag{6}$$

$$\sum_{j=1}^{G}\delta_{jt=1}, \quad \mathit{for\ all\ t} \tag{7}$$

$$x_{ijt} \leq \delta_{jt}, \quad for\, all\, i, j\, and\, t \tag{8}$$

The objective function (1) represents the discounted cash flow. Constraints (2) and (3) enforce the mining and processing limits on average. Constraints (4)–(6) enforce the panel extraction precedence constraints, and constraints (7) and (8) ensure that the same COG is applied to all panels extracted in any given time period.

This MIP formulation is solved by the commercially available software package CPLEX version 9.0, by ILOG Inc.

Case Study

To test the algorithm we have chosen ten conditional simulations of a block model containing one type of metal and using one processing plant. Because of confidentiality requirements, all the economic parameters were rescaled and do not represent reality. All of the relative characteristics which demonstrate the potential of this new method are not affected by this rescaling. The ultimate pit for the design is chosen by applying the Lersch-Grossmann algorithm (Lersch and Grossmann 1965) in a procedure similar to that used in Whittle Four-X software. The ultimate pit contains 191 million tonnes of rock and 62.9 ± 2.7 million tonne of ore (above the marginal COG = 0.6%). The undiscounted value in the ultimate pit (if processed with the marginal COG) is $(1316 ± 99) million. It was divided into six mining phases and scheduled over 12 years. The mining rate was set to 30 Mtpa and the processing rate to 5 Mtpa. The initial capital investment was assumed to be $300 million, and the discount rate 10%. The base case optimisation was done using the marginal COG applied individually to all conditional simulation. The NPV for this case was $(404 ± 31) million. The mining schedule is shown in Fig. 1. The second optimisation was done using the variable COG, but was based on the mean grade block model, i.e. it was similar to an optimisation generated by using a single deterministic model. This schedule was evaluated against all ten realisations of orebody model and produced the NPV = $(485 ± 40) million, an increase of 20% over the base case. This mining schedule is shown in Fig. 2. The third optimisation was done using the algorithm described in earlier, using all orebody realisations as input to the optimisation and produced the NPV = $(505 ± 43) million, a further increase of 4.1% over the case of mean grade based optimisation. This mining schedule is shown in Fig. 3. The relative variability of NPV in all cases was roughly the same, about 8%. The cumulative NPV graphs for the three different schedules are shown in Fig. 4, and the comparison between expected NPVs and their variability is shown in Fig. 5. Another important result of the variable COG policy is that the pay-back period (defined here as the time when the cumulative NPV becomes equal to zero) is decreased from five to three years (see Fig. 4).

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Chapter No.: 8 | Date: 14-10-2017 Time: 2:29 pm | Page: 7/10

