Chapter 7
Feedback Control for Distributed Massive MIMO Communication

S. Dasgupta, R. Mudumbai and A. Kumar

Abstract We present a distributed nullforming algorithm where a set of transmitters transmit at full power to minimize the received power at a designated receiver. Each transmitter adjusts the phase and frequency of its transmitted RF signal in a purely distributed fashion as it uses only an estimate of its own channel gain to the receiver, and a feedback signal from the receiver, that is common across all the transmitters. This assures its scalability; in contrast any noniterative approach to the nullforming problem requires that each transmitter know every other transmitter's channel gain. We prove that the algorithm practically, globally converges to a null at the designated receiver. By practical convergence we mean that the algorithm always converges to a stationary trajectory, and though some of these trajectories may not correspond to a minimum, those that do not are locally unstable, while those that do are locally stable. Unlike its predecessors the paper does not assume prior frequency synchronization among the transmitters, but asymptotically secures frequency consensus.

7.1 Introduction

Multiple Input Multiple Output (MIMO) techniques [1–4] have played an important part in the remarkable explosion of wireless communication systems in the past two decades, and MIMO is now an integral component of all recent WiFi and cellular

---

This work was partly supported by US NSF grants CNS-1239509, CAREER award ECCS-1150801, and CCF-1302456, and ONR grant N00014-13-1-0202.

S. Dasgupta (✉) · R. Mudumbai · A. Kumar
Department of Electrical and Computer Engineering, University of Iowa,
Iowa City, IA 52242, USA
e-mail: dasgupta@engineering.uiowa.edu

R. Mudumbai
e-mail: rmudumbai@engineering.uiowa.edu

A. Kumar
e-mail: amy-kumar@uiowa.edu

R. Tempo et al. (eds.), *Emerging Applications of Control and Systems Theory*, Lecture Notes in Control and Information Sciences - Proceedings,
https://doi.org/10.1007/978-3-319-67068-3_7

standards e.g. 802.11ac [2] and 5G [3]. Specifically, MIMO communication involves transceivers equipped with multiple antennae. This allows directional transmission, permitting control of the interference produced by wireless transmitters, and orders of magnitude increases in energy and spectral efficiency.

Yet the applicability of MIMO is in practice severely limited by constraints such as form factors, and the size and the number of antennae that can be realistically supported. An attractive alternative, springing from the pioneering work of [5], is *distributed MIMO (DMIMO)*, [6] where instead of deploying centralized antenna systems, groups of single antennae transceivers collaborate to form a *virtual antennae* system that mimics the functionality of a centralized multi-antennae system. Studied extensively by theoreticians, until recently, this concept has been dismissed by practioners as being beyond the realm of practicality for several reasons. Among these are issues engendered by uncertain geometries and the fact that unlike centralized systems, by its very definition, each constituent of a DMIMO system carries its own clock and oscillator. These suffer significant and rapid drift modeled as a second-order stochastic process, [7] that induces DMIMO units to migrate from synchrony to complete incoherence within mere hundreds of milliseconds.

Over the last decade several authors, [8–22], have sought to realize this decades-old concept, by addressing two salient components of DMIMO: distributed beamforming, [8–16] and distributed nullforming, [17–22]. In the former, groups of transmitters, collaborate to form a beam of maximum possible power using constructive interference at a receiver. In the latter their transmissions cancel each other at the designated receiver. On the other hand [21] simultaneously forms nulls at some receivers and beams at others. Physical implementations of both beam and nullforming algorithms on software defined radio (SDR) platforms, are described in [12, 13, 20]. Apart from being important constituents of the overall DMIMO architecture, beamforming is key to power efficient communication, just as nullforming has applications in interference avoidance for increased spatial spectrum reuse [23], cognitive radio [24] and cyber security [25].

This brings us to the role of *feedback control* in these schemes. Ultimately in all these applications all transmissions must arrive at their target receiver synchronized in frequency and with precise phase, and for nullforming, amplitude relationships. Uncertain geometries and mobility, make it impossible for the transmitters to determine the phases, and amplitudes of their transmission at the receiver. Thus all the references cited above rely on some receiver cooperation. This takes the form of feedback from the receiver to the transmitters, and possibly between the transmitters, using which the transmitters must adjust the phase, amplitude and frequency of their transmissions to achieve synchronization at the receiver. A key issue, of course, is what type of feedback is needed, the minimum information exchange required to achieve one's objective.

In this light, among the DMIMO papers we have cited here, barring [11, 19], all assume, prior frequency synchronization, presumably through information exchange among the transmitters. The earliest among the DMIMO papers, [8], assumes prior frequency synchronization, and requires that the receiver feed back to *each* *transmitter a separate feedback signal throughout its operation.* Such an algorithm is thus not scalable.

A breakthrough idea introduced in [9], and used in several subsequent papers, is the notion of *common aggregate feedback*. This involves the receiver broadcasting to all the transmitters either the complex baseband version of its total received signal or some attribute there of. In either case the burden of repeatedly feeding back a separate signal to each transmitter is alleviated. To wit [9], assumes prior frequency synchronization among the nodes and executes a *1-bit feedback* algorithm to achieve beamforming. The algorithm itself is in the mold of randomized ascent. Each transmitter updates its phase according to a preselected distribution. The 1-bit feedback is whether or not the net received power declines as a result of these updates. If the power declines, the updates are discarded. Else they are retained. Under mild assumptions on the distribution from which the phase updates are chosen, this algorithm is provably convergent.

