

Evaluating Vibration Serviceability Using Experimental Modal Testing

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Abstract. Predicting the dynamic response of pedestrian structures such as building floors and footbridges to human-induced activity is a complex problem that involves estimation of the dynamic properties of the structure and estimating its response to a loading that varies in intensity and location with respect to time. Even though the problem is complex, researchers have developed guides to aid designers in avoiding vibration serviceability issues by providing simple analytical tools for evaluating a proposed design. The strength of these guides lies in prevention of serviceability problems at the design stage, although they also serve as an informative measure when in-service floors or pedestrian bridges are reported to have excessive vibrations.

Current methods of on-site serviceability evaluation typically involve heel-drop tests to determine natural frequencies and walking tests to record sinusoidal peak acceleration response, generally as response-only single channel measurements. A general understanding of the floor response is achieved this way by looking at the resulting acceleration traces and autospectra resulting from these unmeasured excitations. The most accurate method for estimating the dynamic properties of a structure is experimental modal testing to acquire acceleration frequency response functions (FRF).

This paper proposes a method for evaluation of vibration serviceability using the mid-bay/span driving point acceleration FRFs of low-frequency (<9 Hz) pedestrian structures derived from experimental modal testing. The method proposes using an acceleration limit curve generated from a contemporary design guide to represent a tolerance limit of vibration serviceability. On-site evaluation can be performed with modal testing by comparing the peaks of a measured acceleration FRF with the acceleration limit curve. The method is demonstrated using a set of mid-bay driving point acceleration FRF measurements from an in-situ building floor and comparing with a widely recognized design guide-based acceleration limit curve.

Keywords: Modal analysis · Floor · Human-induced vibration · Evaluation · Serviceability

1 Introduction

Evaluating the vibration serviceability of low-frequency (<9 Hz) pedestrian structures such as building floors and pedestrian bridges subjected to human-induced activity is a complex problem. The first step is the estimation of the static and dynamic properties of a relevant portion of a structure. The second step involves estimating a structure's response to a human loading that varies in both intensity and location with respect to time. Lastly, the predicted response is compared to a human tolerance threshold, which can also vary with frequency, to determine whether or not the vibration level is acceptable. Even though the problem is complex, researchers have developed guides to aid designers in avoiding vibration serviceability issues by providing simple analytical tools for evaluating a proposed design. The strength of these guides lies in prevention of serviceability problems at the design stage, although they also serve as an informative measure when the occupants of an in-service building floor or other pedestrian structure report excessive vibrations and a consultant is asked to evaluate.

Current methods of on-site serviceability evaluation typically involve a series of heel-drop tests to determine natural frequencies and walking tests to record sinusoidal peak acceleration response. This testing is generally accomplished using response-only single channel measurements, from which a general understanding of the floor response is achieved by looking at the resulting acceleration traces and autospectra from these unmeasured excitations. The most accurate method for estimating the dynamic properties of a structure, however, is experimental modal testing. Experimental modal testing measures both the input force and resulting acceleration response to develop an acceleration frequency response function (FRF). This paper proposes a direct method for evaluation of vibration serviceability by comparing measured FRFs of low-frequency pedestrian structures derived from experimental modal testing to an acceleration response tolerance threshold derived from contemporary design guidance.

2 On-Site Evaluation Using Response-Only Testing

The second edition of AISC's *Steel Design Guide Series 11: Vibrations of Steel-Framed Structures Due to Human Activity* [1], hereafter referred to as DG11, has a chapter on evaluation of vibration problems, including recommended vibration measurement techniques. Though experimental modal testing is discussed in the chapter, the nature of on-site serviceability evaluation lends itself to portable testing equipment, and consequently most on-site evaluation is accomplished using equipment that typically only measures the acceleration response to some human-induced excitation such as a heel drop or paced walking. Davis et al. [2] demonstrated a simplified procedure using a portable response-only measurement system for evaluating a problem floor and developing a retrofit solution. A heel drop is an impact force caused by a person assuming a natural stance, maintaining straight knees, shifting their weight to the balls of the feet, rising approximately 65 mm on their toes, and then suddenly relaxing to allow their full weight to freefall and strike the floor with their heels. As a human-induced impulse load requiring no equipment other than sturdy shoes, a heel drop serves as naturally portable and effective input force to a pedestrian structure from

