# A New Methodology for Flexible Mine Design

**B. Groeneveld, E. Topal and B. Leenders**

**Abstract** Uncertainty and risk are invariably embedded in every mining project. Mining companies endeavouring to maximise their return for shareholders make important strategic decisions which take years or even decades to 'play out'. Therefore, developing a model that analyses the potential payoff of a decision based on current fixed assumptions is severely flawed. A model that incorporates uncertainty and is able to adapt, almost certainly will help deliver a design with a better risk-return profile. In this paper, a new approach is developed in order to have a design that is flexible and able to adapt with change. This is achieved by developing a mixed integer programming model that determines the optimal design for simulated stochastic parameters. This research has incorporated optionality (flexibility) in relation to mining, stockpiling, processing plant and port capacity. The results are promising and are helping decision makers to think in terms of value, risk and frequency of execution.

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## Introduction

Mining projects are characterised as being highly uncertain and variable mainly due to the volatile nature of commodity prices and uncertainty around geological conditions encountered in ore bodies. Uncertainty can arise from many different sources including; market prices, grade distribution, ground conditions, equipment reliability, recovery of ore, human capital and legislative change (Topal 2008). The mining industry will be more sustainable if projects are developed in a manner that increases flexibility to respond to uncertainties the business cycle. For example, the global minerals industry has seen an unprecedented demand for its products in recent years, however the industry has struggled to change its level of supply in response to price movements. Being able to design an operation that has flexibility to respond to this change quickly will deliver better returns to stakeholders.

Geological uncertainty and risk have been incorporated in optimum mine planning and design by a few studies to date. Ramazan and Dimitrakopoulos (2004) develop a stochastic based mixed integer programming (MIP) model for multiple element that uses several simulated orebodies in order to minimise the grade uncertainty in the life of the mine schedule. The model also takes into account risk quantification, equipment access and mobility and other operational requirement such as blending, mill capacity and mine production capacity. Godoy and Dimitrakopolus (2004) develop a new set of way to generate a mine production schedule under geological uncertainty. The first stage of the method generates a stable solution domain which shows the possible ore and waste extraction rates for a given open pit. The second stage generates optimum ore production and waste removal under uncertainty. The third stage generates a series of physical schedules which obey slope constraints, maximise the equipment utilisation and meet mill requirements while matching the mining rates previously derived by the optimisation. The last stage generates a single mining sequence from alternative sequences produced in the third stage by using a new algorithm based on the simulated annealing method. Leite and Dimitrakopoulos (2007) develop a stochastic based optimisation model for open pit mines and apply it to a copper deposit for risk analysis. The study shows the stochastic approach generates 26% higher NPV than the conventional schedule. Also, the study suggests that life of mine schedules which incorporate geological uncertainty lead to more informed investment decisions and improved mining practice.

A developing decision making tool aimed at increasing the flexibility of an engineering system is Real Options ‘in’ projects. Significant research into this method has been undertaken by Wang and de Neufville (2005, 2006) and his colleagues with applications in various industries. This method is located midway between financial Real Options analysis (which does not deal with system flexibility) and traditional engineering approaches (which does not to deal with financial flexibility). A popular example used to explore the concept of Real Options ‘in’ projects is that of a multi-story car park. Flexibility in this situation is in the design of the footing and columns of the building so that additional levels can be added at a later date. This flexibility comes at a cost, and the designer must determine if this is warranted. An example of the opportunity this technique poses is applied by Cardin et al. (2008) in conjunction with Codelco, to a Chilean mine in the 'Cluster Toki' region. In this example, a staged development of the Real Option 'in' projects methodology is used where different operating plans are designed to respond to changing prices. Truck fleet capacity and crusher size were altered in the different operating plans. The application of this method resulted in approximately 30–50% more accurate project value than current estimates. This approach provides a strong basis on which to grow Real Options 'in' projects theory for mining. However, there are several deficiencies in the current model. First, the initial scenario construction used in the model, limits the flexibility up front in the model and prevents the optimal design being chosen. Therefore, how useful is this technique for valuing flexibility? Secondly, the model fails to deal with variations in grade and recovery in a transparent manner; one of the key drivers. Finally, the model does not incorporate options at all stages of a typical mine value chain (de Neufville et al. 2005; Wang and de Neufville 2005, 2006; Cardin 2007; Cardin et al. 2008). This paper outlines a new methodology to evaluate the flexibility of strategic mine design under uncertainty, using Mixed Integer Programming (MIP) and Monte Carlo Simulation (MCS). An application of this methodology to a hypothetical case study will be undertaken in order to show the power of the model to handle complex strategic decisions.

## Methodology

In order to evaluate the flexibility in strategic mine design, this research employs MIP and MCS. In particular, MIP allows for 'go' or 'no go' decisions to be modelled for the optimal execution under a set of uncertainties. Uncertainties (or stochastic parameters) can be simulated using MCS. In this way, each model (or trial) represents a single path of a lattice tree (or binomial tree).

### *Description of Model Components*

The model consists of three main components which feed the MIP model; resource model, design options and stochastic parameters (Fig. 1). Running the model multiple times generates a database of optimal designs for a given 'state-of-the-world'. This dataset then provides a pathway to determine the flexibilities that provide the best risk-return profile.

