Two-dimensional massless light-front fields and conformal field theory

L'ubomír Martinovič

Abstract A consistent quantization of two-dimensional (2D) massless light-front fields (scalar and fermion) is formulated. Their two-point functions exactly reproduce the massless limit of the two-point functions of the corresponding massive fields. The novel formalism incorporates bosonization in a natural way and also provides us with elements needed for an independent light-front (LF) study of the exactly solvable models (the Thirring or Thirring-Wess model, e.g.). Moreover, it displays closeness of the 2D massless LF quantum fields to conformal field theory (CFT). We calculate a few correlators including those between the components of the LF energy-momentum tensor and derive the Virasoro algebra in the LF operator form. Going over to the euclidean time, we can direcly transform all calculated quantities to the (anti)holomorphic form, in agreement with those from CFT.

1 Introduction

The light front (LF) form of quantum field theory (QFT) has been praised for its potential for decades. Its features that are superior to the conventional ("space-like" - SL) form of QFT, include the minimal number (3) of dynamical Poincaré generators [1], the status of the vacuum state, and a reduced number of independent field components. The most fundamental aspect is the equality of the physical vacuum state (= the lowest energy eigenstate of the full generic Hamiltonian) to the Fock vacuum (= state without field quanta). This property follows from the positivity and conservation of the LF momentum p^+. Only the field zero modes, carrying $p^+ = 0$, and a narrow set of (symmetry) operators [2,3], depending on the details of the specific dynamics, can transform the LF Fock vacuum into a more complex object. The

L'ubomír Martinovič
BLTP JINR Dubna and Institute of Physics, Slovak Academy of Sciences Dúbravská cesta 9, 845 11 Bratislava, Slovakia, e-mail: fyziluma@savba.sk

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latter will however be much simpler than its SL counterpart, which in principle has to be obtained by (unrealistically complicated) dynamical calculations.

Availability of the consistent Fock expansion based on the LF vacuum, with the amplitudes having direct probabilistic interpretaion makes the LF approach attractive from the point of view of phenomenological applications. On the other hand, proliferation of the non-dynamical field variables complicates the theory by the need to invert operator constraint equations. There still exist some concerns pertaining to the validity of the LF theory. The typical question is how the LF scheme can cope with the isue of vacuum condensates and the symmetry breaking with underlying vacuum degeneracies, given its greatly simplified, structureless, ground state.

What is then the relation between the SL and LF theory? Could it be that the LF version conceptually as well as technically simplifies the structure of QFT while still maintaining potential for reliable predictions? The area of 2D solvable relativistic models represents a very suitable environment to study these questions [4].

Surprisingly however, the 2D massless LF fields, being the essential elements for exact operator solutions of the models, have not been understood and correctly quantized until nowadays. Not even the simplest (and prototypic) gauge theory, the massless Schwinger model, has been solved in the LF version of the theory [5].

Recently, a simple and natural way of quantizing the two-dimensional massless LF fields has been suggested [6]. In our contribution, we shall first give a brief exposition of this quantization scheme. Its validity will be demonstrated by by the LF bosonization of the massless fermion field. In the second part, the closeness of the massless LF quantum fields to conformal field theory (CFT) will be demonstrated by calculating several correlation functions of elementary and composite operators. Going over to the euclidean time, one immediately reproduces the CFT results. Virasoro algebra is also obtained directly in the LF operator formalism.

Throughout this paper, we will use the following LF notation: $x^\mu = (x^+, x^-) = (x^0 + x^1, x^0 - x^1)$. The momentum is designed as k^μ (or p^μ), $k^\mu = (k^+, k^-)$,

$$\partial_\pm = \frac{\partial}{\partial x^\pm}, \;\; \hat{k}\cdot x = \frac{1}{2}k^+ x^- + \frac{1}{2}\hat{k}^- x^+, \;\; k^2 = \mu^2 \Rightarrow \hat{k}^- = \frac{\mu^2}{k^+}. \tag{1}$$

$\hat{k}^-$ is the on-shell LF energy. in the LF form. Both k^+, k^- can be taken positive.

