On Travel Time Functions for Mixed Traffic Systems Dominated by Motorcycles

Tuan Nam Nguyen and Gerhard Reinelt

Abstract Mixed traffic systems dominated by motorcycles (MTSDM) are characteristic for developing countries like Taiwan, Vietnam, The Philippines, Thailand, or India. The present paper investigates a new travel time function including not only the driving time but also the waiting time at the signal line on a link in such a traffic environment. Foremost, a review of popular travel time functions with discussion about their advantages as well as disadvantages when applied in a traffic assignment (TA) model is discussed. Suitable parameters for applying the Bureau of Public Road (BPR) function to an MTSDM are calibrated after introducing a method to estimate the practical capacity of a link based on the certain saturation flow. The travel times calculated by the new BPR function turned out to be highly accurate when compared to the simulated travel time in VISSIM—a popular traffic visualization software. Moreover, the User Equilibrium (UE) model using the new function could forecast traffic flows close to the real flows collected in Hanoi,Vietnam.

1 Introduction

A key ingredient of successful traffic planning is the ability of forecasting the traffic flows on the links of a given traffic network whenever it is attempted to change the traffic system, e.g., by building new links, closing links or installing a new signal light system, etc. Project managers must be able to evaluate in a reliable way which effects on the traffic system are expected to result from the modifications. The problem of forecasting the number of vehicles traveling on each link is known as *traffic assignment problem* (TA) or *traffic assignment modeling* (TAM).

In modern traffic planning, various traffic assignment models have been proposed, such as the All or Nothing (AON) model, the Incremental (ICM) model, the System Equilibrium (SE) model, or the UE model and its variations, etc. Each

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H.G. Bock et al. (eds.), *Modeling, Simulation and Optimization of Complex Processes HPSC 2015*, DOI 10.1007/978-3-319-67168-0_12

model is based on certain assumptions, e.g., about driving behaviors or about traffic infrastructure. One of the most common routing behaviors of drivers is to choose paths regarding to the travel time, that is strongly related to the traffic flow on links on the paths. The relationship between the travel time and the traffic flow on a link is called under different names, such as *travel time function (TTF)*, *travel cost function*, or *speed-flow equation*, etc. The mathematical formula of a travel time function depends not only the geographical characteristics of links but also the characteristics of traffic systems and some others. In this paper, we investigate a travel time function for *mixed traffic systems dominated by motorcycles* (MTSDM) that is characterized by a mixture of various kinds of vehicles with the majority of motorcycles. Our function also takes the possible average waiting time at signal line into account, which can be as complicated as driving time. Our purpose is to develop a simple function for estimating total travel time on a link in an MTSDM that can be easily applied to TAM.

The work flow of the paper is as follows. In Sect. 2 we review proposed travel time functions with the details of the conical function and the BPR function. The method for calibrating the parameters of the BPR function when apply to a link in an MTSDM is explained in Sect. 3. The computational results, including not only the comparison of the new BPR function to the simulated time in VISSIM but also its application to traffic assignment with a UE model, are shown in Sect. 4.

2 Review of Travel Time Functions

The *traffic flow* on a link corresponds to the number of vehicles passing a certain point on the link within a specified time interval, e.g., 12 motorcycles and 3 cars passing the entrance of the link within every 10 s. The *free flow* is a possible traffic flow when vehicles do not interact with each other, i.e., when they can drive as freely as driving without other vehicles. The traffic flow is measured in a *traffic flow unit* that usually corresponds to one vehicle of the most common kind in the traffic system. For example, in a traffic system containing mostly cars, the traffic flow unit should be *Passenger Car Unit* (PCU). The *capacity of a link* is characterized in two ways. The *steady capacity* is the maximum steady-state flow on the link, i.e., the capacity of the point on the link with the minimum capacity. This point is usually the end point (at an intersection) or a bottle-neck point on the link. The *practical capacity* is defined as the maximum flow that can go through the link when there is no dense traffic or congestion.