#### Overview of Resource Model

Resource characteristics are a driving force in mine design. The MIP model uses a resource model to provide a representation of material that is available for

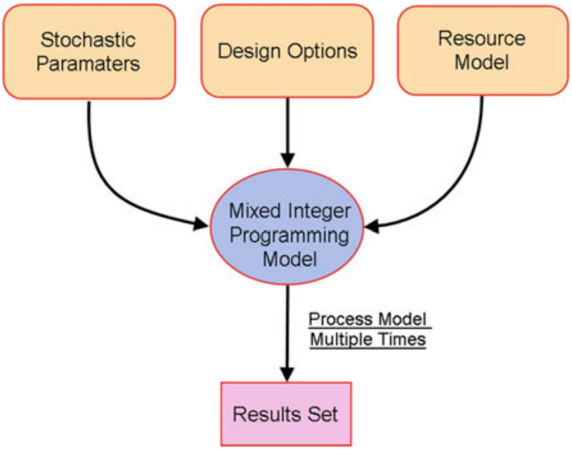


**Fig. 1** Conceptual diagram of how the various model components feed the Mixed Integer Programming model and the result set

processing through the life of mine (both ore and waste is considered). The representation of the resource is carried out by parcels of material. A parcel of material can be defined as a quantity of material with an average grade determined by the weighted average of the grade bins contained within the parcel. A parcel may be made up of one or more grade bins. A grade bin represents a quantity of material at a specified grade. This is incorporated to provide a higher level of detail to the model which will alter the decisions on how material is processed, whilst minimising the number of integer variables. These parcels are designed to represent a physical constraint on the resource. The most common physical constraint is the vertical mining constraint which is included in the model through parcel dependency. Mining of the grade bins within a parcel can occur in any order as long as the average parcel grade (within a nominal deviation) is extracted each period. This forces the model to take waste and ore in the same proportion.

## Overview of the Design Options

Flexibility is included through various design options in the MIP model. Solving the MIP models will determine which options are executed and when. A full set of design option are dynamically incorporated in the model which determine if and when these options should be executed. These options are broken into four categories; mine, pre-processing stockpiling, processing plants and port capacity. More than one option type can be executed in each period, hence these are not mutually exclusive decisions. An illustration of the material flow and points where design options may occur is shown in Fig. 2. Some assumptions have been made to simplify the model at this early stage of development. These assumptions can be removed with further refinement to the model; one type of circuit exists in each plant with one set of beneficiation characteristics; port stockpiles are not available in the model. This means the model must ship material as soon as it is processed.

## Available Mine Options in the Model

Mine options are incorporated in the model to reflect mining capacity constraints that exist in an operation. It is not feasible to have unlimited mining capacity, due to the high capital cost associated with additional capacity and/or technical pit constraints (geotechnical and equipment interaction). Mine options can be modelled to reflect truck capacity or shovel capacity. This type of decision is repeatable many times in each period (i.e. you can purchase more then one truck of the same type), thus mine options are represented as integers. This allows for one or more trucks of the same type to be purchased in each period.

## Available Stockpile Options in the Model

Stockpiling is used in mine operations for many reasons including; blending of material, storage of excess mine production and storage of low grade ore for future production. Long-term stockpiling is included in the model allowing material to be stored on a stockpile in time ($t$) and removed in subsequent periods ($t + 1 \ldots t + N$). A further ability of the stockpile option is its ability to represent long-term waste dumps. This functionality allows the model to consider waste movement and dynamically changes the cut-off grade. Waste dumps are developed by entering an option that is similar to a stockpile but has no plants for the material to flow too, forcing it to remain on the waste dump.

## Available Plant Options in the Model

Plant flexibility is incorporated to model the options managers have around processing of mined ore through varying plant designs. A plant option is characterised by its capacity, capital cost, fixed operating cost, recovery characteristics and grade limits. A processing plant is the link between the mine and the port in the flow of

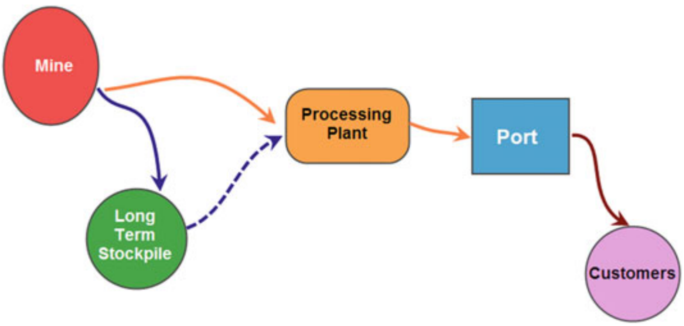


**Fig. 2** Material flow in the model and the location of the design options

material in the model. Plant options can also be dependent on other plant options being built, allowing the idea of modular plant capacity to be modelled. That is where a high initial capital cost is incurred to allow for expansion at a later date with a smaller capital cost.

### Available Port Options in the Model

Port options allow the sale of material to customers in the model. Enough port capacity must exist in a period in order for any plant production to occur in that period. For example, if we have two million tonnes of plant capacity and no port capacity in the first period then the production from the plant is forced to be zero.

### Overview of how Mine Scheduling work in the Model

Resource characteristics are a driving force in mine design. The MIP model uses a resource model to provide a representation of material that is available for processing through the life of mine (both ore and waste is considered).

The representation of the resource is carried out by parcels of material. A parcel of material can be defined as a quantity of material with an average grade determined by the weighted average of the grade bins contained within the parcel. A parcel may be made up of one or more grade bins. A grade bin represents a quantity of material at a specified grade. This is incorporated to provide a higher level of detail to the model which will alter the decisions on how material is processed, whilst minimising the number of integer variables. These parcels are designed to represent a physical constraint on the resource. The most common physical constraint is the vertical mining constraint which is included in the model through parcel dependency.

Mining of the grade bins within a parcel can occur in any order as long as the average parcel grade (within a nominal deviation) is extracted each period. This forces the model to take waste and ore in the same proportion.

### Options in the Model to test best Plant or Stockpile location through varying Mining Costs

The optimal location for a processing plant varies with time as the resource is mined in different regions. Therefore, determining the best location for a plant or a stockpile is not a simple case and must consider these multiple uncertainties as it will most likely change over the life of the project. Different plant locations and stockpile locations can be tested by developing a mining cost which varies by parcel and destination in the model.

