Game Theory Based Spectrum Sharing

1 Introduction

As depicted in the Visual Networking Index (VNI) by Cisco [1], 5.5 billion mobile users are expected by 2021, with an average mobile connection speed of 20.4 Mbps. Compared with the 4.9 billion mobile users and 6.8 Mbps speed from 2016, the increasing number of mobile users and the threefold growth on speed motivate the exploration and expansion of other possible spectrum resources, including the unlicensed spectrum bands which are dominantly presently used by Wi-Fi networks.

Accordingly, the coexistence of Cellular Networks (CNs) and Wi-Fi networks is expected, provided that the mutual interference between CNs and Wi-Fi networks is properly under control. To address the above issue, many existing works have proposed solutions and algorithms to ensure possible coexistence of U-LTE and Wi-Fi in the unlicensed spectrum. In [2], the authors introduce the spectrum sharing problems when cellular network operators are allowed to access the unlicensed spectrum. The authors propose a hybrid method where cellular base stations can simultaneously offload traffic to Wi-Fi networks and occupy certain number of time slots on unlicensed bands. Practical strategies have been proposed to maximize the minimum average per-user throughput of each small cell. In [3], the authors introduce a network architecture where small cells can share the unlicensed spectrum with the performance guarantee of Wi-Fi systems. An almost blank subframe (ABS) scheme is employed to mitigate the co-channel interference from small cells to Wi-Fi systems, and an interference avoidance scheme is proposed based on small cell estimation of the density of nearby Wi-Fi access points. The authors in [4] evaluate and compare several existing licensed and unlicensed user coexisting mechanisms. The appropriate coexistence mechanisms, such as static muting and sensing-based adaptive, are required to achieve a balance between the performance of LTE and WLAN systems. In [5], the authors propose a cap-limited water-filling method for the U-LTE users to regulate the interference to Wi-Fi users

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in the unlicensed spectrum. In [6], the authors propose a novel proportional fair allocation scheme which guarantees fairness when both U-LTE and Wi-Fi coexist in the unlicensed spectrum. In [7], the authors propose a spectrum etiquette protocol to restrict the priority of U-LTE and balance the unfair competition between LTE and Wi-Fi in the unlicensed spectrum. In [8], the authors propose an "intelligent" power allocation strategy to optimize the utility of users with U-LTE and the social welfare simultaneously. In [9], an improved power control method is proposed for uplink transmissions, and thus both Wi-Fi and LTE are able to coexist with acceptable interference levels. Moreover, in order to guarantee the performance of Wi-Fi users, the strategies in cognitive radio networks can also be applied in the relations between U-LTE and Wi-Fi. In [10], the authors model the cognitive users' network access behavior as a two-dimensional Markov decision process and propose a modified value iteration algorithm to find the best strategy profiles for cognitive users. In [11], the authors jointly consider the spectrum sensing and access problems as an evolutionary game, where each secondary user senses and accesses the primary channel with the probabilities learned from its history. In [12], a Dynamic Chinese Restaurant Game is proposed to learn the uncertainties of networks and make optimal strategies. In [13], the authors propose a dynamic spectrum access protocol for the secondary users to deal with unknown behaviors of primary users. In [14], the authors investigate resource allocation problems for the uplink transmission of a spectrum-sharing-enabled femtocell network. A Stackelberg game with one leader and multiple followers is applied where the macrocell base station, i.e., the leader, sets prices to the femtocell users, i.e., the follower, to control its interference on the macrocell users. As the macrocell users and femtocell users share the licensed spectrum, each femtocell user determines and optimizes the transmit power on each sub-band only. In [15], the authors propose a fair and Quality-of-Service (QoS) based unlicensed spectrum splitting strategy to realize the joint operation of femtocell networks and Wi-Fi networks in the unlicensed spectrum band. In [16], an analytical model is developed for evaluating the baseline performance of the coexistence of Wi-Fi networks and LTE networks. In [17], a practical algorithm, which takes into account the real-time channel, interference and traffic conditions of licensed and unlicensed bands, is proposed for the integrated femto-WiFi and the dual-band femtocell to balance their traffic in both spectrum bands.

Moreover, the presence of multiple operators in a common unlicensed spectrum band makes the coexistence problem more challenging. Spectrum sharing among multiple operators has been studied in many works. In [18], the potential network efficiency gain from spectrum sharing between operators is investigated. In [19], the authors look into the problem of inter-operator sharing of radio resources, including capacity, spectrum and base stations sharing. From their work, the realistic sharing architecture and process are supported in the testbed network. However, in the unlicensed spectrum, how to jointly operate multiple wireless cellular networks and Wi-Fi networks remains a critical technical problem. Not only should we consider the competitions among all operators, but each operator is also required to ensure the performance of its users and Wi-Fi networks users at the same time. In [20], two general ideas are put forward to solve the problem. One is applying the orthogonal/exclusive use of the unlicensed spectrum for each operator. The other is to propose dynamic schemes for shared use of unlicensed radio resources. The use of unlicensed spectrum depends on the instantaneous/semi-static traffic load of U-LTE. However, the first solution lacks flexibility and the second solution requires perfect central control mechanisms.

Different from the above mentioned literature, we consider in this chapter the power control problem in a multi-operator spectrum-sharing scenario. Considering the distributive behaviors of the Wi-Fi Access Point (WAP) and each operator, game theory is introduced and applied in this scenario, so as to provide optimal strategies for each operator and Wi-Fi, to achieve high revenues. We model the interactions among all the operators and the WAP as a layered game. We first propose the zero-determinant power control strategy for a considered operator during the interaction with the WAP, by fixing the behaviors of all the other operators. With the predicted strategies of other operators, all operators play a non-cooperative game and determine their optimal power control strategies to achieve the Nash equilibrium results. Simulation results verify the theoretical analysis carried out in this chapter and show that a high performance can be achieved by applying the proposed zero-determinant strategies.

