Chapter 15
Grid Integration of Renewable Electricity and Distributed Control

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Abstract Motivated by climate change and sustainability, and the resulting need to decarbonize the electricity sector, there is a major global movement toward large-scale integration of renewable energy, i.e., wind and solar, into the existing power grid. The inherent variability of wind and solar energy production poses a major challenge in achieving these goals. The problem becomes more challenging as we consider issues of competitive markets, low cost and high reliability. In the last few years, we have been working on new systems and control problems that arise from these considerations. In this paper, we will present some highlights of our work on developing demand response methods using distributed control and bounding the loss of efficiency in these methods.

15.1 Introduction

Carbon emissions leading to climate change and sustainability are some of the major reasons motivating adoption of renewable energy sources such as wind and solar into the electric energy system. Large-scale integration of wind and solar electric energy poses significant technological challenges. These energy sources are inherently uncertain (power generation not known in advance), intermittent (large fluctuations and

This work was supported by NSF Grants ECCS-1723849 (previously ECCS-1129061) and CNS-1723856 (previously CNS-1239274).

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R. Tempo et al. (eds.), *Emerging Applications of Control and Systems Theory*, Lecture Notes in Control and Information Sciences - Proceedings,
https://doi.org/10.1007/978-3-319-67068-3_15

ramps) and non-dispatchable (unable to follow a command). The term *variability* is used to represent these three characteristics [17] and is a significant hurdle in the large-scale integration of renewables. A promising solution to address the variability is to deploy *demand side management* (DSM) or *demand response* (DR) programs that adjust the consumption to match the predicted generation.

A paradigm shift in the power system operations is underway where consumers will be incentivized to manage their demand by leveraging the flexibility of their loads such as electric vehicles (EV), air conditioning, heat pumps, water heaters, etc. [2, 18]. DSM or DR programs in power systems operations exploit this flexibility in power consumption loads. Distributed control has been used as a major tool to solve problems where a central authority sends a control signal, e.g., price of electricity, and consumers decide their consumption schedules according to some private utility function [13]. A strategy for assigning quantities in a distributed price-based framework is the *proportional allocation mechanism* where the central authority calculates a price for all the consumers in such a way that the assigned quantity to each agent is proportional to the monetary value that the agent is willing to pay [12]. This mechanism has been used to formulate EV charging problems [10, 22]. However, it has not been examined in broader smart grid settings.

Consumer behavior plays a critical role in the implementation of demand response programs in distributed mode and the assumption of price-taking consumers might not always be true. Selfish behavior of agents in a non-cooperative game leads to inefficiency with respect to the solution that maximizes system welfare. Consequently, it is crucial to design distributed control systems in such a way that the efficiency loss due to selfish behavior is bounded. The term *price of anarchy* (PoA) has been coined as a measure of efficiency of distributed control as compared with centralized optimal solution. Bounds on the PoA for various cost-sharing games, congestion games and payoff maximization games have been derived in [11, 16, 20].

In the smart grid scenario, non-cooperative game theoretic methods have been used to model problems [14, 15, 23], however the loss of efficiency by selfish behavior has not been widely investigated. The Nash equilibrium has been shown to be efficient in an infinite population game when the charging rates of all the EVs are equal [14] and in a DR problem with different consumers [15], however in the first case the assumptions are rather impractical and in the second one, the consumers' utility functions were ignored. Regarding wind variability, a game has been formulated among various power consumers in [23] where the PoA bound has been calculated for an example case.

During the last few years, we have been addressing various challenging problems involving both technical and economic issues of smart grid [3–9]. We present here a few salient results on two demand response methods. In the first one, the available power is limited and the control authority designs a price signal aiming to maximize social welfare subject to supply-demand balancing. We show that if proportional allocation is used to design the price signal, then the lower bound of the price of anarchy is 75%. We also develop some strategies for improving efficiency further. In the second case, we consider a demand response problem where the available power is not limited and the price signal is set by the load consumption. Under some conditions of the utility functions of the consumers with respect to the price, we obtain a robust lower bound of the price of anarchy of 50%.

15.2 A Demand Response Program Using Proportional Allocation Mechanism with Tight PoA Bound

In this section, we develop a distributed method for controlling the consumers' flexible demand with intra-day supply forecasts. Flexible consumers are modeled as individually rational agents that maximize their net utilities in presence of load consumption constraints. The consumers bid the monetary value they are willing to pay for each time interval and the central authority obtains a price signal based on a proportional allocation mechanism. Two scenarios are considered, *price taking* and *price anticipating* consumers. In the first case, the proportional allocation method provides a competitive equilibrium that maximizes the system welfare. In the second case, the consumers' selfish behavior is modeled using a non-cooperative game. A Nash equilibrium always exists for this game but it is not efficient. We are able to obtain a lower bound on the PoA of 75% and we develop some strategies to improve the game efficiency.

15.2.1 Problem Formulation

Let us consider a residential area where electric power is supplied by thermal and renewable generators. The power consumption in the area is controlled by a central *control authority*. Two types of residential consumers are considered: *fixed consumers* and *flexible consumers*. Only flexible consumers are willing to adjust their consumption schedules in response to some signal from the authority.