**Overview of what Stochastic Parameters Are Included**

Uncertainty in the mining process is incorporated through the market price, cost (capital and operating), utilisation of equipment, plant recovery and time to build an option. Values for these various inputs are simulated through a MCS process.

## *Model Formulation*

The developed MIP model optimises the available mine, stockpile, plant and port flexibility for a simulated scenario. These various design options dictate how the system is configured and consequently the amount of production that can occur. They also dictate the financial viability of the operation and drive both revenue and operating costs. An outline of the mathematical formulation is provided below.

## *Objective Function*

The objective function seeks to maximise before tax net present value (NPV):

$$\sum_{t=1}^{T}\frac{1}{(1+r^*)^t}\Bigg[S_t\sum_{l=1}^{L}M_{l,t}-\sum_{m=1}^{M}C_mI_{m,t}-\sum_{s=1}^{S}C_sY_{s,t}-\sum_{l=1}^{L}C_lY_{l,t}$$
$$-\sum_{o=1}^{O}C_oY_{o,t}-\sum_{m=1}^{M}D_mID_{l,t}-\sum_{l=1}^{L}D_lID_{l,t}-\sum_{o=1}^{O}D_oID_{o,t}$$
$$-\sum_{m=1}^{M}V_{m,t}X_{m,t}-\sum_{l=1}^{L}V_{l,t}X_{l,t}-\sum_{s=1}^{S}V_{s,t}X_{s,t}-\sum_{o=1}^{O}V_{o,t}X_{o,t}$$
$$-\sum_{m=1}^{M}F_{m,t}I_{m,t}-\sum_{l=1}^{L}F_{l,t}Y_{l,t}-\sum_{o=1}^{O}F_{o,t}Y_{o,t}-\sum_{m=1}^{M}FR_{m,t}ID_{m,t}$$
$$-\sum_{l=1}^{L}FR_{l,t}ID_{l,t}-\sum_{o=1}^{O}FR_{o,t}ID_{o,t}-\sum_{l=1}^{L}L_l-\sum_{p=1,b=1}^{P,B}X_{p,b,l,t}$$
$$-\sum_{s=1}^{S}L_s\sum_{p=1,b=1,l=1|l\in s}^{P,B,L}XI_{p,b,s,l,t}\Bigg]$$

Where;

| | |
|---|---|
| $r^*$ | is the rate of return on the project |
| $S_t$ | is sale price in time period $t$ (in \$/metal unit) |
| $M_{l,t}$ | is the metal units exiting plant $l$ in time $t$ |

$C_m, C_s, C_l, C_o$ is the capital cost of mine $m$ or stockpile $s$ or plant $l$ or port $o$
$I_{m,t}$ is the execution integer on mine option $m$ in time $t$.
0 is no execution of option;
otherwise, is the number of times the option is executed in time $t$.

$Y_{s,t}, Y_{l,t}, Y_{o,t}$ is the execution binary on stockpile $s$ or plant $l$ or port $o$.
0 is no execution of option;
otherwise, the option is executed in time $t$.

$D_m, D_s, D_l, D_o$ is the disposal cost of mine $m$ or stockpile $s$ or plant $l$ or port $o$
$ID_{m,t}$ is the disposal integer on mine option $m$ in time $t$.
0 is no disposal occurs in time $t$;
otherwise, is the number of options disposed of in time $t$.

$ID_{l,t}, ID_{o,t}$ is the disposal integer on plant $l$ or port $o$.
0 is no disposal occurs in time $t$;
otherwise, is option is disposed in time $t$.

| | |
|---|---|
| $V_{m,t}, V_{s,t}, V_{l,t}, V_{o,t}$ | is the variable cost of mining a tonne of ore from mine $m$ or stockpile $s$ or plant $l$ or port $o$ in time $t$ |
| $X_{m,t}, X_{s,t}, X_{l,t}, X_{o,t}$ | is the tonnage processed through mine $m$ or stockpile $s$ or plant $l$ or port $o$ in time $t$ |
| $F_{m,t}, F_{l,t}, F_{o,t}$ | is the fixed cost of mining from mine $m$ or plant $l$ or port $o$ in time $t$ |
| $FR_{m,t}, FR_{l,t}, FR_{o,t}$ | is the reduction in fixed cost of disposing of an option in time $t$ |
| $L_l$ | is the cost of mining a tonne of ore to plant $l$ |
| $L_s$ | is the cost of mining a tonne of ore to stockpile $s$ |
| $X_{p,b,l,t}$ | is the tonnage mined from parcel $p$ bin $b$ to plant $l$ in time $t$ |
| $XI_{p,b,s,l,t}$ | is the tonnage mined from parcel $p$ bin $b$ to stockpile $s$ at plant $l$ in time $t$ |

The objective function represents the following:

> The revenue from the sale of the ore less the capital cost of building an option less the disposal cost of reducing capacity less the variable cost of processing ore less the fixed cost of maintaining an option; all multiplied by the relevant discount factor for the cash flow in time t. The model seeks to maximise this relationship.

The constraints in the model can be divided into five categories: production, mining, stockpiling, processing plant and port constraints.

