

# Modelling of Pot Bearings – A Preliminary Study

Mahir Ülker-Kaustell<sup>1</sup>(✉), Gabriel F. Boschmonar<sup>2,3</sup>,  
Pablo B. Isusi<sup>2,3</sup>, Stefan Trillkott<sup>2</sup>, Claes Kullberg<sup>2</sup>,  
and Raid Karoumi<sup>2</sup>

<sup>1</sup> Tyréns AB, Stockholm, Sweden

mahir.ulker-kaustell@tyrens.se

<sup>2</sup> KTH, Royal Institute of Technology, Structural Engineering and Bridges,  
Stockholm, Sweden

<sup>3</sup> Polytechnic University of Madrid, E.T.S.I., Caminos, Canales y Puertos,  
Madrid, Spain

**Abstract.** Previous research indicates that roller and pot bearings may give rise to considerable non-linear effects in certain bridges. These effects appear as variations in natural frequency, mode shape and modal damping ratio, depending on the amplitude of vibration. At small amplitudes of vibration, it seems reasonable to assume that the rolling or sliding mechanism is inactive, thus yielding a stiffer structure and no additional dissipation of energy due to friction. At slightly larger amplitudes of vibration, although still remaining small with respect to geometrical non-linearity, the rolling or sliding mechanism activates, whereby the corresponding constraints are relieved. At the same time, because of the rolling or sliding friction, a certain amount of energy dissipates to the surroundings. In order to improve our understanding of these mechanisms and their practical implications, a preliminary experimental study has been performed with the aim of developing a simple model of these mechanisms, which can be included in theoretical models of bridges and other structures. In this paper, we give a short description of the outcome of our laboratory tests and the status of our model development process.

**Keywords:** Bridges · Pot bearings · Friction · Laboratory tests · Non-linear modes of vibration

## 1 Introduction

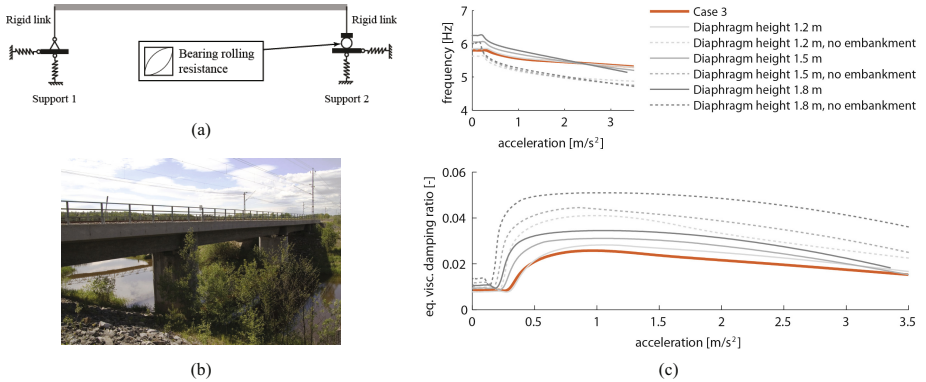
This preliminary study was motivated by an analysis of the free vibrations after passing trains. Rebelo et al. [1] found a shift in the natural frequency of a ballasted railway bridge when they used the short time Fourier transform to analyze their free vibration data. A faster analysis, including an estimate of the instantaneous equivalent viscous damping ratio was obtained by applying the wavelet transform to the free vibration signals [2]. This analysis revealed that the natural frequency of the fundamental mode of vibration was reduced as the amplitude of vibration increased, while the modal damping ratio increased.

The dynamic properties of civil engineering structures are often analyzed under the assumption that the structural response is linear. This holds true in most applications in the serviceability limit state, but there are conditions under which this assumption does not hold true. One such condition is the presence of friction mechanisms such as pot bearings, spherical bearings, disk bearings and roller bearings. These mechanisms all have in common that there exists a limit force, which must be overcome in order to activate the sliding or rolling mechanism. Thus, for responses which are too small to activate the sliding or rolling motion over moveable bearings, the dynamic properties may differ substantially from those obtained under responses which are large enough to activate the motion.