2 Quantization of free massless light-front fields in 2D

Our quantization of the massless **LF scalar field** starts from the massive field. Its Lagrangian and the field equation takes in terms of the LF variables the form

$$\mathscr{L} = 2\partial_+\phi\partial_-\phi - \frac{1}{2}\mu^2\phi^2, \quad \left(4\partial_+\partial_- + \mu^2\right)\phi(x) = 0. \tag{2}$$

The solution of the field equation (2) is expressed in terms of Fock operators as

$$\phi(x) = \int_0^{+\infty} \frac{dk^+}{\sqrt{4\pi k^+}} \left[a(k^+) e^{-\frac{i}{2}k^+x^- - \frac{i}{2}\frac{\mu^2}{k^+}x^+} + a^\dagger(k^+) e^{\frac{i}{2}k^+x^- + \frac{i}{2}\frac{\mu^2}{k^+}x^+}\right], \tag{3}$$

$\left[a(k^+), a^\dagger(l^+)\right] = \delta(k^+ - l^+)$, $a(k^+)|0\rangle = 0$. The LF Hamiltonian and momentum operator is given in terms of densities $T^{++} = 4 : \partial_-\phi\partial_-\phi :$, $T^{+-} = \mu^2 : \phi^2 :$,

$$P^\nu = \frac{1}{2}\int_{-\infty}^{+\infty} dx^- T^{+\nu}(x) = \int_0^{+\infty} dk^+ \hat{k}^\nu a^\dagger(k^+) a(k^+),\ \hat{k}^\nu = (\frac{\mu^2}{k^+}, k^+). \tag{4}$$

From (3) we calculate the conjugate momentum $\pi(x) = 2\partial_-\phi(x)$ and the time derivative $\theta(x) = 2\partial_+\phi(x)$. In the following, we shall need the correlation functions

$$D_0^{(+)}(z) = \langle 0|\phi(x)\phi(y)|0\rangle,\ D_1^{(+)}(z) = \langle 0|\phi(x)\pi(y)|0\rangle,\ D_2^{(+)}(z) = \langle 0|\phi(x)\theta(y)|0\rangle, \tag{5}$$

$$D_i^{(+)}(z) = i\int_0^{\infty} \frac{dk^+}{4\pi} f_i(k^+) e^{-\frac{i}{2}k^+(z^- - i\varepsilon^-) - \frac{i}{2}\frac{\mu^2}{k^+}(z^+ - i\varepsilon^+)},\ \ z = x - y. \tag{6}$$

Here $f_0(k^+) = -\frac{i}{k^+}$, $f_1(k^+) = 1$, $f_2(k^+) = \frac{\mu^2}{k^{+2}}$. The small imaginary parts in the exponents are necessary for the existence of the integrals, which are evaluated in terms of the (modified) Bessel functions $J_\nu(z), N_\nu(z), K_\nu(z)$, $\nu = 0, 1$:

$$D_1^{(+)}(z) = -\theta\left(z^2\right)\frac{\mu}{4}\sqrt{\frac{z^+}{z^-}}\,i\left[J_1\left(\mu\sqrt{z^2}\right) - i\,\mathrm{sgn}(z^+)N_1\left(\mu\sqrt{z^2}\right)\right] + \tag{7}$$
$$-\ \theta\left(-z^2\right)\mathrm{sgn}(z^+)\frac{\mu}{4\pi}\sqrt{-\frac{z^+}{z^-}}K_1\left(\mu\sqrt{-z^2}\right),\ \ D_2^{(+)} = D_1^{(+)}(x^+ \leftrightarrow x^-).$$

Now, one observes that both $D_1^{(+)}$ and $D_2^{(+)}$ have a non-vanishing massless limit,

$$D_1^{(+)}(z;\mu^2 = 0) = \frac{1}{2\pi}\frac{1}{(z^- - i\varepsilon^-)},\ D_2^{(+)}(z;\mu^2 = 0) = \frac{1}{2\pi}\frac{1}{(z^+ - i\varepsilon^+)}. \tag{8}$$

