Multi-Decision Players in R&D Investment Games

Mouna Ben Brahim

Abstract We focus on research and development investment decision in market with multi players and focus the influence of free riding on investment behavior. We consider a dynamic triopoly with two innovating firms investing in R&D, however, the third is a free rider. Innovating firms optimize both investment and outputs levels, while, free rider only determines its optimal output. Our purpose is to establish the investment level in both cooperative and non cooperative cases then compare. Our main result is that, despite of the presence of a free rider, cooperation always increases investment level. Moreover, free rider' presence is not decreasing investment in dynamic settings as in earlier static results.

Keywords Dynamic games • Multi-decision makers • R&D investment

1 Introduction

R&D investment decision is one of crucial topics that industrial literature extensively dealt with last decades. Such markets are characterized by innovation externalities and focused on spillovers as a strategic variable in decision making. A main purpose is to determine the optimal market structure that improves R&D investment and total knowledge level in the industry. Static models, since the very influential paper of D' Aspremont and Jacquemin (1988), established that for sufficiently high externalities or spillovers, firms' cooperation implies higher R&D investment than firms' competition. Some authors developed dynamic versions and showed that previous results are still hold. Nevertheless, they considered that all firms are investing in R&D. We develop a heterogenous market where we assume the existence of a free rider. We consider both cooperative and non cooperative cases for innovating firms and check, if in such conditions, the previous results hold. The

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reminder of this chapter is as follow: Section 1 reviews main dynamic R&D models, Sect. 2 describes our cooperative and non cooperative cases, Sect. 3 establishes some relevant comparisons and Sect. 4 concludes.

2 Dynamic Strategic R&D Models: A Review

We review, in this section, main dynamic R&D games. Our review of literature allows to classify major dynamic games into three issues: Some models dealt with the relationship between R&D investment and market structure (number of firms). Others compared R&D cooperation and R&D competition efficiency, and finally, some others considered endogenous spillovers. We also have noted that most of works in this area were form Cellini and Lambertini.

For our literature review, we start by defining notations and assumptions we will use.

2.1 *Notation and Assumptions*

- N is the firms number in the market conducting R&D.
- All firms compete on an infinite time horizon.
- The inverse demand function is given by:

$$P_i(t) = A - Bq_i(t) - D(t)\sum_{j\neq i} q_j(t) \tag{1}$$

  where $A, B > 0$.
- $D(t)$ is the substitutability degree.
- k_i is the R&D investment level of firm i.
- q_i is the output level of firm i.
- $F(k_i(t))$ is the R&D cost function which can be linear or quadratic.
- β is the spillovers rate.
- c is the unit marginal cost.
- $C_i(t)$ represent the production cost.
- Production costs are given by:

$$C_i(t) = cq_i(t) \tag{2}$$

- The instantaneous cost function is given by:

$$\frac{dc_i(t)}{dt} = c_i(t)[-k_i(t) - \beta k_j(t) + \delta] \tag{3}$$

  where δ is the discount rate.

– The instantaneous profit is given by:

$$\pi_i(t) = P_i(t)q_i(t) - C_i(t)q_i(t) - F(k_i(t)) \tag{4}$$

– Each firm tries to maximize its discounted profit and to determine both of their optimal R&D expenditures and output levels given its costate equation such that

$$\pi_i = \int_0^{\infty} \{q_i(t)[A - c - Bq_i(t) - D(t)\sum_{j \neq i} q_j(t)] - k_i(t)\}dt \tag{5}$$

s.t

$$\frac{dc_i(t)}{dt} = c_i(t)[-k_i(t) - \beta k_j(t) + \delta] \tag{6}$$

2.2 Dynamic R&D Models

A direct extension of D&J model is Lambertini and Cellini model (Cellini and Lambertini 2005). They studied the relationship between R&D investment intensity and market structure. They developed a Cournot game and established that R&D increases with the number of firms. This Arrowian result contrasts with static well-established result.

A similar work of the authors Cellini and Lambertini (2011) dealing with the same model but based on Bertrand framework was developed in (2005). The authors established similar previous results but also showed that increasing product substitutability decreases R&D efforts.

In another paper, the authors Cellini and Lambertini (2004) considered that firms invest in R&D in order to increase product differentiation. They developed both open-loop and closed-loop equilibriums and studied the relationship between the number of firms and the optimal degree of differentiation comparing private and social optima. Closed-loop equilibrium leads to a Schumpeterian conclusion while Open-loop equilibrium leads to Arrowian one. Individual, industry profits and welfare are higher under open-loop than closed-loop equilibrium. For the steady state, the differentiation degree is closer to social optimum in open-loop than in closed loop equilibriums. Indeed, an increase in product differentiation benefits to both firms and consumers. Firms'profits, consumer surplus and welfare are positively related by contrast to earlier results. However, overall welfare analysis is ambiguous.

Cellini and Lambertini (2008) focused on optimal managerial choices and developed both cases of product and process innovation in managerial firms. Process innovation aimed to reduce marginal cost of production.

The authors established that the higher is the managerial incentive, the lower is the steady cost. So, the optimal R&D effort increases with the degree of delegation.

Managerial incentives leads to underinvest in product differentiation and overinvest in process innovation. They also (Lambetini and Palestini 2014) developed a similar dynamic game model where they change the linear demand function by a hyperbolic demand function and allows capacity accumulation. The profitability of mergers in both static and dynamic settings with this form of demand function encourages mergers to occur more than with linear one.