A simply supported beam bridge will serve as an example. Figure 1(a) shows a model of a simply supported bridge consisting of an Euler-Bernoulli beam, resting on some form of (more or less) flexible supports. Due to the eccentricity between the elastic line of the beam and the bearings, the dynamic properties of the beam will be highly dependent on the ability of the moveable bearing to move [3, 4]. If the friction force in the bearing is not overcome by the internal forces, the beam will be clamped and if it is, the beam will be (close to) simply supported. This will influence the modes of vibration and their frequencies. Furthermore, when the motion over the bearings is active, a certain amount of frictional damping will contribute to the overall damping of the structure. That this contribution to the overall damping is in fact relevant can be shown by simple calculations, e.g. by comparing the strain energy in the beam, at given amplitude of vibration, to the work performed by the friction forces in the sliding bearings. Figure 1(b) shows a photo of a continuous 3-span post-tensioned concrete beam bridge. From the free vibrations, only very small variations in the dynamic properties were found. However, when a simple hysteretic bearing model based on the Bouc-Wen model, was used to simulate free vibrations of larger amplitudes of vibration, the results shown in Fig. 1(c) were obtained. Case 3 is the best fit of the Bouc-Wen model used model the bearings to the available experimental data. The rest of the curves in Fig. 1(c) are the result of a small parametric study, reported in reference [4]. These results have justified further studies within this field and have the following implications when analyzing experimental data from existing structures:

1. Estimates of natural frequencies can be overestimated since we often use free vibrations from traffic loads or ambient sources of excitation, which only produce relatively small amplitudes of vibration.
2. Estimates of the mode shapes may deviate substantially from the expected.
3. Estimates of modal damping ratios can be underestimated.

In model updating procedures, one may need to compensate for these effects. In terms of stiffness, the extreme cases of either sliding or not sliding may serve as useful approximations. However, in order to quantify the additional damping and exploit it, in a safe and robust way, for assessments of existing bridges and in design of new bridges, more detailed knowledge is required. Therefore, this investigation aims at defining a simple bearing mechanism element, suitable for use in finite element models of bridges and other structures. A non-linear spring-dashpot element based on the Bouc-Wen model [5, 6] has been chosen as the basis for such a bearing mechanism element.



**Fig. 1.** (a) Principles for 2D models of beam bridges. (b) The Sagån bridge. (c) Theoretical natural frequency and equivalent viscous damping ratio of the fundamental mode of the Sagån bridge (see reference [4]).

## 2 Steel-PTFE Contact Interfaces

Most modern bearing types, i.e. pot bearings, spherical bearings and disk bearings, rely on Polytetrafluoroethylene (PTFE), to provide bearings with a translational capacity either in one or in two directions. Bridge bearings have not been studied so much in the context of structural dynamics, but naturally, seismic devices have been studied rather extensively in this context. In particular, Dolce et al. [7] have performed some rather extensive studies on the kinetic coefficient of friction of steel—PTFE interfaces. From these studies, one may conclude that the steel-PTFE contact interface is highly dependent on the following three factors:

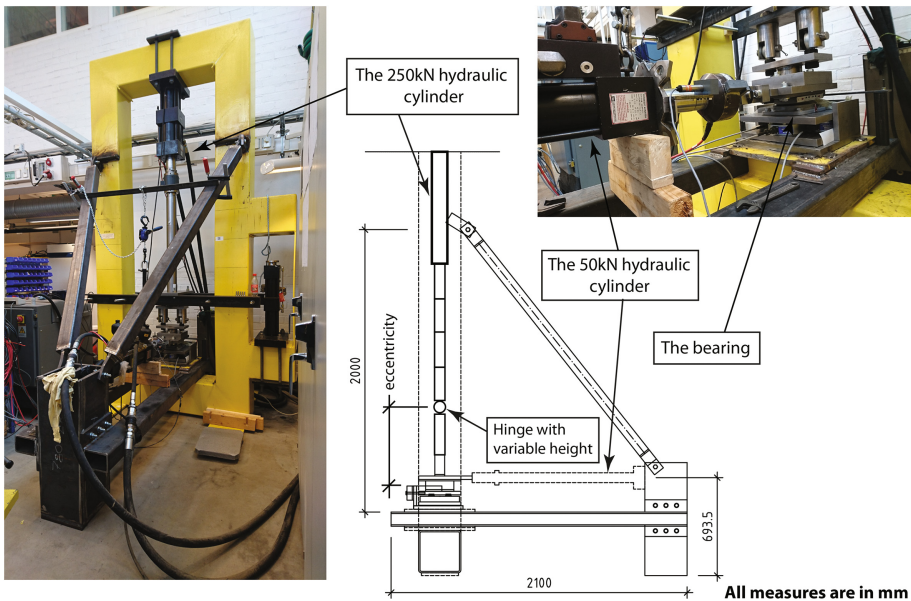
1. Sliding speed (the coefficient of friction increases asymptotically with increasing sliding speed).
2. Contact pressure (the coefficient of friction decreases with increasing contact pressure).
3. Contact temperature (the coefficient of friction decreases with increasing contact temperature).

Apart from the above listed factors, there exist some cyclic effects which tend to reduce the coefficient of friction as the number of cycles increase. This effect is more pronounced for higher sliding speeds which imply that it is somehow related to the heat generated at the steel-PTFE interface. More recently, Lomiento et al. [8] have provided a friction model which accounts for the cyclic effects reasonably accurately. In the present context however, it is assumed that the cyclic effects can be discarded. The influence of break-away and stick-slip is also discarded.

### 3 Laboratory Tests

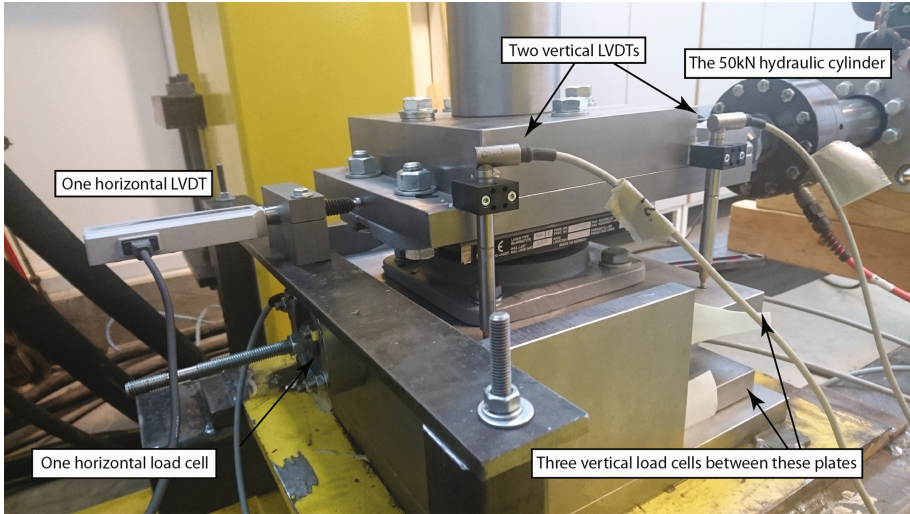
We have performed some preliminary tests with the purpose of obtaining a direction for the development of a model of bearing mechanisms with steel-PTFE sliding contacts.