Technically, this is due to the behaviour of the Bessel function $K_1(z) \sim \frac{1}{z}$ for the small value of z. These results suggest that there must exist massless analogs of the fields $\phi(x), \pi(x), \theta(x)$ reproducing (8). Indeed, from the LF massless Klein-Gordon equation $\partial_+\partial_-\tilde{\phi}(x) = 0$, one expects a general solution of the form

$$\tilde{\phi}(x) = \tilde{\phi}(x^+) + \tilde{\phi}(x^-). \tag{9}$$

Since the integration measure of the LF field is mass-independent [7], the massless limit ($\mu = 0$ in the plane-wave factors) of the massive solution (3) gives just $\tilde{\phi}(x^-)$. The piece $\tilde{\phi}(x^+)$ can be recovered from (3) by the change of variables (done more correctly at the classical level) $k^+ = \frac{\mu^2}{k^-}$. x^+ and x^- interchange their places in (3), and the Fock operators in terms of the new variable should satisfy [6]

$$\Big[\frac{\mu}{k^-}a\big(\frac{\mu^2}{k^-}\big),\frac{\mu}{l^-}a^\dagger\big(\frac{\mu^2}{l^-}\big)\Big]=\frac{\mu^2}{k^-l^-}\delta\big(\frac{\mu^2}{k^-}-\frac{\mu^2}{l^-}\big)=\delta(k^- - l^-). \tag{10}$$

The rhs of (10) survives the massless limit, hence $\lim_{\mu\to 0}\frac{\mu}{k^-}a\big(\frac{\mu^2}{k^-}\big)\equiv\tilde{a}(k^-)\neq 0$, with the commutators $\big[\tilde{a}(k^-),\tilde{a}^\dagger(l^-)\big]=\delta(k^- - l^-)$, $\big[\tilde{a}(k^+),\tilde{a}^\dagger(l^-)\big]=0$. After the change of variables, the massless limit in (3) yields

$$\tilde{\phi}(x^+)=\int_0^{+\infty}\frac{dk^-}{\sqrt{4\pi k^-}}\big[\tilde{a}(k^-)e^{-\frac{i}{2}k^-x^+}+\tilde{a}^\dagger(k^-)e^{\frac{i}{2}k^-x^+}\big], \tag{11}$$

and similarly for $\theta(x^+)$ and $\pi(x^-)$. The basic field commutators are consequently

$$\big[\tilde{\phi}(x^-),\tilde{\phi}(y^-)\big]=-\frac{i}{4}\varepsilon(x^- - y^-),\ \ \big[\tilde{\phi}(x^+),\tilde{\phi}(y^+)\big]=-\frac{i}{4}\varepsilon(x^+ - y^+). \tag{12}$$

The variables k^+ and k^- actually coincide, in complete analogy with the SL case $k^0=|k^1|$. Also, one verifies that the two-point functions calculated from the massless fields coincide with the massless limits (8) of the massive functions. Using similar reasoning and the above Fock commutators, the operators

$$P^+=\int_0^{+\infty}dk^+k^+a^\dagger(k^+)a(k^+),\quad P^-=\int_0^{+\infty}dk^+k^-a^\dagger(k^-)a(k^-) \tag{13}$$

are shown to generate the correct Heisenberg equations $2i\partial_\pm\phi(x^\pm)=-[P^\mp,\phi(x^\pm)]$.