Breton et al. (2004) developed a differential R&D cost reducing game where they compared Bertrand and Cournot equilibria efficiency in a dynamic framework. They considered a two stage game where firms either to maximize their outputs or prices and then determine their optimal R&D expenditures levels. Numerical simulations established that Cournot equilibrium is more efficient even if firms are not symmetric.

The second category of works reviewed a crucial topic that compares between R&D cooperation and R&D competition and studied their social efficiency. In their model, Cellini and Lambertini (2002) studied the relationship between R&D investment and substitutability degree. As our work is close to Cellini and Lambertini (2002), we present in the following both cooperative and non cooperative equilibria.

The Model

– **Competition case:**

$$\pi_i = \int_0^{\infty} e^{-\rho t}\{q_i(t)[A - c - Bq_i(t) - D(t)\sum_{j\neq i} q_j(t)] - k_i(t)\}dt \tag{7}$$

The Hamiltonian function is given by:

$$H(t) = e^{-\rho t}(A - c)q_i(t) - B(q_i(t))^2 - D(t)q_i(t)\sum_{j\neq i} q_j(t) - k_i(t)$$

$$+\lambda - \frac{k_i(t) + \sum_{j\neq i} k_j(t)}{1 + [k_i(t) + \sum_{j\neq i} k_j(t)]}D(t)] \tag{8}$$

The symmetric Nash equilibrium is given for:

$$q(t) = \frac{A - c}{2B + (n-1)D(t)} \tag{9}$$

The degree of substitutability is:

$$D^* = \frac{(A-c)^2 - 4B\rho - (A-c)\sqrt{(A-c)^2 - 8B\rho}}{2(n-1)\rho} \tag{10}$$

- **Cooperation case:**

  In this case, firms maximize industry profits w.r.t aggregate R&D investment:

$$\pi_i = \int_0^{\infty} e^{-\rho t}\{q_i(t)[A - c - Bq_i(t) - D(t)\sum_{j\neq i} q_j(t)] - k(t)\}dt \tag{11}$$

s.t

$$\frac{dD(t)}{dt} = \frac{-nk(t)}{1 + nk(t)}D(t) \tag{12}$$

with $k(t) = \frac{K(t)}{n}$.

The Hamiltonian function for a cartel member is given by:

$$H(t) = e^{-\rho t}\{(A - c)q_i(t) - B(q_i(t))^2 - D(t)q_i(t)\sum_{j\neq i} q_j(t) - k(t)$$

$$+ \lambda(t)[\frac{-nk(t)}{1 + nk(t)}D(t)]\} \tag{13}$$

In this second case, the substitutability degree equilibrium is given by:

$$D^* = \frac{n(A - c) - 4B\rho - (A - c)\sqrt{n^2(A - c)^2 - 8B\rho n}}{2(n - 1)\rho} \tag{14}$$

Firms invest to increase product differentiation. The higher the substitution degree is, the larger is the number of firms. Cooperation implies higher R&D investment than competition and leads to a decrease of the substitutability degree.

In another more recent paper, Cellini and Lambertini (2009) developed a dynamic duopoly where R&D is cost reducing and considered both competition and cooperation cases. They also proceed to the comparison of private and social incentives. They established that both of them coincide and that divergences does not emerge in a dynamic framework.

The Model

- R&D functions differ in competition and in cooperation. In the competitive case, they are given by:

$$\Gamma(k_i(t)) = b[k_i(t)]^2 \tag{15}$$

In the cooperative case, firms undertake R&D in a single lab:

$$\Gamma(k(t)) = b[k(t)]^2 \tag{16}$$

– **The competitive case:**
Each firm maximizes its payoff to determine output level and R&D effort

$$Max_{q_i(t),k_i(t)}\pi_i = \int_0^{\infty} \pi_i(t)e^{-\rho t}dt \tag{17}$$

where

$$\pi_i(t) = [A - q_i(t) - q_j(t) - c_i(t)]q_i(t) - b[k_i(t)]^2 \tag{18}$$

s.t

$$\frac{c_{i(t)}}{c_i(t)} = -k_i(t) - \beta k_j(t) + \delta \tag{19}$$

$$\frac{c_{j(t)}}{c_j(t)} = -k_j(t) - \beta k_i(t) + \delta \tag{20}$$

The Hamiltonian function is given by:

$$H_i(q,k,c) = e^{-\rho t}[A - q_i(t) - q_j(t) - c_i(t)]q_i(t) - b[k_i(t)]^2 -$$

$$\lambda_{ii}(t)c_i(t)[k_i(t) + \beta k_j(t) - \delta] - \lambda_{ij}(t)c_j(t)[k_j(t) + \beta k_i(t) - \delta]] \tag{21}$$

The authors began by demonstrating that the open-loop Nash equilibrium is subgame perfect. They determine outputs, costs and investments steady state points.

In the case of independent ventures, the marginal cost is given for:

$$c^{IV} = \frac{A(1+\beta) - \sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{2(1+\beta)} \tag{22}$$

The investment level is given for:

$$k^{IV} = \frac{\delta}{1+\beta} \tag{23}$$

The output level is given by:

$$q^{IV} = \frac{A(1+\beta) + \sqrt{(1+\beta)^2[A^2(1+\beta) - 24b\rho\delta]}}{6(1+\beta)} \tag{24}$$