The test rig (see Fig. 2) was designed for the smallest available commercial pot bearing (provided by KB Spennteknikk, [www.spennteknikk.no](http://www.spennteknikk.no)). We mounted the bearing in a stiff box, facilitating three vertical load cells under the box to measure the vertical force and the retaining moment of the bearing and one horizontal load cell to measure the friction force. A number of displacement transducers were used to measure the horizontal displacement of the bearing and the vertical displacements at two different locations to measure the rotation of the bearing. The vertical load was applied using a 250 kN hydraulic cylinder and a 50 kN hydraulic cylinder was used to apply the motion in a displacement-controlled cyclic test-procedure.



**Fig. 2.** Description of the test rig used for the laboratory tests.

The tested bearing (TOBE, Type E10) is shown in Fig. 3. The figure also shows how the bearing was instrumented with load cells and how we mounted the bearing in order to be able to measure the vertical force, the moment around the transversal axis, the horizontal displacement and the horizontal force over the bearing. The area of the PTFE plate was  $A_{\text{PTFE}} = 9859 \text{ mm}^2$ . Therefore, the maximum contact pressure is  $p = 250 \text{ kN}/A_{\text{PTFE}} = 25 \text{ MPa}$  although we chose to limit the applied vertical load to 200 kN. Typically, the PTFE stress is limited to around 25–30 MPa which is near the ultimate compressive stress for PTFE at room temperature. However, in the present context, where a serviceability limit state is considered, this is not a severe limitation.



**Fig. 3.** A photograph of the bearing as mounted in the test rig.

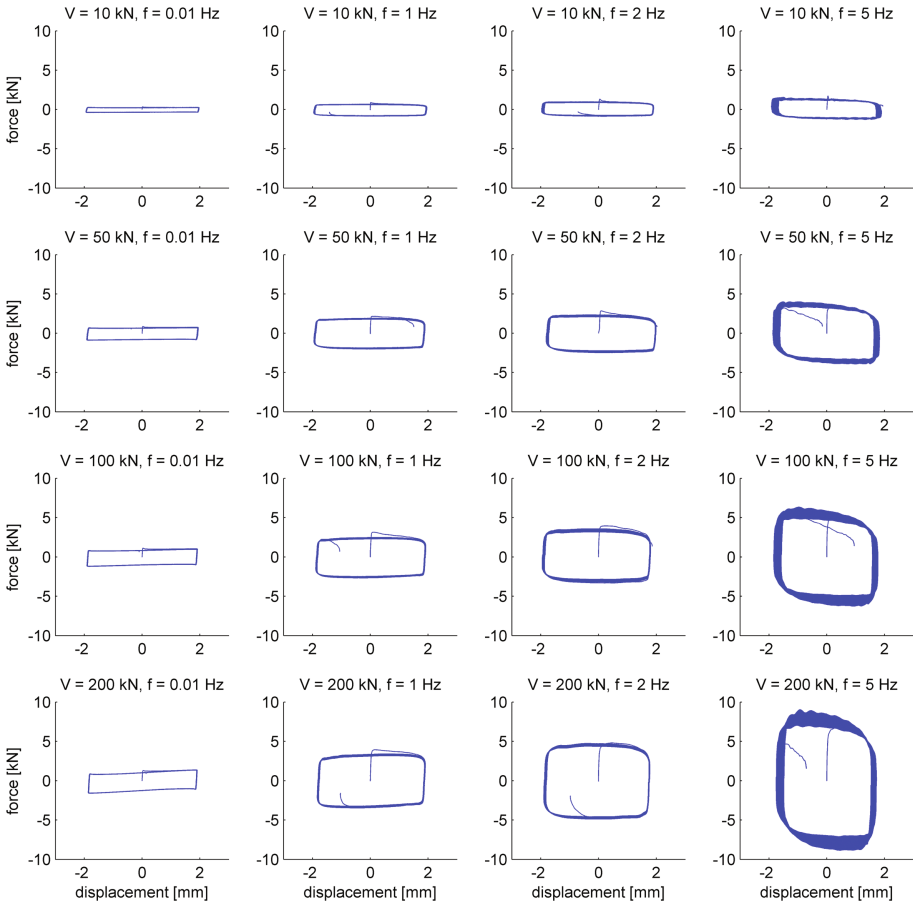
The available pump could produce fairly consistent results for load frequencies  $f < 5$  Hz. For faster motions, the pump required a large number of cycles to reach the programmed amplitude of the motion, as can be seen in Fig. 4.

