Evaluation of Vulnerability and Seismic Risk Parameters Through a Fuzzy Logic Approach

Lorenzo J. Cevallos-Torres[(✉)], Alfonso Guijarro-Rodriguez,
Nelly Valencia-Martínez, Jorge Tapia-Celi,
and Wilmer Naranjo-Rosales

Faculty of Mathematical and Physical Sciences, University of Guayaquil,
Guayaquil, Guayas, Ecuador
{lorenzo.cevallost,Alfonso.guijarror}@ug.edu.ec

Abstract. Due to the geographical location of the Pacific fire belt, Ecuador is the scenery of natural phenomena such as earthquakes, which might be the main threat source on national territory and their intensity is likely to cause disasters, to facilitate the identification process of the risk level these disaster cause, we apply techniques of fuzzy logic, because related studies in this area have given more accurate results when working directly with the qualitative values of the data, allowing to deal with the uncertainty of the information on physical and social damages in a given area. For this study, the information was classified considering loss and immediate response scenarios, obtaining variables to which the Analytic Hierarchy Process (AHP) model and the fuzzy set theory were applied using the software ¨Matlab¨ for the evaluation of the results.

Keywords: Earthquakes · Uncertainty · Decision making · Risk · Fuzzy logic · Fuzzy sets · AHP

1 Introduction

Natural disasters, in the case of earthquakes, have been over the last years a very broad discussion topic as these events unleashed great negative effects, especially in those urban regions constituting the basic poles of a city system and densely populated [1]. Because of these continuous telluric movements due to the sliding of the tectonic plates, a high level of uncertainty is induced, followed by great social, environmental, infrastructure consequences, as well as a destabilization in the economy and finally human losses; these factors depend on the intensity of the natural phenomenon [2–4]. Few works have been carried out regarding seismic disasters, amongst which we can find a study carried out by the Geophysical Institute of the National Polytechnic School of Ecuador. It is important to indicate that these works were carried out in an empirical way since prevention methods were used rather than prediction ones to minimize any damage from seismic hazard, which means an accurate seismic risk assessment cannot be achieved. Fuzzy logic yet offers significant advantages over such approaches thanks to its ability to represent qualitative aspects of examination data in a natural way and to apply flexible inference rules [5].

One of the main tools for computational intelligence is fuzzy logic, which is why this tool has been very helpful in the field of seismic engineering, allowing dealing with uncertainty due to imprecision of data, achieving favorable results and providing a wide knowledge. For instance, there are many uncertainties in the assessment of seismic hazard during calculation phases, such as the definition of seismic sources and wave attenuation models that present a greater degree of imprecision in the data, unlike other phases, while these data are more relevant in their result, as shown in [6–9]. Therefore, it has been considered in this paper to use a fuzzy approach with fuzzy sets to evaluate earthquakes and to use it to study the earthquake registered in Ecuador on April 16, 2016, having a magnitude of 7.8 degrees on the Richter scale. [1, 10–12].

2   Fuzzy Logic Techniques

Fuzzy logic techniques, initiated by [13], are used to define processes that are imprecise and ambiguous. Fuzzy sets are used to define the membership of data that don't belong to a particular group, but are rather part of a set. An important point within fuzzy logic and specifically fuzzy sets is the so-called membership function of a set, i.e. determining the fact of "belonging to a set" or "not belonging to a set" in a gradual way, where a membership function of a set A on a universe X is defined as follows: μA: $X \rightarrow [0,1]$ where $\mu A (x) = r$ if r is the degree to which x belongs to A, whose characteristic will have values whose set is $\{0,1\}$, whereas, if it is fuzzy, it will have them in the closed interval of [0,1]. If $\mu A (x) = 0$ the element does not belong to the set, otherwise if $\mu A (x) = 1$ the element does belong completely to the set [14].

Fuzzy logic allows transferring sophisticated precepts from the natural language to a mathematical formulation, which means they give flexibility to modeling using linguistic expressions such as "much", "little", "severe", "scarce", and so on. As an example, we can determine that fuzzy sets are used to define magnitudes observed when an earthquake occurs and that can be considered as "mild", "moderate" or "serious"; otherwise when distances from the epicenter where the earthquake originates, which are "near field", "intermediate field" and "far field". The degree of membership of a set describes the level by which the data belong to that particular group. As for instance, the distance from the source where the earthquake occurs. [15], is given by.

$$r_{close}(x) \in [0, 1] \tag{1}$$

Where $r_{close}(x)$ is the degree of membership that X has in the fuzzy site set "near the source of the earthquake" and x is the distance between the site and the epicenter (3). The fuzzy set of the distance from the source is shown on Fig. 1

Determining seismic sources is very important because of the vagueness degree of the information, since spatial locations of earthquakes can be grouped by fuzzy grouping analysis, making it possible to specify the extent of each seismic source [14, 16].
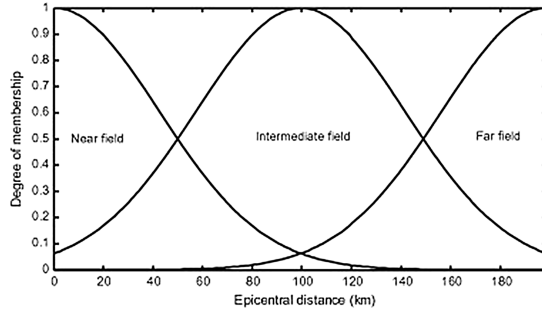
Fig. 1. Distance fuzzy sets

2.1 Fuzzy Sets

Fuzzy sets are collections of elements whose characteristics are defined by linguistic values having a degree of membership between {0, 1} within a discourse universe. The notation defined for fuzzy sets is the one established by Lofti Zadeh, which combines the concepts of logic and the Lukasiewicz set by means of the degree of relationship [13]

$$A = \{(x, u_A(x))/x \in \cup \}$$ (2)

Where A is the ordered pairs of x and the result of the membership function $u_A(x)$ for every element x of the discourse universe U.