– **The cooperative case:**

Firms choose non cooperatively outputs while they maximize their joint profits to determine R&D efforts. The authors imposed a priori symmetric conditions $k_i(t) = k_j(t) = k(t)$ and $c_i(t) = c_j(t) = c(t)$. In this case, the state equation is given by:

$$c(t) = c(t)[-(1+\beta)k(t) + \delta] \tag{25}$$

The problem of a firm in this case is given by:

$$Max_{q_i(t),k(t)}\pi_i = \int_0^\infty (A - q_i(t) - q_j(t) - c(t)]q_i(t) - b[k(t)]^2)e^{-\rho t}dt \tag{26}$$

s.t

$$c(t) = c(t)[-(1+\beta)k(t) + \delta]] \tag{27}$$

The Hamiltonian in this case is given by:

$$H_i(q_i, k, c) = e^{-\rho t}[A - q_i(t) - q_j(t) - c(t)]q_i(t) - b(k(t))^2 +$$

$$\lambda(t)c(t)[-(1+\beta)k(t) + \delta]] \tag{28}$$

In this case, the steady state for the marginal cost is given by:

$$c^{IV} = \frac{A(1+\beta) - \sqrt{A^2(1+\beta)^2 - 24b\rho\delta}}{2(1+\beta)} \tag{29}$$

The steady state for the investment level is given by:

$$k^{CI} = \frac{\delta}{1+\beta} \tag{30}$$

The output level at the steady state point is given by:

$$q^{CI} = \frac{A(1+\beta) + \sqrt{A^2(1+\beta)^2 - 24b\rho\delta}}{6(1+\beta)} \tag{31}$$

Comparing both previous cases, they established that by contrast to static models, private and social incentives coincide under a dynamic framework. Costs are lower under R&D cooperation.

Cellini et al. (2015) developed a dynamic model of price and quality competition in order to study the effects of competition on quality. The authors used a Hotelling model and developed open-loop and closed loop equilibria. Closed loop equilibrium is lower than open-loop equilibrium and for steady state quality. It also increases with competition level. However, open loop equilibrium is independent of competition and is more socially efficient.

Kobayachi (2015) developed a dynamic game where R&D is accumulated over time. They established a stable open-loop Nash equilibrium for small spillovers, R&D competition implies higher investment level than R&D cooperation and for low discount value the equilibrium approaches two-stage static game equilibrium. Thus, the author considered that the model constitutes the dynamic version of D' Aspremont and Jacquemin model (D' Aspremont and Jacquemin 1988). However, when he established Markovian perfect equilibrium, R&D investment are higher than R&D competition for all spillovers levels.

Finally, some authors focused on the nature of spillovers. Confessore and Mancuso (2002) introduced absorptive capacity into effective R&D effort function and developed a dynamic non cooperative feedback game. They studied the impact of both spillovers and absorptive capacity on firm's profitability and considered that the absorptive capacity depends on the stock of accumulated investments. They established, through numerical simulation, firms'decisions on R&D expenditures depends more on absorptive capacity than on spillovers and that inter-temporal profit and social welfare are negatively related to absorptive capacity.

Hasnas et al. (2014) developed a dynamic Cournot duopoly where knowledge spillovers are determined endogenously. The model allows asymmetric solution when a firm can absorb from its rival instead of investing itself. The major result is that the firm with higher open innovation absorption has lower output and higher profits than its rival.

On the basis of this literature review, we have noted that many issues on R&D investment decision were addressed in a dynamic framework. However, even some works assumed some firms' asymmetries, none considered free riders in the market. Cournot equilibrium is more socially efficient than Bertrand equilibrium when firms are asymmetric. In addition, open-loop equilibrium is closer than closed-loop one to social optimum. We develop in the next section our open-loop Cournot-Nash dynamic game considering a free rider in the market. Our purpose is to check if R&D cooperation still dominates R&D competition in a dynamic framework.

3 Our Oligopoly with a Surfer

After reviewing R&D dynamic games and studying major results? We develop a heterogeneous triopoly with two innovating firms and a free rider. It is a more general framework that better describes some markets realities. We consider both cooperative and non cooperative cases open-loop Cournot Nash equilibrium and study different issues in presence of a free rider. First, we compare our both cases in order to check if cooperation is still leading to higher aggregate investment than competition. Second, we proceed to a benchmark with Cellini and Lambertini (2009) to highlight the influence of the surfer and finally we compare our dynamic results to static ones.

3.1 The Model

Consider two innovators and a free rider in the market. Innovators conduct cost reducing R&D. They can either compete or cooperate on R&D investment while all firms compete on the product market. The surfer can decrease its unit production cost by free-riding innovators'effort through some absorption rate. We, first, prove that even with surfer'presence, the open-loop equilibrium remains subgame perfect. Then, we establish the Nash open-loop equilibrium for both R&D competition and R&D cooperation cases and determine steady state values of R&D expenditures, marginal costs and outputs.

Notation and Assumptions

- Consider an oligopoly with two innovators and a surfer.
- R&D is cost reducing.
- q_i is firm i output level.
- k_i is innovator i R&D investment level.
- The inverse demand function is $P = A - Q$ where $Q = \sum_{i=1}^{3} q_i$ with $A > 0$.
- The instantaneous cost function for the production of the final good is given by:

$$C_i(c_i(t), q_i(t)) = c_i(t)q_i(t) \tag{32}$$

- The marginal cost for an innovator is given by:

$$\frac{dc_i(t)}{dt} = c_i(t)[-k_i - \beta k_j + \delta] \tag{33}$$

  where δ is a depreciation rate measuring the instantaneous decrease in productivity efficiency.
- β is the exogenous spillover rate between innovators such that $0 \leq \beta \leq 1$.
- The marginal cost for the surfer is given by:

$$\frac{dc_s(t)}{dt} = c_i(t)[-\psi(k_i + k_j) + \delta] \tag{34}$$

- where ψ is the surfer absorption level such that $0 \leq \psi < \beta \leq 1$.
- The instantaneous R&D cost is given by:

$$f(k_i(t)) = \frac{\gamma}{2}k_i^2 + \theta k_i + L \tag{35}$$

  with $\gamma > 0$, $\theta > 0$ and $L > 0$.
- We also consider a constant discount rate ρ.

