A New Methodology for the Balancing of Mechanisms Using the Davies' Method

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Abstract. A general method for balance of planar mechanisms is presented in this paper. In order to determine the force that causes unbalanced, called shaking forces, the dynamics equations of motion for mechanisms are formulated systematically using the Davies' method. The formulation leads to an optimization scheme for the mass distribution to improve the dynamic performances of mechanisms. The method is illustrated with a slider-crank mechanism. Balancing of shaking forces shows a significant improvement in the dynamic performances compared to that of the original mechanisms.

Keywords: Dynamics · Shaking force · Balancing · Davies' method

1 Introduction

Despite large research effort in the field of balance of mechanisms, modelling the behaviour of unbalanced forces are still considered important and a major challenge in recent investigations [13]. The development of models using Fourier series in analytical way can be difficult to interpretate, due to the complexity of the equations. Also, an approach that considered the behaviour of others forces in the model, as example, gravitational force, can be difficult to integrate with analytical models.

In order to address the issues previously discussed we proposed a method based in a synthetical view which can not only be used to solve the internal unbalanced forces, but integrated also external forces which are applied to the structure. The method is validated using a planar slider-crank mechanism (Fig. 1), but it has the potential to be expanded in order to use it in spatial mechanisms.

An overview on balancing techniques is presented in Sect. 2. Section 3 describe the proposed methodology and summarises scientific contributions of this paper. The mechanical design of the slider-crank mechanism is presented in Sect. 4 along

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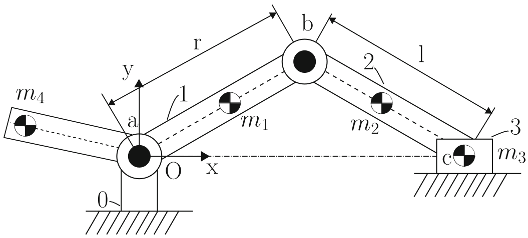


Fig. 1. Slider-crank mechanism

the equivalent masses model. Section 5 present the solution of the accelerations center of mass and Sect. 6 the solution of the inertial equations. The optimization algorithm used is briefly introduced in Sect. 7. Section 8 present the results for the shaking forces and Sect. 9 the conclusions of the paper.

2 Background

Different approaches has been developed for the balance of mechanisms. In [2,11] a method based on the conditions of the angular and linear moment being equal to zero was used. In [3,13] a closed analytical condition has been found to determine the position of the center of mass of a mechanism and the conditions for being stationary. Both methods are convenient for the analytical solution of the problem but sometimes leads to complex solutions, requiring significant algebraic effort to the derivation of the balance conditions.

In [4,6] a fast Fourier transform analysis is derived and investigated to the partial balancing of high-speed machinery by the use of counterweights mounted on shafts. This solution has found wide application as it may be accomplished by attaching balancing elements to the crank. However, for mechanisms with more degrees of freedom (DOFs), the dynamic balance conditions become increasingly to find as the number of bodies increase. In [8] a screw-based dynamic balancing approach has been developed to obtain the balancing conditions directly from the momentum equations. Using Screw theory it has been found that a simplification of the balancing process is obtained. On overview of the theory of balancing mechanism is given in [1].

In this paper, the Davies' method is used to solve the dynamic balance for a planar slider-crank mechanism. The method is based on Graph theory, Screw theory and an adaptation of the Kirchhoff's laws and is applied to solve both differential kinematics and statics of mechanism [5,7,9]. Recently, the method was used to solve the rollover of long combination vehicles, treating the acceleration of the last unit (trailer) and the stiffness of the suspension system [10]. Because of this combination of powerful mathematical tools, as Graph theory and Screw theory, and also for the adaptation from electric circuits, the Davies' method have been proved to be more general and straight forward than other tools.

3 Methodology

In this section, a new methodology is derived and presented to solve the shaking forces in a slider-crank mechanism. The methodology is based on the Davies' method to solve the inertial equations and after an optimization method to describe the position of the counterweights to reduce the shaking forces in the mechanism.