The rest of this chapter is organized in the following way. Game theory is preliminarily introduced first in Sect. 2. Then we model the system and formulate the power control problem in Sect. 3. Based on the formulated problem, we analyze the interactions between one operator and one WAP by fixing the behaviors of all other operators in Sect. 4.1. Then according to the predicted strategies between each operator and the WAP, we consider a non-cooperative game among all operators in Sect. 4.2. We present our simulation results in Sect. 5, and finally summarize our works in Sect. 6.

2 Preliminaries of Game Theory

Game theory is introduced as a powerful tool to analyze the distributive strategies in competitive or coordinative scenarios, which have been widely applied in economics, politics, psychology, biology, computer science, engineering, etc. With tremendous contributions, eleven game-theorists have won economics Nobel Prizes and have applied a wide range of behavioral relations among humans, animals and computers efficiently and beneficially. In game theory, there are three main characteristics, i.e., player, action and utility.

- **Player**: Players indicate the set of rational individuals which can make decisions autonomously. In the game, the conflicts normally exist among players and each player is required to make proper behaviors to either compete or coordinate with other players.

- **Action**: Actions denote the behaviors and strategies of each player during its interaction with other players. Due to conflicts, the action of one player will affect the optimal actions of other players.
- **Utility**: Utilities refer to the revenues or penalties the action brings to each player. Based on the actions of other players, each player is required to set up the optimal actions in order to achieve maximum utility for itself. Moreover, in the distributive network, as the action of other players is related to the action of the player itself, each player is required to predict and consider the possible reactions of other players, as well as determine its optimal actions to maximize its utility.

With the definition of player, action and utility, a game can be played either statically or sequentially. In the static game, all players play the game simultaneously. Accordingly, each player is required to analyze the optimal strategies of other players before determining the strategy for itself. In order to achieve stable results for all players, the Nash equilibrium concept is put forward.

Definition 1 Let $(\mathbf{X}, \mathbf{u})$ denote the static game with m players. $\mathbf{X} = \mathbf{X}_1 \times \mathbf{X}_2 \times, \ldots, \times \mathbf{X}_m$ refers to all sets of strategy profiles of all players. $\mathbf{u} = (u_1(\mathbf{x}), \ldots, u_m(\mathbf{x}))$ is the utility profile of all players. Let $\mathbf{x}_i$ be a strategy profile of player i, $\mathbf{x}_{-i}$ be a strategy profile of other players except for player i. A set of strategy profiles $\mathbf{x}^* \in X$ is able to achieve the Nash equilibrium if $\forall i$, $\mathbf{x}_i \in \mathbf{X}_i$,

$$u_i(\mathbf{x}_i^*, \mathbf{x}_{-i}^*) \geq u_i(\mathbf{x}_i, \mathbf{x}_{-i}^*). \tag{1}$$

Apart from the static game, a game can also be played sequentially. In the sequential game, the players can be divided into leaders and followers, where the leaders act first and the followers behaves correspondingly. Accordingly, the first-mover advantage exists, where the leader is able to predict the corresponding reactions of followers and make actions firstly for high utilities. In the sequential game, the stable results can be achieved with Stackelberg equilibrium, which is defined as follows.

Definition 2 Let $((\mathbf{X}, \mathbf{A}), (g, f))$ be the general sequential game with m leaders and n followers. $\mathbf{X} = \mathbf{X}_1 \times \mathbf{X}_2 \times, \ldots, \times \mathbf{X}_m$ and $\mathbf{A} = \mathbf{A}_1 \times \mathbf{A}_2 \times, \ldots, \times \mathbf{A}_n$ are all sets of strategy profiles of all leaders and all followers, respectively. $g = (g_1(\mathbf{x}), \ldots, g_m(\mathbf{x}))$ is the payoff function of leaders for $\mathbf{x} \in \mathbf{X}$, and $f = (f_1(\boldsymbol{\alpha}), \ldots, f_n(\boldsymbol{\alpha}))$ is the payoff function of followers for $\boldsymbol{\alpha} \in \mathbf{A}$. Let $\mathbf{x}_i$ be a strategy profile of leader i, $\mathbf{x}_{-i}$ be a strategy profile of all leaders except for leader i, $\boldsymbol{\alpha}_j$ be a strategy profile of follower j, and $\boldsymbol{\alpha}_{-j}$ be a strategy profile of all other followers except for leader j. A set of strategy profile $\mathbf{x}^* \in X$ and $\boldsymbol{\alpha}^* \in \mathbf{A}$ is the equilibrium of the multi-leader multi-follower game if $\forall i, \forall j, \mathbf{x}_i \in \mathbf{X}_i, \boldsymbol{\alpha}_j \in \mathbf{A}_j$,

$$g_i(\mathbf{x}_i^*, \mathbf{x}_{-i}^*, \boldsymbol{\alpha}^*) \geq g_i(\mathbf{x}_i, \mathbf{x}_{-i}^*, \boldsymbol{\alpha}^*) \geq g_i(\mathbf{x}_i, \mathbf{x}_{-i}, \boldsymbol{\alpha}^*),$$

$$f_j(\mathbf{x}, \boldsymbol{\alpha}_j^*, \boldsymbol{\alpha}_{-j}^*) \geq f_j(\mathbf{x}, \boldsymbol{\alpha}_j, \boldsymbol{\alpha}_{-j}^*).$$

In the following sections, as all operators and the WAP are autonomous individuals, which try to optimize their own utilities based on the behaviors of others, we consider all operators and the WAP as the players in one game [21]. With the established system model and formulated problems for each player, we analyze the optimal strategies of each operator and WAP pair, and the optimal strategies among all operators, respectively, in a game-theoretical perspective.