In this paper we consider distributed nullforming without prior frequency synchronization with only phase and frequency updates. The algorithm we formulate is akin to *Lyapunov redesign* in the controls and adaptive systems literature, [26, 27]. We observe that distributed nullforming algorithms can be found in [17–22]. Each of these, however assumes prior frequency synchronization at the outset of operation, presumably through information exchange among transmitters. While this is reasonable, oscillator frequencies also undergo drift modeled as Brownian motion, albeit at orders of magnitude slower rates than oscillator phases. On the other hand drift in oscillator frequencies has a more dramatic impact on performance than has phase drift. Furthermore Doppler shift occurs at receivers, thus receiver feedback should be used to guide the adjustment of frequencies at transmitters.

Nullforming is fundamentally more difficult than is beamforming, [22], for two reasons. First, it is much more sensitive to phase drift. Because of this, a 1-bit algorithm like in [9] is unable to adjust quickly enough to achieve a decent null. Second, while beamforming only requires frequency and phase alignment at the receiver, nullforming requires the precise control of the amplitude and phase of the radio-frequency signal transmitted by each cooperating transmitter to ensure that they cancel each other. Accordingly, [17, 18], requires that in addition to the common aggregate feedback of the total baseband received signal, each transmitter must also learn the *channel state information* of *every* transmitter to the receiver. This latter requirement is substantially relaxed in [22], where each channel, at the point of setup requires only the knowledge of its channel to the receiver. Simulations in [22] show that the gradient descent algorithm it employs is robust to substantial channel estimation errors, while capable of tracking significant oscillator drift.

This paper extends [22] to the case where the transmitters even if initially frequency synchronized undergo small frequency drifts. These frequency drifts even if small, can destroy a good null very quickly. As in [22], we assume that each user knows its complex channel gain to the receiver. Unlike [17, 18] it does not have the CSI seen by the other transmitters. Like [22], the receiver feeds back the net baseband signal. In [22] this feedback is used by each transmitter to perform a distributed gradient descent minimization of the total received power. The minimization is distributed, as each transmitter can perform it using only the aggregate feedback and its own complex channel gain to the receiver. Similarly, in this paper each transmitter adjusts its phase and *frequency* knowing only its CSI to the receiver and the aggregate feedback signal, to asymptotically drive the received signal to zero. However, we show that the lack of frequency synchronization precludes the use of gradient descent. Instead a Lyapunov redesign is needed.

We observe that [11, 19] also eschew the assumption of prior frequency synchronization. Among these, [11] uses a 1-bit type algorithm to beamform. We have however, discovered a conceptual error in the algorithm. On the other hand the preliminary paper, without proofs, [19], has the important difference that it critically assumes that each transmitter equalizes its complex channel gain. In this paper we equalize only the phase and *not the magnitude* of the channel. This is in the vein of [22] and permits a key application of distributed nullforming: *Namely, permitting transmission at full power, thus maximizing incoherent power pooling gains, while protecting the null target.* As explained in [22] this opens up the prospect of both *Space Division Multiple Access (SDMA)* and cyber security.

Section 7.2 provides the algorithm. Section 7.3 has some preliminary analysis. Section 7.4 presents a stability analysis. Received power which must be minimized is a nonconvex function of the transmitter phases and frequencies. Unsurprisingly our Lyapunov redesign yields a distributed algorithm that has multiple stationery points/trajectories that may not correspond to a minimum. Yet we show that only those stationary points are locally stable that do correspond to global minima. Section 7.5 is the conclusion.

7.2 The Algorithm

We now describe a scalable gradient descent algorithm for distributed nullforming in a node. As in [22] and unlike [17, 18], we assume that at the beginning of a nullforming epoch, each transmitter has access only to its own complex channel gain to the receiver, using which it equalizes the *phase rather than also the magnitude* of the channel to the receiver. Assume there are N transmitter nodes.

Denote $\theta_i(t)$ to be the equalized phase of the i-th transmitter and $r_i > 0$ is the magnitude of the received signal. Assume that $\omega_i(t)$, is a frequency offset of the i-th transmitter, from a nominal frequency to which each transmitter should ideally have been synchronized, but oscillator drift prevents the maintenance of such synchronization.