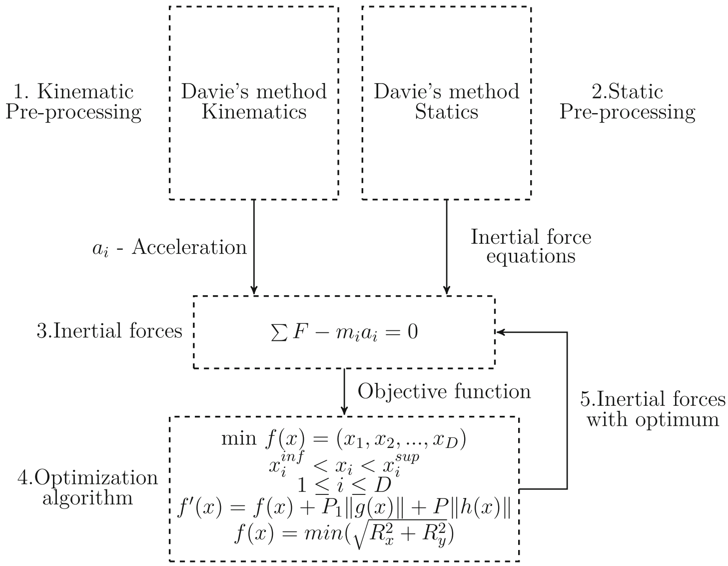


Fig. 2. Flowchart of the methodology

Figure 2 shows the flow chart of the methodology: the first step is solving the accelerations of the center of mass of each link of the mechanism and the inertial forces acting in each link, as we call the kinematic pre-processing and static pre-processing. Once the accelerations and the inertial forces are known, to solve the shaking forces, an objective function is built, using a convenient number of optimization variables. In this paper, the differential evolution optimization method was used to solve the minimization of the shaking forces.

4 System Modeling

The slider-crank mechanism is divided by a crankshaft (1), connecting rod (2) and slider block (3). The respectively total mass of the crankshaft, connecting rod and slider block is m_1, m_2 and m_3. The length of the crankshaft and connecting rod is, respectively, r and l. To balance the shaking forces in the mechanism, a counterweight with total mass m_4 is add in the crankshaft (Fig. 1). The weight and the position of the counterweight it will be determined by the optimization method.

To solve the inertial forces in the slider-crank it is necessary to know the position of each center of mass of each link and the counterweight. For this solution, it is better to represent the masses of the links as equivalent masses. For the dynamic equivalence, it is necessary to assume three conditions [2].

- The mass of the model must be equal that of the original body.
- The center of gravity must be in the same location as that of the original body.
- The mass moment of inertia must equal that of the original body.

Figure 3 shows the equivalent mass of the crankshaft. The total mass of the crankshaft m_1 can be represented as an equivalent mass in the point b.

$$\begin{aligned} m_{1_b}\, r &= m_1\, r_{G_2} \\ m_{1_b} &= m_1 \frac{r_{G_2}}{r} \end{aligned} \tag{1}$$

For the connecting rod, the total mass can be derived in three masses: m_{2_b}, m_{2_g} and m_{2_c} (Fig. 3).

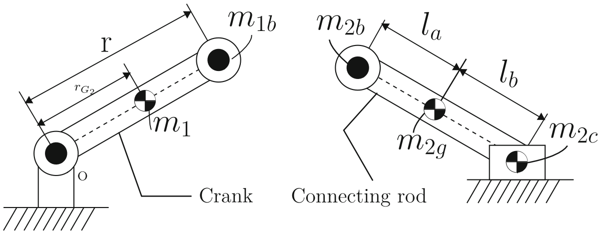


Fig. 3. Equivalent mass model

The equivalent mass system is obtained by the Eqs. 2, 3 and 4.