3.1.1 R&D Competition

The two innovators decide on their R&D investment and market output levels while the surfer determines only its output. $k(t)$, $q(t)$ and $c(t)$ are the vectors of controls and states. Firms maximize their discounted profits.

The optimization problem for each of the two innovators is given by:

$$Max\pi_i = \int_0^{\infty} e^{-\rho t}[A - q_i(t) - q_j(t) - q_s(t) - c_s(t)]q_i(t) - (L + \theta k_i + \frac{\gamma}{2}k_i^2)dt \quad (36)$$

And for a surfer

$$Max\pi_s(k_i(t), q(t), c_i(t)) = \int_0^{\infty} e^{-\rho t}[A - q_i(t) - q_j(t) - q_s(t) - c_s(t)]q_s(t)dt \quad (37)$$

Given the following dynamic constraints for the innovators:

$$\frac{dc_i(t)}{dt} = c_i[-k_i - \beta k_j + \delta] \quad (38)$$

And for a surfer:

$$\frac{dc_s(t)}{dt} = c_i[-\psi(k_i + k_j) + \delta] \quad (39)$$

The corresponding Hamiltonian functions are given by:

For an innovator:

$$H_i(k_i(t), q(t), c_i(t)) = e^{-\rho t}\{[(A - Q)q_i(t) - c_i(t)q_i(t) - (L + \theta k_i(t) + \frac{\gamma}{2}k_i(t)^2]$$

$$-\lambda_{ii}c_i(t)(k_i(t) + \beta k_j(t) - \delta)-$$

$$\lambda_{ij}c_j(t)(k_j(t) + \beta k_i(t) - \delta) - \lambda_{is}c_s(t)(\psi(k_i(t) + k_j(t)) - \delta) \quad (40)$$

For a surfer:

$$H_s = e^{-\rho t}\{(A - Q)q_s(t) - c_s(t)q_s(t) - \lambda_{ss}c_s(t)(\psi(k_i(t) + k_j(t)) - \delta)-$$

$$\lambda_{si}(k_i(t) + \beta k_j(t) - \delta)c_i(t) - \lambda_{sj}(k_j(t) + \beta k_i(t) - \delta)c_j(t)\}dt \quad (41)$$

Proposition 1 *The Nash open-loop equilibrium is subgame perfect.*

Proof Our objective is to show that the Nash open-loop equilibrium is subgame perfect even when we consider a surfer in the industry. We are looking to show that $\frac{\partial k_i}{\partial c_j} = 0$, this proves that first order conditions are independent from initial conditions. To do so, we use a similar approach to that developed by Cellini and Lambertini (2009).

The first order conditions are given by for an innovator:

$$\frac{\partial H_i}{\partial q_i} = 0 \quad (42)$$

$$A - 2q_i - q_j - q_s - c_i = 0 \quad (43)$$

$$\frac{\partial H_i}{\partial k_i} = 0 \quad (44)$$

$$-\theta - \gamma k_i - \lambda_{ii} c_i - \beta \lambda_{ij} c_j - \lambda_{is} c_s \psi = 0 \quad (45)$$

The optimal investment level from the first order condition is:

$$k_i^* = \frac{\lambda_{ii} c_i + \beta \lambda_{ij} c_j + \lambda_{is} c_s \psi + \theta}{-\gamma} \quad (46)$$

The adjoint costate equations are given by:

$$-\frac{\partial H_i}{\partial c_i} - \frac{\partial H_i}{\partial k_j} \times \frac{\partial k_j^*}{\partial c_i} - \underbrace{\frac{\partial H_i}{\partial k_s} \times \frac{\partial k_s^*}{\partial c_i}}_{=0} = \frac{\partial \lambda_{ii}}{\partial t} - \rho \lambda_{ii} \quad (47)$$

$$\Leftrightarrow \frac{\partial \lambda_{ii}}{\partial t} = q_i + \lambda_{ii}(k_i + \beta k_j - \delta + \rho) - \frac{\beta}{\gamma} \lambda_{ji} (\beta \lambda_{ii} c_i + \lambda_{ij} c_j + \psi \lambda_{is} c_s) \quad (48)$$

$$\frac{\partial H_i}{\partial c_j} - \frac{\partial H_i}{\partial k_i} \times \frac{\partial k_i^*}{\partial c_j} - \underbrace{\frac{\partial H_i}{\partial k_s} \times \frac{\partial k_s}{\partial c_j}}_{=0} = \frac{\partial \lambda_{ij}}{\partial t} - \rho \lambda_{ij} \quad (49)$$

$$\frac{\lambda_{ij}}{\partial t} = \lambda_{ij}(k_j + \beta k_i - \delta + \rho) -$$

$$\frac{\beta \lambda_{ij}}{\gamma}(-\theta - \gamma k_i - \lambda_{ii} c_i - \beta \lambda_{ij} c_j - \lambda_{is} c_s \psi)$$

$$-\frac{\partial H_s}{\partial c_s} - \frac{\partial H_s}{\partial k_j} \times \frac{\partial k_j^*}{\partial c_s} - \frac{\partial H_s}{\partial k_i} \times \frac{\partial k_i^*}{\partial c_s} = \frac{\partial \lambda_{ss}}{\partial t} - \rho \lambda_{ss} \quad (50)$$