3 System Model and Problem Formulation

We consider an indoor environment where there is a set $\mathscr{N} = \{1, \ldots, N\}$ of operators trying to serve their MUs in the unlicensed spectrum. However, as shown in Fig. 1, the WAP already serves Wi-Fi users (WUs) in the unlicensed spectrum, so all N operators are required to guarantee the performance of the WUs while increasing the QoS for their MUs. We suppose the WAP adopts Frequency Division Multiple Access (FDMA) and there are totally S sub-bands in the unlicensed spectrum, each labeled as $s \in \mathscr{S} = \{1, \ldots, S\}$. As each sub-band $s \in \mathscr{S}$ of the unlicensed spectrum is independent of other sub-bands. Thus, in the following sections, we analyze the strategies of the WAP and all operators in one sub-band, say s, and hence drop the sub-band index to simplify notational expressions. The strategies in other sub-bands can be analyzed in a similar way. Accordingly, when the WAP shares the sub-band with all N operators, the spectrum efficiency of the WAP can be expressed as

$$R^{(\mathrm{w})} = \log_2\left(1 + \frac{p^{(\mathrm{w})} g^{(\mathrm{w})}}{\sum_{n \in \mathscr{N}} p_n^{(\mathrm{m})} h_n^{(\mathrm{m})} + \sigma^2}\right), \tag{2}$$

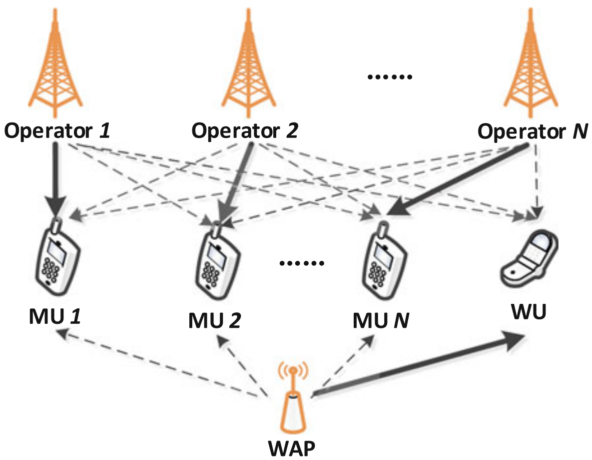

Fig. 1 System architecture when multiple wireless operators implement LTE unlicensed in the same spectrum band (MU: mobile user; WU: Wi-Fi user)

where $p^{(w)}$ is the transmit power allocated by the WAP for a scheduled WU in the sub-band. $g^{(w)}$ is the path gain from the WAP to the WU. $p_n^{(m)}$ is the transmit power allocated by one base station (BS) of operator n for a scheduled MU. $h_n^{(m)}$ is the path gain from the BS of operator n to the WU. Thus, $p^{(w)}g^{(w)}$ is the signal strength that the WU receives from the WAP, and $\sum_{n\in\mathcal{N}} p_n^{(m)} h_n^{(w)}$ is the total interference from BSs of all operators. σ is the power of the additive white noise in the sub-band.

Correspondingly, we assume that each operator serves one MU with the closest BS in the sub-band. Without causing any confusion, we shall thus interchangeably use an operator and a BS in the following analysis. The spectrum efficiency of each operator $n \in \mathcal{N}$ in the sub-band can be expressed as

$$R_n^{(m)} = \log_2\left(1 + \frac{p_n^{(m)} g_n^{(m)}}{p^{(w)} h_n^{(w)} + \sum_{n'\in\mathcal{N}\setminus\{n\}} p_{n'}^{(m)} h_{n'n}^{(m)} + \sigma^2}\right), \tag{3}$$

where $g_n^{(m)}$ is the path gain from the BS of operator n to the scheduled MU of operator n. $h_n^{(w)}$ is the path gain from the WAP to the MU. $h_{n'n}^{(m)}$ is the path gain from an operator $n' \in \mathcal{N} \setminus \{n\}$ to the MU. Accordingly, $p_n^{(m)} g_n^{(m)}$ is the signal strength the MU gets from its associated BS of operator n, $p^{(w)} h_n^{(w)}$ is the interference the MU receives from the WAP, and $\sum_{n'\in\mathcal{N}\setminus\{n\}} p_{n'}^{(m)} h_{n'n}^{(m)}$ is the interference the MU receives from other operators in the sub-band.

Furthermore, the data transmissions from the WAP and all N operators to their WU and MUs consume transmit power. To encourage minimizing power consumption, we suppose the transmit power cost of the WAP is

$$c^{(w)} = p^{(w)} r^{(w)}, \tag{4}$$

where $r^{(w)}$ is the price of unit transmit power of the WAP. The cost of transmit power for each operator $n \in \mathcal{N}$ is

$$c_n^{(m)} = p_n^{(m)} r_n^{(m)}, \tag{5}$$

where $r_n^{(m)}$ is the price of unit transmit power of the base station of operator n.

Therefore, in line with the above discussions, the utility of the WAP can be denoted as the achieved capacity by serving the WU minus its transmit power cost, i.e.,

$$U^{(w)}\left(p^{(w)} \middle| \mathbf{p}^{(m)}\right) = BR^{(w)} - c^{(w)}, \tag{6}$$

where B is the bandwidth of the considered sub-band s of the unlicensed spectrum. The transmit powers of both the WAP and all WOs can affect the final utility of the WAP due to spectrum sharing. We choose $\mathbf{p}^{(m)}$ to denote the transmit powers from all operators.

Similarly, the utility of an operator $n \in \mathcal{N}$ is the achieved capacity by serving the MU minus the corresponding transmit power cost, that is,

$$U_n^{(\mathrm{m})}\left(p_n^{(\mathrm{m})}\middle|p^{(\mathrm{w})}, \mathbf{p}_{-n}^{(\mathrm{m})}\right) = BR_n^{(\mathrm{m})} - c_n^{(\mathrm{m})}, \tag{7}$$

where $\mathbf{p}_{-n}^{(\mathrm{m})}$ denotes the transmit powers from all operators except for the operator n.