$$m_{2_b} + m_{2_g} + m_{2_c} = m_2 \tag{2}$$

$$m_{2_b}\, l_a + m_{2_g}\, 0 + m_{2_c}\, l_b = 0 \tag{3}$$

$$m_{2_b}\, l_a^2 + m_{2_g}\, 0 + m_{2_c}\, l_b^2 = J_S \tag{4}$$

Solving m_{2_b} in Eq. 3, we obtain,

$$m_{2_b} = \frac{m_{2_c}\, l_b}{l_a} \tag{5}$$

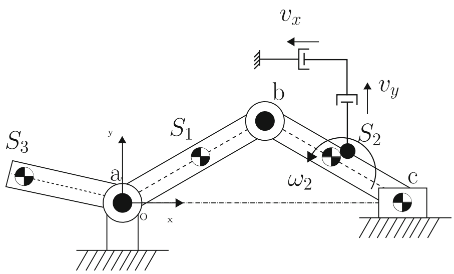
Now, applying Eq. 5 in Eq. 4,

$$m_{2_c} = \frac{J_S}{l_b\, l_a + l_b^2} \tag{6}$$

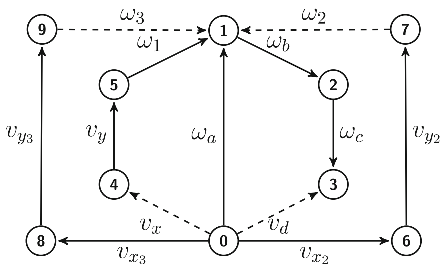
Knowing the moment of inertia J_S, it is possible to describe the equivalent masses of the link. These simplifications lead to the equivalent masses model of the slider-crank.

5 Solving the Acceleration

As stated previously, the Davies' method is used in this paper to solve the kinematics of the slider-crank. The first step in the solution of the shaking forces, it is necessary to know the accelerations of the center of mass of each link of the mechanism. For the slider-crank, it is necessary to know the acceleration of the counterweight (S_3), joints b and c of the connecting rod, and the center of mass S_2 of the connecting rod, as shown in Fig. 4(a).



(a) Kinematic parameters



(b) Motion graph

Fig. 4. Kinematic parameters and motion graph of the slider-crank

To describe the acceleration of the connecting rod center of mass it is applied an assur virtual chain, composed by prismatic kinematic pairs at x and y axis, and one revolute joint about the z axis, as we called a PPR assur virtual chain. The prismatic joint is used in these case to solve the linear velocities of the center of mass v_x and v_y.

Using graph theory we can represent the whole velocities in the slider-crank as a graph called the motion graph G_M. The slider-crank is composed by three revolute joints a, b and c and one prismatic joint d. The DOF of the revolute joint are the angular velocities about the z axis ω_a, ω_b and ω_c. And the DOF of the prismatic joint is the velocity in the x axis called v_d. In the motion graph, each edge represents the degrees of freedom f of the coupling (joint). The motion graph G_M of the slider-crank mechanism is shown in Fig. 4(b).

In the motion graph, we represent also the degrees of freedom of the assur virtual chain, in this case 3 virtual chains to describe the velocities of the center of mass b, S_2 and S_3. A set of f independent motions represented by twists is written in the circuit matrix $\{B_M\}_{l,F}$. The number of circuits l is 4 and the gross network degree of freedom F is 13, so the circuit matrix G_M is

$$[B_M]_{4\times 13} = \begin{bmatrix} -1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad (7)$$

The solution of the kinematic magnitudes of the slider-crank is obtained applying the circuit-law in the unit motion matrix [5]. The solution of the secondary variables $[\Psi_p]$ is obtained in Eq. 8.

$$\begin{Bmatrix} v_{b_x} \\ v_{b_y} \\ v_{S_{2x}} \\ v_{S_{2y}} \\ v_{c_x} \\ v_{S_{3x}} \\ v_{S_{3y}} \end{Bmatrix} = \begin{Bmatrix} -y_b \\ x_b \\ \frac{-(y_{S_2}\, x_b - y_b\, x_c)}{x_b - x_c} \\ \frac{x_b\,(x_{S_2} - x_c)}{x_b - x_c} \\ \frac{-(x_b\, y_c - y_b\, x_c)}{x_b - x_c} \\ -y_{S_3} \\ x_{S_3} \end{Bmatrix} \omega_a \tag{8}$$

where $[x_i, y_i]$ are the coordinates of each joint and center of mass illustrated in Fig. 4. Once the velocities are known, the accelerations are easily obtained applying the derivative.

6 Solving the Inertial Forces

We can apply the Davies' method to solve the reactions forces at the frame of the slider-crank. The reactions forces R_x and R_y can be represented by the inertial forces acting in each link. The inertial forces due to the acceleration of the link act in a straight line passing through the center of mass of the link. The inertial forces F_i acting in each link are shown in Fig. 5.

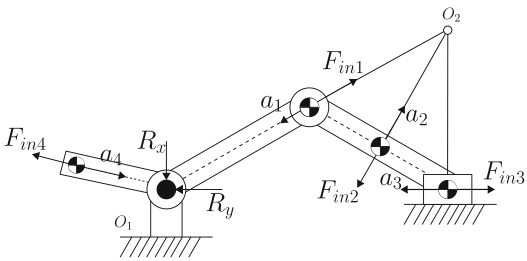


Fig. 5. Inertial forces acting in the slider-crank.