which the acceleration response can be analyzed to determine the natural frequencies. Knowing the frequencies from the heel drop tests, paced walking tests using a metronome at a subharmonic of the structure's dominant frequencies can incite a resonant response from which the accelerations are compared to human tolerance limits. The time-history decay from a heel drop (or bouncing at a resonant frequency) can also be used to estimate damping in the structure, although it has been shown this can result in misleading damping estimates in floors with closely spaced modes [3]. The cost of equipment required for response-only testing is lower than other methods; however a proper serviceability evaluation involving walking tests often requires more than one person and is still time and labor intensive.

3 Experimental Modal Testing

Experimental modal testing, which measures both the input force and acceleration response to construct the acceleration FRF, is unmatched in estimating the dynamic properties of a structure. For low-frequency floors or pedestrian bridges, modal testing allows estimation of the most relevant dynamic parameters such as dominant natural frequencies, damping, and with a set of measurements, the mode shapes of vibration over a tested area. Using modal analysis software, these parameters can be estimated even when modes are very closely spaced, which is often the case in continuous-slab multi-bay floors. Modal testing is typically accomplished to validate or tune finite element (FE) models, as measured frequencies and mode shapes can be used to guide FE model adjustments to bring predicted frequencies and mode shapes into agreement with measured values. Much of DG11's new chapter on FE modeling of floors is based on techniques refined through this type of experimental testing [3–5].

Input force is often applied using an electrodynamic shaker, which provides the most controllable source of dynamic loading to a structure. An electrodynamic shaker has the ability to provide an input force at a relatively constant magnitude and within a very specific frequency range of interest by using a swept sine signal. A swept sine signal, also known as a chirp signal, is a sinusoidal function with a changing frequency over time to provide force input to the floor or pedestrian bridge structure over the specified range of frequencies. The input force generated by the shaker is measured either indirectly or directly. Attaching an accelerometer to the armature mass allows an indirect computation of input force by multiplying the armature's measured acceleration by its mass. Davis et al. [6] cautioned against this indirect method for measuring input force due to shaker-structure interaction, especially for light structures, which can lead to lower quality FRFs that inaccurately predict the peak acceleration values. The preferred method for directly measuring input force from the shaker is accomplished by placing a force transducer between the shaker and the structure. Acceleration response is measured using one or more accelerometers, and with the input force measured as described above, a digital signal processor analyzes the signals to estimate the acceleration FRF.

One of the distinct advantages of experimental modal testing is the ability to estimate the mode shapes of a structure, though this typically requires a considerable number of measurements, leading to extended times required for testing. Obtaining mode shapes

are generally of most interest for validating FE models and not required for the evaluation method presented here. For evaluating vibration serviceability using modal testing, a focus on just acquiring driving point FRF measurements from the middle of each bay/span significantly reduces the time required on site for testing. A driving point FRF is the measurement taken with an accelerometer at the same location as the input force. A mid-bay/span driving point FRF is acquired by exciting the structure at the center of a bay of a floor or span of a pedestrian bridge and has been shown to be the most informative measurement for parameter estimation [9]. The dominant peaks of the driving point FRFs identify frequencies most susceptible to a harmonic of walking excitation, and the magnitude of those peaks directly represent the expected acceleration response per input force applied at that frequency. The mid-bay/span location is the point that most design guides use for their estimation of peak acceleration response for comparison to a human tolerance limit, making the mid-bay/span driving point FRF the ideal experimental modal testing measurement for directly evaluating vibration serviceability.

Contemporary design guidance is not easily compared to modal measurements in the field. The following section develops an accelerance limit curve based on the tolerance limits and walking force representation in DG11, whose evaluation accuracy has been vetted extensively against a large database of problem floors [10]. Once established, an accelerance limit curve could serve as a convenient threshold for direct on-site evaluation using experimental modal testing.