The same procedure can be applied to the **light front fermion field**. The massive (two-dimensional Dirac) field equation $i\gg^\mu\ \partial_\mu\psi(x)=m\psi(x)$ decomposes as

$$2i\partial_+\psi_2(x)=m\psi_1(x),\ 2i\partial_-\psi_1(x)=m\psi_2(x) \tag{14}$$

$$\Rightarrow\ \psi_2(x)=\tilde{\psi}_2(x^-),\ \psi_1(x)=\tilde{\psi}_1(x^+),\ \text{ if } m=0. \tag{15}$$

For the correct quantization, we again start from the two components of the massive field in the momentum representation that solve the field equations (14):

$$\psi_2(x)=\int_0^{+\infty}\frac{dp^+}{4\pi}\big[b(p^+)e^{-\frac{i}{2}p^+x^- -\frac{i}{2}\frac{m^2}{p^+}x^+}+d^\dagger(p^+)e^{\frac{i}{2}p^+x^- +\frac{i}{2}\frac{m^2}{p^+}x^+}\big], \tag{16}$$

$$\psi_1(x)=\int_0^{+\infty}\frac{dp^+}{4\pi}\frac{m}{p^+}\Big[b(p^+)e^{-\frac{i}{2}p^+x^- -\frac{i}{2}\frac{m^2}{p^+}x^+}-d^\dagger(p^+)e^{\frac{i}{2}p^+x^- +\frac{i}{2}\frac{m^2}{p^+}x^+}\Big], \tag{17}$$

where $\{b(p^+),b^\dagger(q^+)\}=\{d(p^+),d^\dagger(q^+)\}=\delta(p^+ - q^+)$, and study their massless limit. For ψ_2, this again is straightforward. The limits of the fermion two-point functions $S_{11}(z), S_{22}(z)$ coincide up to the factor $(-i)$ with that of $D_1^{(+)}$ and $D_2^{(+)}$. Hence we change the variables for $\psi_1(x)$ and repeat all the steps from the scalar-field case. This results in the massless field expansions and their Fock algebra:

$$\tilde{\psi}_2(x^-) = \int_0^{+\infty} \frac{dp^+}{\sqrt{4\pi}} \left[\tilde{b}(p^+)e^{-\frac{i}{2}p^+x^-} + \tilde{d}^\dagger(p^+)e^{\frac{i}{2}p^+x^-}\right],$$

$$\tilde{\psi}_1(x^+) = \int_0^{+\infty} \frac{dp^-}{\sqrt{4\pi}} \left[\tilde{b}(p^-)e^{-\frac{i}{2}p^-x^+} - \tilde{d}^\dagger(p^-)e^{\frac{i}{2}p^-x^+}\right], \quad (18)$$

$$\{\tilde{b}(p^+), \tilde{d}^\dagger(q^+)\} = \delta(p^+ - q^+),\ \{\tilde{b}(p^-), \tilde{b}^\dagger(q^-)\} = \delta(p^- - q^-). \quad (19)$$

The $\tilde{d}$-operators satisfy the same anticommutation relations. As a consequence,

$$\left\{\tilde{\psi}_1(x^+), \tilde{\psi}_1{}^\dagger(y^+)\right\} = \delta(x^+ - y^+), \quad \left\{\tilde{\psi}_2(x^-), \tilde{\psi}_2{}^\dagger(y^-)\right\} = \delta(x^- - y^-). \quad (20)$$

The two kinds of modes decouple: $\{\tilde{b}(p^-), \tilde{b}^\dagger(q^+)\} = \{\tilde{b} \to \tilde{d}\} = 0$. The two-point function of the massless $\tilde{\psi}_1(x^+)$ coincides with the massless limit of the massive 2-point function. From the expansions (18), one constructs the bilinear operators (the current $j^\mu = (: \tilde{\psi}_1{}^\dagger \tilde{\psi}_1 :, : \tilde{\psi}_2{}^\dagger \tilde{\psi}_2 :)$ and the scalar densities $\tilde{\psi}_2{}^\dagger \tilde{\psi}_1 \pm \tilde{\psi}_1{}^\dagger \tilde{\psi}_2$).

Thus, the quantum theory of the massless LF fermion field has been established. The necessary information is contained in the original massive solutions. Since solvable models are based on free Heisenberg fields, the above derivation opens the road to the genuine LF solution of the class of models with massless fermions [8].