2.2 Fuzzy Sets Operations

The basic operations in fuzzy logic and fuzzy sets are Intersection, Union and Complement; these operations are performed in the membership function of the fuzzy sets [17]. The operations of fuzzy sets are used for the following properties (Table 1 and Fig. 2):

Table 1. Fuzzy sets properties

Properties	Definition
Associative	$A \cup (B \cap C) = (A \cup B) \cap C$ $A \cup (B \cup C) = (A \cup B) \cup C$
Commutative	$A \cap B = B \cap A$ $A \cup B = B \cup A \cup$
Involution	$\bar{\bar{A}} = A$
Identity	$A \cap X = A$ $A \cup \emptyset = A$
Morgan law	$A \cap B = \overline{A \cup B} = \bar{A} \cup \bar{B}$

2.3 Membership Functions Characteristics

$$\text{Core}(A) = \{x \in X / u_A(x) = 1\}$$ (3)
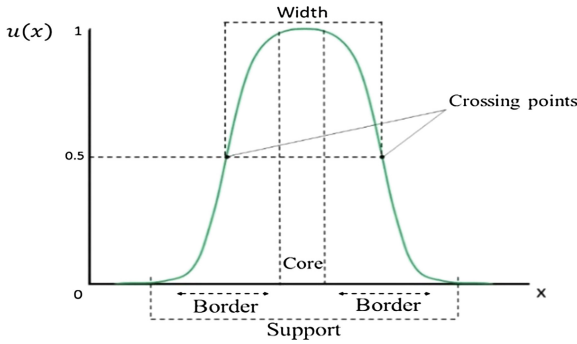
Fig. 2. Membership functions characteristics

$$\text{Border}\,(A) = \{x \in X/0 < u_A(x) < 1\} \tag{4}$$

$$\text{Support}\,(A) = \{x \in X/u_A(x) > 1\} \tag{5}$$

$$\text{Crossing}\,(A) = \{x \in X/u_A(x) = 0.5\} \tag{6}$$

$$\text{Width}\,(A) = |x_2 - x_1| \tag{7}$$

2.4   Integration of AHP with Fuzzy Sets

In order to justify the relationship and the support of the AHP model to the theory of fuzzy sets, we can indicate that a fuzzy inference model, which allowed us to incorporate AHP, to obtain a value of membership with regard to set variables used. Using this methodology, we give account altering the process established by the model of fuzzy inference, i.e. AHP change the numbers employed in the scale of assessment of Saaty (Table 2) by fuzzy numbers, since this usually gets the value of membership by

Table 2. Saaty's evaluation scale.

Saaty scale	
9	A is extremely better than B
7	A is markedly better than B
5	A is better than B
3	A is slightly better than B
1	A is equal to B
1/3	B is slightly better than A
1/5	B is better than A
1/7	B is markedly better than A
1/9	B is extremely better than A
2, 4, 6, 8	**Intermediate values - used in case of evaluation with different judgments**

the cut of the function in the calculation of abscissas While the model AHP does so by its evaluation, allowing that the fuzzy inference only gets a level of risk.

3 Study of the Seismic Case Through the Analytic Hierarchy Process (AHP)

For this research study, we chose the cantons of Manabí that were affected by the earthquake in April 2016, whose epicenter was located in the city of Pedernales. The seismic risk will be assessed from a physical and social damage approach since the degree of seismic threat in those areas is high; in addition, the information obtained from the reports of the SGR (Secretariat for Risk Management) and the INEC (National Institute of Statistics and Censuses), both for damages and for resources used to solve the emergency, are available. It is important to emphasize that during the process to determine the exact number of casualties situation reports were generated by the risk management Secretariat, in his web page (http://www.gestionderiesgos.gob.ec/informes-de-situacion-actual-terremoto-magnitud-7-8/), from the number 1 status report until the number 71, in order to be able to clearly demonstrate the official results of the most relevant variables, which caused inconsistencies during the analysis of weights of importance. The results obtained will help us identify areas of greater risk and thus contribute to the decision making helping to mitigate the seismic threat (Fig. 3).
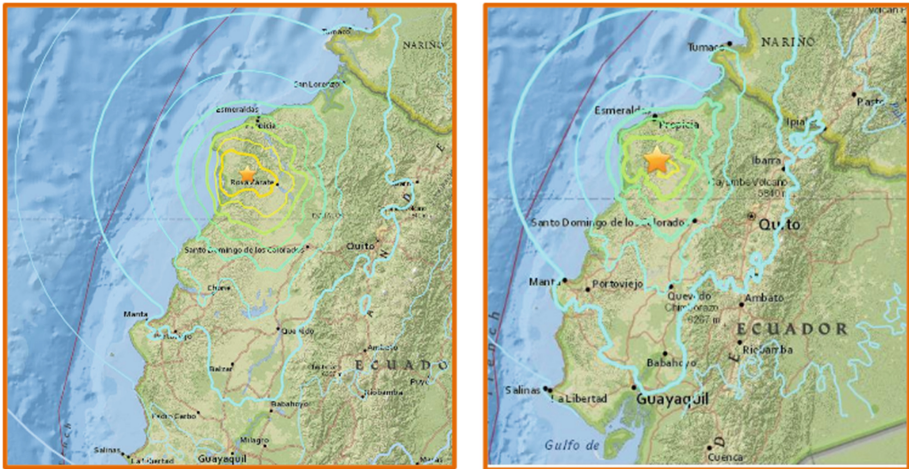
Fig. 3. Seismic intensities registered in the Ecuadorian coastline

The technique to make decisions using the ranking proposed by Saaty (Table 2), it is in essence the reduction of the complexity of the evaluation of multiple attributes, replacing it with a series of comparisons by pairs, which are grouped into a matrix reciprocal positive.