As in the kinematic solution, the actions in the mechanism can be represented by a graph called action graph G_A. In the action graph, each constraint in a joint is represented as a set of edges in parallel. The inertial forces acting in each link of the slider-crank mechanism can be represented as a screw with six coordinates. Here we call the screw as and inertial screw. The inertial screw is composed by one unitary vector with six coordinates and an scalar representing its magnitude. The magnitude is the properly inertial force, as estimated by the second Newton law: $\Psi = -m\,a$.

The negative in the inertial force is written in the vector of six coordinates. Written the inertial force in each axis of the coordinate system O_{xyz}, the inertial screw is

$$\$^{in} = \begin{Bmatrix} \boldsymbol{M}^{in} \\ -- \\ \boldsymbol{R}^{in} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{S}_0 \times \boldsymbol{R}^{in} \\ --- \\ f_x^{in} \\ f_y^{in} \\ f_z^{in} \end{Bmatrix} = \begin{Bmatrix} m\,(z_i\,a_y - y_i\,a_z) \\ m\,(x_i\,a_z - z_i\,a_x) \\ m\,(y_i\,a_x - x_i\,a_y) \\ -------- \\ -m\,a_x \\ -m\,a_y \\ -m\,a_z \end{Bmatrix}$$

$$\$^{in} = \begin{Bmatrix} 0 \\ -z_i \\ y_i \\ -- \\ -1 \\ 0 \\ 0 \end{Bmatrix} m\,a_x + \begin{Bmatrix} z_i \\ 0 \\ -x_i \\ -- \\ 0 \\ -1 \\ 0 \end{Bmatrix} m\,a_y + \begin{Bmatrix} -y_i \\ x_i \\ 0 \\ -- \\ 0 \\ 0 \\ -1 \end{Bmatrix} m\,a_z \tag{9}$$

As the slider-crank mechanism is in the planar case ($\lambda = 3$), it is possible to eliminate the rows 1, 2 and 6 of the vector. The set of inertial screws and wrenches it is written in the cutset matrix $\{Q_A\}_{k,C}$. The number of cut-sets k is 3 and the gross network degree of freedom C is 17, so the cutset matrix will be in the form $\{Q_A\}_{3,17}$. For brevity, the cut-set matrix it is not showed here.

Using the cut-set law the algebraic sum of the normalized wrenches that belong to the same cut must be equal to zero [5]. So it is possible to obtain the action unit matrix $[\hat{A}_N]_{\lambda k,C} = [\hat{A}_N]_{9,17}$. Using as primary variables the magnitudes of the inertial screws, and solving the action unit matrix using the Gauss-Jordan elimination method, the solution of the system provides the following equations,

$$R_x = -F_{in_{1_x}} - F_{in_{2_x}} - F_{in_{3_x}} - F_{in_{4_x}} \tag{10}$$

$$R_y = -F_{in_{1_y}} - F_{in_{2_y}} - F_{in_{4_y}} \tag{11}$$

Equations 10 and 11 describe the shaking forces at the frame of the slider-crank in function of the inertial forces acting in each link. In the next section, the construction of the objective function to minimize these shaking forces will be explained.

7 Objective Function

The strategy used in this paper to solve the optimization problem was the Differential Evolution (DE) proposed in [12]. The objective function to the problem of minimization of shaking forces in the slider-crank was built using three variables. The first variable is the mass of the counterweight m_4. The two others variables describe the position of the counterweight relative to the coordinate system O_{xyz}. r_2 is the length of the link counterweight and γ is the angle between the crankshaft. Here the objective function is written in the way to minimize the force (R) of the shaking force,

$$minimize : R(m_4, r_2, \gamma) = R = min(\sqrt{R_x^2 + R_y^2}) = \dots$$
$$\dots = \sqrt{-F_{in_{1_x}} - (m_A - \overline{m}_b)\, a_{2_x} - F_{in_{3_x}} - F_{in_{4_x}}}$$
$$+ \sqrt{-F_{in_{1_y}} - (m_A - \overline{m}_b)\, a_{2_y} - F_{in_{4_y}}} \tag{12}$$