Although limited in many ways, the tests were judged to provide us with the information we need to motivate further studies. Given the limitations in vertical load (contact pressure) and load frequency (sliding speed), a test series consisting of four discrete force values; 10, 50, 100 and 200 kN and 4 discrete load frequencies; 0.01, 1, 2 and 5 Hz was chosen. Thus sixteen combinations of vertical force and load frequency are obtained for each value of the eccentricity.

A uni-directional bearing is unstable in the direction of the bearing motion, since only the friction force in the bearing provides lateral support for the structure that applies the vertical load. This was dealt with by stabilizing the system using the horizontal cylinder that was used to apply the motion over the bearing. For the same reason, the test must be displacement-controlled.

The relation between the horizontal displacement and the rotation over the bearing is highly dependent on the eccentricity between the elastic line of the structure and the rotation center of the bearing. This distance is typically in the order of 1 m. If the eccentricity is small, the influence of the bearing friction on the dynamic properties of the bridge appears less pronounced than if the eccentricity is large [2]. In the tests performed this far, only one eccentricity has been studied, namely  $e = 1$  m. The reason for this is that as the eccentricity increases, the bearing mechanisms approaches a linear sliding mechanism since the rotation for a given displacement amplitude is reduced. In the vertical section shown in Fig. 2, it is illustrated how the eccentricity of the bearing can be varied in the test rig.

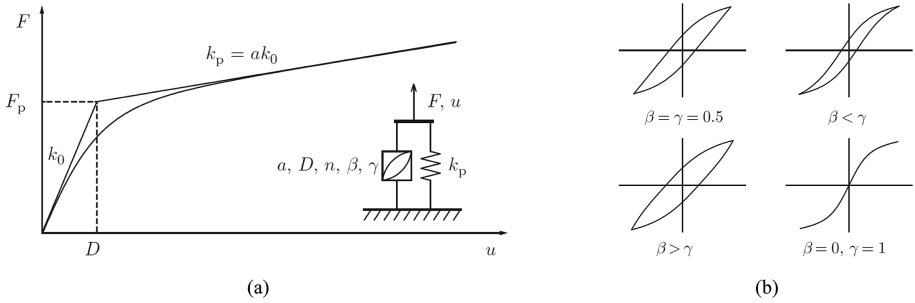
Figure 4 shows one set of hysteresis loops obtained from the tests with the displacement amplitude 2 mm and an eccentricity of 1 m. The tests were performed twice,



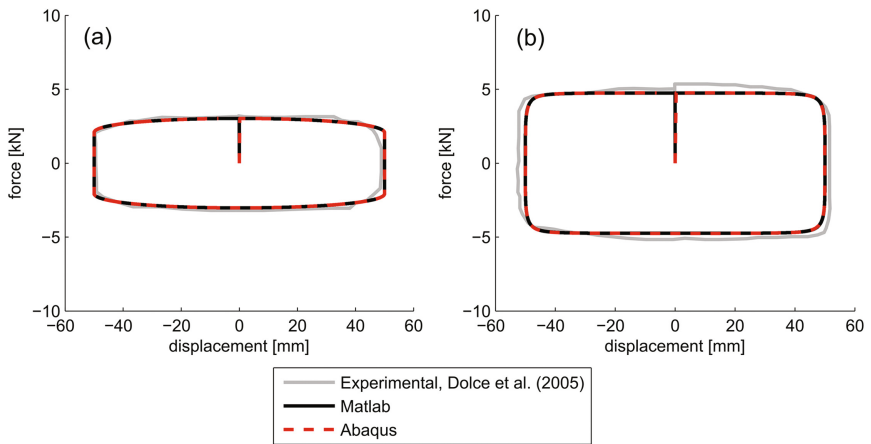
**Fig. 4.** Hysteresis loops for a uni-directional pot bearing.

directly after each other, for each parameter set (horizontal load frequency and vertical load), because the amount of break-away friction depends on the time between the tests (Fig. 5).