The values in the array will be numeric and the method to guarantee the results imposed by a couple of logical constraints: (i) when you compare an alternative against itself, you are given "equal importance", and (ii) if option A is assigned a number x to compare it with the B option, then when it is compared against the option B option value that will be assigned to this comparison is 1, in our case study when comparing the variable A variable B, for a certain period of time, the variable A, was "more important". In any case this value may vary when analyzing another period of time.

During the process of evaluating seismic risk, the use of fuzzy sets by weighting the input variables through the Analytic Hierarchy Process (AHP) allows us to identify the level of risk in a geographic area with respect to a level of physical risk caused by the telluric event and a level of social aggravation [18]. Fuzzy inference systems help to interpret human knowledge through a set of rules helping the decision making and as an alternative to determine the digital processing of seismic signals, and in this study we decided to use the programming language of "MATLAB", a mathematical calculation platform; it shall allow us to develop fuzzy systems that streamline the process of estimating the total risk of a given area.

The Analytic Hierarchy Process (AHP) is a basic analysis model for decision making allowing to represent a problem through a hierarchical structure in order to establish the criteria importance of a level with respect to a higher level and thus to select the best alternative [19]. The steps to follow are:

3.1   Structuring the Problem

This step is the most important one because the problem must be broken down into components that are relevant to it. The basic hierarchy is established by a goal or objective, criteria and alternatives. The Fig. 4 shows the variables of physical risk and social aggravation used in the total risk calculation process.
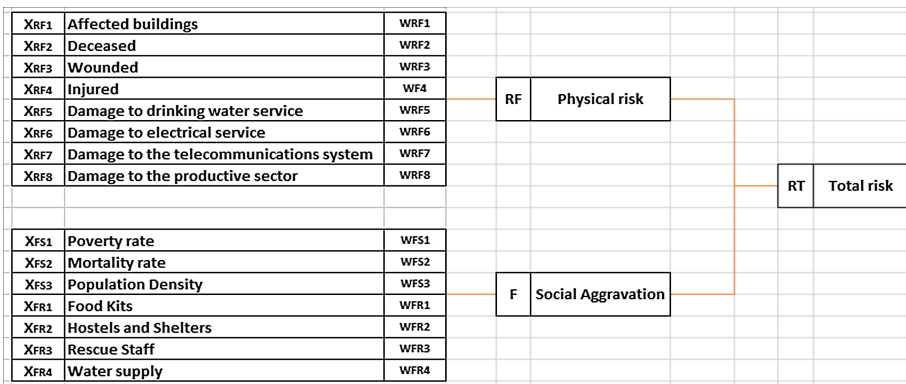
Fig. 4. Calculation of total risk, variables of physical risk and aggravation factors (social fragility and lack of resilience) with their respective weights.

3.2   Construction of the Comparison Matrix

Once the criteria and alternatives have been defined, they are hierarchized through the design of a square matrix called peer comparison matrix that is based on a relative importance where the paired comparison is made between the criteria and alternatives established.

$$A = \begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix} \tag{8}$$

To calculate the priority level between the paired comparisons we use a scale of proportions or intensities denominated by Saaty, shown in Table 2. This numerical scale is used effectively for qualitative interpretation in many applications requiring a weighting of its elements through their homogeneous comparison [19]. In addition, a type of comparison must be considered. The existing ones are:

- Importance: Appropriate when comparing criteria with each other.
- Preference: Appropriate when comparing alternatives.
- Most likely: Used when you compare the probability of the results, either with criteria or alternatives.

3.3   Estimation of Relative Weights

It is being done using the eigenvalues method where weights are assigned amongst n alternatives, to do so, one only needs to perform n-1 estimates.

Once the values of the comparison matrix are calculated, the columns are normalized to 1 by dividing each element by the total sum of the columns. The eigenvector is obtained by calculating the average of each row of the normalized matrix.

$$p = \begin{pmatrix} \frac{1}{n}\sum_{1}^{n} a_{1j} \\ \frac{1}{n}\sum_{1}^{n} a_{2j} \\ \cdots \\ \frac{1}{n}\sum_{1}^{n} a_{nj} \end{pmatrix} \tag{9}$$

Vector of criteria priorities is obtained as shown in 10

$$p = \begin{pmatrix} p_{c11} \\ p_{c12} \\ \cdots \\ p_{c1n} \end{pmatrix} \tag{10}$$

3.4   Results Analysis

The AHP method allows measuring the non-consistency and sensitivity of the judgments by calculating the consistency ratio, to this extent the consistency index must first be obtained.

Calculation of the consistency index.
It is the index measuring the consistency of the comparison matrix, its formula is the following:

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{11}$$

Where n is the size of the matrix and λ_{max} is the eigenvalue, which is obtained through the multiplication of matrices between the elements of the eigenvector and the original matrix, thus obtaining a quotient for each element that we must add and then divide for n elements.