$$\frac{\partial \lambda_{ss}}{\partial t} = q_s + \lambda_{ss}[\psi(k_i + k_j) - \delta + \rho] - \frac{\psi \lambda_{is}}{\gamma}[\lambda_{ss} c_s \psi + \lambda_{si} c_i + \lambda_{sj} \beta c_j]$$

$$-\frac{\psi\lambda_{js}}{\gamma}[\lambda_{ss}c_s\psi + \beta\lambda_{si}c_i + \lambda_{sj}c_j] \tag{51}$$

$$\frac{\partial H_s}{\partial c_j} - \underbrace{\frac{\partial H_s}{\partial c_s} \times \frac{\partial k_s^*}{\partial c_j}}_{=0} - \frac{\partial H_s}{\partial k_i} \times \frac{\partial k_i}{\partial c_j} - \frac{\partial H_s}{\partial k_j} \times \frac{\partial k_j}{\partial c_i} = \frac{\partial \lambda_{sj}}{\partial t} - \rho\lambda_{sj} \tag{52}$$

$$\frac{\partial \lambda_{sj}}{\partial t} = -\lambda_{sj}(k_j + \beta k_i - \delta + \rho) \tag{53}$$

These conditions must be evaluated along with the initial conditions $c(0) = \{c_{0i}\}$ and the transversality conditions $lim_{t\to\infty}e^{-\rho t}\lambda_{ij}c_j = 0$.

We also impose symmetry $\lambda_{ij} = \lambda_{ji}$.

From the adjoint equation for the innovator, we establish that $\frac{\partial \lambda_{ij}}{\partial t} = 0$ in $\lambda_{ij}(t) = 0$.

And also for the surfer, we have $\frac{\partial \lambda_{sj}}{\partial t} = 0$ in $\lambda_{sj}(t) = 0$. given this information, the optimal investment level becomes:

$$k_i^* = -\frac{\lambda_{ii} + \theta}{\gamma} \tag{54}$$

And then we can establish that:

$$\frac{\partial k_i^*}{\partial c_j} = 0 \; j \neq i \tag{55}$$

The property whereby the first order conditions are independent of the co-state variables is state redundancy. So, we demonstrated that first order conditions are independent of states and initial conditions and that the subgame is perfect.

Proposition 2 *Provided that* $(1+\beta)^2[A^2 - 16\rho\theta] - 16\rho\gamma(1+\beta)\delta) \geq 0$*, the steady state point is given by:*

$$c^{ss} = \frac{A(1+\beta) - \sqrt{(1+\beta)^2[A^2 - 16\rho\theta] - 16\rho\gamma(1+\beta)\delta}}{2(1+\beta)} \tag{56}$$

$$k^{ss} = \frac{\delta}{1+\beta} \tag{57}$$

is the unique saddle point equilibrium.

Proof The costate equations simplify as follows:

$$-\frac{\partial H_i}{\partial c_i} = \frac{\partial \lambda_{ii}}{\partial t} - \rho\lambda_{ii} \tag{58}$$

$$\frac{\partial \lambda_{ii}}{\partial t} = q_i + \lambda_{ii}(k_i + \beta k_j - \delta + \rho) \tag{59}$$

$$-\frac{\partial H_i}{\partial c_j} = \frac{\partial \lambda_{ij}}{\partial t} - \rho\lambda_{ij} \tag{60}$$

$$\frac{\partial \lambda_{ij}}{\partial t} = \lambda_{ij}[k_i + \beta k_j + \rho - \delta] \tag{61}$$

$$-\frac{\partial H_s}{\partial c_s} = \frac{\partial \lambda_{ss}}{\partial t} + \rho\lambda_{ss} \tag{62}$$

$$\frac{\partial \lambda_{ss}}{\partial t} = q_s + \lambda_{ss}[\psi(k_i + k_j) - \delta + \rho] \tag{63}$$

$$-\frac{\partial H_s}{\partial c_j} = \frac{\partial \lambda_{sj}}{\partial t} + \rho\lambda_{sj} \tag{64}$$

$$\frac{\partial \lambda_{sj}}{\partial t} = -\lambda_{sj}(k_j + \beta k_i - \delta + \rho) \tag{65}$$

The first order conditions become:

$$\frac{\partial H_i}{\partial q_i} = 0 \tag{66}$$

$$A - 2q_i - q_j - q_s - c_i = 0 \tag{67}$$

$$\frac{\partial H_i}{\partial k_i} = 0 \tag{68}$$

$$-\theta - \gamma k_i - \lambda_{ii}c_i - \beta\lambda_{ij}c_j - \lambda_{is}c_s\psi = 0 \tag{69}$$

$$\frac{\partial H_s}{\partial q_s} = 0 \tag{70}$$

$$A - q_i - q_j - 2q_s - c_i = 0 \tag{71}$$

These first order conditions lead to the following optimal outputs and investment level:

$$q_i^* = \frac{A - Q_{-i} - c_i}{2} \tag{72}$$

$$q_s^* = \frac{A - Q_{-i} - c_s}{2} \tag{73}$$

$$k_i^* = -\frac{\lambda_{ii} c_i + \theta}{\gamma} \tag{74}$$

We differentiate w.r.t time to get dynamic expression of k_i:

$$\dot{k} = \frac{\partial k_i}{\partial t} = -\frac{1}{\gamma}(\dot{\lambda}_{ii} c_i + \dot{c}_i \lambda_{ii}) \tag{75}$$