According to [3], the first research on travel time functions started in the 1920s and Schaar published the first proposal in 1925. Branston gave a good review of travel time functions by 1976 in [1]. His review is briefly summarized in Table 1 with an addition of the *conical volume-delay function* proposed by Spiess [15].

We go to the details of the last two functions in the table since they are widely used in modern traffic assignment models. The *BPR function* was developed by the

Table 1 Overview of some popular travel time functions

Type	Authors	Comment
n-line	Irwin et al. [7]	Simple, but hard to identify without data
Curvilinear	Smock [13]	Expensive in computation
	Soltman [14]	
	Overgaard [11]	
Logarithmic exponential	Mosher [10]	Not for iterative assignment procedures
BPR	Bureau of Public Roads [2]	Simple, easily and quickly integrable
	Steenbrink [16]	Suitable for UE models
Conical	Spiess [15]	Simple, easily and quickly integrable Suitable for UE models

Bureau of Public Roads (US) [2] with the formula as

$$t(x) = T_0\Big(1 + a\,\big(\frac{x}{C_p}\big)^b\Big)\,, \tag{1}$$

where a and b are parameters suggested by BPR engineers to be 0.15 and 4 without any explanation. T_0 is the travel time at free flow, and C_p is the practical capacity of the link. The BPR function is continuous, easily integrable, and simple. These advantages make the BPR function very suitable for applying to traffic assignment models like UE models. But, the default parameters suggested by BPR engineers were questioned by various researchers. In [16] Steenbrink applied the BPR function with new parameters $a = 2.62$, $b = 5$, and he indicated that the new set of parameters is most suitable for the BPR function. However, the data for the paper was collected in regions with low ratio $\frac{x}{C_s}$ of flow and capacity. Nevertheless, Florian and Nguyen [5] showed that the original BPR function with $a = 0.15$ and $b = 4$ generally gives a better travel time estimation than the one proposed by Steenbrink.

In [15] Spiess formulated the *conical volume-delay function* (or *conical function* for short) as

$$t(x) = T_0\left(2 + \sqrt{\alpha^2(1-y)^2 + \beta^2} - \alpha(1-y) - \beta\right)\,, \tag{2}$$

where α is the parameter of the function, $y = \frac{x}{C_p}$, and $\beta = \frac{2\alpha-1}{2\alpha-2}$. Although the formula of the conical function is different from the formula of the BPR function, it still has all the advantages of the BPR function, i.e., simple, easily and quickly integrable. Figure 1 shows the values of the BPR function and the conical function with some sets of parameters. It points out that the conical function seems to be closer to the BPR function with the parameter $b = 4$ than to the BPR function with $b = 5$, especially when the flow is greater than the capacity of the link. This is in agreement with the paper of Florian and Nguyen [5], mentioned above.

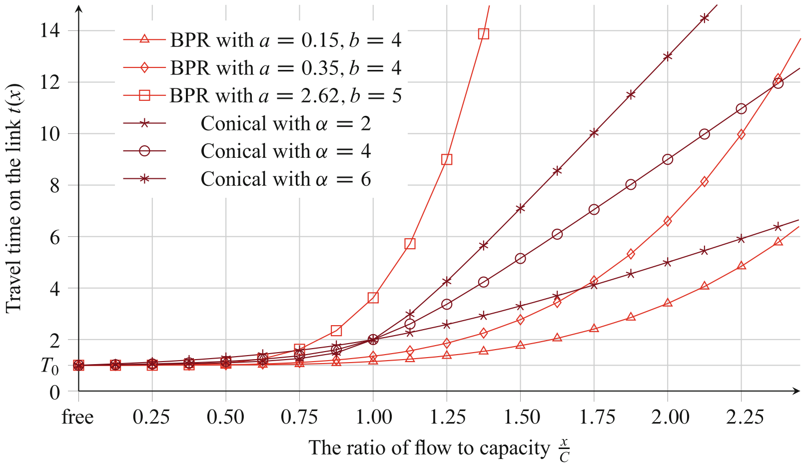

Fig. 1 Comparison of the BPR function and conical function using different sets of parameters

In summary, both the BPR and the conical functions have lot of advantages when applied to a traffic assignment model like the UE model. However, to the best of our knowledge, a version of the BPR function, including the possible waiting time at the signal line, on a link in an MTSDM is still missing. This is the main motivation of our work, and the method for calibrating parameters of the BPR function is presented in the next section.