4 Developing an Accelerance Limit

In North America, the leading publication used for evaluation of vibration serviceability of structures due to walking excitation is the previously discussed AISC DG11, which is used to demonstrate the proposed evaluation method in this paper. Though there are various other design guides currently in use throughout the world [11–13], they address vibration serviceability in the same general manner as DG11:

- (1) Estimate the dynamic properties of the floor.
- (2) Estimate a dynamic loading to simulate the applied forces of human activities.
- (3) Compute the acceleration response of the floor for comparison with an established level of acceptability.

Like DG11, other publications offer similar simplified methods for manually computing the dynamic properties of fundamental frequency and effective mass/weight of the floor and provide recommended values of damping. These publications recognize the complexity of representing the forces from human activities such as walking and take the approach of representing this complex loading as a Fourier series. Using the computed properties of the floor and assumed loading, the publications compute an acceleration response to compare to acceptability criteria. For office floors, the acceptability criteria suggested by DG11 is a constant 0.005 g for frequencies between 4 Hz and 8 Hz, although this level can also be conservatively applied to frequencies outside that range. Most publications recognize the stepped nature of the dynamic load coefficients within certain frequency ranges (corresponding to the various harmonics of

step frequency). DG11 offers a simplified exponentially decreasing curve fit expression, $0.83e^{-0.35f}$.

The computed acceleration using DG11 applies at the fundamental frequency, and no consideration is given for the response at (or contribution from) other frequencies. In this respect, using the acceleration FRF differs because it describes the response (and allows evaluation) over a range of frequencies and includes the contribution of other modes.

The fundamental premise of the presented evaluation method, and its ability to represent an acceleration response tolerance limit (serviceability threshold) over a range of frequencies, lies within the definition of the acceleration frequency response. The value of acceleration is best described by:

$$\text{Measured Accelerance FRF Magnitude} = \frac{\text{measured steady state acceleration response}}{\text{measured input force}} \quad (1)$$

Because DG11 defines a steady state acceleration limit for human comfort and estimates the applied loading from walking excitation as a steady state harmonic force, a design acceleration magnitude is defined:

$$\text{Design Accelerance FRF Magnitude} = \frac{\text{steady state acceleration response tolerance limit}}{\text{design input force}} \quad (2)$$

The acceleration FRF is a function of frequency, f . The acceleration response tolerance limit can be taken as a constant value over the frequency range of interest. The input force from walking excitation is a function of frequency and the magnitude of the simulated sinusoidal force is based on the harmonic of walking that will likely correspond with frequency of the floor. Thus, an acceleration limit as a function of frequency, $A_o(f)$, is defined in general terms as

$$\begin{aligned} A_o(f) &= \frac{\text{steady state acceleration response limit}}{\text{design input force}} \\ &= \frac{a_o}{F(f)} \text{ (acceleration units)/(force units)} \end{aligned} \quad (3)$$

Using the form of Eq. (3), an acceleration limit can be developed from design guidance to accommodate both the suggested acceleration tolerance limits and the curve-fit harmonic force representation of walking excitation. In this form, the design guide-based acceleration limit is directly comparable to on-site acceleration measurements from modal testing.

For DG11, the general form of the acceleration limit is:

$$A_o(f) = \frac{a_o}{R\alpha_i P} = \frac{a_o}{P_o e^{-0.35f}} = \left(\frac{a_o}{P_o} \right) e^{0.35f} \quad (4)$$

where:

$A_o(f)$ = accelerance limit as a function of frequency, f , in units of acceleration per unit of input force

a_o = acceleration limit for human comfort

= 0.005g (0.5%g) for office floors

= 0.05g (5%g) for outdoor pedestrian bridges

$\alpha_i = 0.83e^{-0.35f}$ (DG11 simplified dynamic load coefficient)

P = person's weight (taken as 698 N for DG11)

R = reduction factor (taken as 0.5 for office floors, 0.7 for footbridges in DG11)