Note that from Eq. (74), we obtain:

$$\lambda_{ii} = -\frac{\gamma k_i + \theta}{c_i} \tag{76}$$

Remind that from Eq. (59) we obtain:

$$\dot{\lambda}_{ii} = q_i + \lambda_{ii}(k_i + \beta k_j - \delta) \tag{77}$$

and

$$\dot{c}_i = c_i[-k_i - \beta k_j + \delta] \tag{78}$$

Replacing both previous expressions in $\dot{k}_i$, we have:

$$\dot{k}_i = \frac{1}{\gamma}[c_i(q_i + \lambda_{ii}(k_i + \beta k_j + \rho - \delta)] + c_i(-k_i - \beta k_j + \delta)\lambda_{ii}] \tag{79}$$

$$= \frac{1}{\gamma}[c_i q_i - c_i \rho \lambda_{ii}] \tag{80}$$

$$= \frac{1}{\gamma}[c_i q_i - \rho(k_i \lambda + \theta)] \tag{81}$$

Imposing symmetry for innovators $c_i = c_j$, $k_i = k_j$ and $q_i = q_j = q$, and solving the system of best reply functions, we obtain the following optimal output levels:

$$q^* = \frac{A - 2c + c_s}{4} \tag{82}$$

$$q_s^* = \frac{A - 2c_s + c}{4} \tag{83}$$

Then the instantaneous investment level is given by:

$$\dot{k} = \frac{1}{\gamma}[c_i(\frac{A - 2c + c_s}{4}) - \rho(k\gamma + \theta)] \tag{84}$$

Imposing the stationary condition $\dot{k} = 0$, we have:

$$k^N = \frac{1}{\gamma}[\frac{c(A - 2c + c_s)}{4\rho} - \theta] \tag{85}$$

The steady state for the marginal cost $c(t)$ is obtained by solving:

$$\frac{\partial c(t)}{\partial t} = 0 \tag{86}$$

$$\frac{\partial c_s(t)}{\partial t} = 0 \tag{87}$$

Equivalent to:

$$\frac{dc_i(t)}{dt} = c_i[-(1 + \beta)K^N + \delta] \tag{88}$$

$$\frac{dc_s(t)}{dt} = c_i[-2\psi K^N + \delta] \tag{89}$$

Given the stationarity conditions, we have:

$$-c[k^N(1 + \beta) - \delta] = 0 \tag{90}$$

$$-c[2\psi k^N - \delta] = 0 \tag{91}$$

Note that we should have $1 + \beta = 2\psi$, thus the equilibrium holds for $\psi = \beta = 1$ and in this case we obtain $c = c_s$. Replacing k^N in the first equation gives:

$$(1 + \beta)\frac{1}{\gamma}[\frac{c(A - c)}{4\rho} - \theta] - \delta = 0 \tag{92}$$

Replacing k^N by its expression, we obtain the next F.O.C:

$$(1+\beta)c(A-c) - 4\rho\theta(1+\beta) - 4\rho\delta\gamma = 0 \tag{93}$$

$$(1+\beta)c^2 - (1+\beta)Ac + 4\rho\theta(1+\beta) + 4\rho\delta\gamma = 0 \tag{94}$$

This yields $c = 0$ and

$$c = \frac{A(1+\beta) \pm \sqrt{A^2(1+\beta)^2 - 16\rho[\theta(1+\beta) + \gamma\delta](1+\beta)}}{2(1+\beta)} \tag{95}$$

Previous equation exists if $\gamma\rho\delta \leq \frac{(1+\beta)[A^2-16\rho\theta]}{16}$, and the solution should verify that $c \in [0, A]$, thus we have:

$$c = \frac{A(1+\beta) - \sqrt{A^2(1+\beta)^2 - 16\rho[\theta(1+\beta) + \gamma\delta](1+\beta)}}{2(1+\beta)} \tag{96}$$

3.1.2 R&D Cooperation

Now the two innovators coordinate their R&D expenditures and maximize their joint profit for the R&D decisions. All the three firms compete on the product market. We also establish open-loop equilibrium and determine steady state points for unit cost, outputs and R&D investments. We impose, as already assumed in the literature, the symmetry such that $k_i(t) = k_j(t) = k(t)$ and $c_i(t) = c_j(t) = c(t)$.

The state equation becomes:

$$c(t) = c_i[-(1+\beta)k(t) + \delta] \tag{97}$$

And for the surfer:

$$\frac{dc_s(t)}{dt} = c_i[-\psi(k_i + k_j) + \delta] \tag{98}$$

The optimization problem for an innovator is:

$$Max\pi_i = \int_0^{\infty} e^{-\rho t}[A - q_i(t) - q_j(t) - q_s(t) - c(t)]q_i(t) -$$

$$(L + \theta k(t) + \frac{\gamma}{2}k(t)^2]dt] \tag{99}$$

And for the surfer:

$$Max\pi_s(k_i(t), q(t), c_i(t)) = \int_0^\infty e^{-\rho t}[A - q_i(t) - q_j(t) - q_s(t) - c_s(t)]q_s(t) \tag{100}$$

The Hamiltonian for an innovator is given by:

$$H_i(k(t), q(t), c(t)) = e^{-\rho t}[(A - Q)q_i - c(t)q_i(t) -$$

$$(L + \theta k(t) + \frac{\gamma}{2}k(t)^2 L] - \lambda(t)c(t)[(1+\beta)k(t) - \delta]] \tag{101}$$

And for a surfer, it is given by:

$$H_s = e^{-\rho t}\{(A - Q)q_s - c_s q_s - \lambda_{ss} c_s(t)(\psi k(t) - \delta) -$$

$$\lambda_{si}[(1+\beta)k(t) - \delta]c(t)dt] \tag{102}$$

Given the initial conditions $c(0) = c_{i0}$ and the transversality conditions $lim_{\to\infty}\lambda(t)c(t) = 0$.