Equation 12 shows that the two components of the shaking force are written in function of the optimization variables (m_4, r_2, γ). To solve the problem it is necessary to write a set of inequality constraints for each optimization variables, indicating the limits of each variable. For this problem the inequality constraints is,

$$subject\,to: \quad g_1(m_4): \quad 0 \leqslant m_4 \leqslant m_{4_{max}} \tag{13}$$
$$g_2(r_2): \quad 0 \leqslant r_2 \leqslant r_{2_{max}} \tag{14}$$
$$g_3(\gamma): \quad 0 \leqslant \gamma \leqslant 2\pi \tag{15}$$

8 Results

The methodology is validated by calculating the shaking forces at the frame of the slider-crank using Matlab®. The data used for the slider-crank is: $r = 0.4$ [m], $l = 0.15$ [m], the masses and moment of inertia of the crank, connecting rod and slider, respectively $m_1 = 0.5$ [kgf] and $J_{S_1} = 1.8e^{-5}$ [kg m^2], $m_2 = 0.1$ [kgf] and $J_{S_2} = 9.50e^{-7}$[kg m^2], $m_3 = 0.5$ [kgf] and $J_{S_3} = 5.05e^{-7}$[kg m^2]. The angular velocity is $\omega_a = 300$ [rpm]. Figures 6 and 7 shows the shaking force R_x and R_y at the frame of the slider-crank without counterweight. Figures 8 and 9 shows the maximum force and the forces R_x and R_y for the case when the counterweight is add to the crankshaft.

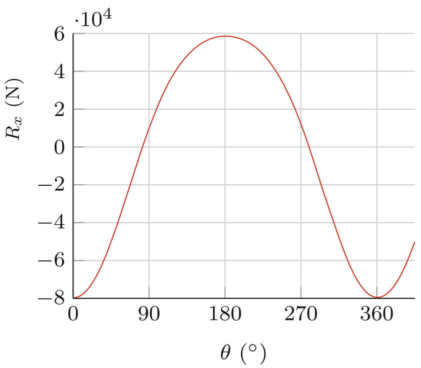


Fig. 6. Unbalanced shaking force R_x

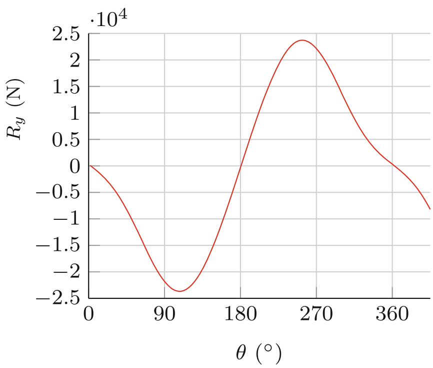


Fig. 7. Unbalanced shaking force R_y

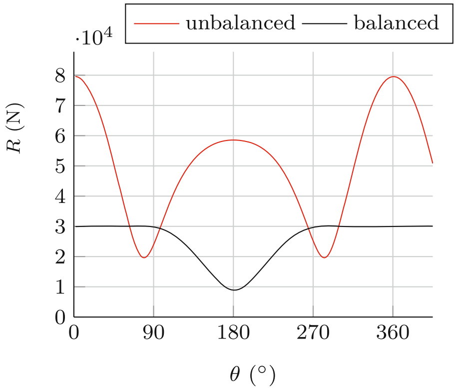


Fig. 8. Maximum shaking force

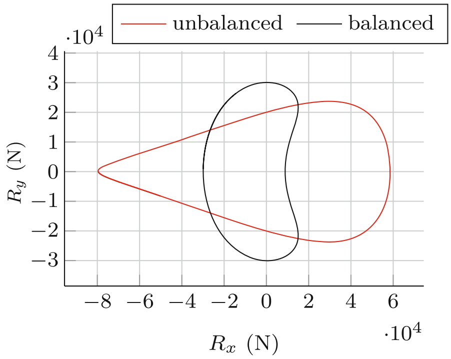


Fig. 9. Shaking forces Rx and Ry

9 Conclusion

In this paper a new methodology to derive the equations of the shaking forces in a slider-crank mechanism by using the Davies' method was presented. The results using the method show that the equations are obtained with a minimum effort. The method shows a great potential to be applied in spatial mechanisms and also to include external forces within the model. Compared to another methods that use Fourier analysis this method treats the shaking forces as a unique approximation. Another advantage is that the approach is formulated as a general mathematical optimization problem. Future works involve including other variables in the optimization algorithm, such as dimensions and weight of the connecting rod.

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