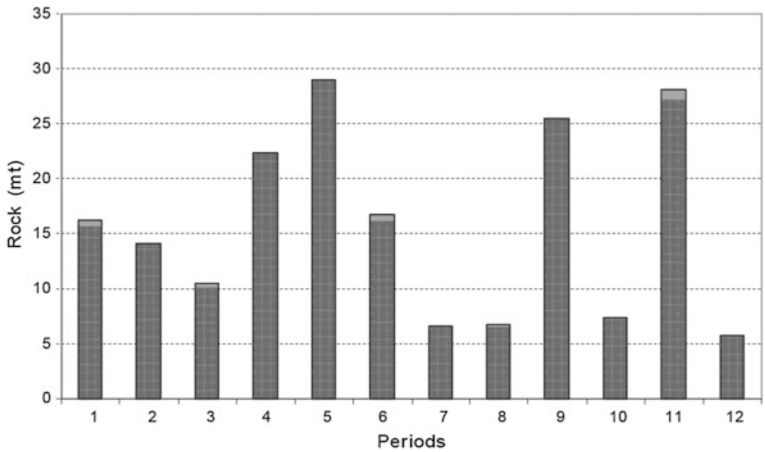

Fig. 1 Mining schedule optimised with the marginal COG

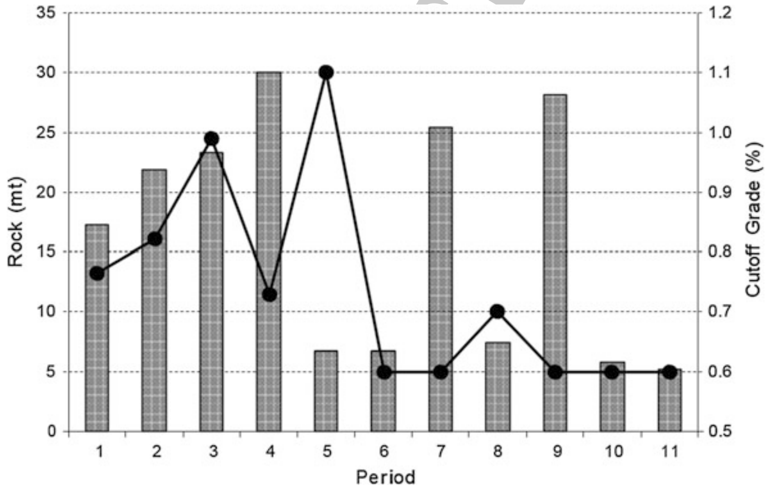

Fig. 2 Mining schedule optimised with the mean grade model

The increase of 4.1% in NPV may be not seen as a very substantial, but it should be mentioned that the block model considered does not have a high variability. The relative variance in the undiscounted value of the ultimate pit is only 7.6%. There are many deposits that have variability of the order of 20–30%. For these kind of deposits the potential improvement in the expected NPV may be substantially higher.

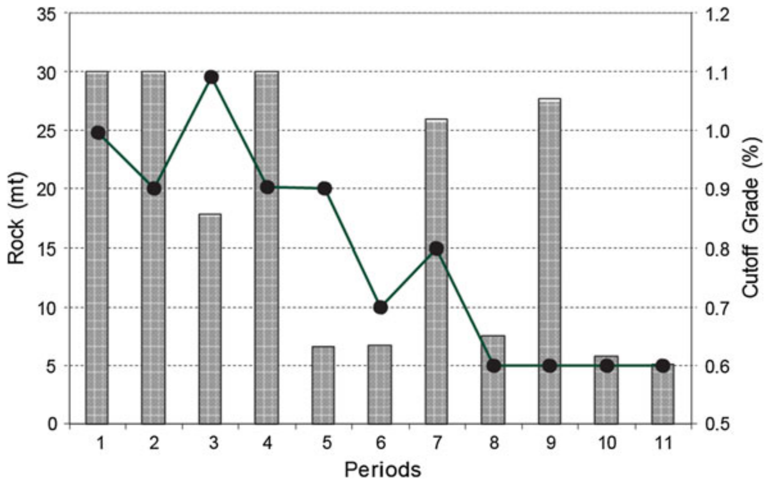

Fig. 3 Mining schedule optimised with the set of conditional simulations

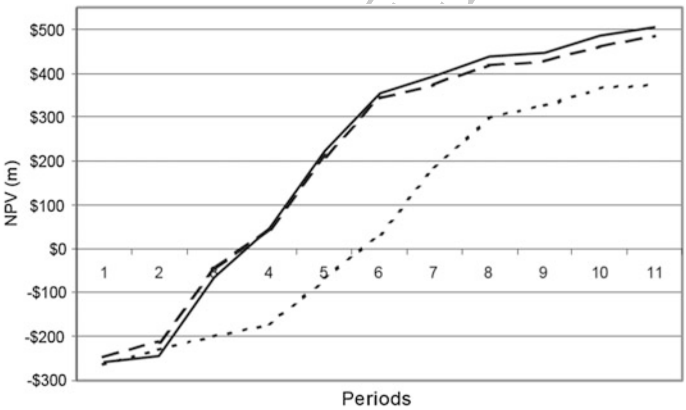

Fig. 4 Cumulative NPV for different missing schedules (solid line-variable COG on conditional simulations; dashed line-variable-COG on the mean grade model; dotted line-marginal COG)