Then the complex baseband signal received at the cooperating receiver is:

$$s(t) = R(t) + jI(t) \tag{7.1}$$

where

$$R(t) = \sum_{i=1}^{N} r_i \cos(\omega_i(t)t + \theta_i(t)) \tag{7.2}$$

and

$$I(t) = \sum_{i=1}^{N} r_i \sin(\omega_i(t)t + \theta_i(t)). \tag{7.3}$$

As in [22], the transmission process is thus, equivalent to each transmitter transmitting, the phase equalized complex baseband signal, $e^{j((\omega_i(t)t+\theta_i(t))}$. The baseband signal the receiver sees is

$$s(t) = \sum_{i=1}^{N} r_i e^{j((\omega_i(t)t+\theta_i(t))} \tag{7.4}$$

see (7.1). Indeed it is $s(t)$ that the receiver broadcasts to all transmitters, which they must use to adjust their frequency and phase. Define $\theta(t) = [\theta_1(t), \cdots, \theta_N(t)]^\top$ and $\omega(t) = [\omega_1(t), \cdots, \omega_N(t)]^\top$. We note the key difference with [19], which apart from not providing any proofs, assumes that all $r_i = 1$. The received power is:

$$J(\theta, \omega, t) = I^2(t) + R^2(t). \tag{7.5}$$

The receiver feeds back $s(t)$. The i-th transmitter uses $s(t)$ to adjust its $\theta_i(t)$ and $\omega_i(t)$ to asymptotically force $J(\theta, \omega, t)$ to zero. In reality both the adjustment and feedback will be discrete time. However, should the continuous time algorithm be uniformly asymptotically stable (u.a.s), then averaging theory, [28] ensures that for high enough feedback rates, and small enough gains, an obvious discretized version will also be u.a.s.

As noted above, [22] where $\omega = 0$, uses a gradient descent algorithm. The resulting J is autonomous in [22]. The frequency offsets here make the cost function nonautonomous, as it may change its value even if the phases and frequencies are not adjusted. To amplify this point observe that a pure gradient descent algorithm would take the form:

$$\dot{\theta}(t) = -\frac{\partial J(\theta, \omega, t)}{\partial \theta}; \ \dot{\omega}(t) = -\frac{\partial J(\theta, \omega, t)}{\partial \omega}. \tag{7.6}$$

Now observe that under (7.6)

$$\begin{aligned} \dot{J} &= \frac{\partial J}{\partial t} + \frac{\partial J}{\partial \theta}\dot{\theta} + \frac{\partial J}{\partial \omega}\dot{\omega} \\ &= \frac{\partial J}{\partial t} - \left\| \frac{\partial J}{\partial \theta} \right\|^2 - \left\| \frac{\partial J}{\partial \omega} \right\|^2. \end{aligned}$$

Should $\dfrac{\partial J}{\partial t}$ be zero, as is the case is in [22], this guarantees a nonincreasing $J(\theta, \omega, t)$. However, with ω_i potentially nonzero under (7.2), $\dfrac{\partial J}{\partial t}$ need not be zero, preventing guaranteed decrescence of $J(\theta, \omega, t)$. Thus a Lyapunov redesign of the nullforming algorithm is needed.

In Sect. 7.3 we present a Lyapunov function, and show that its decrescence is guaranteed by the following algorithm.

$$\dot{\theta} = -\frac{\partial J}{\partial \theta} - \frac{\omega}{2} \tag{7.7}$$

$$\dot{\omega} = -\frac{1}{2}\frac{\partial J}{\partial \theta}. \tag{7.8}$$

Thus the i-th node can implement these as long as it has access to its frequency, phase, CSI to the receiver and the common feedback signals $I(t)$ and $R(t)$ permitting a totally distributed implementation, as

$$\frac{\partial J}{\partial \theta_i} = -2R(t)r_i \sin(\omega_i(t)t + \theta_i(t)) + 2I(t)r_i \cos(\omega_i(t)t + \theta_i(t)), \tag{7.9}$$

and

$$\frac{\partial J}{\partial \omega} = t\frac{\partial J}{\partial \theta}. \tag{7.10}$$

Also observe that unlike (7.6) there is an additional corrective term $\frac{\omega}{2}$ in (7.7), that accounts for the frequency offsets. In keeping with our mandate for phase and frequency only updates the gains r_i are not updated.

7.3 Preliminaries of the Stability Analysis

In this section, we present certain preliminary results that among other things show the uniform convergence of the gradient of J with respect to θ, and explore the properties of the stationary points of (7.7, 7.8).

But first, a result used in [29].

Lemma 7.1 *Suppose on a closed interval $\mathscr{I} \subset \mathbb{R}$ of length T, a signal $w : \mathscr{I} \to \mathbb{R}$ is twice differentiable and for some ε and M'*

$$|w(t)| \leq \varepsilon_1 \text{ and } |\ddot{w}(t)| \leq M' \quad \forall \quad t \in \mathscr{I}.$$

Then for some M independent of ε_1, $\mathscr{I}$ and M', and $M'' = \max(M', 2\varepsilon_1 T^{-2})$ one has:

$$|\dot{w}(t)| \leq M(M''\varepsilon_1)^{1/2} \quad \forall \quad t \in \mathscr{I}.$$

We begin by deriving a preliminary relation.

$$\frac{\partial J}{\partial t} = -2R(t)\sum_{i=1}^{N}\omega_i(t)r_i\sin(\omega_i(t)t+\theta_i(t)) + 2I(t)\sum_{i=1}^{N}r_i\omega_i(t)\cos(\omega_i(t)t+\theta_i(t)),$$
$$= \omega^\top(t)\frac{\partial J}{\partial \theta}. \tag{7.11}$$

We now show that under (7.7, 7.8) there is a Lyapunov function that is nonincreasing and the gradient of J with respect to θ converges uniformly to zero.