Calculation of the consistency ratio.
It is obtained by dividing the consistency index for an already established random value that depends on n elements used, it is recommended not to use more than nine elements so that the method can maintain consistent (Table 3).

$$CR = \frac{IC}{ICA} \tag{12}$$

According to Saaty, a good accuracy of the consistency value is recommended, with CR less than 0.1, however, values ranging up to 0.2 are mentioned; if the condition is not met, the whole process must be re-run from the comparison matrix until it can have an acceptable consistency.

Table 3. Random consistency according to the number of criteria

	3	4	5	6	7	8	9	10	11	12	13	14	15
ICA	0.525	0.882	1.115	1.252	1.341	1.404	1.1452	1.1484	1.1513	1.1535	1.555	1.570	1.583

In this case several tests for the evaluation, in order to obtain a better accuracy of the values corresponding to each of the variables using the AHP model, with respect to the weights that were used for the evaluation by fuzzy sets, these will indicate the degree of membership of each variable were considering their level of risk. The method gives us an analysis of consistency, which if it is greater than 0.1, will indicate that the evaluation is inconsistent.

Regarding the seismic risk assessment, this methodology will help us obtain the weights of the input variables belonging to the risk factors for physical damage and

social aggravation, they will be used in the application of fuzzy sets during the estimation process. Below are shown the tables of the analytic hierarchy process for the weighting of each group of factors ($F_{RFk}, F_{FSk,}, F_{FRk,}$) shown before on Table 4 (Tables 5, 6, 7, 8 and 9).

Table 4. Comparison matrix of physical risk.

	F_{RF1}	F_{RF2}	F_{RF3}	F_{RF4}	F_{RF5}	F_{RF6}	F_{RF7}	F_{RF8}
F_{RF1}	1	3	3	3	5	5	7	5
F_{RF2}	0,33	1	1	1	3	3	5	3
F_{RF3}	0,33	1,00	1	1	3	3	5	4
F_{RF4}	0,33	1,00	1,00	1	4	4	5	3
F_{RF5}	0,20	0,33	0,33	0,25	1	3	5	3
F_{RF6}	0,20	0,33	0,33	0,25	0,33	1	5	3
F_{RF7}	0,14	0,20	0,20	0,20	0,20	0,20	1	1/2
F_{RF8}	0,20	0,33	0,25	0,33	0,33	0,33	2,00	1

Table 5. Normalized matrix of physical risk.

	F_{RF1}	F_{RF2}	F_{RF3}	F_{RF4}	F_{RF5}	F_{RF6}	F_{RF7}	F_{RF8}
F_{RF1}	0,3646	0,4167	0,4215	0,4265	0,2964	0,2560	0,2000	0,2222
F_{RF2}	0,1215	0,1389	0,1405	0,1422	0,1779	0,1536	0,1429	0,1333
F_{RF3}	0,1215	0,1389	0,1405	0,1422	0,1779	0,1536	0,1429	0,1778
F_{RF4}	0,1215	0,1389	0,1405	0,1422	0,2372	0,2048	0,1429	0,1333
F_{RF5}	0,0729	0,0463	0,0468	0,0355	0,0593	0,1536	0,1429	0,1333
F_{RF6}	0,0729	0,0463	0,0468	0,0355	0,0198	0,0512	0,1429	0,1333
F_{RF7}	0,0521	0,0278	0,0281	0,0284	0,0119	0,0102	0,0286	0,0222
F_{RF8}	0,0729	0,0463	0,0351	0,0474	0,0198	0,0171	0,0571	0,0444

Table 6. Priority Vector and Weight Allocation

Factor	Variables	Weights	Priority vector
F_{RF1}	Affected buildings	w_{RF1}	0,33
F_{RF2}	Deceased	W_{RF2}	0,14
F_{RF3}	Attended people	w_{RF3}	0,15
F_{RF4}	Injured	W_{RF4}	0,16
F_{RF5}	Effect on drinking water service	W_{RF5}	0,09
F_{RF6}	Effect on electric system	W_{RF6}	0,07
F_{RF7}	Effect on telecommunications system	W_{RF7}	0,03
F_{RF8}	Effect on productive sector	W_{RF8}	0,04

Eigenvalue = 8,6185
CI = 0,0884
CR = 0,0629

Table 7. Comparison matrix of social aggravation.

	F_{FS1}	F_{FS2}	F_{FS3}	F_{FR1}	F_{FR2}	F_{FR3}	F_{FR4}
F_{FS1}	1	3	4	1/3	1	1	1/2
F_{FS2}	0,33	1	3	1/5	1/5	1/5	1/3
F_{FS3}	0,25	0,33	1	1/3	1	1/3	1/2
F_{FR1}	3,00	5,00	3,00	1	1	2	2
F_{FR2}	1,00	5,00	1,00	1,00	1	2	1
F_{FR3}	1,00	5,00	3,00	0,50	0,50	1	1
F_{FR4}	2,00	3,00	2,00	0,50	1,00	1,00	1

Table 8. Normalized matrix of social aggravation.