$P_o = 0.83RP$

The DG11 accelerance limit for an office floor is then:

$$A_{o, \text{ office floor}}(f) = \frac{a_o}{R\alpha_i P} = \frac{0.5\% \text{ g}}{(0.5)(0.83e^{-0.35f})(698 \text{ N})} = \left(\frac{0.5\% \text{ g}}{290 \text{ N}}\right)e^{0.35f} = 0.00173e^{0.35f} \% \text{ g/N} \quad (5)$$

Accelerance limits can also be developed for pedestrian bridges using the same approach, though DG11 suggests using a reduction factor $R = 0.7$ and an acceleration tolerance limit of 0.015 g or 0.05 g for indoor and outdoor pedestrian bridges, respectively. For an outdoor pedestrian bridge, the DG11 accelerance limit is:

$$A_{o, \text{ outdoor footbridge}}(f) = \frac{a_o}{R\alpha_i P} = \frac{5\% \text{ g}}{(0.7)(0.83e^{-0.35f})(698 \text{ N})} = \left(\frac{5\% \text{ g}}{406 \text{ N}}\right)e^{0.35f} = 0.0123e^{0.35f} \% \text{ g/N} \quad (6)$$

The DG11-based accelerance limit curves of Eqs. (5) and (6) are plotted in Fig. 1.

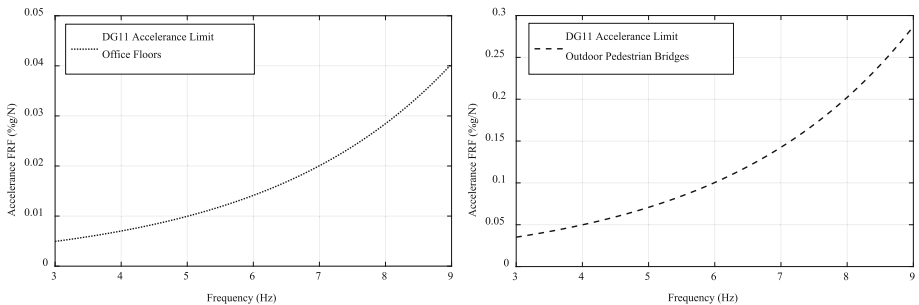


Fig. 1. DG11-based accelerance limit curves

The acceleration limit a_o and forcing terms $\alpha_i P$ presented in DG11 are based on established theories of human tolerance to vibration and the effective harmonic force due to walking, however R represents a catch-all reduction factor used to account for less-than-full steady state resonant response and because the individual walking and the individual subject to vibration are not located at the point of maximum response within a bay. The subjective nature of its suggested value means that other values of R may be considered.

Although the variables a_o , R , and $\alpha_i P$ in the basic expression of Eq. (4) are from DG11, they represent the general terms used in some form within all of the evaluation methods (acceleration limit, reduction factor, and effective forcing amplitude, respectively). The simplified dynamic load coefficient term used by DG11 makes each acceleration limit curve a smooth increasing exponential. Some other methods of evaluation use a stepped dynamic load coefficient (or stepped function with slight slope), and thus the form of the design acceleration “curve” would also be stepped. Although specific terms of the other methods differ from DG11, they are highlighted to demonstrate that they are not insurmountable for developing acceleration limits based on the evaluation method fundamentals.

5 Evaluating Vibration Serviceability Using Experimental Modal Testing

This section demonstrates the evaluation of vibration serviceability of an in-situ building floor using measurements obtained by experimental modal testing and the above-developed acceleration limit curve. Originally presented by Davis [4], the tested floor is part of a four story, 5,100 m² building that was under construction at the time of testing (Fig. 2(a)). Though under construction at the time of testing, the floor was mostly clear of construction material (Fig. 2(b)) and the underside supported only minimal piping and ductwork.

The framing plan for the tested floor is shown in Fig. 2(c). Most bays of the building are 9.14 m square, and the floors were constructed using conventional steel framing beneath 325 mm total thickness composite slabs comprised of normal weight concrete on 50 mm metal deck. Experimental modal testing was accomplished using a 108 kg electrodynamic shaker atop a force transducer to measure input force and an array of accelerometers measuring acceleration response. Driving point measurements (accelerometer and force input at same location) were taken with the shaker located at mid-bay of each of the four numbered bays shown in Fig. 2(c).