Lemma 7.2 *Consider (7.7, 7.8) with (7.2–7.5) initial time $t_0 \geq 0$. Then the following hold:*

(a) For all $t \geq t_0$,

$$V(t) = J(t) + \frac{\|\omega(t)\|^2}{2} \tag{7.12}$$

is nonincreasing.

(b) The following occurs uniformly in t_0.

$$\lim_{t\to\infty}\frac{\partial J}{\partial \theta}(t) = 0. \tag{7.13}$$

Proof Because of (7.5), (7.9–7.10) and (7.7, 7.8), there holds:

$$\begin{aligned}\dot{J} + \frac{d}{dt}\left\{\frac{\omega^\top\omega}{2}\right\} &= \frac{\partial J}{\partial t} + \dot{\theta}^\top\frac{\partial J}{\partial \theta} + \dot{\omega}^\top\frac{\partial J}{\partial \omega} + \omega^\top\dot{\omega}\\ &= \omega^\top\frac{\partial J}{\partial \theta} - \left\|\frac{\partial J}{\partial \theta}\right\|^2 - \frac{\omega^\top}{2}\frac{\partial J}{\partial \theta}\\ &\quad -\frac{\omega^\top}{2}\frac{\partial J}{\partial \theta} - \frac{t}{2}\left\|\frac{\partial J}{\partial \theta}\right\|^2\\ &= -\left(1+\frac{t}{2}\right)\left\|\frac{\partial J}{\partial \theta}\right\|^2\end{aligned} \tag{7.14}$$

Consequently (a) holds. Further, ω is uniformly bounded. Consequently from (7.9) there is an M_1, independent of t_0, such that for all $t \geq t_0$

$$\left\|\frac{d}{dt}\left\{\frac{\partial J}{\partial \theta}(t)\right\}\right\| \leq M_1.$$

Equally, there exists an M_2, also independent of t_0, such that for all $t \geq t_0$,

$$\left\| \frac{\partial J}{\partial \theta}(t) \right\| \leq M_2.$$

Further, since the initial time $t_0 \geq 0$, from (7.14) and $V(t)$ is nonnegative, one obtains that for all $t \geq t_0$:

$$\begin{aligned}\int_{t_0}^{t} \left\| \frac{\partial J}{\partial \theta}(s) \right\|^2 \mathrm{d}s &\leq \int_{t_0}^{t} \left(1 + \frac{s}{2}\right) \left\| \frac{\partial J}{\partial \theta}(s) \right\|^2 \mathrm{d}s \\ &\leq V(t_0).\end{aligned}$$

Thus, for every $\varepsilon > 0$, there exists a T independent of t_0 such that for all $t \geq T + t_0$,

$$\int_{T+t_0}^{t} \left\| \frac{\partial J}{\partial \theta}(s) \right\|^2 \mathrm{d}s \leq \varepsilon.$$

Then from Lemma 7.1, there is a K independent of t_0 such that for all $\varepsilon > 0$, there exists a T independent of t_0 such that for all $t \geq T + t_0$,

$$\left\| \frac{\partial J}{\partial \theta}(s) \right\|^2 \leq \varepsilon, \quad \forall s \geq T + t_0.$$

Thus indeed (b) holds uniformly in t_0.

Thus, (7.7, 7.8) converges uniformly to a trajectory where:

$$\frac{\partial J}{\partial \theta} = 0. \tag{7.15}$$

Because of (7.2, 7.3), (7.9) there holds:

$$\begin{aligned}\frac{1}{2}\frac{\partial J}{\partial \theta_i} &= -R(t)\sum_{i=1}^{N} r_i \sin(\omega_i(t)t + \theta_i(t)) + I(t)\sum_{i=1}^{N} r_i \cos(\omega_i(t)t + \theta_i(t)) \\ &= \sum_{i=1}^{N}\sum_{l=1}^{N} r_i r_l \{\cos(\omega_i(t)t + \theta_i(t)) \sin(\omega_l(t)t + \theta_l(t)) \\ &\quad - \sin(\omega_i(t)t + \theta_i(t)) \cos(\omega_l(t)t + \theta_l(t))\} \\ &= \sum_{i=1}^{N}\sum_{l=1}^{N} r_i r_l \sin((\omega_l(t) - \omega_i(t))t + \theta_l(t) - \theta_i(t)).\end{aligned} \tag{7.16}$$

Thus (7.15) also implies that for some constant ω^* on this trajectory the frequency offsets

$$\omega_i = \omega^*, \ \forall i \in \{1, \cdots, N\}. \tag{7.17}$$

Observe from (7.5), (7.2) and (7.3),

$$\begin{aligned} J(\theta, \omega, t) &= \left(\sum_{i=1}^{N} r_i \cos(\omega_i(t)t + \theta_i(t))\right)^2 + \left(\sum_{i=1}^{N} r_i \sin(\omega_i(t)t + \theta_i(t))\right)^2 \\ &= \sum_{i=1}^{N} r_i^2 + 2\sum_{i=1}^{N}\sum_{\substack{l=1\\ l\neq i}}^{N} r_i r_l \cos((\omega_l(t) - \omega_i(t))t + \theta_l(t) - \theta_i(t)) \end{aligned} \tag{7.18}$$

This shows that these are in fact *manifolds* rather than points.