3 BPR Function in an MTSDM

In this section, we calibrate parameters for the BPR function including waiting time at the signal line of a link. The new BPR function (the BPR function with the new parameters) will be evaluated by comparing its predicted travel times on a link with those of the original BPR function (using default parameters) and with the simulated times in VISSIM.

3.1 Link Capacity in Motorcycle Unit

Since the present paper is aiming at an MTSDM containing about 80% motorcycle traffic without dedicated lanes, it is reasonable to define a traffic flow unit based on motorcycles, i.e., *Motorcycle Unit* (MCU) rather than on the *passenger car unit* (PCU). In order to estimate the practical capacity of a given link in an MTSDM, we use the certain saturation flow of the link that is investigated by Hien and Montgomery [6]. In the research, they also used MCU as the flow unit, and utilized the regression model to develop a formula of the saturation flow within 4 s of the green phase of the signal light on the link. The case study in the research is in Hanoi, Vietnam, and the formula is given as

$$4S = 12.08 + 2.13\,(W - 3.5) - 47.12\,\frac{Prt}{Rrt} - 36.15\,\frac{Plt}{Rlt}, \tag{3}$$

where S is the average number of out-going vehicles in 1 s of green phase, W is the width of the link in meters, $\frac{Prt}{Rrt}$ and $\frac{Plt}{Rlt}$ are the proportions between the number of turners and the turning radius on the right side and on the left side, respectively. In one cycle of the signal light the maximum number of vehicles, that can go though the link, is the number of vehicles go though the signal line at the saturation flow in the green phase; therefore, the practical capacity of a link can be calculated as

$$C_p = \frac{t_g S}{t_g + t_r}, \tag{4}$$

where t_g, t_r are the green time and red time of the traffic light, respectively, and C_p is the capacity flow, i.e., the maximum vehicles can go through the link in 1 s without making congestion is C_p (MCU).

To evaluate the proposed practical capacity formula in Eq. (4), the traffic simulation software *VISSIM* is utilized, see [4, 9]. In the simulations for a number of typical kinds of links in Hanoi, we took most of typical driving behaviors as well as the shares of kinds of vehicles into account. The results show that the differences between the predicted capacity and the simulated capacity are in the range $[-2.67\%, 8.70\%]$. For a complicated traffic system like the MTSDM in Hanoi—where motorcycles and cars use the same lanes, and drivers, sometimes, break the traffic rules—this narrow range really makes sense.

3.2 Parameter Setting for the BPR Function

In Sect. 2 we have presented a review of travel time function with an emphasis on the two most popular functions, namely the BPR and the conical functions. The comparison between these functions using different sets of parameters showed that the BPR function with the parameter $b = 5$ grows rapidly when the flow is greater than the capacity of the link, and it is much bigger than the conical function. Moreover, some research have indicated that the original value from literature of the parameter b, i.e., $b = 4$, is generally better than $b = 5$. With respect to these research and literature, in this paper we use the parameter $b = 4$ in the BPR function. The other possible values of b, however, can be investigated in further research. We consider the BPR function as the following formula

$$t(x) = T_0\left(1 + a(\frac{x}{C_p})^4\right), \tag{5}$$

where T_0 and a are two parameters being calibrated. We consider a link in an MTSDM with the length L and the maximum allowed speed V_0. The driving time is denoted as t^d and the average waiting time is denoted as t^w. Assume that one cycle of the traffic light consists of two phases: green with duration time t_g seconds and red with duration time t_r seconds. The yellow phase is normally quite short, and it is added to the previous phase (red or green). The signal cycle starts with the red phase at time 0. Variable t is the time that the considering vehicle arrived at the signal line. We consider two cases when the flow is free and when the flow is equal to capacity of the link.