## *Production Constraints*

### Resource Constraint

This constraint makes sure the total amount of material extracted from a mining pit has an upper bound based on the resource. This constraint is applied at a parcel and bin level in the model:

$$\sum_{t=1}^{T} X_{p,b,t} - R_{p,b} \leq 0 \quad \forall p, b$$

Where;

$X_{p,b,t}$ is the tonnage mined from parcel $p$ bin $b$ in time $t$
$R_{p,b}$ is the resource of parcel $p$ bin $b$

### Sequencing Constraint 1

This constraint in conjunction with the next constraint forces the binary value to be one in the period the parcel is fully mined. This then allows the model to mine any successor parcels of ore:

$$\sum_{b=1,tt=1}^{B,t} X_{p,b,tt} \geq R_p * Y_{p,t} \quad \forall p, t$$

Where;

$R_p$ is the resource of parcel $p$

### Sequencing Constraint 2

This constraint ensures that a parcel's predecessor is mined before the successor is mined:

$$\sum_{b=1,tt=1}^{B,t} X_{p+1,b,tt} \leq R_{p+1} * \sum_{tt=1}^{t} Y_{p,tt} \quad \forall p, t$$

Where;

$R_{p+1}$ is the resource of the successor parcel $p + 1$
$X_{p+1,b,tt}$ is the tonnage mined from the successor parcel $p + 1$ bin $b$ in time $tt$

**Sequencing Constraint 3**

This constraint is a set packing constraint that forces a parcel to only be fully mined once:

$$\sum_{t=1}^{T} Y_{p,t} \leq 1 \qquad \forall p$$

**Equal Mining of a Parcel**

This constraint forces the model to take high grade ore, waste and low grade ore in equal proportions. This prevents the model taking high grade in the first period, followed by low grade in the second period and waste in the following period. A minimal deviation ($\gamma$) of 2% was allowed to prevent an infeasible solution:

$$\sum_{b=1}^{B} X_{p,b,t} G_{p,b} \geq \left[ \sum_{b=1}^{B} X_{p,b,t} \right] G_p * (1 - \gamma\%) \qquad \forall p, t$$

$$\sum_{b=1}^{B} X_{p,b,t} G_{p,b} \leq \left[ \sum_{b=1}^{B} X_{p,b,t} \right] G_p * (1 + \gamma\%) \qquad \forall p, t$$

Where;

$G_{p,b}$ is the grade of parcel $p$ and bin $b$
$G_p$ is the grade of parcel $p$

**Flow Balance Constraint**

This constraint links the flow paths in the model and ensures that the material available to the processing plant and stockpiling options originates from the resource:

$$X_{p,b,t} = \sum_{l=1}^{L} X_{p,b,l,t} + \sum_{s=1,k=1,l=1}^{S,K,L} XI_{p,b,s,k,l,t} \qquad \forall p, b, t$$

## *Mining Constraints*

### Mining Requirements

The constraint makes sure that mining includes all movement to plant options, stockpile options and movement off stockpiles:

$$\sum_{m=1}^{M} X_{m,t} = \sum_{p=1,b=1}^{P,B} X_{p,b,t} + \sum_{s=1,l=1}^{S,L} XO_{s,l,t} \qquad \forall t$$

Where;

$XO_{s,l,t}$ is the tonnage sent from stockpile $s$ at location $l$ in time $t$

### Mining Capacity Limit

This constraint ensures that mining only occurs if there is sufficient capacity in a period to handle the movement. Capacity is determined dynamically based on when mining options are executed:

$$\sum_{u=1}^{t} A_{m,u,t} I_{m,u} - \sum_{u=2}^{t} A_{m,u,t} ID_{m,u} \leq X_{m,t} \qquad \forall m, t$$

Where;

$A_{m,u,t}$ is the capacity of mine option $m$ that was executed in period $u$ in time $t$

### Mine Option Disposal Constraint

This constraint ensures disposal of an option can only occur if the option has been built. For example, if a mine option is built in period one and in period five there is no more material to mine, then the model can dispose of this capacity in order to reduce the fixed cost incurred:

$$ID_{m,t} \leq \sum_{tt=2}^{t} I_{m,tt-1} \qquad \forall m, t$$

$$\sum_{tt=2}^{t} ID_{m,tt} \leq \sum_{tt=2}^{t} I_{m,tt-1} \qquad \forall m, t$$

### Maximum Execution

Since the mine option can be modelled as an integer variable, limits on how many times it can be applied in the model may be included. This is an optional constraint which can be turned on or off when running the model.

### Period Constraint

This constraint restricts the number of mine options built in a period to the period constraint maximum:

$$I_{m,t} \leq PC_m \qquad \forall m, t$$

### Overall Constraint

This constraint restricts the total number of mine options built over the life of a mine:

$$\sum_{t=1}^{T} I_{m,t} \leq OC_m \qquad \forall m$$

Where;

*PC* is the period constraint limit or the maximum number of times an option can be executed in any period;

*OC* is the overall constraint which is the maximum number of times an option can be executed over the life of the project.

## *Stockpiling Constraints*

### Total Inflow Constraint

This constraint makes the total amount of material that is entering a stockpile equal the material entering each stockpile bin:

$$XI_{s,l,t} = \sum_{k=1}^{K} XI_{s,k,l,t} \qquad \forall s, l, t$$

Where;

$XI_{s,l,t}$ is the total tonnage sent into stockpile $s$ at plant $l$ in time $t$

## Flow Balance Constraint

This constraint restricts the total amount of material coming into each bin in the stockpile to be equal to the material sent from the each parcel to the bin:

$$XI_{s,k,l,t} = \sum_{p=1,b=1}^{P,B} XI_{p,b,s,k,l,t} \qquad \forall s,k,l,t$$

Where;

$XI_{s,k,l,t}$ is the tonnage sent into stockpile $s$ grade bin $k$ at plant $l$ in time $t$
$XI_{p,b,s,k,l,t}$ is the tonnage from parcel $p$ bin $b$ sent into stockpile $s$ grade bin $k$ at plant $l$ in time $t$

## Stockpile Capacity Constraint

This constraint makes sure the tonnage of material stockpiled across all plant locations does not exceed the stockpile capacity:

$$\sum_{l=1,tt=1}^{L,t} XI_{s,l,tt} - \sum_{l=1,tt=2}^{L,t} XO_{s,l,tt} \leq \sum_{tt=1}^{t} A_s Y_{s,tt} \qquad \forall s,t$$