Consistency of the scheme is further confirmed by **LF bosonization**. Bosonization is a remarkable property of the 2D field theory: fermion fields can be represented in terms of boson variables [9, 10]. Our derivation of its LF version is based on the natural decomposition of the massless $\phi(x)$ and $\psi(x)$ fields (9),(15).

Consider first $\tilde{\psi}_2(x^-)$. Assume that it can be represented as

$$\varphi_2(x^-) = C : e^{i\alpha\phi(x^-)} : \ = Ce^{i\alpha\phi^{(-)}(x^-)}e^{i\alpha\phi^{(+)}(x^-)}. \quad (21)$$

The constants C and α can be adjusted in such a way that two φ_2 with different arguments anticommute and $\varphi_2(x^-)$, $\varphi_2^\dagger(y^-)$ satisfy the anticommutation relation (20). The first condition fixes α to the value $\hat{\alpha} = 2\sqrt{\pi}$. The second determines the constant C as $\hat{C} = \left(\frac{\lambda e^{\gg_E}}{4\pi}\right)^{1/2}$ (λ is the infrared cutoff associated with the massless $D_0^{(+)}$ function [6] and $\gg_E$ is the Euler's constant). It follows that the operators $\hat{\varphi}(x^-)$ and the analogously obtained $\hat{\varphi}(x^+)$ represent the bosonized form of the fields $\tilde{\psi}_2(x^-)$ and $\tilde{\psi}_1(x^+)$. Forming their appropriate point-split products, the bosonized vector current is found to be $\hat{j}^+(x^-) = 2\pi^{-1/2}\partial_-\phi(x^-)$, $\hat{j}^-(x^+) = 2\pi^{-1/2}\partial_+\phi(x^+)$. It correctly reproduces the Schwinger term in the current-current commutators, $\left[\hat{j}^\mp(x^\pm), \hat{j}^\mp(y^\pm)\right] = i\pi^{-1}\partial_x\delta(x^\pm - y^\pm)$. Similarly, for the scalar densities, one gets

$$\overline{\psi}(x)\psi(x) = \frac{\lambda e^{\gg_E}}{4\pi}\cos\left(2\sqrt{\pi}\phi(x)\right),\ \overline{\psi}(x) \gg^5 \psi(x) = i\frac{\lambda e^{\gg_E}}{4\pi}\sin\left(2\sqrt{\pi}\phi(x)\right). \quad (22)$$

Thus the LF version of bosonization yields the results known from the SL theory.

3 Conformal properties of the 2D massless LF fields

The massless 2D fields exhibit conformal symmetry, whose (anti)holomorphic formulation was developed in [11]. Here we shall show that after switching to the euclidean time, our formalism generates results in agreement with CFT.

The Hamiltonian density $T^{+-}(x)$ of the free massless scalar field vanishes, as required by conformal symmetry (the massless limit (13) of the massive $P^- \neq 0$, however). The other components of the energy-momentum tensor are nonvanishing:

$$T^{++}(x^-) =: \pi(x^-)\pi(x^-) :, \; T^{--}(x^+) =: \theta(x^+)\theta(x^+) : . \tag{23}$$

Note that the LF Hamiltonian (13) can also be obtained as the x^+-integral of the density $T^{--}(x^+)$, analogously to P^+ which is the x^--integral of $T^{++}(x^-)$.