We suppose that the WAP and all operators are autonomous individuals. In order to achieve a high utility for itself, the WAP should determine its transmit power $p^{(\mathrm{w})}$ based on the transmit powers from all operators in the sub-band. For each operator, however, based on the behaviors of all other operators and the WAP, it is supposed to determine the transmit power $p_n^{(\mathrm{m})}$ in order to improve its utility while guaranteeing the performance of the WU at the same time. As the WAP and all operator are able to make decisions in an iterated way, for simplicity of the analysis, we suppose the WAP and all operators have two power level choices, namely, $p^{(\mathrm{w})} \in \{p_1^{(\mathrm{w})}, p_2^{(\mathrm{w})}\}$, $p_n^{(\mathrm{m})} \in \{p_1^{(\mathrm{m})}, p_2^{(\mathrm{m})}\}$, where 1 stands for the low power level and 2 refers to the high power level. Accordingly, in the current iteration, if the probability of $p^{(\mathrm{w})} = p_i^{(\mathrm{w})}$ is $v_i^{(\mathrm{w})}$, and the probability of $p_n^{(\mathrm{m})} = p_{j_n}^{(\mathrm{m})}$ is $v_{j_n}^{(\mathrm{m}),n}$, $\forall i, j_n \in \{1, 2\}$, $\forall n \in \mathcal{N}$, the expected utility of the WAP and the operator n can be, respectively, shown as

$$E^{(\mathrm{w})} = \sum_{i,\{j_n|n\in\mathcal{N}\}} \left[v_i^{(\mathrm{w})} \prod_{n\in\mathcal{N}} v_{j_n}^{(\mathrm{m}),n} U^{(\mathrm{w})}\left(p_i^{(\mathrm{w})}\middle|\left(p_{j_n}^{(\mathrm{m})}\middle|n \in \mathcal{N}\right)\right)\right], \tag{8}$$

and

$$E_n^{(\mathrm{m})} = \sum_{i,\{j_n|n\in\mathcal{N}\}} \left[v_i^{(\mathrm{w})} \prod_{n\in\mathcal{N}} v_{j_n}^{(\mathrm{m}),n} U_n^{(\mathrm{m})}\left(p_{j_n}^{(\mathrm{m})}\middle|p_i^{(\mathrm{w})}, \left(p_{j_{n'}}^{(\mathrm{m})}\middle|n' \in \mathcal{N} \setminus \{n\}\right)\right)\right]. \tag{9}$$

For each pair of an operator $n \in \mathcal{N}$ and the WAP, if in the current iteration the transmit power of the operator n is in level x_n, and the transmit power of the WAP is in level y, we define the expected probability that in the next iteration the operator n decides the power in level x_n' is $z_{yx_nx_n'}$, $\forall y, x_n, x_n' \in \{1, 2\}$. Correspondingly, we define the probability that in the next iteration the WAP transmits in level y' is $a_{yx_1x_2\ldots x_Ny'}$, $\forall y, x_n, y' \in \{1, 2\}$, $\forall n \in \mathcal{N}$, given that in the current iteration the transmit power of each operator n is in level x_n, and the transmit power of the WAP is in level y. Accordingly, in the current iteration, the strategy profile for the operator n can be given by $\mathbf{z}_n = \{z_{yx_nx_n'}, \forall y, x_n, x_n' \in \{1, 2\}\}$, $\forall n \in \mathcal{N}$. The strategy profile for the WAP can be denoted as $\mathbf{a} = \{a_{yx_1x_2\ldots x_Ny'}, \forall y, x_n, y' \in \{1, 2\}, \forall n \in \mathcal{N}\}$.

In the iterated scenario, for an operator $n \in \mathcal{N}$, to guarantee the performance of the WU, it is required to maximize the total utility accumulated over both itself and the WAP in the same sub-band of the unlicensed spectrum, without knowing the strategy of the WAP. Furthermore, to achieve a high utility performance for the MUs, the utility of operator n should be k times larger than the utility of the WAP, where $k > 0$ is a constant. Eventually, the optimization problem for operator n can be formulated as follows,

$$\begin{aligned} &\max_{\mathbf{z}_n} \quad E_n^{(\mathrm{m})} + E^{(\mathrm{w})} \\ &\text{s.t.} \begin{cases} \mathbf{0} \leqslant \mathbf{z}_n \leqslant \mathbf{1}, \\ E_n^{(\mathrm{m})} \geq kE^{(\mathrm{w})}. \end{cases} \end{aligned} \tag{10}$$

Based on the formulated problem, in the following sections, game-theoretical analysis is adopted to determine the optimal strategies for each operator or WAP so as to achieve its optimal utility, respectively.

4 Game Analysis

In this section, we analyze the optimal power control strategies for each operator and the WAP. As the strategy of an operator is affected by all other operators, we first fix the behaviors of all the other operators and discuss the optimal strategies for one operator and WAP pair in Sect. 4.1. Furthermore, by predicting the behaviors of every operator and WAP pair, each operator n, $\forall n \in \mathcal{N}$, is able to adjust its strategy and compete with other operators. Accordingly, in Sect. 4.2, we formulate the competition among all operators as a non-cooperative game, and find out the Nash equilibrium of the game where each of the operators cannot unilaterally change its behaviors for a higher utility.