	F_{FS1}	F_{FS2}	F_{FS3}	F_{FR1}	F_{FR2}	F_{FR3}	F_{FR4}
F_{FS1}	0,1165	0,1343	0,2353	0,0862	0,1754	0,1327	0,0789
F_{FS2}	0,0388	0,0448	0,1765	0,0517	0,0351	0,0265	0,0526
F_{FS3}	0,0291	0,0149	0,0588	0,0862	0,1754	0,0442	0,0739
F_{FR1}	0,3495	0,2239	0,1765	0,2586	0,1754	0,2655	0,3158
F_{FR2}	0,1165	0,2239	0,0588	0,2586	0,1754	0,2655	0,1579
F_{FR3}	0,1165	0,2239	0,1765	0,1293	0,0877	0,1327	0,1579
F_{FR4}	0,2330	0,1343	0,1176	0,1293	0,1754	0,1327	0,1579

Table 9. Priority vector and weight allocation

Factor	Variables	Weights	Priority vector
F_{FS1}	Poverty rate	W_{FS1}	0,14
F_{FS2}	Mortality rate	W_{FS2}	0,06
F_{FS3}	Population density	W_{FS3}	0,07
F_{FR1}	Food Kits	W_{FR1}	0,25
F_{FR2}	Hostels and Shelters	W_{FR2}	0,18
F_{FR3}	Rescue staff	W_{FR3}	0,15
F_{FR4}	Water supply	W_{FR4}	0,15

Eigenvalue = 7,78
CI = 0,1307
CR = 0,0975

4   Seismic Risk Assessment - Set Method

For the case study, the theory of fuzzy sets was applied in order to identify the level of seismic risk of a given geographical area, considering risk variables for physical damage and social aggravation extracted from a database provided by the INEC (http://www.ecuadorencifras.gob.ec/institucional/home/) and the Secretariat for Risk Management (http://www.gestionderiesgos.gob.ec/).

Once the variables are established, we proceed to obtain their weights (W_{RFk}, W_{FSi}, W_{FRj}) corresponding to the variables of physical risk and social aggravation (social fragility and lack of resilience) (X_{RFk}, X_{FSi}, X_{FRj}) through the AHP (Analytic Hierarchy Process) method applied in Sect. 4.

With this method, the problem will be structured, i.e. the physical risk factor F_{RFk} and aggravation factor F_{FSk}, then a matrix will be designed for each factor; and then values will be assigned through the Saaty comparison scale.

It is important to define the linguistic values for the calculation of the total risk. For this study five linguistic values will be used (very low, low, medium, high, very high) and using fuzzy sets, one obtains the membership functions of each variable; in this process, it is necessary to define the range of the abscissa next to their grades corresponding to the degree of membership as shown in Table 10.

Next are presented the following nomenclatures:

- L_{RFi} Physical risk level.
- L_{Fj} Aggravation level.
- $u(x)$ Membership function.
- u_{LRF} Membership function of physical risk.
- u_{LF} Membership function of aggravation.

It is essential to establish the membership functions well because the fact of having a good estimate of the total seismic risk depends on them; the variables input values for the evaluation process can be considered of good quality since the fuzzy logic deals with this imprecision in the data. Once the incoming information is established, it is compared with the values of the abscissas previously defined in the membership functions in order to obtain a level of physical risk or aggravation for each variable. This process is called fuzzification and it is used in the methods of inference to obtain a fuzzy value from said comparison.

Following the sequence of the fuzzy inference model, a new output fuzzy set must be generated using the implication where a cut is made, choosing the minimum membership degree between the fuzzy value obtained and the fuzzy set of the output variable. Table 11 shows the membership functions of the variable outputs of physical risk and aggravation.

By using the fuzzy output sets we perform the aggregation through the union operation of the membership functions and then with the centroid method we defuzzify, in order to generate an index and linguistic value of physical risk or aggravation. For these two processes, the following formulas are defined:

Physical Risk

Union

$$u_{RF}(X_{RF}) = \max(W_{RF1}u_{LRF1}(L_{RF1}), \ldots, W_{RFk}u_{LRFk}(L_{RFk})) \tag{1}$$

It is the union (max) of the output sets relative to the physical risk input membership functions, weighted with their weights and risk level of each variable.

Table 10. Code developed on Matlab for the design of membership functions for deceased, poverty rate and hostels/shelter

`%%—> Dead people <—%%`
`%Abscissa`
`rf2 = 0:paso:3000;`
`%Membership function`
`fall_mb = sigmf(rf2, [–0.03 220]);`
`fall_b = gbellmf(rf2, [160 3 400]);`
`fall_m = gbellmf(rf2, [160 3 760]);`
`fall_a = gbellmf(rf2, [160 3 1100]);`
`fall_ma = sigmf(rf2, [0.03 1300]);`
`%Graph`
`subplot(1,1,1), plot(RF, rf_mb, RF, rf_b, RF, rf_m, RF, rf_a, RF, rf_ma,`
`'LineWidth',1)`
`set(gca, 'FontSize',10), legend('Very low', 'low', 'Medium', 'high',`
`'Very high')`
`xlabel('Level of physical risk'), ylabel('\nu(Membership)')`
`axis([0 1 0 1])`	

`%%—> Poverty rate (%) <—%%`
`%Abscissa`
`afs1 = 0:paso:100;`
`%Membership function`
`pob_mb = sigmf(rf6, [–0.5 26]);`
`pob_b = gbellmf(rf6, [8 3 35]);`
`pob_m = gbellmf(rf6, [8.5 3.5 52.7]);`
`pob_a = gbellmf(rf6, [10 3.7 72]);`
`pob_ma = sigmf(rf6, [0.4 82]);`
`%Graph`
`subplot(1,1,1), plot(afs1, pob_mb, afs1, pob_b,`
`afs1, pob_m, afs1, pob_a,afs1,pob_ma,'LineWidth', 1)`
`set(gca, 'FontSize', 10), legend('Very low', 'low', 'Medium', 'high',`
`'Very high')`
`xlabel('Poverty rate (%)'), ylbel('\nu(Membership)')`	