Where;

$XO_{s,l,t}$ is the total tonnage sent from stockpile $s$ to plant $l$ in time $t$
$A_s$ is the total capacity of stockpile $s$

## Stockpile Grade Constraint on Bins

This constraint applies the grade limits of the stockpile bins to material entering each stockpile bin:

$$\sum_{tt=1}^{t} G_{p,b} XI_{p,b,s,k,l,tt} \leq \sum_{tt=1}^{t} GU_{s,k} XI_{p,b,s,k,l,tt} \qquad \forall p,b,s,l,t$$

$$\sum_{tt=1}^{t} G_{p,b} XI_{p,b,s,k,l,tt} \geq \sum_{tt=1}^{t} GL_{s,k} XI_{p,b,s,k,l,tt} \qquad \forall p, b, s, l, t$$

Where;

$GU_{s,k}$ is the upper grade limit of stockpile $s$ bin $k$
$GL_{s,k}$ is the lower grade limit of stockpile $s$ bin $k$

#### Bin Removal Constraint

The constraint ensures material moved from the stockpile has been added to the stockpile at least one period ago and that material removed from the stockpile is not removed again:

$$XO_{s,k,l,t} \leq \sum_{tt=1}^{t} XI_{s,k,l,tt} + \sum_{tt=2}^{t} XO_{s,k,l,tt-1} \qquad \forall s, k, l, t$$

Where;

$XO_{s,k,l,t}$ is the tonnage removed from stockpile $s$ bin $k$ to plant $l$ in time $t$

#### Bin Extraction

This constraint makes the total tonnage of material that is extracted from each stockpile bin equal the overall extraction from the stockpile:

$$\sum_{k=1}^{K} XO_{s,k,l,t} = XO_{s,l,t} \qquad \forall s, l, t$$

#### Metal Extraction

This constraint ensures the total amount of metal units extracted from a stockpile equals the metal units extracted from the individual grade bins:

$$MO_{s,l,t} = \sum_{k=1}^{K} GA_{s,k} XO_{s,k,l,t} \qquad \forall s, l, t$$

Where;

$MO_{s,l,t}$ is the metal units removed from stockpile s at location $l$ in time $t$
$GA_{s,k}$ is the average grade of stockpile $s$ grade bin $k$

### Opening Limit

This constraint ensures that a stockpile can only be opened once:

$$\sum_{t=1}^{T} Y_{s,t} \leq 1 \qquad \forall s$$

## *Processing Plant Constraints*

### Grade Limits (Upper and Lower)

This constraint applies the grade limits on a given plant in each time period. This ensures every plant processes material it can handle:

$$\sum_{s=1|\delta\varepsilon l}^{S} MO_{s,l,t} + \sum_{p=1,b=1}^{P,B} G_{p,b} X_{p,b,l,t} \leq GU_l \sum_{s=1}^{S} XO_{s,l,t} + GU_l \sum_{p=1,b=1}^{P,B} X_{p,b,l,t} \qquad \forall l, t$$

$$\sum_{s=1|\delta\varepsilon l}^{S} MO_{s,l,t} + \sum_{p=1,b=1}^{P,B} G_{p,b} X_{p,b,l,t} \geq GL_l \sum_{s=1}^{S} XO_{s,l,t} + GL_l \sum_{p=1,b=1}^{P,B} X_{p,b,l,t} \qquad \forall l, t$$

Where;

$GL_l$ is the lower grade limit of plant $l$
$GU_l$ is the upper grade limit of plant $l$

### Plant Capacity Constraint

The constraint ensures that the total tonnage of material processed in a period shall is less than the capacity of plant options built and disposed:

$$\sum_{s=1}^{S} XO_{s,l,t} + \sum_{p=1,b=1}^{P,B} X_{p,b,l,t} \leq \sum_{u=1}^{T} A_{l,u,t} Y_{l,u} - \sum_{u=2}^{T} A_{l,u,t} ID_{l,u} \qquad \forall l, t$$

Where;

$A_{l,u,t}$ is the capacity of plant option $l$ built in $u$ in time $t$

## *Plant Disposal Constraint*

This constraint ensures that a plant option is only disposed if the plant has been built in a previous period. This will result in a fixed cost saving, however an additional disposal cost will be incurred in the objective function:

$$YD_{l,t} - \sum_{t=1}^{t} Y_{l,t} \leq 0 \qquad \forall l, t$$

$$\sum_{t=1}^{t} YD_{l,tt} - \sum_{tt=1}^{t} Y_{l,tt} \leq 0 \qquad \forall l, t$$

### Tonnage Produced

This constraint restricts the tonnage exiting the plant to be equal to the material entering the plant multiplied by the plant recovery:

$$X_{l,t} = \sum_{p=1,b=1}^{P,B} E_{l,t} X_{p,b,l,t} + \sum_{s=1,t=2|s \in l}^{S,T} E_{l,t} XO_{s,l,t} \qquad \forall l, t$$

Where;

$E_{l,t}$ is the recovery of plant $l$ in time $t$

### Metal Units Produced

This constraint calculates the metal production of a plant option by multiplying the metal units into the plant by the recovery and grade multiples for the plant. This is used to calculate the revenue of the mine:

$$M_{l,t} = \sum_{p=1,b=1}^{P,B} E_{l,t} GM_{l,t} X_{p,b,l,t} + \sum_{s=1,t=2|s \in l}^{S,T} E_{l,t} GM_{l,t} MO_{s,l,t} \qquad \forall l, t$$

Where;

$GM_{l,t}$ is the grade multiple of plant option $l$ in time $t$

### Plant Option Dependency

Plant option dependency dictates the relationships that occur between options. Two types of relationships are available; one-for-one and one-for-many.