4.1 Game Analysis Between an Operator and WAP

In order to better analyze the relationship between an operator n and the WAP, we fix the transmit powers of all other operators, i.e., $\mathbf{p}_{-n}^{(\mathrm{m})}$, in each iteration of the game. When both operator n and the WAP transmit in different power levels, they receive the following utilities,

$$W_{yx_n}^{(\mathrm{m})} = U_n^{(\mathrm{m})}\left(p_{x_n}^{(\mathrm{m})} \middle| p_y^{(\mathrm{w})}, \mathbf{p}_{-n}^{(\mathrm{m})}\right), \tag{11}$$

$$W_{yx_n}^{(\mathrm{w})} = U^{(\mathrm{w})}\left(p_y^{(\mathrm{w})} \middle| p_{x_n}^{(\mathrm{m})}, \mathbf{p}_{-n}^{(\mathrm{m})}\right), \tag{12}$$

$\forall y, x_n \in \{1, 2\}$. For a better understanding, we illustrate the utilities in Fig. 2, which is basically a 2×2 static game. According to the property of the utility functions, when operator n increases its power level while the WAP keeps its transmit power unchanged, the utility function of operator n increases and the utility function of the

Fig. 2 Game analysis between the operator n and the WAP in one iteration

WAP \ OPERATOR n	$p_1^{(m)}$	$p_2^{(m)}$
$p_1^{(w)}$	$W_{11}^{(w)}, W_{11}^{(m)}$	$W_{12}^{(w)}, W_{12}^{(m)}$
$p_2^{(w)}$	$W_{21}^{(w)}, W_{21}^{(m)}$	$W_{22}^{(w)}, W_{22}^{(m)}$

WAP decreases, and vice versa. Thus, we have

$$\begin{cases} W_{y2}^{(\mathrm{m})} > W_{y1}^{(\mathrm{m})}, & \forall y \in \{1,2\}; \\ W_{y2}^{(\mathrm{w})} < W_{y1}^{(\mathrm{w})}, & \forall y \in \{1,2\}; \\ W_{2x_n}^{(\mathrm{w})} > W_{1x_n}^{(\mathrm{w})}, & \forall x_n \in \{1,2\}; \\ W_{2x_n}^{(\mathrm{m})} < W_{1x_n}^{(\mathrm{m})}, & \forall x_n \in \{1,2\}. \end{cases} \tag{13}$$

Based on (13) above, $p^{(\mathrm{w})} = p_2^{(\mathrm{w})}$ and $p_n^{(\mathrm{m})} = p_2^{(\mathrm{m})}$ is the Nash equilibrium of the game. If $W_{22}^{(\mathrm{w})} + W_{22}^{(\mathrm{m})} > W_{11}^{(\mathrm{w})} + W_{11}^{(\mathrm{m})}$, $p^{(\mathrm{w})} = p_2^{(\mathrm{w})}$ and $p_n^{(\mathrm{m})} = p_2^{(\mathrm{m})}$ also achieve the Pareto optimality, which constitute the optimal strategies for both operator n and the WAP.

However, if $W_{22}^{(\mathrm{w})} + W_{22}^{(\mathrm{m})} < W_{11}^{(\mathrm{w})} + W_{11}^{(\mathrm{m})}$, the game becomes a prisoner's dilemma where the social optimal point is not the Nash equilibrium solution. In order to achieve high and stable social welfare while guaranteeing the performance of the WU, we suppose the game is played in an iterated way. Thus, zero-determinant strategy can be applied by operator n to unilaterally set a ratio relationship between the operator n and the WAP, no matter what the strategy of the WAP is [22, 23]. Thus, when the WAP maximizes its individual utility, the social welfare can be optimized.

In the iterated game, as we do not consider the strategies of other operators, the strategy of the WAP can be defined as

$$q_{yx_n y'} = \sum_{\{x_{n'} | n' \in \mathcal{N} \setminus \{n\}\}} a_{yx_1 x_2 \ldots x_N y_w}. \tag{14}$$

Thus, the transition matrix of the iterated process can be given as

$$\mathbf{H} = \begin{bmatrix} q_{111}z_{111} & q_{111}z_{112} & q_{112}z_{111} & q_{112}z_{112} \\ q_{121}z_{121} & q_{121}z_{122} & q_{122}z_{121} & q_{122}z_{122} \\ q_{211}z_{211} & q_{211}z_{212} & q_{212}z_{211} & q_{212}z_{212} \\ q_{221}z_{221} & q_{221}z_{222} & q_{222}z_{221} & q_{222}z_{222} \end{bmatrix}, \tag{15}$$

where $q_{yx_n1} + q_{yx_n2} = 1$ and $z_{yx_n1} + z_{yx_n2} = 1$, $\forall y, x_n \in \{1,2\}$.

In each iteration of the game, we assume the probability that the WAP transmits in power level y while the operator n transmits in power level x_n is d_{yx_n}. Thus,

$$d_{yx_n} = v_y^{(\mathrm{w})} v_{x_n}^{(\mathrm{m})}, \tag{16}$$

$\forall y, x_n \in \{1, 2\}$. Denote $\mathbf{d} = [d_{11}, d_{12}, d_{21}, d_{22}]^\top$, we model the iterated process as a Markov chain. If

$$\mathbf{d}^\top \mathbf{H} = \mathbf{d}^\top, \tag{17}$$

can be established, the process achieves a stationary result. Define $\mathbf{H}' = \mathbf{H} - \mathbf{I}$, where $\mathbf{I}$ is the unit diagonal matrix. We then have

$$\mathbf{d}^\top \mathbf{H}' = \mathbf{0}. \tag{18}$$

Moreover, according to Cramer's rule, $\mathrm{adj}(\mathbf{H}')\mathbf{H}' = \det(\mathbf{H}')$, where $\mathrm{adj}(\mathbf{H}')$ is the adjugate matrix of $\mathbf{H}'$. Following the properties of the matrix determinant, we derive $\det(\mathbf{H}') = 0$. Thus,