`%%—> Hostels and Shelters <—%%`
`%Abscissa`
`afr2 = 0:paso:100;`
`%Función de pertenecia`
`alre_mb = sigmf(afr2, [–0.9 6]);`
`alre_b = gbellmf(afr2, [6 3.5 13]);`
`alre_m = gbellmf(afr2, [7 3.5 27]);`
`alre_a = gbellmf(afr2, [7 3.5 42]);`
`alre_ma = sigmf(afr2, [0.7 50]);`
`%Graph`
`subplot(1,1,1), plot(afr2, alre_mb,`
`afr2, alre_b, afr2, alre_m, afr2,`
`alre_a, afr2, alre_ma, 'LineWidth', 1)`
`set(gca, 'FontSize', 10), legend('Very low', 'low', 'Medium', 'high',`
`'Very high')`
`xlabel('Numbers of hostels/shelters'), ylabel('\nu(Membership)')`	

Defuzzification

$$.R_F = [\max(W_{RF1}u_{LRF1}(L_{RF1}), \ldots, W_{RFk}u_{LRFk}(L_{RFk}))]_{centroid} \qquad (2)$$

It is the center calculation of the area under the union curve of the membership functions of the physical risk R_F.

Table 11. Code developed on Matlab for the design of membership output functions, physical risk and social aggravation.

|	

`%%— > Level of physical risk < —%%`
`%Abscissa`
`RF = 0:0.01:1;`
`%Función de pertenecia`
`rf_mb = sigmf(RF, [–110 0.075]);`
`rf_b = gbellmf(RF, [0.065 3.9 0.145]);`
`rf_m = gbellmf(RF, [0.075 3.5 0.29]);`
`rf_a = gbellmf(RF, [0.095 4 0.47]);`
`rf_ma = sigmf(RF, [60 0.57]);`
`%Graph`
`subplot(1,1,1), plot(RF, rf_mb, RF, rf_b, RF, rf_m,`
`RF, rf_a, RF, rf_ma, 'LineWidth',1)`
`set(gca, 'FontSize',10), legend('Very low', 'low',`
`'Medium', 'high', 'Very high')`
`xlabel('Level of physical risk'), ylabel('\nu`
`(Membership)')`
`axis([0 1 0 1])`	
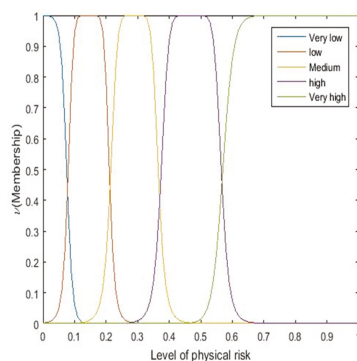
`%%— > Level of social aggravation < —%%`
`%Abscissa`
`A = 0:0.01:1;`
`%Función de pertenecia`
`a_mb = sigmf(A, [–100 0.11]);`
`a_b = gbellmf(A, [0.062 4 0.17]);`
`a_m = gbellmf(A, [0.075 5.2 0.31]);`
`a_a = gbellmf(A, [0.09 4.2 0.48]);`
`a_ma = sigmf(A, [46 0.575]);`
`%Graph`
`subplot(1,1,1), plot(A, a_mb, A, a_b, A, a_m, A, a_a,`
`A, a_ma, 'LineWidth', 1)`
`set(gca, 'FontSize', 10),legend('Very low', 'low',`
`'Medium', 'high', 'Very high')`
`xlabel('Level of social aggravation'), ylabel('\nu`
`(Membership)')`
`axis([0 1 0 1])`	
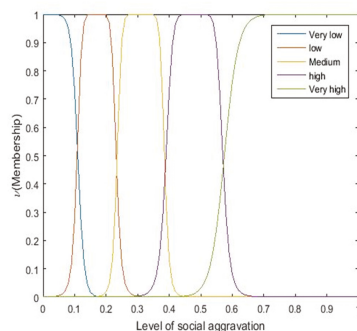
Aggravation

Union

$$u_F(X_{FS}, X_{FR}) = \max\left(W_{FS1}u_{LF1}(L_{F1}), \ldots, W_{FRj}u_{LFj}(L_{Fj})\right) \tag{3}$$

It is the union (max) of the output sets relative to the social fragility and lack of resiliency input membership functions, weighted with their weights and aggravation level of each variable.
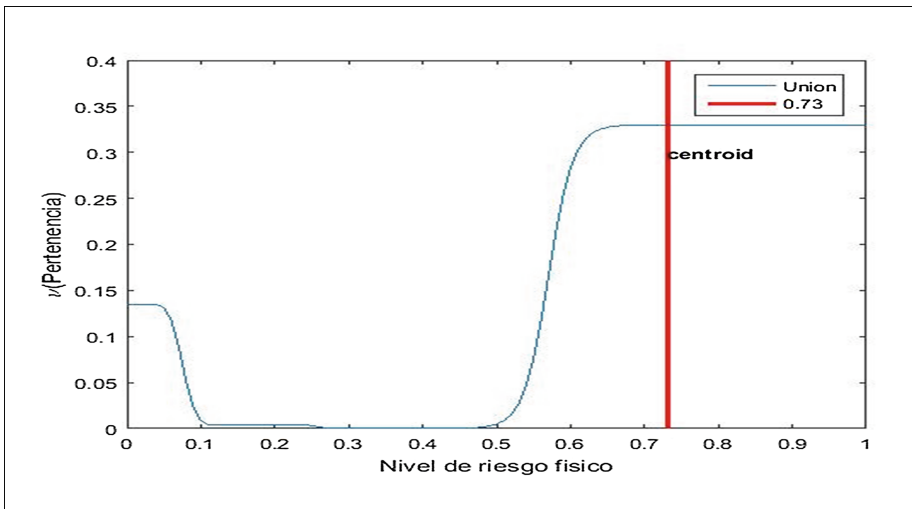
Defuzzification

$$.F = [\max(W_{FS1}u_{LF1}(L_{F1}), \ldots, W_{FRi}u_{LFi}(L_{Fi}))]_{\text{centroid}} \qquad (4)$$

It is the center calculation of the area under the union curve of the membership functions of the aggravation F.