We now turn to a major nontrivial consequence of having potentially nonunity r_i, a problem absent in [19] where all r_i are 1. There are sets of positive r_i for which a null may not be possible. It is thus useful to first characterize conditions on the r_i that ensure that a null is possible. The theorem below provides this characterization. We note it is similar to a result in [22] where the ω_i are all fixed to zero. The theorem also characterizes the global minimum value

$$J^* = \max_{t\geq 0} \min_{\theta,\omega} J(\theta, \omega, t). \tag{7.19}$$

Theorem 7.1 *Consider $J(\theta, \omega, t)$ defined in (7.2, 7.3, 7.5) and J^* as in (7.19), with all $r_i > 0$. Without loss of generality label r_i, so that $r_i \geq r_{i+1}$. Then the following hold:*

(i) $J^* = 0$ *iff*

$$r_1 \leq \sum_{i=2}^{N} r_i. \tag{7.20}$$

(ii) *If (7.20) is violated*

$$J^* = \left(r_1 - \sum_{i=2}^{N} r_i\right)^2. \tag{7.21}$$

Proof Observe $J(\theta, \omega, t) = |s(t)|^2$, with $s(t)$ defined in (7.4). Suppose (7.20) is violated. Clearly, under (7.4), as $r_i > 0$,

$$J(\theta, \omega, t) \geq \left(r_1 - \sum_{i=2}^{N} r_i\right)^2.$$

Thus $J^* > 0$. Further this minimum is attained by choosing $\omega_i = \omega_l$, for all i, l, $\theta_1 = 0$ and $\theta_i = \pi$, $\forall\, i > 1$. This proves (ii) and the "only if" part of (i).

To prove the "if" part of (i), set all ω_i to zero and assume that (7.20) holds. We use induction. Consider $N = 2$. Then $r_1 = r_2$. Thus with $\theta_1 = 0, \theta_2 = \pi$, $J(\theta, 0, t) = |r_1 - r_2|^2 = 0,\ \forall t$. Now suppose the result holds for some $N = n \geq 2$. Consider $N = n + 1$.

Observe with

$$J(0, 0, t) = \sum_{i=1}^{n+1} r_i > 0. \tag{7.22}$$

Consider two cases.

Case I: $r_2 > \sum_{i=3}^{n+1} r_i$: In this case, by hypothesis $0 < r_2 - \sum_{i=3}^{n+1} r_i < r_1$. Thus,

$$J(\pi[1, 0, [1, \cdots, 1]]^T, 0, t) = -r_1 + r_2 - \sum_{i=3}^{n+1} r_i < 0.$$

As for every t, $J(\theta, 0, t)$ is continuous in θ, and (7.22) holds, there exist a θ for which $J(\theta, 0, t) = 0,\ \forall t$. Thus $J^* = 0$.

Case II: $r_2 \leq \sum_{i=3}^{n+1} r_i$: From the induction hypothesis, there exist $\hat{\theta}_2, \cdots, \hat{\theta}_{n+1}$ such that $\sum_{i=2}^{n+1} r_i e^{j\hat{\theta}_i} = 0 < r_1$. Moreover, by assumption $\sum_{i=2}^{n+1} r_i \geq r_1$. Since $\left|\sum_{i=2}^{n+1} r_i e^{j\theta_i}\right|$ is continuous in θ, moving continuously between $[\theta_2, \cdots, \theta_{n+1}]$ between 0 and $[\hat{\theta}_2, \cdots, \hat{\theta}_{n+1}]$ one can find a set of phases $[\theta_2^*, \cdots, \theta_{n+1}^*]$ such that $\left|\sum_{i=2}^{n+1} r_i e^{j\theta_i^*}\right| = r_1$. Thus, for some δ, $\sum_{i=2}^{n+1} r_i e^{j\theta_i^*} = r_1 e^{j\delta}$. Then $J([\pi + \delta, \theta_2^*, \cdots, \theta_{n+1}^*]^T, 0, t) = 0$, completing the proof.

Returning to the stationary trajectories, given by (7.15) and (7.17), some of these correspond to the desired null, or in their absence (7.19). Others do not, and will be called *false stationary* points. We show below that the latter are locally unstable and are thus, rarely attained, and even if attained cannot be practically maintained as noise would drive the trajectories away from them. Thus, by showing the local stability of the global minimum, we will demonstrate the practical uniform convergence of the algorithm to the global minimum.

In the stationary trajectory, (7.15) and (7.17) are not nulls, i.e., $s(t) \neq 0$, then from (7.9) for all i, l, and $t > 0$, $\tan(\omega^* t + \theta_i) = \tan(\omega^* t + \theta_l)$. Consequently on stationary trajectories that are not nulls,

$$\theta_i - \theta_l = k_{il}\pi,\ \forall \{i, l\} \subset \{1, \cdots, N\} \tag{7.23}$$

where k_{il} are integers.