$$\mathrm{adj}(\mathbf{H}')\mathbf{H}' = 0. \tag{19}$$

Based on (18) and (19), we deduce that each column of the $\mathrm{adj}(\mathbf{H}')$ is proportional to $\mathbf{d}^\top$. Accordingly, the dot product of $\mathbf{d}$ with any vector $\mathbf{f} = [f_1, f_2, f_3, f_4]^\top$ can be expressed as

$$\mathbf{d}^\top \cdot \mathbf{f} = \tag{20}$$
$$\det \begin{pmatrix} -1 + q_{111}z_{111} & -1 + q_{111} & -1 + z_{111} & f_1 \\ q_{121}z_{121} & -1 + q_{121} & z_{121} & f_2 \\ q_{211}z_{211} & q_{211} & -1 + z_{211} & f_3 \\ q_{221}z_{221} & q_{221} & z_{221} & f_4 \end{pmatrix},$$

where the second and third column of the determinant is only related to the strategies of the WAP and operator n, respectively. We set $\mathbf{z}_n = [-1 + z_{111}, z_{121}, -1 + z_{211}, z_{221}]^\top$ and $\mathbf{f} = \mathbf{W}^{(w)} - \beta \mathbf{W}_n^{(m)}$, where $\mathbf{W}^{(w)} = [W_{11}^{(w)}, W_{12}^{(w)}, W_{21}^{(w)}, W_{22}^{(w)}]$ and $\mathbf{W}_n^{(m)} = [W_{11}^{(m)}, W_{12}^{(m)}, W_{21}^{(m)}, W_{22}^{(m)}]$.

If

$$\mathbf{z} = \lambda \mathbf{f}, \tag{21}$$

we have

$$\begin{aligned} \mathbf{d}^\top \cdot \mathbf{f} &= \mathbf{d}^\top \cdot \left(\mathbf{W}^{(w)} - \beta \mathbf{W}_n^{(m)}\right) \\ &= F^{(w)} - \beta F_n^{(m)} = 0, \end{aligned} \tag{22}$$

namely,

$$F_n^{(m)} = \frac{1}{\beta} F^{(w)}. \tag{23}$$

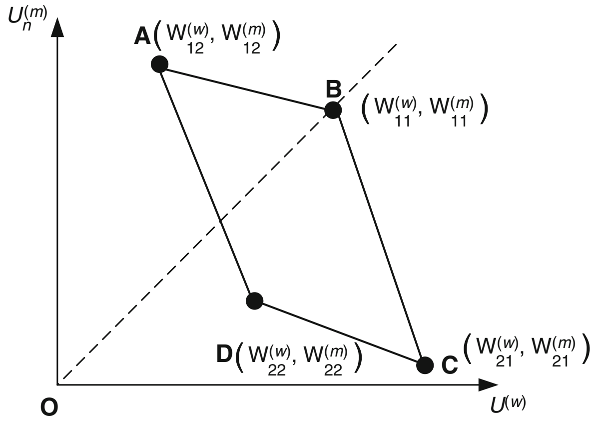

Fig. 3 The utility of the operator n vs the utility of the WAP when the operator n adopts the zero-determinant strategy

where $F^{(\mathrm{w})}$ and $F_n^{(\mathrm{m})}$ are the expected utility of the WAP and operator n in the 2×2 game, respectively.

Accordingly, the zero-determinant strategy for operator n is calculated as,

$$\begin{cases} z_{111} = 1 + \lambda \left(W_{11}^{(\mathrm{w})} - \beta W_{11}^{(\mathrm{w})} \right), \\ z_{121} = \lambda \left(W_{12}^{(\mathrm{w})} - \beta W_{12}^{(\mathrm{w})} \right), \\ z_{211} = 1 + \lambda \left(W_{21}^{(\mathrm{w})} - \beta W_{21}^{(\mathrm{w})} \right), \\ z_{221} = \lambda \left(W_{22}^{(\mathrm{w})} - \beta W_{22}^{(\mathrm{w})} \right). \end{cases} \tag{24}$$

Moreover, as depicted in Fig. 3, the feasible region of the prisoner's dilemma is *ABCD*. The zero-determinant strategy of operator n is characterized by a line starting at O as shown in Fig. 3, i.e., as long as operator n adopts the proposed zero-determinant strategy, no matter what the strategy of the WAP is, the final results of the game fall on one determined line [24]. In order to achieve the maximum utility for both operator n and the WAP, operator n should determine the line OB. Taking into account the constraint that $E_n^{(\mathrm{m})} \geq kE^{(\mathrm{w})}$, the value of β satisfies

$$\frac{1}{\beta} = \max \left\{ k, \frac{W_{11}^{(\mathrm{m})}}{W_{11}^{(\mathrm{w})}} \right\}. \tag{25}$$

4.2 Game Analysis Among Operators

According to the analysis performed in the previous subsection, when the transmit powers of all other operators are fixed, the utility profiles of an operator $n \in \mathcal{N}$ as well as the WAP, namely, $\mathbf{W}_n^{(\mathrm{m})} = [W_{11}^{(\mathrm{m})}, W_{12}^{(\mathrm{m})}, W_{21}^{(\mathrm{m})}, W_{22}^{(\mathrm{m})}]$ and $\mathbf{W}^{(\mathrm{w})} =$

$[W_{11}^{(\mathrm{w})}, W_{12}^{(\mathrm{w})}, W_{21}^{(\mathrm{w})}, W_{22}^{(\mathrm{w})}]$, are fixed. Therefore, operator n is able to configure the proposed zero-determinant strategy by setting a ratio between its own utility and the utility of the WAP. However, when the transmit powers from other operators are changed, the utility profiles of the operator n and the WAP vary, and so does the game between operator n and the WAP. That is, the behaviors of each operator will affect the utility functions of other operators. As each operator would like to increase its utility in a selfish way, we model the competitions among the operators as a non-cooperative game. The probability that each operator n transmit in power level x_n, $\forall x_n \in \{1, 2\}$, $\forall n \in \mathcal{N}$, and the WAP transmits in power level y can be expressed in the following form

$$\pi_{yx_1 \ldots x_N} = v_y^{(\mathrm{w})} \prod_{n \in \mathcal{N}} v_{x_n}^{(\mathrm{m}),n}. \tag{26}$$