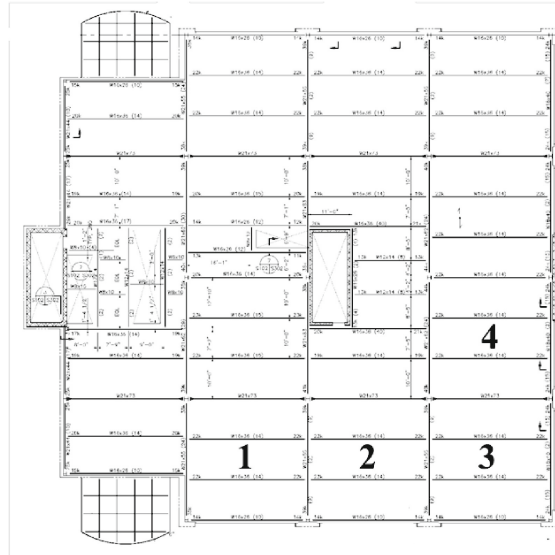
Presented elsewhere in detail [4], FRF curve-fitting of measurements taken with the shaker located in these four bays identified six mode shapes with frequencies ranging from 6.4 Hz to 8.2 Hz and viscous modal damping ratios between 0.51% and 0.61% of critical. The driving point FRFs for the shaker located at the center of each of the four bays are shown in Fig. 3. Also shown on each plot is the DG11-derived acceleration limit curve. A comparison of the acceleration FRF peaks and tolerance limit curve forms the basis of modal testing-based evaluation of vibration serviceability. Peaks that exceed the acceleration limit curve provide an immediate indication of a serviceability problem for the condition of the building at the time of testing.



(a) As-Tested Building Condition



(b) As-Tested Floor Condition



(c) Tested Floor Framing Plan

Fig. 2. Evaluated building

As shown in plots of Fig. 3, this floor is anticipated to have serviceability issues because peaks within the mid-bay driving point FRFs exceeded the acceleration limit curve in three out of the four bays.

- Bay 1's 0.0339% g/N FRF peak exceeds the 0.0297% g/N serviceability limit at 8.125 Hz (Fig. 3(a)).
- Bay 2's 0.0172% g/N FRF peak exceeds the 0.0163% g/N serviceability limit at 6.40 Hz (Fig. 3(b)).
- Bay 3's 0.0265% g/N FRF peak exceeds the 0.0201% g/N serviceability limit at 7.00 Hz (Fig. 3(c)).
- Per Fig. 3(d), Bay 4 should not have a serviceability issue as no FRF peak exceeds the acceleration limit curve. The highest FRF peak, 0.0173% g/N, is below the 0.0233% g/N serviceability limit at 7.425 Hz.

An unsatisfactory serviceability evaluation should not be surprising given the extremely low levels of damping that were measured for this floor under construction (0.5 to