The local analysis of these stationary trajectories, will require the examination of the Hessian with respect to θ on these trajectories. Consider again (7.16). Due to

(7.17), along (7.15) and (7.17) the il-th element of the Hessian along the stationary trajectory is given by:

$$[H(\theta)]_{il}|_{\omega_i=\omega_l} = \frac{\partial^2 J(\theta)}{\partial\theta_i \partial\theta_l} = \begin{cases} -2\sum_{i\neq l} r_i r_l \cos(\theta_i - \theta_l) & i = l \\ 2r_{il}\cos(\theta_i - \theta_l) & i \neq l \end{cases} \tag{7.24}$$

7.4 Stability Analysis

Armed with the preliminary results in Sect. 7.3 we now complete our stability analysis. Lemma 7.2 shows that uniform convergence to a stationary trajectory is guaranteed. Some of these trajectories correspond to a null. Other do not. In this section, we show that *only those that correspond to a null are locally stable. The others are not.* Consequently, one is assured of practical uniform convergence in the sense that stationary trajectories that do not correspond to the desired nulls if at all attained, cannot be practically maintained. Thus for all practical purposes, the algorithm defined in (7.7, 7.8) achieves a desired null.

First we demonstrate the local instability of spurious stationary trajectories. To this end we present need the following lemma.

Lemma 7.3 *The linear system below with scalar $a > 0$ is unstable:*

$$\dot{\eta} = \begin{bmatrix} a & ta + \frac{1}{2} \\ \frac{a}{2} & \frac{at}{2} \end{bmatrix} \eta \tag{7.25}$$

Proof Consider the initial condition $\eta(0) = [0, 1]^\top$. Then it is evident that both elements of the state are nonegative for all $t > 0$. Then the first element of the state vector is

$$\eta_1(t) \geq e^{at}.$$

Thus the system is unstable.

Consider next the Hessian with respect to θ at a critical trajectory. As given in (7.24) this is identical to the Hessian studied in [22]. From [22] we have the following lemma.

Lemma 7.4 *Assume all $r_i > 0$. Consider a false stationary manifold i.e., a stationary manifold on which $J \neq 0$ and $J \neq J^*$. Then $H(\theta)|_{\forall i,\omega_i=\omega^*}$ has at least one negative eigenvalue.*

We now prove that a false stationary trajectory is unstable.

Theorem 7.2 *Assume all $r_i > 0$. Consider (7.7, 7.8), and a stationary manifold defined by (7.15), (7.17) such that along this trajectory $J \neq 0$ and $J \neq J^*$. Then this trajectory is unstable.*

Proof Observe, that for all i, l

$$\frac{\partial^2 J(\theta)}{\partial\theta_i \partial\omega_l} = t\frac{\partial^2 J(\theta)}{\partial\theta_i \partial\theta_l}$$

(7.7, 7.8), linearized around (7.15), (7.17) is given by:

$$\dot{x} = \begin{bmatrix} -H(\theta) & -tH(\theta) + \frac{I}{2} \\ -\frac{H(\theta)}{2} & -\frac{t}{2}H(\theta) \end{bmatrix} x. \tag{7.26}$$

In view of Lemma 7.4 and the symmetry of $H(\theta)$, there is an orthogonal matrix Ω and real λ_i, with $\lambda_1 > 0$, such that with

$$\Lambda = \text{ diag } \{-\lambda_1, \cdots, \lambda_N\},$$

$$H(\theta) = \Omega\Lambda\Omega^T$$

Define $\beta = \text{ diag } \{\Omega, \Omega\}x$. Then the linearized systems is equivalent to:

$$\dot{\beta} = \begin{bmatrix} -\Lambda & -t\Lambda + \frac{I}{2} \\ -\frac{\Lambda}{2} & -\frac{t}{2}\Lambda \end{bmatrix} \beta.$$

Then instability follows from Lemma 7.3.

Thus indeed false stationary manifolds are unstable. To complete the result, it suffices to show that the global minima, nulls or otherwise, are locally stable. As the Hessian is indefinite this is a nonhyperbolic system and a linearization approach will be inconclusive. Thus we use a more direct approach to proving local stability.

Because of (7.18) and (7.23) at false stationary points the cost function takes the values

$$\sum_{i=1}^{N} r_i^2 + 2\sum_{i=1}^{N}\sum_{\substack{l=1 \\ l\neq i}}^{N} r_i r_l \mu_{il}, \;\; \mu_{il} \in \{-1, 1\}. \tag{7.27}$$

Assume again $r_i \geq r_{i+1} > 0$. With $r = [r_1 \cdots, r_N]^T$, define:

$$f(r) = \begin{cases} \min\left\{\left\{\sum_{i=1}^{N} r_i^2 + 2\sum_{i=1}^{N}\sum_{\substack{l=1 \\ l\neq i}}^{N} r_i r_l \mu_{il} | \mu_{il} \in \{-1, 1\}\right\} \setminus \{0\}\right\} & \text{if } r_1 \leq \sum_{i=2}^{N} r_i \\ \min\left\{\left\{\sum_{i=1}^{N} r_i^2 + 2\sum_{i=1}^{N}\sum_{\substack{l=1 \\ l\neq i}}^{N} r_i r_l \mu_{il} | \mu_{il} \in \{-1, 1\}\right\} \setminus \{r_1 - \sum_{i=2}^{N} r_i\}\right\} & \text{else} \end{cases} \tag{7.28}$$

In other words $f(r)$ is the smallest value that J can take at a false stationary point. We can now prove local stability of the null manifold.