And inversely, it's straightforward to get

$$v_{x_n}^{(\mathrm{m}),n} = \sum_{y, \{x_{n'} | n' \in \mathcal{N} \setminus \{n\}\}} \pi_{yx_1 \ldots x_N}, \tag{27}$$

$\forall x_n \in \{1, 2\}$, and

$$v_y^{(\mathrm{w})} = \sum_{\{x_n | n \in \mathcal{N}\}} \pi_{yx_1 \ldots x_N}, \tag{28}$$

$\forall y \in \{1, 2\}$.

Therefore, in the 2×2 game between each operator $n \in \mathcal{N}$ and the WAP, the probability of a situation that all other operators $n' \in \mathcal{N} \setminus \{n\}$ transmits in power level $x_{n'}$ is

$$\kappa^n_{x_1 \ldots x_{n-1} x_{n+1} \ldots x_N} = \sum_{y, x_n} \pi_{yx_1 \ldots x_N}, \tag{29}$$

$\forall x_n \in \{1, 2\}$.

In each situation, there is a corresponding utility profile for the 2×2 game between operator n and the WAP. Following the game analysis in Sect. 4.1, we are able to obtain a stationary vector $\mathbf{d}(x_1, \ldots, x_{n-1}, x_{n+1}, \ldots, x_N)$ for each situation. Accordingly, we attain

$$\sum_{\{x_{n'} | n' \in \mathcal{N} \setminus \{n\}\}} \mathbf{T}_{x_1 \ldots x_{n-1} x_{n+1} \ldots x_N} = \boldsymbol{\Psi}_n, \quad \forall n \in \mathcal{N}, \tag{30}$$

where

$$\begin{aligned} &\mathbf{T}_{x_1 \ldots x_{n-1} x_{n+1} \ldots x_N} = \\ &= \kappa^n_{x_1 \ldots x_{n-1} x_{n+1} \ldots x_N} \mathbf{d}\left(x_1, \ldots, x_{n-1}, x_{n+1}, \ldots, x_N\right), \end{aligned} \tag{31}$$

and

$$\boldsymbol{\Psi}_n = \left[v_1^{(\mathrm{w})} v_1^{(\mathrm{m}),n}, v_1^{(\mathrm{w})} v_2^{(\mathrm{m}),n}, v_2^{(\mathrm{w})} v_1^{(\mathrm{m}),n}, v_2^{(\mathrm{w})} v_2^{(\mathrm{m}),n} \right]. \tag{32}$$

Moreover, based on the above definitions, we have

$$\begin{cases} \sum_{x_n=1}^{2} v_{x_n}^{(\mathrm{m}),n} = 1, \forall n \in \mathcal{N}; \\ \sum_{y=1}^{2} v_y^{(\mathrm{w})} = 1. \end{cases} \tag{33}$$

Accordingly, when all the values of $\pi_{yx_1 \ldots x_N}$ satisfy (30) and (33), all operators achieve a Nash equilibrium, where each operator cannot change its strategy unilaterally for a higher utility. Based on the value of $\pi_{yx_1 \ldots x_N}$, each operator $n \in \mathcal{N}$ plays an 2×2 game with the WAP. The expected utility profile for operator n is

$$\boldsymbol{\Omega}_n^{(\mathrm{m})} = \sum_{\{x_{n'} | n' \in \mathcal{N} \setminus \{n\}\}} \kappa_{x_1 \ldots x_{n-1} x_{n+1} \ldots x_N}^{n} \mathbf{W}_n^{(\mathrm{m})} \left(x_1, \ldots, x_{n-1}, x_{n+1}, \ldots, x_N \right). \tag{34}$$

The expected utility profile for the WAP is

$$\boldsymbol{\Omega}^{(\mathrm{w})} = \sum_{\{x_{n'} | n' \in \mathcal{N} \setminus \{n\}\}} \kappa_{x_1 \ldots x_{n-1} x_{n+1} \ldots x_N}^{n} \mathbf{W}^{(\mathrm{w})} \left(x_1, \ldots, x_{n-1}, x_{n+1}, \ldots, x_N \right). \tag{35}$$

Finally, the optimal zero-determinant power control strategy for operator n is obtained as

$$\bar{\mathbf{z}}_n = \mathbf{z}_n \sum_{\{x_{n'} | n' \in \mathcal{N} \setminus \{n\}\}} \kappa_{x_1 \ldots x_{n-1} x_{n+1} \ldots x_N}^{n}. \tag{36}$$

5 Simulation Results

In this section, we evaluate the performance of the operators and WAP with MATLAB. Without loss of generality, we assume that there are two operators trying to share the unlicensed spectrum with the WAP in a two-dimensional area. The operators are located at coordinates $(50, 0)$ and $(25, 43)$, and their scheduled MUs are located at coordinates $(90, 0)$ and $(-5, 43)$. The WAP is assumed to be located at the origin, and it serves a WU at coordinates $(0, 10)$. We assume two power levels for both the operators and the WAP, i.e., the power levels for both operators are, respectively, $\{600, 1200\}$ and $\{450, 900\}$. And the power levels for the WAP are chosen from $\{400, 800\}$. We set the price of unit transmit power for the WAP to be 0.001 and that for the WO to be 0.002. The power of the additive white noise is $\sigma = -105\,\mathrm{dBm}$.