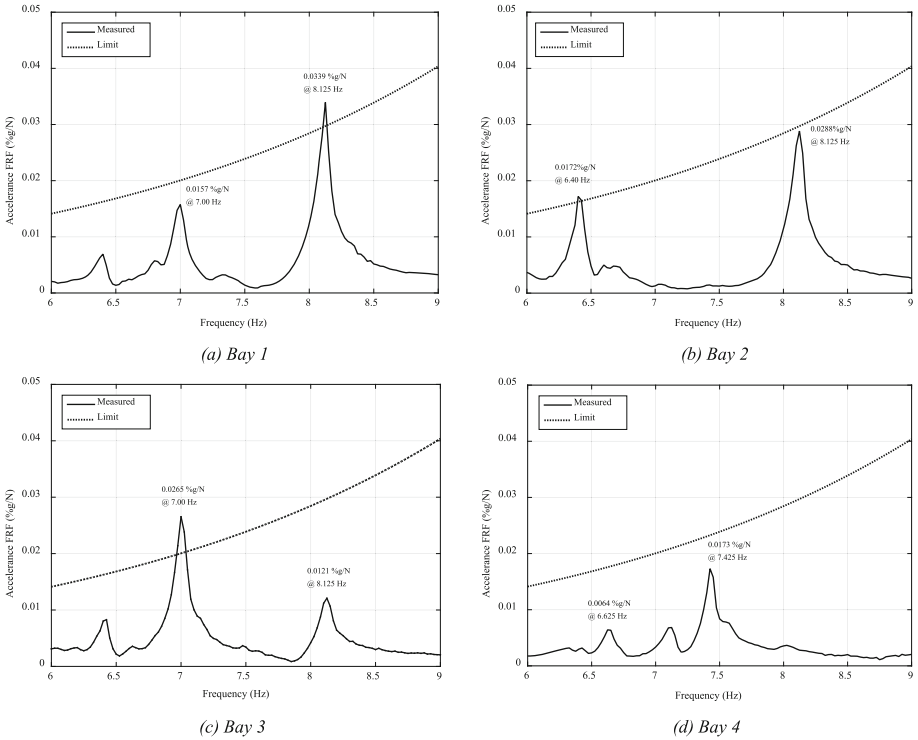


Fig. 3. Mid-bay driving point accelerance FRF magnitudes with accelerance limit curve overlays

0.6% of critical), which was essentially a bare structural system. Even though the approach presented is meant for evaluating serviceability of in-service structures, the floor presented here that was tested while under construction still serves as a useful demonstration of the evaluation method.

6 Comparison of Accelerance-Based Evaluation Results with Walking Tests

In addition to modal testing, walking tests were conducted in each of the four bays by an individual using a metronome set at a pace within the normal range of human walking speeds corresponding to an integer division of a natural frequency identified from the FRF. The upper limit of the normal range of human walking speeds is around 2.2 Hz [1]. The 6.4 to 8.2 Hz range of identified frequencies for the tested floor indicates it could experience a resonant response due to walking at a pace corresponding to the third or fourth subharmonic of one of these frequencies. For Bays 1, 2, and 3, the walking pace frequency selected corresponded to the frequency of the FRF peaks shown to exceed the 0.5% g accelerance limit in Fig. 3(a)–(c). For Bay 1,

a walking pace of 122 beats per minute (bpm) was selected, corresponding to the fourth subharmonic of the 8.125 Hz frequency. For Bay 2, a walking pace of 128 bpm was selected, corresponding to the third subharmonic of the 6.425 Hz frequency. For Bay 3, a walking pace of 105 bpm was selected, corresponding to the fourth subharmonic of the 7.00 Hz frequency. For Bay 4, which did not have an FRF peak exceeding the acceleration limit, a 133 bpm walking pace frequency was selected to correspond with the first notable peak in Fig. 3(d) at 6.625 Hz even though it had a lower magnitude than the dominant peak at 7.425 Hz. This frequency was selected because it could be tested by an achievable pace corresponding to the third subharmonic.

The acceleration response time histories were recorded at mid-bay during each walking test and are shown for each of the four bays in Fig. 4. Tolerance limits of DG11 are in terms of sinusoidal peak accelerations, thus single peak accelerations in measured waveforms are not directly comparable. The calculation procedures for estimating the peak acceleration due to walking in DG11 are based on a single degree of freedom system subjected to a steady state excitation. Thus, a single peak in a measured acceleration record is not the same as was assumed in the development of the criterion. To evaluate typical measured building floor acceleration waveforms, the waveforms are first filtered to 1 Hz to 15–20 Hz (the range that humans feel accelerations). A rolling two-second root mean square (RMS) acceleration is then computed and the center acceleration for each interval converted to peak acceleration by multiplying the RMS value by $\sqrt{2}$ times 100%. The resulting acceleration is referred to as the equivalent sinusoidal peak acceleration (ESPA), which is then compared to the specified tolerance limit for evaluation. The computed ESPA values for each bay are annotated in their respective plots in Fig. 4.