Proposition 3 *Provided that* $(1+\beta)^2A^2 - 16\rho(\theta + \gamma\delta) \geq 0$, *the steady state point given by*

$$c^{ss} = \frac{A(1+\beta) - \sqrt{(1+\beta)^2A^2 - 16\rho(\theta+\delta)}}{2(1+\beta)} \tag{103}$$

$$k^{ss} = \frac{\delta}{(1+\beta)} \tag{104}$$

is the unique saddle point.

Proof The open loop equilibrium first order conditions are:

$$\frac{\partial H(.)}{\partial q_i(t)} = A - 2q_i - q_j - q_s - c(t) \tag{105}$$

$$\frac{\partial H_s}{\partial q_s(t)} = A - q_i - q_j - 2q_s - c_s(t) \tag{106}$$

$$\frac{\partial H(.)}{\partial k(t)} = -\gamma k(t) - \theta - \lambda(t)(1+\beta)c(t) \tag{107}$$

$$-\frac{\partial H(.)}{\partial c(t)} = \frac{\partial \lambda(t)}{\partial t} - \rho\lambda \tag{108}$$

$$-\frac{\partial H_s}{\partial c_s(t)} = q_s + \lambda_{ss}(\psi k(t) - \delta) \tag{109}$$

$$-\frac{\partial H_s}{\partial c(t)} = \lambda_{si}[(1+\beta)k(t) - \delta] \tag{110}$$

From first order condition, the optimal output level is given by:

$$k(t) = \frac{(1+\beta)\lambda(t)c(t)}{-\gamma} \tag{111}$$

The instantaneous investment level of firm i is given by:

$$\dot{k} = \frac{\partial k}{\partial t} = \frac{(1+\beta)}{\gamma}[\dot{c}\lambda + \dot{\lambda}c] \tag{112}$$

Note that we have:

$$\dot{c} = \frac{\partial c}{\partial t} = c[-(1+\beta)k_i + \delta] \tag{113}$$

And,

$$\dot{\lambda} = \frac{\partial \lambda(t)}{\partial t} = q_i(t) + \lambda(t)[(1+\beta)k(t) - \delta + \rho] \tag{114}$$

Thus, the instantaneous investment level is then given by:

$$\dot{k} = \frac{(1+\beta)}{\gamma}[c(q_i + \lambda(1+\beta)k(t) - \delta + \rho] + c[-(1+\beta)k(t) + \delta]\lambda] \tag{115}$$

$$\dot{k} = \frac{(1+\beta)}{\gamma}[cq_i + c\rho\lambda] \tag{116}$$

Note that:

$$\lambda = \frac{-\gamma k_i - \theta}{(1+\beta)c_i} \tag{117}$$

Given that:

$$q_i^* = \frac{A - 2c + c_s}{4} \tag{118}$$

The optimal investment level is given by:

$$\dot{k} = \frac{(1+\beta)cq - (\gamma k + \theta)\rho]}{\gamma} \tag{119}$$

Imposing the stationary condition such that $\dot{k} = 0$, we establish that

$$k^C = \frac{1}{\gamma}[\frac{c(1+\beta)(A - 2c + c_s)}{4\rho} - \theta] \tag{120}$$

The steady state for the marginal cost $c(t)$ is obtained by solving:

$$\frac{\partial c(t)}{\partial t} = 0 \tag{121}$$

$$\frac{\partial c_s(t)}{\partial t} = 0 \tag{122}$$

Equivalent to:

$$\frac{dc_i(t)}{dt} = c_i[-(1+\beta)K^C + \delta] \tag{123}$$

$$\frac{dc_s(t)}{dt} = c_i[-2\psi K^C + \delta] \tag{124}$$

Given the stationarity conditions, we have:

$$-c[k^C(1+\beta) - \delta] = 0 \tag{125}$$

$$-c[2\psi k^C - \delta] = 0 \tag{126}$$

Note that the previous system solution implies that $1 + \beta = 2\psi$. The equilibrium holds for $\psi = \beta = 1$ and in this case we obtain $c = c_s$.

$$k = (1+\beta)^2 \frac{c(A - c) - 4\theta}{4\gamma\rho} \tag{127}$$

Replacing k^C by its expression, the first order condition becomes:

$$(1+\beta)^2 c(A - c) - 4\rho\theta - 4\rho\delta\gamma = 0 \tag{128}$$

$$-(1+\beta)^2 c^2 - A(1+\beta)^2 c - 4\rho[\theta + \gamma\delta] = 0 \tag{129}$$

and then the steady state for marginal cost is given by:

$$c^{ss} = \frac{A(1+\beta) \pm \sqrt{[(1+\beta)^2 A^2 - 16\rho(\theta + \gamma\delta)}}{2(1+\beta))} \tag{130}$$

Previous equation exists if $[(1+\beta)^2A^2 - 16\rho(\theta+\gamma\delta) \geq 0$, and the solution should verify that $c \in [0, A]$, thus we have:

$$c = \frac{A(1+\beta) - \sqrt{[(1+\beta)^2A^2 - 16\rho(\theta+\gamma\delta)}}{2(1+\beta))} \tag{131}$$

4 Comparisons

We begin first by comparing competitive and cooperative cases in presence of surfers. Second, in order to determine the impact of surfers in the market, we proceed to a benchmark with Cellini and Lambertini (2009). Finally, we compare our dynamic results to major static ones.