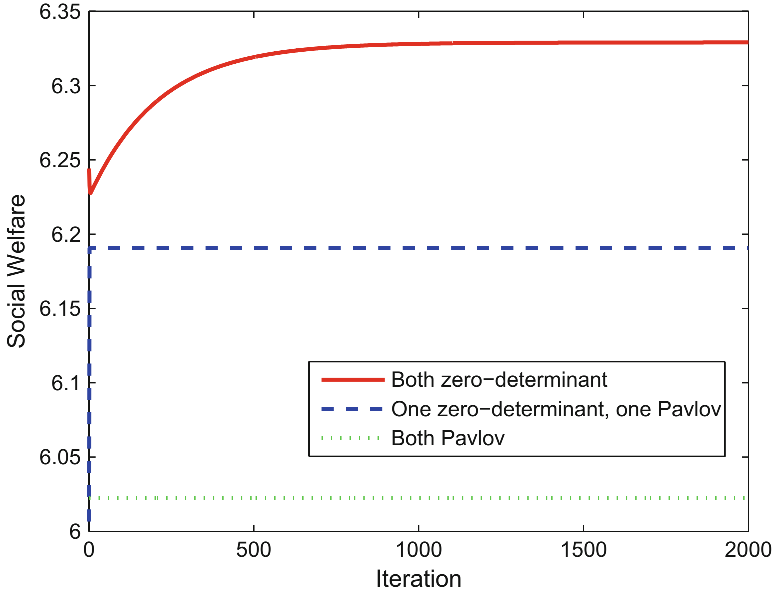

Fig. 4 The social welfare vs. iteration when two WOs and the WAP share unlicensed spectrum at the same time

For better analysis, we compare our proposed zero-determinant strategy with the Pavlov strategy in the game. In the case of an operator choosing to implement the Pavlov strategy, if the received utility is higher than a predefined threshold, operator keeps the current transmit power level. If the received utility is smaller than the threshold, the operator switches to the other power level. Thus, the Pavlov strategy for an operator n can be simply denoted by $\mathbf{z}_n = [1, 0, 0, 1], \forall n \in \{1, 2\}$.

From the curves in Fig. 4, we discover that the social welfare of the game finally converges as the number of iterations increases. The converged value when both the operators adopt the proposed zero-determinant strategy is larger than the value when the first operator applies the proposed zero-determinant strategy and the second operator applies the Pavlov strategy. Moreover, the converged value when the first operator applies the proposed zero-determinant strategy and the second operator applies the Pavlov strategy is larger than the value when both the operators adopt the Pavlov strategy.

Furthermore, we evaluate the influence that the transmit power of the WAP can make to the system in Fig. 5. As the low power level of the WAP increases, we discover that the social welfare of the system gradually increases, but the increasing speed decreases. The reason behind this is that when the low power level of the WAP increases, the WU is able receive a higher data rate from the WAP. However, increasing the transmit power of the WAP also increases the interference to the operators coexisting in the unlicensed spectrum, which indicates the decrease in the increasing speed. We can also see from the plot that the social welfare when both

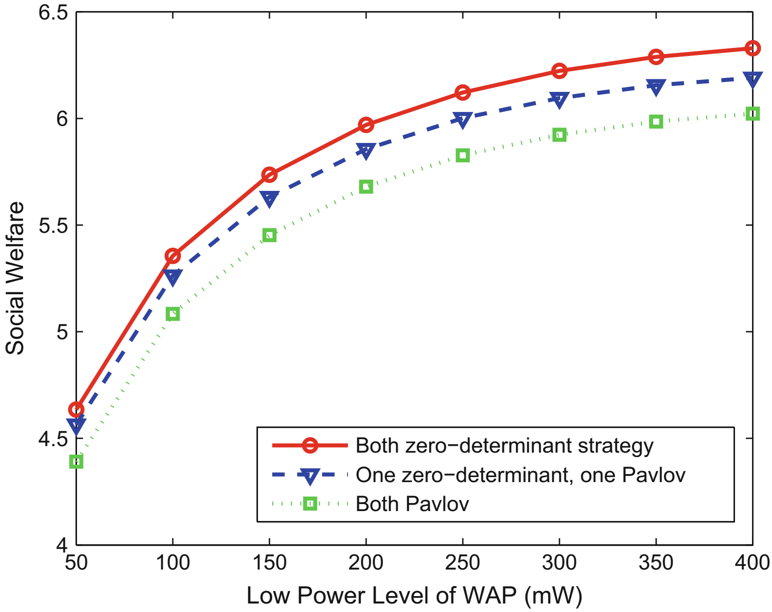

Fig. 5 The social welfare vs. low power level of the WAP in the game

the operators adopt the proposed zero-determinant strategy is always larger than the social welfare when the first operator applies the proposed zero-determinant strategy and the second operator applies the Pavlov strategy. The social welfare when the first operator applies the proposed zero-determinant strategy and the second operator applies the Pavlov strategy is always larger than the social welfare when both the operators adopt the Pavlov strategy.

6 Summary

In this chapter, we formulate a layered power control game among all the operators and the WAP which jointly operate over a common unlicensed spectrum band. Each operator aims to maximize its own utility in a distributed manner with the protection of performance achieved by the WU in the Wi-Fi network. In the layered game, we first fix the transmit powers of all other operators and propose a zero-determinant strategy for the power control of each considered operator. The advantage of implementing the zero-determinant strategy is that operators can optimize the social welfare on their own, no matter what power control strategy is chosen by the WSP. To deal with the competition among the non-cooperative operators, we propose that each operator explores the predicted strategies from all other operators in all situations and hence determines its optimal zero-determinant strategy to reach the

Nash equilibrium results. The provided simulation results validate the correctness of the analysis in this chapter, and confirm that the high performance gain can be realized from the proposed zero-determinant strategies.

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