We compute a few additional correlation functions ($z^\pm = x^\pm - y^\pm$),

$$\langle 0|\theta(x^+)\theta(y^+)|0\rangle = \frac{\pi^{-1}}{(z^+ - i\delta^+)^2}, \langle 0|\pi(x^-)\pi(y^-)|0\rangle = \frac{\pi^{-1}}{(z^- - i\delta^-)^2}, \tag{24}$$

as well as those between components of the energy-momentum tensor,

$$\langle 0|T^{\pm\pm}(x^\mp)T^{\pm\pm}(y^\mp)|0\rangle = \frac{2}{\pi^2}\frac{1}{(x^\mp - y^\mp - i\delta^\mp)^4}. \tag{25}$$

In the holomorphic form of 2D CFT [11,12], the Laurent expansion in the variables

$$z = e^{\frac{2\pi}{L}\zeta}, \;\; \bar{z} = e^{\frac{2\pi}{L}\bar{\zeta}}, \;\; \text{where} \;\; \zeta = \tau - ix, \;\; \bar{\zeta} = \tau + ix, \tag{26}$$

is commonly used. It is based on radial quantization with the euclidean time τ, $t \to -i\tau$. We need to reformulate our results for $\phi(x)$ in the form of infinite series to conform with the discrete picture of [11]. Thus, we consider the massive field in a finite box of length $2L$ in x^- or $2T$ in x^+ with periodic boundary conditions $\phi(x^+, x^- = -L) = \phi(x^+, x^- = L)$, $\phi(x^+ = -T, x^-) = \phi(x^+ = T, x^-)$. Performing the change of variables and the massless limit as before, we arrive at

$$\phi(x^-) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{2Lk_n^+}}\left[a_n e^{-\frac{i}{2}k_n^+ x^-} + H.c.\right] = \frac{1}{\sqrt{4\pi}}\sum_{n=\pm 1,\dots}\frac{1}{\sqrt{|n|}} a_n e^{-i\frac{\pi}{L}nx^-}, \tag{27}$$

$$\phi(x^+) = \phi_0 + \sum_{n=1}^{\infty} \frac{1}{\sqrt{2Lk_n^-}}\left[\bar{a}_n e^{-\frac{i}{2}k_n^- x^+} + H.c.\right] = \frac{1}{\sqrt{4\pi}}\sum_{n=\pm 1,\dots}\frac{1}{\sqrt{|n|}} \bar{a}_n e^{-i\frac{\pi}{L}nx^+} \tag{28}$$

with $\left[a_m, a_n^\dagger\right] = \left[\bar{a}_m, \bar{a}_n^\dagger\right] = \delta_{m,n}$, $\left[\bar{a}_m, a_n^\dagger\right] = 0$ and $a_{-n} \equiv a_n^\dagger$, $\bar{a}_{-n} \equiv \bar{a}_n^\dagger$.

Since $\mu = 0$, ϕ_0 can be non-zero. It is however just a constant whose conjugate momentums vanishes. The 2-point functions $D_0^{(+)}$ are evaluated for $L >> 1$ as

$$D_0^{(+)}(z^\pm) = \langle 0|\phi(x^\pm)\phi(y^\pm)|0\rangle = \frac{1}{4\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-i\frac{\pi}{L}n(z^- - i\varepsilon)} \approx \frac{1}{4\pi}\ln\left[\frac{i\pi}{L}(z^- - i\varepsilon)\right]. \tag{29}$$

L plays the role of the infared regularization parameter. It also introduces the necessary dimension to (29). L drops out of all the other correlation functions (due to the derivatives present). The results match the continuum results (24–25).

The components of the energy-momentum tensor in the discrete form read

$$T^{++}(x^-) = K\sum_{m,n}\varepsilon(m)\varepsilon(n)\sqrt{|m||n|} : a_m a_n : e^{-i\frac{\pi}{L}(n+m)x^-}, \tag{30}$$
$$T^{--}(x^+) = K\sum_{m,n}\varepsilon(m)\varepsilon(n)\sqrt{|m||n|} : \bar{a}_m \bar{a}_n : e^{-i\frac{\pi}{L}(n+m)x^+},\ K = -\frac{\pi}{L^2}.$$

They can be transformed to a "Virasoro form" by simply taking a Fourier transform. Indeed, assume that $T^{++}(x^-)$ can be represented as

$$T^{++}(x^-) = \frac{1}{4L^2}\sum_{l=0,\pm1,\ldots} L_l e^{-i\frac{\pi}{L}lx^-}, L_l = 2L\int_{-L}^{+L} dx^- e^{i\frac{\pi}{L}lx^-}T^{++}(x^-). \tag{31}$$