Theorem 7.3 *Suppose $r_i \geq r_{i+1} > 0$ and with $f(r)$ defined in (7.28). Then with positive initial time t_0, (7.7, 7.8) uniformly converges to a global minimum of $J(\theta, \omega, t)$*

if $V(t_0) < f(r)$. *Further for a constant* ω^* *and all* $i \in \{1, \cdots, N\}$,

$$\lim_{t \to \infty} \omega_i(t) = \omega^* \tag{7.29}$$

Proof Item (b) of Lemma 7.2 holds uniformly in t_0. Further for all $t \geq t_0$

$$J(\theta(t), \omega(t), t) \leq V(t) \leq V(t_0) < f(r).$$

As Lemma 7.2 guarantees convergence to a stationary manifold, and only stationary manifold at which $J(\theta(t), \omega(t), t) < f(r)$, convergence occurs to a global minimum. Finally, (7.29) follows from (7.15) and (7.8).

As convergence to a stationary manifold is guaranteed, and all false stationary points are locally unstable, this thus proves practical uniform convergence to a global minimum. Observe in addition the transmitters attain *frequency consensus*.

In principle the phases need not converge to a fixed point. However, there is a subtlety. On a stationary trajectory, (7.7) and (7.8) reduces to, (7.17) and for each i,

$$\dot{\theta}_i(t) = -\frac{\omega^*}{2}.$$

Thus for suitable α_i the i-th transmitted signal becomes

$$e^{j(\frac{\omega^*}{2}t + \alpha_i)}.$$

thus effectively, the attained phases *are constants*, and *de facto* frequency synchronization obtains.

7.5 Conclusion

We have provided a new distributed nullforming algorithm that is guaranteed to achieve a null, through phase and frequency adapatation. Unlike [22] this paper does not assume prior frequency synchronization. Its effect though is the achievement of frequency synchronization. That all this can be achieved with *no cooperation* between the transmitters, a feedback of the aggregate received signal by the receiver, and the knowledge to each transmitter of only its CSI is in our opinion an intriguing fact. Also intriguing is the fact that an otherwise nonconvex cost function has the attractive property that all its local minima are global minima. Further studies that delineate the minimum information needed to achieve distributed nullforming are warranted.

References

1. Foschini, G.J: Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas. Bell Labs Tech. J. **1**, 41–59 (1996)
2. Bejarano, O., Knightly, E.W., Park, M.: IEEE 802.11ac: from channelization to multi-user mimo. IEEE Commun. Mag. **51**, 84–90 (2013)
3. Wang, C.X., Haider, F., Gao, X., You, X.H., Yang, Y., Yuan, D., Aggoune, H.M., Haas, H., Fletcher, S., Hepsaydir, E.: Cellular architecture and key technologies for 5G wireless communication networks. IEEE Commun. Mag. **52**, 122–130 (2014)
4. Larsson, E.G., Edfors, O., Tufvesson, F., Marzetta, T.L.: Massive MIMO for next generation wireless systems. IEEE Commun. Mag. **52**, 186–195 (2014)
5. Cover, T., Gamal, A.E.L.: Capacity theorems for the relay channel. IEEE Trans. Inf. Theory **25**(5), 572–584 (1979)
6. Madhow, U., Brown, D.R., Dasgupta, S., Mudumbai, R.: Distributed massive MIMO: algorithms, architectures and concept systems. In: Proceedings of Information Theory and Applications Workshop (ITA), pp. 1–7 (2014)
7. Brown, D.R., Mudumbai, R., Dasgupta, S.: Fundamental limits on phase and frequency estimation and tracking in drifting oscillators. In: Proceedings of ICASSP, Invited Paper. Kyoto, Japan, March 2012
8. Tu, Y.-S., Pottie, G.J.: Coherent cooperative transmission from multiple adjacent antennas to a distant stationary antenna through AWGN channels. In: Proceedings of IEEE VTC Spring 02. Birmingham, Alabama, May 2002
9. Mudumbai, R., Barriac, G., Madhow, U.: On the feasibility of distributed beamforming in wireless networks. IEEE Trans. Wirel. Commun. **6**(5), 1754–1763 (2007)
10. Mudumbai, R., Brown III, D.R., Madhow, U., Poor, H.V.: Distributed transmit beamforming: challenges and recent progress. IEEE Commun. Mag. **47**(2), 102–110 (2009)
11. Seo, M., Rodwell, M., Madhow, U.: A feedback-based distributed phased array technique and its application to 60-gGHz wireless sensor network. In: Microwave Symposium Digest, 2008 IEEE MTT-S International, pp. 683–686, June 2008
12. Rahman, M.M., Baidoo-Williams, H.E., Mudumbai, R., Dasgupta, S.: Fully wireless implementation of distributed beamforming on a software-defined radio platform. In: Proceedings of the The 11th ACM/IEEE Conference on Information Processing in Sensor Networks, IPSN'12, pp. 305–316. Beijing, China (2012)
13. Quitin, F., Rahman, M.M.U., Mudumbai, R., Madhow, U.: Distributed beamforming with software-defined radios: frequency synchronization and digital feedback. In: IEEE Globecom 2012, Dec 2012 (to appear)
14. Mudumbai, R., Hespanha, J., Madhow, U., Barriac, G.: Scalable feedback control for distributed beamforming in sensor networks. In: IEEE International Symposium on Information Theory (ISIT), pp. 137–141. Adelaide, Australia, Sept 2005
15. Mudumbai, R., Bidigare, P., Pruessing, S., Dasgupta, S., Oyarzun, M., Raeman, D.: Scalable feedback algorithms for distributed transmit beamforming in wireless networks. In: Acoustics, Speech and Signal Processing (ICASSP), 2012 IEEE International Conference on, pp. 5213–5216, Mar 2012
16. Brown, D.R., Wang, R., Dasgupta, S.: Channel state tracking for large-scale distributed MIMO communication systems. IEEE Trans. Signal Process. **63**(10), 2559–2571 (2015). https://doi.org/10.1109/TSP.2015.2407316
17. Brown, D.R., Madhow, U., Bidigare, P., Dasgupta, S.: Receiver-coordinated distributed transmit nullforming with channel state uncertainty. In: Information Sciences and Systems (CISS), 2012 46th Annual Conference on, pp. 1–6, Mar 2012
18. Brown, D.R., Bidigare, P., Dasgupta, S., Madhow, U.: Receiver-coordinated zero-forcing distributed transmit nullforming. In: Statistical Signal Processing Workshop (SSP), 2012 IEEE, pp. 269–272, Aug 2012
19. Kumar, A., Dasgupta, S., Mudumbai, R.: Distributed nullforming without prior frequency synchronization. In: Proceedings of Australian Control Conference, Nov 2013