4.1 Competition vs Cooperation in Dynamic Framework

Comparing k^C and k^N expressions from Eqs. (86) and (121), one can easily note that cooperation leads to higher R&D investment than competition this confirms Cellini and Lambertini (2009) results even in our heterogeneous market structure. Comparing Eqs. (96) and (132), we establish that competition leads to a higher marginal cost and it converges to different steady states given the difference in investment flows between competition and cooperation. The steady state of R&D effort is the same in both cases. As output level is inversely related to the marginal cost, R&D cooperation also leads to a higher market output, a whole industry output level and then to a lower market price. Thus, R&D cooperation is the more desirable situation whatever the spillovers level is even in the presence of a surfer. Intuitively, innovators coordinate their R&D investment decisions, they improves unit production cost avoiding effort duplication and beneficiate from synergies.

Our analysis confirms earlier dynamic results and also static ones which support the idea for a sufficiently high spillovers. However, our result contradicts with Kobayachi (2015) who assumed R&D accumulation instead of marginal cost and who established that competition dominates cooperation for all spillovers values.

4.2 A Benchmark with Cellini and Lambertini Model (Cellini and Lambertini 2005)

We compare our oligopoly composed by two innovators and a surfer to Cellini and Lambertini (2009) oligopoly model with three innovating firms in the market. Note that if we assume that $\theta = 0$ and that $\gamma = 2b$, we obtain the same R&D cost function and compare with Cellini and Lambertini (2009) competitive case for n= 3.

Remind that the aggregate investment level at the steady state in our case as we have only two innovators in a three firms market is give by:

$$K = \frac{2\delta}{1+\beta} \tag{132}$$

For Cellini and Lambertini (2009) competitive oligopoly with three innovators, the steady state for aggregate investment is given by:

$$K = \frac{3\delta}{(1+2\beta)} \tag{133}$$

Aggregate R&D investment level is lower in our case than in Cellini and Lambertini (2009), this confirms Arrowian conclusion in Cellini and Lambertini (2009) that innovators number positively varies with investment level.

For the marginal cost, in our case and for $\theta = 0$ and if we posit $\gamma = 2b$, then

$$c^{ss} = \frac{A}{2} - \sqrt{\frac{A^2}{4} - \frac{8b\delta\rho}{(1+\beta)}} \tag{134}$$

For Cellini and Lambertini (2009) model, the steady state is given for:

$$c^{ss} = \frac{A}{2} - \sqrt{\frac{A^2}{4} - \frac{6b\delta\rho}{(1+2\beta)}} \tag{135}$$

The marginal cost is are higher as the number of innovating firms is decreasing in our case. This is obvious given that R&D investment is lower in our case because of the Arrowian nature of the model.

4.3 *Dynamic vs Static Results*

Our result confirms, at some extent, a major volume of static games literature as in D' Aspremont and Jacquemin (1988), Kamien et al. (1992) and many others assuming that all firms conduct R&D. Our model established that R&D cooperation leads to higher investment than R&D competition for all spillovers levels, the same result holds for sufficiently high spillovers in static settings. Ben Abdelaziz et al. (2008) showed that innovators tend to reduce their R&D investment level in the presence of surfers. However, when we compare individual R&D investment level in both cases, we note that an innovator increases its investment in a dynamic framework. So, innovating firms tend to increase their R&D effort in the presence of a free rider in a dynamic framework as innovators can smooth their investment efforts over a long time horizon.

5 Conclusion

Our purpose is to develop a heterogeneous oligopoly where we introduce a free rider in the market. Indeed, given many industrial market, free riding phenomenon could no longer be ignored. We studied the innovative behavior in such conditions. Earlier dynamic models assumed that all firms are investing in R&D and studied different issues as R&D investment and market structure, social and private incentives to invest or also product differentiation and investment. Main works in this area are developed by Cellini and Lambertini (2009) who proposed a dynamic version of D&J (D' Aspremont and Jacquemin 1988) model and who already studied most of the previous issues. However, free riding phenomenon is still ignored in such literature. We, then, propose a general dynamic game with a more complex R&D cost function and include a free rider in the market. We determined open-loop Nash equilibriums as it was showed that they are more efficient than closed loop ones in Lambertini and Cellini works. We established both cooperative and non cooperative cases and proceed to many conclusions.

Some relevant conclusions were established. First, the R&D cooperation is still leading to a higher level of aggregate investment level and this independently of spillovers level. This result confirms Cellini and Lambertini (2009) results but contrasts with Kobayachi (2015) who assumed R&D accumulation. Second, the Arrowian conclusion of the homogeneous oligopoly is also maintained in our framework, investment level increases with active firms' number. Third, the free rider presence does not lead innovators to reduce their investment level as in static model as innovators can smooth their investment over time in dynamic models. Finally, research joint venture or cartel formation should be encouraged even when there are free riders in the market. The presence of free rider is more aggressive in static models than in a dynamic ones given the possibility of smoothing expenses over time. This also shows that different frameworks lead to divergent results and strategic decisions should be taken carefully before engaging in some market policies.

Finally, our model assumed the heterogeneous nature of industrial sectors and take into account of free riding phenomenon. It brings some answers to R&D investment decision making in infinite time horizon either when they compete or cooperate. Some further researches are worthy to be studied as including absorptive capacities, uncertain research outputs or product innovation. Surfers introduction would be worth considering in future research as they exist and play an important role in the investment and innovation process.

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