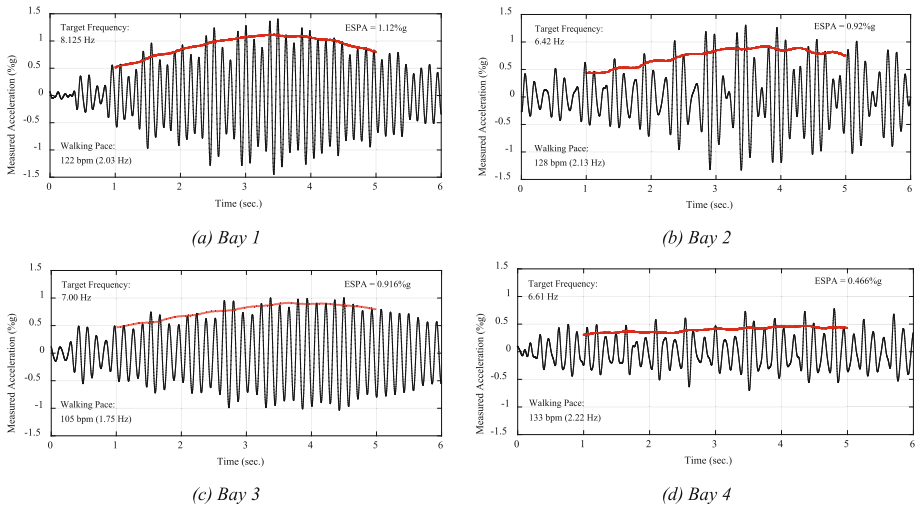


Fig. 4. Paced walking test acceleration time histories

As shown in the paced walking test acceleration waveforms in Fig. 4(a) through (c), Bays 1, 2 and 3 each experienced a resonant build-up, resulting in ESPA values exceeding the 0.5% g serviceability limit. For Bay 4, the walking pace of 133 bpm, corresponding to the third subharmonic of 6.625 Hz, did not result in an ESPA exceeding the 0.5% g tolerance limit as shown in Fig. 4(d). Though not shown, walking tests were performed at a 105 bpm pace, corresponding to the fourth subharmonic of the dominant 7.425 Hz frequency, however these tests resulted in accelerations less than those shown in Fig. 4(d). For all four bays, the results of walking tests agreed with evaluation results obtained by comparing the DG11-derived accelerance limit curve to the mid-bay driving point FRFs obtained using modal testing. Note that the results of these walking tests were only presented to support validity of the accelerance-based evaluation method. In practice, accelerance-based evaluation would alleviate the need to perform walking tests, potentially even making experimental modal testing evaluation quicker than traditional response-only testing and evaluation.

7 Conclusions

This paper described the formulation of an accelerance limit based on a contemporary design guide's acceleration tolerance limits and estimated dynamic loading from human activities. Establishing a tolerance limit in terms of accelerance allows direct on-site serviceability evaluation using the high-quality accelerance FRFs acquired with modal testing. A demonstration evaluating serviceability using an accelerance limit was presented using the floor system of an in-situ building. By comparing of the accelerance limit to experimental modal testing-derived mid-bay driving accelerance FRFs, the floor was shown to exceed recommended limits, which was confirmed from the results of paced walking tests.

It should be noted that there is currently a limited database of high quality modal testing-derived driving point FRF measurements for in-situ floor and pedestrian bridge structures in the published literature. Although the database of floors and pedestrian bridges with these accelerance measurements is limited, the methods of DG11 have been successfully vetted against an extensive database of problem floors evaluated using response-only testing. The accelerance limit presented in this paper is based on DG11 and capitalizes on its prediction accuracy. Though modal testing has traditionally been prohibitive for on-site evaluation, testing focused on only acquiring mid-bay/span driving point FRFs significantly reduces testing time, making this evaluation method comparable to traditional response-only testing. It is possible that future research could make modal testing more accessible, from which the database of pedestrian structures with FRF measurements and their corresponding subjective serviceability evaluations (human surveys of whether the vibration levels are acceptable or unacceptable) would expand. The strength in the evaluation method lies in its ability to be verified by field measurements. As more lively floors are evaluated via modal testing, the calibration of the presented accelerance limit can be verified or adjusted.

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