20. Peiffer, B., Mudumba, R., Goguri, S., Dasgupta, S., Kruger, A.: Experimental demonstration of nullforming from a fully wireless distributed array. In: Proceedings of ICASSP. New Orlans, LA, Mar 2017
21. Kumar, A., Mudumbai, R., Dasgupta, S., Madhow, U., Brown, D.R.: Distributed MIMO multicast with protected receivers: a scalable algorithm for joint beamforming and nullforming. IEEE Trans. Wirel. Commun. **16**(1), 512–525 (2017). https://doi.org/10.1109/TWC.2016.2625784
22. Kumar, A., Mudumbai, R., Dasgupta, S., Rahman, M.M., Brown, D.R., Madhow, U., Bidigare, P.: A scalable feedback mechanism for distributed nullforming with phase-only adaptation. IEEE Trans. Signal Inf. Process. over Netw. **1**, 58–70 (2015). https://doi.org/10.1109/TSIPN.2015.2442921
23. Ozgur, A., Lévêque, O., Tse, D.N.C.: Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks. IEEE Trans. Inf. Theory **53**(10), 3549–3572 (2007)
24. Yucek, T., Arslan, H.: A survey of spectrum sensing algorithms for cognitive radio applications. IEEE Commun. Surv. Tutor. **11**(1), 116–130 (2009)
25. Dong, L., Han, Z., Petropulu, A.P., Poor, H.V.: Cooperative jamming for wireless physical layer security. In: Statistical Signal Processing, 2009: SSP'09. IEEE/SP 15th Workshop on, pp. 417–420. IEEE (2009)
26. Dasgupta, S., Anderson, B.D.O., Kaye, R.J.: Output error identification methods for partially known systems. Int. J. Control **43**, 177–191 (1986)
27. Fu, M., Dasgupta, S., Soh, Y. C.: Integral quadratic constraint approach vs. multiplier approach. Automatica, 281–287, Feb 2005
28. Anderson, B.D.O., Bitmead, R.R., Jr. Johnson, Kokotovic, P.V., Kosut, R.L., Mareels, I.M.Y., Praly, L., Riedle, B.D.: Stability of Adaptive Systems: Passivity and Averaging Analysis. MIT Press, Cambridge (1986)
29. Dasgupta, S., Anderson, B.D.O., Tsoi, A.C.: Input conditions for continuous time adaptive systems problems. IEEE Trans. Autom. Control, 78–82 (1990)
30. Rahman, M.M., Dasgupta, S., Mudumbai, R.: A scalable feedback-based approach to distributed nullforming. In: Proceedings of WICON. Shanghai, China, Apr 2013
31. Cao, M., Yu, C., Morse, A.S., Anderson, B.D.O., Dasgupta, S.: Generalized controller for directed triangle formations. In: Preprints of IFAC World Congress. Seoul, Korea, July 2008
32. Dasgupta, S., Anderson, B.D.O.: Physically based parameterizations for designing adaptive algorithms. Automatica **23**, 469–477 (1987)
33. Hahn, W.: Stability of Motion. Springer, Heidelberg (1967)