### One for One Relationship

This constraint makes sure a successor option is built prior to the predecessor option being built in an equal ratio. For example, this can be used to model a modular plant design where an initial investment can be made in plant capacity that has the ability to be expanded easily for a lower capital than if the initial investment was not made (this later expansion is optional):

$$\sum_{tt=1}^{t} Y_{l,tt} \leq \sum_{tt=1}^{t-DT} Y_{c,tt} \qquad \forall l, c, t | c \in l$$

### One for Many Relationships

This constraint allows a successor option to be built if its predecessor option has been built at least once. This can be used to model a rail link to a plant location where an initial capital investment is required. However, once this has been built numerous plants can be built at the same location:

$$Y_{l,t} \leq \sum_{t=1}^{T-DT} Y_{c,t} \qquad \forall l, c | c \in l$$

Where;

$C$ is the predecessor plant option of plant option $l$
$DT$ is the lead time on the relationship
$Y_{c,tt}$ is the execution variable of the predecessor plant $c$ of plant $l$ in time $tt$.

## *Port Constraints*

### Port Production Constraint

The constraint ensures the total tonnage of material processed through all plant options is less than or equal to the total port capacity:

$$\sum_{l=1}^{L} X_{l,t} \leq \sum_{o=1}^{O} X_{o,t} \qquad \forall t$$

### Capacity Constraint

The constraint requires the total tonnage of material shipped in a period to be less then the port capacity:

$$X_{o,t} \leq \sum_{u=1}^{t} A_{o,u,t} Y_{o,u} - \sum_{u=2}^{t} A_{o,u,t} ID_{o,u} \qquad \forall o, t$$

Where;

$A_{o,u,t}$ is the capacity of port option $o$ built in $u$ time $t$

### Disposal Constraint 1

Disposal of a port option may only occur if the option has previously been built:

$$ID_{o,t} \leq \sum_{tt=1}^{t} Y_{o,tt} \qquad \forall o, t$$

$$\sum_{tt=1}^{t} ID_{o,t} \leq \sum_{tt=1}^{t} Y_{o,tt} \qquad \forall o, t$$

### Port Option Dependency

Port option dependency may occur in one for one or one for many relationships.

**One for One Relationship**

This constraint makes sure a successor option is built prior to the predecessor option being built in an equal ratio:

$$\sum_{tt=1}^{t} Y_{o,tt} \leq \sum_{tt=1}^{t-DT} Y_{c,tt} \qquad \forall o, t|c \in o$$

## *One for Many Relationships*

This constraint allows a successor option to be built if its predecessor option has been built at least once. This relationship can be used to model a rail link that must be built before any port can be built:

$$Y_{o,t} \leq \sum_{tt=1}^{t-DT} Y_{c,tt} \qquad \forall o, t|c \in o$$

Where;

$DT$ is the lead time on the relationship
$Y_{c,tt}$ is the execution variable of the predecessor port $c$ of port $o$ in time $tt$.

## *Non-negativity, Binary and Integer Restrictions*

**Non-negativity**

The following variables are restricted to taking on positive values as a negative would represent an infeasible situation:

$$X_{p,b,t}, XI_{p,b,s,k,l,t} \geq 0 \qquad \forall p, b, s, k, l, t$$

$$X_{m,t}, X_{l,t}, X_{o,t} \geq 0 \qquad \forall m, l, o, t$$

$$XO_{s,l,t}, XO_{s,k,l,t}, XI_{s,k,l,t} XI_{s,l,t} \geq 0 \qquad \forall s, k, l, t$$

**Integers**

The following variables must take on integer values in the model:

$$I_{m,t}, ID_{o,t}, ID_{m,t}, ID_{l,t} \qquad \forall m, l, o, t$$

**Binaries**

The following variables must take on binary values; integers with an upper bound of one and lower bound of zero:

$$Y_{c,tt}, Y_{p,t}, Y_{o,t}, Y_{l,t}, Y_{s,t} \qquad \forall p, o, l, s, t, c, tt$$

## Case Study: Open Pit Mine

An application of the methodology was implemented to a hypothetical mining scenario. The problem is similar in nature to an iron ore mine, although it could be applied to any open cut mine. A single mine site is used in this example.

### *The Problem*

The operation consists of three mining pits (two high grade and one low grade), two plant locations with associated rail infrastructure, stockpiling and waste storage capabilities at each location and two port options with associated rail requirements (Fig. 3). In this diagram, the rectangular boxes represent different plant locations (note that location A has the shortest haul for pit 1 and location B has the shortest haul for pit 3 whilst pit 2 has an equivalent haul to either location). In order to process material through a plant at location A, a rail link of 35 km with a capital of $65 M needs to be built. Likewise, at location B a rail link of 10 km needs to be built for a capital of $30 M. Finally, in order to process any material through the port, a rail link from the junction of A and B to the coast needs to be built for a capital of $20 M. The analysis will look at the system configuration over five periods.

### *The Model Inputs*

Multiple options were included in the model of this problem as summarised in Table 1. A full list of the fixed costs, variable costs and grade constraints is not provided for simplicity purposes. In order to simulate the different processing plant locations available to the model, a differential mining cost was used based on the destination of material. A summary of the different costs associated with each location is outlined in Table 2.