At free flow: all vehicles can run at the maximum allowed speed V_0, so the running time is given as

$$t_0^d = \frac{L}{V_0}. \tag{6}$$

The dependence of the waiting time at free flow on the arrival time t is described in Fig. 2a. If $0 \leq t < t_r$, the signal light is on the red phase. Because of the free flow, there is no queue of waiting vehicles, so the considering vehicle can go out as soon as the light turns to green. Thus, the waiting time is equal to the time waiting the light changing to green $t_0^w = t_r - t$. If the vehicle arrive at green phase, i.e. $t_r \leq t < t_r + t_g$, the vehicle can go through without stopping, i.e., $t_0^w = 0$. The average waiting time at free flow is computed as

$$\begin{aligned} t_0^w(average) &= \frac{1}{t_g + t_r}\left(\int_0^{t_r} (t_r - t)dt + \int_{t_r}^{t^r + t_g} 0dt\right) \\ &= \frac{t_r^2}{2(t_r + t_g)}. \end{aligned} \tag{7}$$

From Eqs. (6) and (7), the total travel time at free flow T_0 is described as

$$T_0 = \frac{L}{V_0} + \frac{t_r^2}{2(t_r + t_g)}. \tag{8}$$

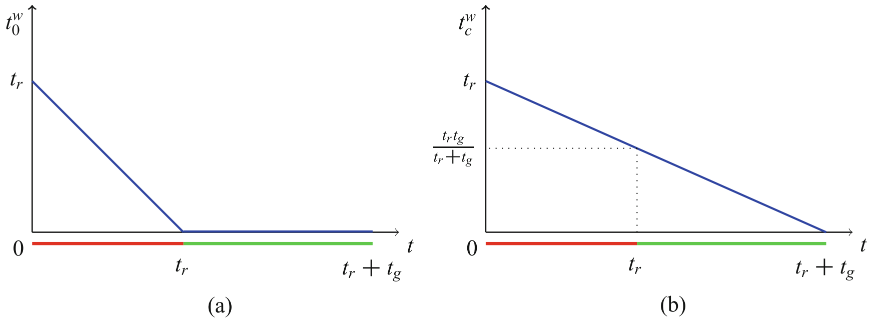

Fig. 2 Waiting time w.r.t. arrival time at free flow (**a**) and at capacity flow (**b**)

At capacity flow: vehicles move on the link smoothly at a speed close to the maximum allowed speed V_0. However, the real driving distance is less than the length of the link L because of a waiting queue at the signal line. In our observation on the traffic in Hanoi, the driving time at capacity flow is similar to the driving time at free flow, i.e., $t_c^d = \frac{L}{V_0}$. However, the waiting time is much bigger than the waiting time at free flow. Figure 2b describes the waiting time at capacity flow as a function of the arrival time t. Two cases have to be considered.

Case 1: $0 \leq t < t_r$. The signal light is now red, and the waiting time for the signal light turning from red to green is $t_r - t$. The number of vehicles in the queue is $t\,C_p$ (MCU) so the queuing time is $\frac{t\,C_p}{S}$. Replace C_p by the right side of Eq. (4) we have the queuing time is $\frac{t\,t_g}{t_r+t_g}$. So, the total waiting time in this case is

$$\begin{aligned} t_c^w(t) =& t_r - t + \frac{t\,t_g}{t_r + t_g} \\ =& t_r - t\frac{t_r}{t_g + t_r}. \end{aligned} \tag{9}$$

Case 2: $t_r \leq t < t_r + t_g$. The signal light is now green, but there is a queue of waiting vehicles which is entered the queue in the previous red phase. Thus, the waiting time in this case is the queuing time. Number of entered vehicle in the range $[0, t]$ is $t\,C_p$, and the number of out-going vehicle in the range $[t_r, t]$ is $(t - t_r)S$. The waiting time in this case is calculated as

$$\begin{aligned} t_c^w(t) &= \frac{t\,C_p - (t - t_r)S}{S} \\ &= t_r - t\frac{t_r}{t_g + t_r}. \end{aligned} \tag{10}$$