Inserting $T^{++}(x^-)$ in the Fock form (30) into (31) gives ($L_0 = 4LP^+$),

$$L_n = -4\pi\sum_{k=\pm1,\ldots}\varepsilon(k)\varepsilon(n-k)\sqrt{|k||n-k|}a_k a_{n-k}. \tag{32}$$

A calculation based on the commutators below Eq.(28) yields the LF version of the Virasoro algebra, including the c-number term, not present at the classical level:

$$\left[L_n, L_m\right] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0},\ c = 1, \tag{33}$$

where c is the "central charge". Taking $T^{--}(x^+)$ in (31) instead of T^{++} generates the algebra (33) with $L_n \to \bar{L}_n$. It follows from $\left[a_n, \bar{a}_m\right] = 0$ that $\left[L_n, \bar{L}_m\right] = 0$.

To give a few details of these calculations, we switch back to the "$a, a^\dagger$" picture:

$$L_n = -\sum_{k=1}^{n-1}\sqrt{k(n-k)}a_k a_{n-k} + 2\sum_{k=n+1}^{\infty}\sqrt{k(k-n)}a^\dagger_{k-n}a_k,\ L_n^\dagger = L_{-|n|}. \tag{34}$$

The "anomaly" comes from the commutator between the first terms:

$$\Big[\sum_{l=1}^{m-1}\sqrt{l(m-l)}a_l a_{m-l}, \sum_{k=1}^{n-1}\sqrt{k(n-k)}a_k^\dagger a_{n-k}^\dagger\Big] = \sum_{l=1}^{m-1}\sqrt{l(m-l)}\sum_{k=1}^{n-1}\sqrt{k(n-k)}\times$$

$$\times\big\{\delta_{m-l,k}\delta_{l,n-k}+\delta_{l,k}\delta_{m-l,n-k}\big\} = 2\delta_{m,n}\sum_{l=1}^{m-1}l(m-l) = \frac{1}{3}m(m^2-1)\delta_{m,n}. \quad (35)$$

This agrees with the CFT result after taking into account the different normalization.

All the LF results can be easily transformed into the conformal ((anti)holomorphic) form by switching to the euclidean time and defining the variables ζ and $\bar{\zeta}$ (26).

With the conventional CFT normalization (factor 2π in the definition of the energy-momentum tensor instead of 4 in the LF case), we get (cf. Eq.(24)):

$$\langle 0|\pi(\zeta)\pi(\zeta')|0\rangle = -\frac{1}{(\zeta-\zeta')^2},\ \langle 0|T(\zeta)T(\zeta')|0\rangle = \frac{c}{2}\frac{1}{(\zeta-\zeta')^4},\ c=1. \quad (36)$$

Our field expansions (28,27) read $(\phi(\bar{\zeta}) = \phi(\zeta)$ with $(\zeta, z, a_n) \to (\bar{\zeta}, \bar{z}, \bar{a}_n))$

$$\phi(\zeta) = \frac{1}{\sqrt{4\pi}}\sum_{n=\pm1,\pm2,\dots}\frac{1}{\sqrt{|n|}}a_n z^n,\ \ [a_m, a_n] = \delta_{m+n,0}. \quad (37)$$

It is analogous to the transition [12] to the conformal field in the conventional treatment. A completely parallel LF analysis can be given for the fermion field.

4 Conclusions

We have formulated the quantum theory of two-dimensional massless light-front fields as a unique limit of the corresponding massive fields. Its consistency is proved by the equality of the two-point functions calculated from the massless fields to the massless limit of the massive two-point functions. Our quantization scheme leads to the LF form of bosonization and to the genuine LF operator solutions of a few exactly solvable models (like the Thirring and Thirring-Wess models). The developed LF operator formalism also reproduces known results of conformal field theory.

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