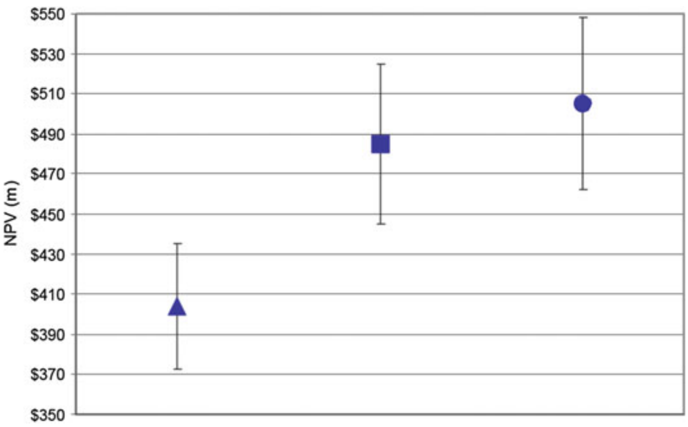

Fig. 5 Comparison of expected NPVs and their variability for different mining schedules (circle-variable COG on the conditional simulations; square-variable COG on the mean grade model; traingle-marginal COG)

Conclusions

A new method for simultaneous optimisation of the extraction sequence and cut-off grade policy for a set of conditionally simulated orebody realisations has been developed and demonstrated. This method is based on the mixed integer programming model and uses the commercially available software package CPLEX by ILOG Inc. The goal of the optimisation is to find the extraction sequence and cut-off grade policy, which, when evaluated through the whole set of conditionally simulated orebodies (representing the range of possible outcomes), will produce the best possible expected NPV. The degree of accuracy of this optimised schedule can be estimated precisely, in contrast to a number of heuristic routines used in current commercially available mining optimisation software packages. A fully functional software prototype that uses the new optimisation method has been developed.

In this study, we were using the expected NPV as the objective function and the mining and processing constraints were applied to the mean rock and ore tonnages. Some of the possible extensions of this method may include some kind of penalty functions in the objective function in order to find a schedule with a reduced variability in NPV, defining hard constraints bounding the NPV from below, or defining a lower bound on the annual cash flows. Another very interesting generalisation may include a stochastic price model for metals and adjustable cut-off grade policy.

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Chapter No.: 8 Date: 14-10-2017 Time: 2:29 pm Page: 10/10

References

Caccetta L, Hill SP (2003) An application of branch and cut to open pit mine scheduling. J Global Optim 27(2–3):349–365

Dimitrakopoulos R (1998) Conditional simulation algorithms for modeling orebody uncertainty in open pit optimization. Int J Surf Min 12(4):173–179

Isaaks EH, Srivastava RM (1989) Applied geostatistics. Oxford University Press, New York, p 561

Jewbali A (2006) Modelling geological uncertainty for stochastic short-term production scheduling in open pit metal mines. Ph.D. thesis, The University of Queensland, Australia, p 280

Lane KF (1988) The economic definition of ore. Mining Journal Books Ltd., London, p 147

Lerchs H, Grossmann L (1965) Optimum design of open-pit mines. Trans CIM, LXVII, pp 17–24

Padberg MW (1995) Linear optimization and extensions. New York, Springer, p 449

Ramazan S (2007) Large-scale production scheduling with the fundamental tree algorithm—model, case study and comparisons. In: Dimitrakopoulos R (ed) Orebody modelling and strategic mine planning, 2nd edn. The Australasian Institute of Mining and Metallurgy, Melbourne, pp 121–127

Ramazan S, Dimitrakopoulos R (2017) Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation. (in this volume)

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