From the test performed by Dolce et al., we know how a linear sliding device based on a steel-PTFE contact behaves. The bearing has two features that differ from a linear sliding device; the rubber, which allows the sliding surface to rotate and a guide, which provides a uni-directional motion. The hysteresis loops of a linear sliding device are close to doubly symmetric (see Fig. 6.) if one ignores the break-away effect in the first cycle. The stick-slip behavior is not very strong. The results shown in Fig. 4 show that the rotation of the bearing leads to a small, approximately linear increase in the friction force, causing the loops to be sheared so that the part of the loop that resides in the first and the fourth quadrants move upwards, while the part of the loop that resides in the second and third quadrants move downwards. However, as the load frequency increases, the loops seem to be sheared in the opposite direction.



**Fig. 5.** The force-displacement relationship (a) and the loop shapes (b) of the Bouc-Wen model.



**Fig. 6.** Comparison between the Bouc-Wen-Dolce-model (black, solid lines: Matlab; red, dashed lines: Abaqus) and experimental results from Dolce et al. [7] (grey solid lines). (a) Maximum sliding speed 15 mm/s. (b) Maximum sliding speed 316 mm/s.

Increasing the load frequency increases the friction force, smoothens the shape of the loop and somewhere between 2 and 5 Hz, the effect of stick-slip becomes clear. However, since stick-slip is dependent on the flexibility of the sliding mechanisms and its surrounding structures, the observed stick-slip behavior must somehow be related to actual structural systems with pot bearings, i.e. this effect can be more or less pronounced in a bridge structure. As stick-slip is likely to increase wear, further studies along this line may improve bridge designs with respect to bearing maintenance.

At low sliding speeds, increasing the vertical load increases the friction force, the break-away friction and the slope of the plastic part of the hysteretic loops. The loops become sharper and the stick-slip becomes more apparent. At higher sliding speeds, assuming that the recorded loops are correct although the displacement amplitude does not reach the programmed value, the stick-slip effect is increased and the break-away friction becomes increasingly less pronounced.

## 4 Theoretical Modelling of Pot Bearings

Within this project, two types of theoretical models of pot bearings are studied; 3D finite element models are used to simulate the behavior of the bearings and to learn more about them and simple macro-elements are being developed in order to include the bearing mechanisms in models of structural systems. In the following, our progress in defining a macro element for bearing mechanisms is shortly described.

As the coefficient of friction is dependent on the sliding speed and the contact pressure, the classical models of friction i.e. the laws of Amonton and Coulomb need to be complemented to include these effects. The coefficient of friction decreases quite substantially with increasing contact pressure. Therefore a conservative model should consider the additional contact pressure from e.g. traffic loads. In modern railway bridges, the traffic load can be in the same order of magnitude as the dead weight.