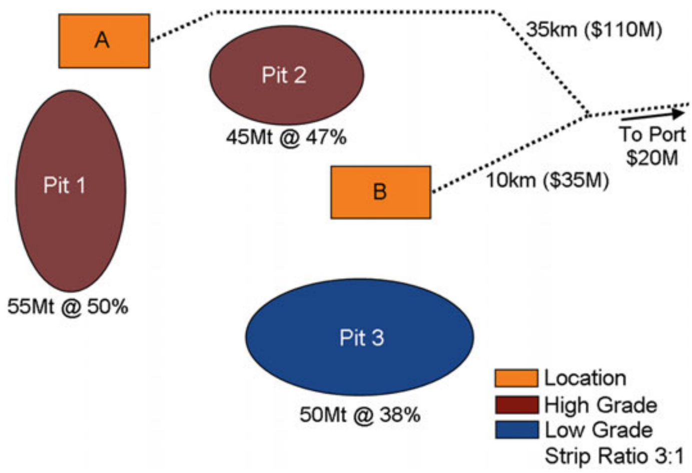


**Fig. 3** Conceptual layout of hypothetical mine

**Table 1** Options available in hypothetical example

| Type | Cost ($M) |
|---|---|
| *Mine options* | |
| 1 Mt/a unit | 3 |
| 2 Mt/a unit | 4.5 |
| *Stockpile options* | |
| Waste stockpile (500 Mt capacity) | 0 |
| Low-grade (30 Mt capacity) | 0 |
| *Plant options* | |
| 5 Mt/a fixed (lead 0.5 yr) | 50 |
| 10 Mt/a fixed (lead 1.5 yr) | 92 |
| 5 Mt/a flexible (modular, lead 0.5 yr) | 75 |
| Additional 5 Mt/a flexible (modular, lead 0.5 yr) | 30 |
| *Port capacity* | |
| 10 Mt/a (lead 0.75 yr) | 100 |
| 20 Mt/a (lead 1.5 yr) | 175 |

## *The Stochastic Variables*

In this problem, it was determined that seven stochastic parameters would be included. These were price, recovery, capital cost, operating cost and utilisation for mine, plant and port options.

Choice of underlying distributions was done through discussions with professionals. No detailed analysis of the underlying nature of the stochastic variables has

**Table 2** Differential mining costs to handle different locations in the model

| Cost ($/t) | Pit 1 | Pit 2 | Pit 3 |
|---|---|---|---|
| Plant location A | 0.2 | 0.6 | 1.8 |
| Plant location B | 2.2 | 0.6 | 0.4 |
| Waste location A | 0.1 | 0.3 | 0.9 |
| Waste location B | 1.1 | 0.3 | 0.2 |

been carried out, as detailed research in other papers is available which was not the primary purpose of this paper (Dimitrakopoulos and Abdel Sabour 2007; Godoy and Dimitrakopoulos 2004; Lima and Suslick 2006; Morley et al. 1999; Topal 2008).

A summary of the values used for each distribution is as follows:

- Price follows a lognormal distribution with a mean of $85, standard deviation of $25 and a correlation of 0.30 between periods;
- Recovery follows a triangular distribution with a maximum value of 90%, likely value of 80%, minimum value of 70% and a correlation of 0.05 between periods;
- Capital cost multiple follows a normal distribution with a mean of 1.08, standard deviation of 0.20 and a correlation of 0.40 between periods;
- Operating cost multiple follows a normal distribution with a mean of 1.03, standard deviation of 0.10 and a correlation of 0.10 between periods;
- Mine equipment utilisation follows a triangular distribution with a maximum value of 95%, likely value of 75%, minimum value of 60% and a correlation of 0.22 between periods;
- Plant utilisation follows a triangular distribution with a maximum value of 95%, likely value of 80%, minimum value of 65% and a correlation of 0.21 between periods; and
- Port utilisation follows a triangular distribution with a maximum value of 95%, likely value of 80%, minimum value of 65% and a correlation of 0.34 between periods.

## *Results Analysis*

Based on the input parameters 200 trials were run, with CPLEX™ used to solve the MIP model. In total it took 3 h to process the model, which was deemed a good solution time for this model size. The raw data from the results exceeds four gigabytes. A results analysis process has been developed which summarises this data. After processing of the model, the frequency of execution for each options was analysed (Table 3). Frequency of execution is calculated by dividing the count of the number of times an option is executed by the maximum number of times it could be executed. Some categories of options (plant, port, mine) sum to more than 100% because multiple expansions of that type can occur in the same time period as the options are not mutually exclusive.

**Table 3** Frequency of execution for all options in the model

| | Period 1 (%) | Period 2 (%) | Period 3 (%) | Period 4 (%) | Period 5 (%) | All periods (%) |
|---|---|---|---|---|---|---|
| Mine 1 Ml/a | 18 | 4 | 5 | 0 | 0 | 5 |
| Mine 2 Mt/a | 98 | 32 | 9 | 3 | 1 | 29 |
| Rail link to A | 94 | 0 | 0 | 0 | 0 | 19 |
| Rail link to B | 45 | 1 | 0 | 0 | 0 | 9 |
| Plant 5 Mt/a (A) | 69 | 26 | 14 | 6 | 2 | 23 |
| Plant 10 Ml/a (A) | 66 | 14 | 5 | 1 | 0 | 17 |
| Plant 5 Mt/a modular (A) | 90 | 41 | 13 | 2 | 0 | 29 |
| Additional 5 Mt/a modular (A) | 0 | 82 | 35 | 16 | 1 | 27 |
| Plant 5 Mt/a (B) | 19 | 4 | 1 | 1 | 0 | 5 |
| Plant 10 Mt/a (B) | 44 | 19 | 6 | 2 | 0 | 14 |
| Plant 5 Mt/a modular (B) | 41 | 15 | 4 | 2 | 0 | 12 |
| Additional 5 Mt/a modular (B) | 0 | 35 | 14 | 3 | 2 | 11 |
| Port 10 Mt/a | 98 | 1 | 0 | 0 | 0 | 20 |
| Port 20 Mt/a | 91 | 84 | 53 | 28 | 8 | 53 |

From examining Table 3 several conclusions can be developed. First, it is evident that larger port capacity options should be investigated as the execution frequency is over 50% for the 20 Mtpa port option (the largest in the model) for three periods. Second, the mine option with 1 Mtpa capacity and the plant with 5 Mtpa at location B are not valuable options as there execution is lower than 20%. Finally, location A is preferred over location B as the rail link options which dictate which locations can be used are executed 94% and 45% for A and B respectively.