Equation (9) is equivalent to Eq. (10), and they are shown in Fig. 2 as the function of waiting time corresponding to the arrival time at signal line. The average of waiting time at capacity flow is calculated as in Eq. (11).

$$\begin{aligned} t_c^w &= \frac{1}{t_r + t_g}\int_0^{t_r+t_g} (t_r - t\cdot\frac{t_r}{t_r + t_g})dt \\ &= \frac{t_r}{2}. \end{aligned} \tag{11}$$

So we have the total travel time at capacity flow is

$$t_c = \frac{L}{V_0} + \frac{t_r}{2}. \tag{12}$$

According to the BPR function as in Eq. (5), the travel time at capacity flow can be computed as

$$t_c = T_0\Big(1 + a\big(\frac{C_p}{C_p}\big)^4\Big). \tag{13}$$

This implies

$$a = \frac{t_c}{T_0} - 1. \tag{14}$$

From Eqs. (8), (12) and (14), the parameter a can be calculated as

$$a = \frac{t_r t_g}{t_r^2 + 2\frac{L}{V_0}(t_r + t_g)}. \tag{15}$$

Replacing T_0, a and b in Eq. (1) by the right side of Eq. (8), the right side of Eq. (15) and value 4, resp., the function of travel time including the driving time and the waiting time on a link in an MTSDM is shown in Eq. (16).

$$t(x) = \Big(\frac{L}{V_0} + \frac{t_r^2}{2(t_r + t_g)}\Big)\Big(1 + \big(\frac{t_r t_g}{t_r^2 + 2\frac{L}{V_0}(t_r + t_g)}\big)\big(\frac{x}{C_p}\big)^4\Big), \tag{16}$$

where C_p is the practical capacity of the link formulated as in Eq. (4).

4 Computational Results

Table 2 shows the values of the parameter a for common kinds of links in Hanoi, that are calculated as the new formula in Eq. (15). The columns are grouped into two blocks. Each block has five columns with the meanings as follows. The first column, labeled $\frac{L}{V_0}(s)$, indicates the running times—in second—at the free flow. The second and the third columns, labeled $t_r(s)$, $t_g(s)$, are the times—in second— of the red phase and the green phase, respectively. The fourth column, labeled $T_0(s)$, indicates the calculated average travel times, including the waiting time at the traffic line, at free flow. The last column, labeled a, shows the values of the parameter a on the links. These values of a range from 0.05 to 0.50 with the average value 0.169, that is close to the default value 0.15 suggested in [2]. However, there is a considerable difference between values of the parameter a for short links and for long links. In details, the values of a for short links, where $15 \leq \frac{L}{V_0} \leq 30$, are considerably bigger than the original value, while for long links, where $90 \leq \frac{L}{V_0} \leq 120$, the values of a are significantly less than the original value.

In order to evaluate the new formula of the parameter a, VISSIM were again used to simulate traffic on a number of common links in Hanoi. In the simulations,