Our own tests and the tests provided by Dolce et al. [7] clearly indicate that the difference between the static and the kinetic coefficients of friction give rise to break-away and stick-slip behavior (see for example [9] and the references therein). In the present context, it is assumed that the stick-slip effect can be neglected since it only causes some small ripples on the hysteresis loops, immediately after the sliding speed (or the force) changes sign. The effect of the break-away force would be to post-poner the motion slightly but typically, the horizontal reaction force resulting from a vertical load on a simply supported beam is considerably higher than the available friction force. As an example, consider the support moment  $M$  of a clamped beam with length  $L$  for a transversal point load  $P$  at the mid-point of the beam,  $M = PL/4$ . Given the eccentricity  $e$ , this moment (on the moveable end of the beam) can also be written  $M = eF_\mu$ , where  $F_\mu$  is the friction force. In order for sliding to occur, we need  $P(1 - 2\mu e/L) > 2\mu mg$ . Here,  $m$  is the distributed mass of the beam and  $g$  is the gravitational constant. Clearly, since  $e \ll L$  and  $\mu \sim 0.1$ , the influence of the external load is negligible. Furthermore, for short to medium span, single track railway bridges, the distributed mass  $m \sim 20000 \text{ kg/m}$ . Assuming a coefficient of friction  $\mu = 0.05$ , the above reasoning gives  $P > 2\mu mg \approx 2 \cdot 0.05 \cdot 1 \text{ m} \cdot 20000 \text{ kg/m} \cdot 10 \text{ m/s}^2 = 20 \text{ kN}$ . Typical axle loads for passenger traffic on modern railways lie around 200 kN. Therefore, an additional break-away friction force of 20–30% will most likely have little effect. However, the dependency of the coefficient of friction on the sliding speed and the contact pressure seem to be highly important. The kinetic coefficient of friction can be modelled with an exponential function [10]

$$\mu = \mu_{\max} - (\mu_{\max} - \mu_{\min})e^{-\alpha v} \quad (1)$$

where  $\mu_{\max}$  and  $\mu_{\min}$  are the temperature and pressure dependent coefficients of friction at high and very low sliding speeds, respectively,  $\alpha$  is a constant depending on temperature, pressure and the condition of the contact interface and  $v$  is the sliding speed. In the following, a generalization of the Bouc-Wen model, which accounts for the dependency on sliding speed and pressure, is proposed.



#### 4.1 The Bouc-Wen Model

The Bouc-Wen model provides a convenient way of modelling hysteretic behavior. This model introduces an internal variable  $z \in [-1, 1]$ , which describes the hysteretic behavior. The internal variable is governed by a non-linear ordinary differential equation:

$$\dot{z} = \frac{\dot{u}}{D} [1 - (\beta + \text{sgn}(\dot{u}z)\gamma)|z|^n] \quad (2)$$

where  $\beta + \gamma \neq 0$  and  $n > 0$  are model parameters. In a single degree-of-freedom system based on the Bouc-Wen model, the force is given by

$$F = ak_0u(t) + (1 - a)Dk_0z(t) \quad (3)$$

Here,  $k_0$  is the elastic stiffness,  $a$  is the ratio between the elastic stiffness and the plastic stiffness  $k_p$  and  $D$  is the plastic displacement limit.

#### 4.2 A Generalization of the Bouc-Wen Model

The hysteretic behavior of the macro-elements is based on a generalization of the Bouc-Wen model. This generalization consists in including the effects of sliding speed and contact pressure in the Bouc-Wen model. For this purpose, we have been using data from Dolce et al. [7] and therefore, we have chosen to refer to this generalized Bouc-Wen model as the Bouc-Wen-Dolce model.

From the results of Dolce et al., we know that:

- The ratio between the elastic stiffness and the plastic stiffness  $a = k_p/k_0 \approx 0$ .
- $k_0 \gg k_p$ .

If the initial stiffness  $k_0$  is assumed to be fixed, the plastic limit displacement  $D$  will have to describe the coefficient of friction. These parameters are related to the plastic limit force  $F_p = k_0D$  which in turn can be related to the friction force by

$$F_p = \mu(v, p) \cdot N = \mu(v, p) \cdot A_{PTFE} \cdot p \quad (4)$$

Thus,

$$D = \frac{F_p}{k_0} = \frac{\mu(v, p) \cdot A_{PTFE} \cdot p}{k_0} \quad (5)$$

We have implemented this in Matlab, using an explicitly integrated, single degree of freedom system and as a general 3D user-defined element in ABAQUS, with one Bouc-Wen element, i.e. a linear spring, a linear dashpot and a Bouc-Wen internal variable corresponding to each degree of freedom (3 translations and 3 rotations). A comparison with some of the data presented by Dolce et al. [7] is shown in Fig. 6 where we used  $n = 1$ ,  $\beta = 0.6$  and  $\gamma = 0.4$ , together with  $a = 1 \cdot 10^{-12}$  and  $k_0 = 30$  GN/m.