A value at risk graph (VARG) shows the risk to return relationship. Figure 4 displays the VARG for this example with the base case representing a fixed mine design with no optionality. The design chosen for the base case was based on the 50th percentile design when the model with optionality was run. This design was then fixed in the MIP model and reprocessed with the same uncertainties. This shows the outcome of management not changing the operating policy of the mine. The mean NPV of the base case was \$702 M and for the case with options was \$1298 M, an 85% increase.

Further to these analysis methods, experimentation is currently underway with using various data mining techniques. An open source software package called Rapid Miner is currently being used. An example (from a different problem set) of the output generated is shown in Fig. 5. This example shows a decision tree with the associated percentages of times the decision paid off highlighted in the Yes/No boxes at the bottom of the nodes.

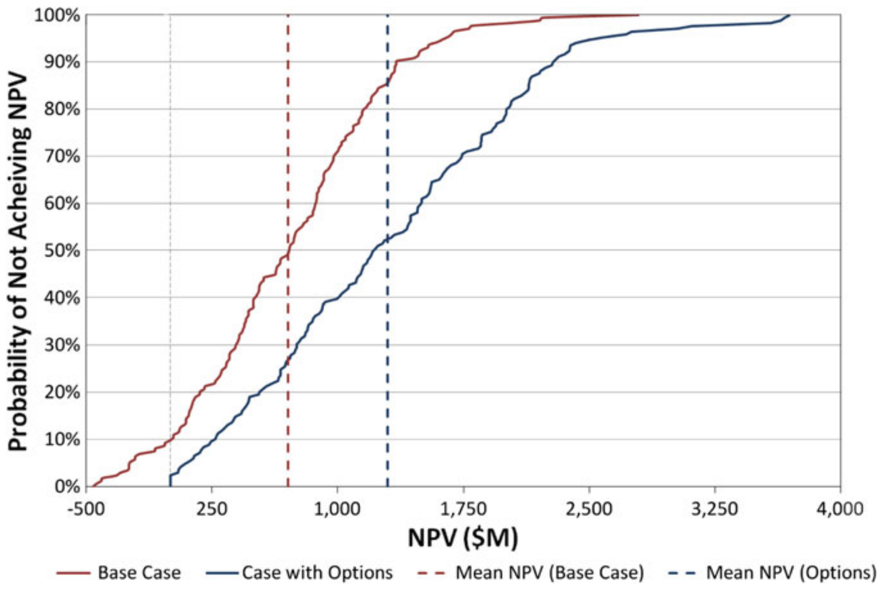


**Fig. 4** Value at risk for hypothetical example

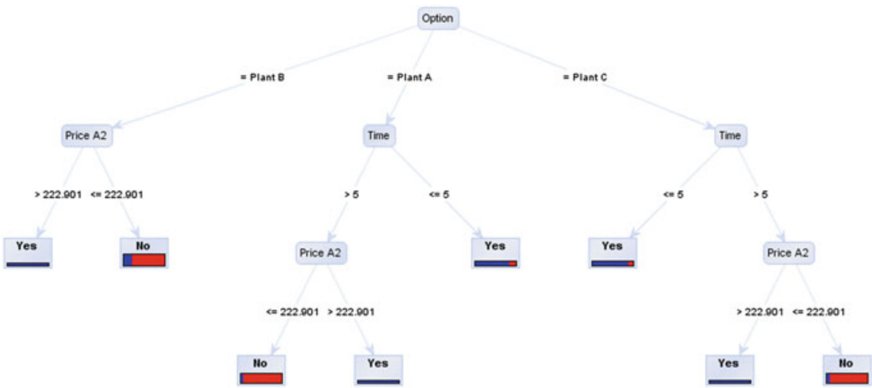


**Fig. 5** Sample of Decision Tree output from Data Mining

## Conclusions

In conclusion, this paper has developed a methodology to evaluate the strategic mine design flexibility under stochastic environment. The proposed methodology is a unique approach that allows flexible mine designs to be justified. The decision maker is supported in their choice of and refinement mine design. Increasing flexibility in mine designs would be advantageous for responding to changing business conditions across the full economic cycle.

For the sake of comparison, the proposed methodology has been implemented to a hypothetical mining scenario. The results demonstrated that the value of expected NPV increases by 85% with flexible mine design compared to without flexibility. The paper illustrates how to incorporate design options (flexibility) into a strategic mine plan in a manner that proactively manages inevitable uncertainties. It is hoped this research will help in justifying more flexible mine designs and further the sustainability of the industry.

## Recommendations

Whilst the model handles a simple case, currently further research and model improvements continue in the following areas:

- More detailed modelling which considers multiple process options and multi product options;
- Handling of grade variability through the use of conditional simulation methods will greatly improve the power of the model (Dimitrakopoulos and Ramazan 2004);
- Further investigation into appropriate results analysis techniques is required to fully understand how the primary question of flexibility is answered;
- MIP performance improvement algorithms need to be investigated, these methods may include reducing the feasible region with additional constraints and/or developing a node selection routine for the branch and bound algorithm that exploits some of the nuances in the model; and
- Application of this technique to underground mining is needed to fully capture the options available to mine management. In particular, incorporating the process to optimise the open cut and underground transition point would be highly beneficial. This would assist in strategic planning for the entire orebody.

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