Table 2 Values of parameter a for some popular links in Hanoi

$\frac{L}{V_0}$ (s)	t_r (s)	t_g (s)	T_0 (s)	a	$\frac{L}{V_0}$ (s)	t_r (s)	t_g (s)	T_0 (s)	a
15	60	30	35.000	0.286	60	60	30	80.000	0.125
15	45	45	26.250	0.429	60	45	45	71.250	0.158
15	30	60	20.000	0.500	60	30	60	65.000	0.154
15	40	20	28.333	0.235	60	40	20	73.333	0.091
15	30	30	22.500	0.333	60	30	30	67.500	0.111
15	20	40	18.333	0.364	60	20	40	63.333	0.105
30	60	30	50.000	0.200	90	60	30	110.000	0.091
30	45	45	41.250	0.273	90	45	45	101.250	0.111
30	30	60	35.000	0.286	90	30	60	95.000	0.105
30	40	20	43.333	0.154	90	40	20	103.333	0.065
30	30	30	37.500	0.200	90	30	30	97.500	0.077
30	20	40	33.333	0.200	90	20	40	93.333	0.071
45	60	30	65.000	0.154	120	60	30	140.000	0.071
45	45	45	56.250	0.200	120	45	45	131.250	0.086
45	30	60	50.000	0.200	120	30	60	125.000	0.080
45	40	20	58.333	0.114	120	40	20	133.333	0.050
45	30	30	52.500	0.143	120	30	30	127.500	0.059
45	20	40	48.333	0.138	120	20	40	123.333	0.054

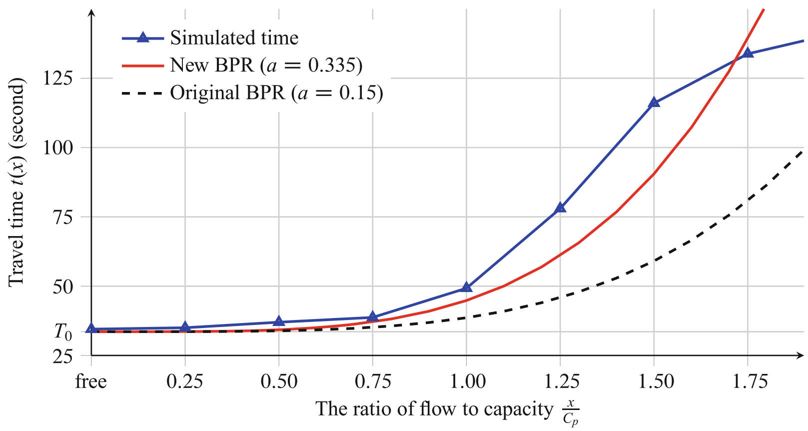

Fig. 3 The computational results on the link "LK1" where the new value of a is bigger than the original value

the input flows vary from free flow to 2 times of the capacity, i.e., $0 \leq \frac{x}{C_p} \leq 2$, since these values are adequate for the real traffic situation in Hanoi. The vehicle composition consists of 80% motorcycles, 19% personal cars, and 1% buses.

Figure 3 shows the computational results on a link, denoted "LK1" with 6 m width and 248 m long. The new value of a on "LK1" is 0.33, that is bigger than

Table 3 Gaps between BPR functions and the simulated times on the link "LK1"

Flow/capacity	New BPR (%)	Original BPR (%)
Free flow	2.70	2.70
0.25	3.97	4.03
0.50	7.38	8.43
0.75	4.11	9.19
1.00	8.99	21.60
1.25	21.79	41.20
1.50	21.98	49.08
1.75	3.89	39.61

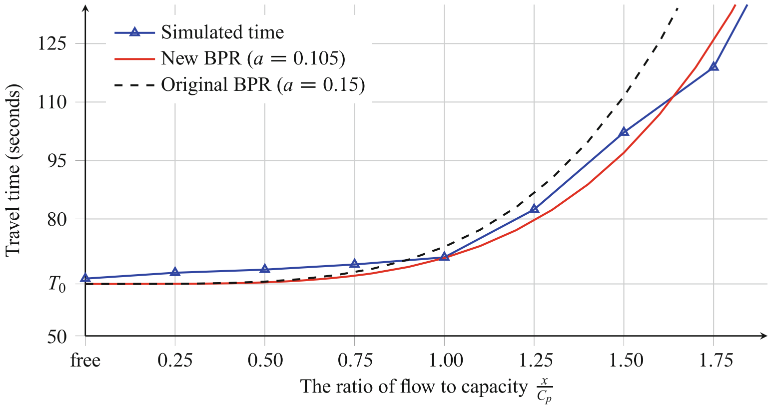

Fig. 4 The computational results on the link "LK2" when the new value of a is less than the original value

the default value, i.e., 0.15. The travel time predicted by the new BPR function and the original function is compared with the simulated travel time in VISSIM. Table 3 indicates the gaps between the travel times predicted by the two BPR functions and the average simulated travel time. The results shows that the new BPR function can estimate the total travel time on the link "LK1" better than the original BPR function does.