## 5 Discussion

This paper aimed at describing the current status of our project. We are beginning to finalize our laboratory tests and in terms of modelling, we have combined the data from Dolce et al. [7] with the friction model from Constantinou et al. [10] and the classical Bouc-Wen model. From the experimental results obtained this far, apart from the fact that more extensive tests on large bearings and also on other bearing types need to be performed, the following observations have been made; The shapes of the hysteresis loops do not vary so much for the studied contact pressures and sliding speeds; Break-away and stick-slip, i.e. effects related to static friction, do not seem very important in the context of studying the dynamic properties of structures. However, this needs to be further studied. When the laboratory tests have been completed, they will be described in a technical report and made public.

By using the Bouc-Wen-Dolce model as a starting point, the remaining steps in defining the sought model do not seem too distant. However, a number of issues remain to resolve in order to define a reliable model for practical purposes:

- Can the Bouc-Wen-Dolce model be adapted to the bearing tests without modification?
- Scale effects. It has to be ascertained that no relevant scale effects exist.
- Annual variations in temperature and freezing can be important factors in cold climates such as the Swedish.
- Other bearing types.
- The influence of the elasticity of the supports. The elasticity of the supports could lead to variations in the stick-slip behavior and reduces the sliding distance over moveable bearings.
- Validation of the model. Full scale testing of bridge structures at relevant amplitudes of vibration appears to be the most robust way to ascertain that the model is correct.

**Acknowledgments.** The authors gratefully acknowledge Tyréns AB and the Swedish Transport Administration for providing the financial support for this project.

## References

1. Rebelo, C., Simões Da Silva, L., Pircher, M.: Dynamic behavior of twin single-span ballasted railway bridges—field measurements and modal identification. *Eng. Struct.* **30**(9), 2460–2469 (2008)
2. Ülker-Kaustell, M., Karoumi, R.: Application of the continuous wavelet transform on the free vibrations of a steel-concrete composite railway bridge. *Eng. Struct.* **33**, 911–919 (2011)
3. Ülker-Kaustell, M., Karoumi, R.: Influence of rate-independent hysteresis on the dynamic response of a railway bridge. *Int. J. Railw. Transp.* **1**(4), 237–257 (2013)
4. Ülker-Kaustell, M.: Essential modelling details in dynamic FE-analyses of railway bridges. TRITA-BKN Bulletin 120, KTH, Royal Institute of Technology (2013)

5. Bouc, R.: Forced vibration of mechanical systems with hysteresis. In: Proceedings of the Fourth International Conference on Nonlinear Oscillations, Prague, 5–9 September 1967
6. Wen, Y.K.: Method for random vibration of hysteretic systems. *J. Eng. Mech. Div. ASCE* **102**(2), 249–263 (1976)
7. Dolce, M., Cardone, D., Croatto, F.: Frictional behavior of steel-PTFE interfaces for seismic isolation. *Bull. Earthq. Eng.* **3**, 75–99 (2005)
8. Lomiento, G., Bonessio, N., Benzoni, G.: Friction model for sliding bearings under seismic excitation. *J. Earthq. Eng.* **17**(8), 1162–1191 (2013)
9. Nikfar, F., Konstantinidis, D.: Effect of the stick-slip phenomenon on the sliding response of objects subjected to pulse excitation. *J. Eng. Mech. ASCE* (2016). doi:[10.1061/\(ASCE\)EM.1943-7889.0001183](https://doi.org/10.1061/(ASCE)EM.1943-7889.0001183)
10. Constantinou, M.C., Mokha, A., Reinhorn, A.M.: PTFE bearings in base isolation: modelling. *J. Earthq. Eng.* **116**(2), 455–472 (1990)