Figures 5 and 6 show the union process of the weighted membership functions with their weights and later defuzzification to obtain the risk index.
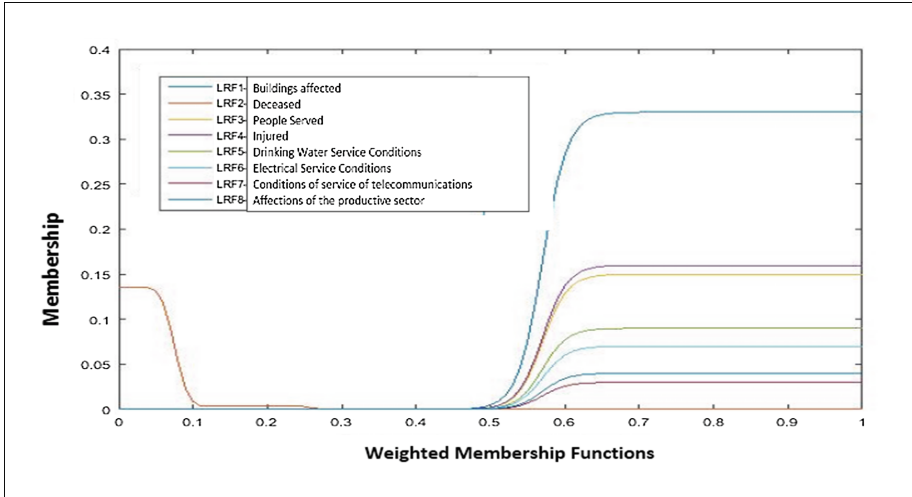
Fig. 5. Weighted membership functions with their respective weights

Fig. 6. Union and defuzzification to obtain physical risk index

Once the physical risk and the aggravation indexes have been calculated, we assign a level to each factor using the linguistic values defined in the membership functions and the comparison of the input values with their respective abscissas. For the final calculation of the total risk assessment, we compare the levels of each factor with the fuzzy rules established in Table 12.

Table 12. Fuzzy rules to estimate total risk.

	Aggravation				
Physical risk	**Low**	**Medium Low**	**Medium high**	**High**	**Very high**
Low	Low	Low	Medium Low	Medium Low	Medium Low
Medium Low	Medium Low	Medium Low	Medium high	Medium high	Medium high
Medium high	Medium high	Medium high	High	High	Very high
High	High	High	Very high	Very high	Very high
Very high	Very high	Very high	Very high	Very high	Very high

4.1 Fuzzy Logic Toolbox - Matlab

Matlab is a mathematical calculation development platform that facilitates the analysis, design and visualization of results. For this study, we used the fuzzy logic toolbox using inference modeling functions, it facilitates the design and simulation of fuzzy systems.

In the estimation of the total seismic risk it shall help us model the membership functions of each variable, as well as execute the inference process (fuzzification, implication, aggregation and defuzzification) allowing us to obtain the resulting indexes. In Fig. 7, we can see that the greatest physical risk are in the cities of Puerto Lopez, Portoviejo, Jama, El Carmen, and the greatest social aggravation is in the cities of Puerto Lopez, El Carmen, Jama, Portoviejo.

Table 13 shows the qualitative value of the total risk obtained through the fuzzy rules established.

Table 13 shows the qualitative value of the total risk obtained through the fuzzy rules. The case in the proposed study, the total risk level is "very high" in almost all of the variables, when comparing the abscissas of the crisp output value from Fig. 6, (which indicates that the rate of physical risk of the canton Portoviejo is 0.73) and Fig. 7 (both physical risk levels as social aggravation of 22 cantons) This indicates the index of the level of risk that will be used in the diffuse tables (Table 12), and in this case it is to us that it is "very high", this same procedure is done for all other situations, where the level of overall risk resulting in "high"; It is worth highlighting that the design of the membership function of fuzzy sets, were analyzed on the basis of reports published on the website of the Secretariat of management of risk, however other potential extension of the proposal is to allow opinions without interaction of several experts and working on a method previously established consensus-based comparisons of experts within the framework of fuzzy systems.
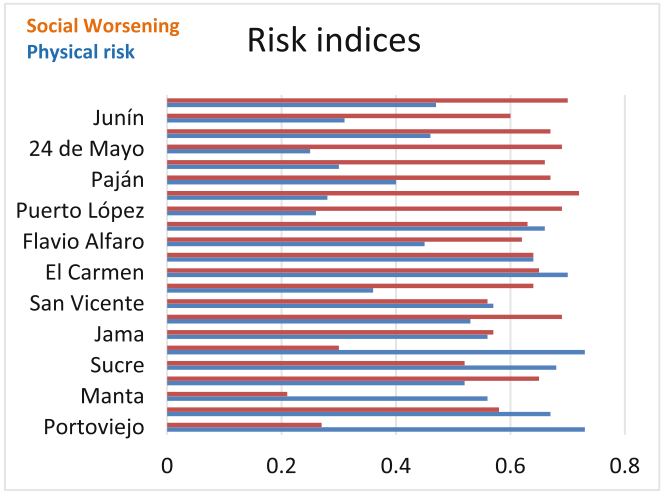
Fig. 7. Indices of the physical risk level and aggravation of the cantons of the province of Manabí.