The computational results on another link, denoted "LK2" with 9 m width and 667 m long, are illustrated in Fig. 4. The value of a for the link "LK2" is 0.105, that is smaller than the original value. It can be seen on the figure that when the flow is less than the capacity of the link, i.e. $0 \leq \frac{x}{C_p} < 1$, the original BPR function is slightly better than the new BPR function, however, in other cases, i.e. $1 \leq \frac{x}{C_p}$, the new BPR function has better results (Fig. 5).

For an application of the new BPR function, i.e. the BPR function using the new value of a, we applied the function to the UE traffic assignment model running on the Hanoi traffic system with the real data collected by HAIDEP project in [8]; of course, with some updates. The output of the model points out the areas with high ratio of the traffic flow to the capacity $\frac{x}{C_p}$. This areas are defined as areas with high

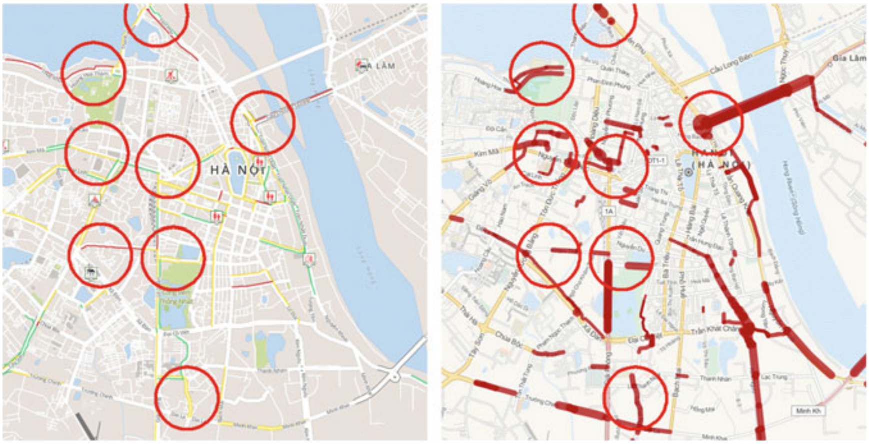

Fig. 5 Comparison of predicted flows and real traffic situation provided by Remon. *Left*: Real traffic situation provided on Remon-hanoi website. Links with *red* or *yellow bar* indicate the locations of traffic congestion. *Right*: Predicted traffic flows with the UE model using the new BPR function. The areas with high value of the ratio of flow to capacity is marked with *red circles*

probability of traffic congestion. They are in agreement with the real traffic situation provided on the website remon-hanoi.net, see [12].

5 Conclusion

The review of travel time functions has pointed out that the BPR function and the conical volume delay function are suitable for modern traffic assignment models, such as the UE model. By defining the practical capacity of a link dominated by motorcycles based on the saturation flow, we proposed step by step the method to calibrate suitable parameters for the BPR function, consisting of both driving time and waiting time at the signal line. With promising results, the present paper proved that the new proposed BPR function was reliable and convenient to be applied to an UE model.

For further work, we are going to investigate other possible value of the parameter b in the BPR function, that is proposed to be four. We are also considering to apply the method for calibrating parameters to other travel time functions, in particular to the conical volume delay function. Furthermore, additional traffic assignment models for MTSDM using new BPR function will be investigated. The final target is to make this research useful not only for other researchers but also for traffic planners dealing with traffic problems in developing countries.

Acknowledgements This document is partly supported by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 101.01-2013.10, and by the HGS MathComp & DFG (Fonds 28445/7811618). We would like to thank Msc. Truong Hoang Hai and all the partners for collecting and providing the data of traffic in Hanoi.

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