Table 13. Total risk level of the Manabí province cantons

Canton	Total risk level
Portoviejo	Very high
Chone	Very high
Manta	Medium
Montecristi	Very high
Sucre	Very high
Pedernales	Very high
Jama	High
Jaramijo	Very high
San Vicente	High
Bolivar	Very high
El Carmen	Very high
Rocafuerte	Very high
Flavio Alfaro	Very high
Tosagua	Very high
Puerto Lopez	Very high
Santa Ana	Very high
Pajan	Very high
Jipijapa	Very high
24 de Mayo	Very high
Olmedo	Very high
Junin	Very high
Pichincha	Very high

5 Conclusions

The study focused on the 22 cantons of the province of Manabí, where data of the potential losses suffered by these areas during the earthquake of April 2016 were obtained. The information was classified for physical risk and social aggravation scenarios, the fuzzy sets theory was applied to it, in order to identify the risk levels of each zone.

The physical risk and social aggravation indices of each canton; according to the results shown in Fig. 7, all of the cantons of Manabí were severely affected: with respect to physical damages, the most affected ones were Pedernales, El Carmen and Portoviejo, while the one with the smallest damage is 24 de Mayo. With respect to the aggravation, the biggest situation was seen in Santa Ana, and the smallest was Manta.

Total risk was assessed using fuzzy rules with the risk levels assigned by the calculation of their indices, and just like in the previous analysis, we observed that the situation in the affected areas was of high gravity. The context of material losses was the most influent one for social impact, as it is the case in the Pedernales canton which was totally destroyed, leaving its population in extreme poverty. Others were impacted by the scarcity of resources and basic services that led the inhabitants to stay in shelters until they found a way to recover in a socio-economic way.

Fuzzy set generations allow obtain high quality pattern easily understood. In future research, it is expected to improve the results by delving deeper in the study of data sets.

References

1. Parra, H., Benito, M.B., Gaspar-Escribano, J.M.: Seismic hazard assessment in continental Ecuador. Bull. Earthq. Eng. **14**(8), 2129–2159 (2016)
2. Saaty, T., Vargas, L.: Models, Methods, Concepts and Applications of the Analytic Hierarchy Process, vol. 175, 2nd edn., p. 5. Springer (2012)
3. Martínez, F.M., Sánchez Meca, J., Lopez, J.: The meta-analysis in the field of Health Sciences: an indispensable methodology for the efficient accumulation of knowledge. Physiotherapy **31**(3), 107–114 (2009)
4. Höhle, U., Rodabaugh, S.E.: Mathematics of Fuzzy Sets: Logic, Topology, and Measure Theory, vol. 3. Springer, Heidelberg (2012)
5. Cardona, O.D.: Indicators of Disaster Risk and Risk Management: Program for Latin America and the Caribbean. Summary Report. Inter-American Development Bank (2005)
6. Laasri, E., Akhouayri, E.S., Agliz, D., Zonta, D., Atmani, A.: A fuzzy expert system for automatic seismic signal classification. Expert Syst. Appl. **42**(3), 1013–1027 (2015)
7. Ahumada, A., Altunkaynak, A., Ayoub, A.: Fuzzy logic-based attenuation relationships of strong motion earthquake records. Expert Syst. Appl. **42**(3), 1287–1297 (2015). doi:10.1016/j.eswa.2014.09.035
8. Carreño, M., Barbat, A., Cardona, O.: Numerical method for the holistic evaluation of the seismic risk using the set theory Diffuses. In: Numerical Methods for Calculation and Design in Engineering (2013)

9. Messick, M.A.: Natural disasters in Latin America: The role of disaster type and productive sector on the urban-rural income gap and rural to urban migration. The University of Southern Mississippi (2016)

10. Toya, H., Skidmore, M.: Economic development and the impacts of natural disasters. Econ. Lett. **94**(1), 20–25 (2007)

11. Espinilla, M., Liu, J., Martínez, L.: An extended hierarchical linguistic model for decisionmaking problems. Comput. Intell **27**(3), 489–512 (2011)

12. Sharma, M.L.: Attenuation relationship for estimation of peak ground horizontal acceleration using data from strong-motion arrays in India. Bull. Seismol. Soc. Am. **88**(4), 1063–1069 (1998)

13. Po-Shen, L., Chyi-Tyi, L.: Ground-motion attenuation relationships for subduction-zone earthquakes in Northeastern Taiwan. Bull. Seismol. Soc. Am. **98**(1), 220–240 (2008)

14. Ambrasseys, N., Douglas, J.: Near field horizontal and vertical earthquake ground motions. Soil Dyn. Earthq. Eng. **23**, 1–18 (2003)

15. Wadia-Fascetti, S., Gunes, B.: Earthquake response spectra models incorporating fuzzy logic with statistics Computer-Aided. Civil Infrastruct. Eng. **15**, 134–146 (2000)

16. Coburn, A., Spence, R., Pomonis, A.: Factors determining human casualty levels in earthquakes: Mortality prediction in building collapse. In: Proceedings of the Tenth, World Conference on Earthquakes Engineering (1992)

17. Maldonado, E., Casas, J., Canas, J.: Use of fuzzy sets in seismic vulnerability models. Monograph CIMNE IS-39 (2000a)

18. Maldonado, E., Casas, J., Canas, J.: Models of seismic vulnerability of bridges based on "Fuzzy sets". Monograph CIMNE IS-39 (2000b)

19. Maldonado, E., Casas, J., Canas, J.: Application of fuzzy sets in the evaluation of the parameters of the seismic vulnerability of bridges. International Journal of Numerical Methods for Calculation and Design in Engineering **18**(2), 209–226 (2000)