Let us consider a set $\mathcal{N} := \{1, 2, \ldots, N\}$ of flexible consumers. Each flexible consumer possesses a smart energy scheduling device with two-way communication capability. We assume that the supply is initially scheduled in a traditional day ahead market, based on demand predictions. The time interval of interest $[t_0, t_f]$, corresponding to the intra-day horizon, is divided into T slots of length $\Delta t = (t_f - t_0)/T$. The set of time slots is $\mathcal{T} := \{1, 2, \ldots, T\}$, and we consider the following variables at time slot $t \in \mathcal{T}$: $q_i(t) \in \mathbb{R}_+$ is the power consumption of all the flexible loads of the i-th flexible consumer (no power transfer from the consumers to the grid is allowed), $c(t) \in \mathbb{R}_+$ is the total scheduled power generation of all the thermal power plants, $\widehat{w}(t) \in \mathbb{R}_+$ is the estimate of the power generation of all the renewable sources, $\widehat{n}(t) \in \mathbb{R}_+$ is the estimate of the total power consumption of all fixed loads of both fixed and flexible consumers. Let $v(t)$ denote the estimated net generation available for flexible demand at time slot $t \in \mathcal{T}$, i.e., $v(t) := c(t) + \widehat{w}(t) - \widehat{n}(t)$.

The control authority obtains a forecast of renewable generation and balances the estimated demand with supply for each time slot of the operating day, i.e.,

$$v(t) = \sum_{i \in \mathcal{N}} q_i(t), \ t \in \mathcal{T}. \tag{15.1}$$

Assuming that net power supply is always sufficient to meet the fixed loads' demand, i.e., $v(t) > 0$ for any $t \in \mathcal{T}$, the supply-demand balancing is accomplished by adjusting the power consumption of the flexible loads. There is always an inevitable mismatch between the estimated power generation and consumptions and their realized values. Ancillary services are implemented to handle this real-time mismatch. However, the use of the intra-day flexible load control mechanism proposed here will reduce the need for ancillary services while making large-scale renewable integration less burdensome.

Let $\mathbf{q}_i, \mathbf{v} \in \mathbb{R}_+^T$ denote vectors of dimension T that collect the consumption of flexible consumer $i \in \mathcal{N}$ and the net power generation available for flexible consumers, respectively, for every time slot $t \in \mathcal{T}$. The output of flexible consumption $i \in \mathcal{N}$ is represented in monetary units by the utility function $U_i(\mathbf{q}_i) : \mathbb{R}^T \to \mathbb{R}$, which is assumed to be non-negative, concave and continuously differentiable. In addition, it is also assumed to be a strictly increasing, i.e., $\nabla U_i(\mathbf{q}_i) > 0$, where $\nabla U_i : \mathbb{R}^T \to \mathbb{R}^T$ denotes the gradient of U_i. The operational constraints of the flexible consumers depending upon the type of loads can be expressed by a set of linear inequalities:

$$\mathbf{H}_i \mathbf{q}_i \leq \mathbf{b}_i, \ i \in \mathcal{N}, \tag{15.2}$$

where $\mathbf{H}_i \in \mathbb{R}^{M \times T}$, $\mathbf{b}_i \in \mathbb{R}^M$ and M is the number of constraints. Let $\mathcal{Q}_i := \left\{\mathbf{q} \in \mathbb{R}^T : \mathbf{b}_i - \mathbf{H}_i \mathbf{q} \geq \mathbf{0}\right\}$ denote the set of consumption vectors satisfying the operational constraints for $i \in \mathcal{N}$ and $\mathcal{S} := \left\{\mathbf{q}_i \in \mathcal{Q}_i : \mathbf{v} - \sum_{i \in \mathcal{N}} \mathbf{q}_i = \mathbf{0}, \ i \in \mathcal{N}\right\}$ the *feasibility set* of the consumption vectors satisfying both the supply-demand power balance constraint and the operational constraints. We assume that the feasibility set $\mathcal{S}$ is nonempty.

15.2.2 Centralized Control

In this idealized scenario, the central control authority dictates how much power is assigned to each flexible consumer by maximizing the social welfare. The centralized control problem is

$$\max_{\mathbf{q}_i} \left\{ \sum_{i \in \mathcal{N}} U_i(\mathbf{q}_i) : \mathbf{q}_i \in \mathcal{S} \right\} \tag{15.3}$$

The existence of a maximum is guaranteed because the objective function is concave and the search space is a nonempty compact convex set. A solution of (15.3) maximizes the social welfare and is referred to as the *centralized optimal solution*. But the consumers may want to control their loads on their own and the central authority may not have computational capability to solve the optimization problem for a large number of residential consumers. A feasible alternative is a distributed control approach.

15.2.3 Distributed Control with Price-Taking Consumers

In a distributed control model, the behavior of the consumers is an important aspect to consider. We begin by considering individually rational flexible consumers that behave as price takers. Let $\mathbf{k}_i \in \mathbb{R}_+$ denote the amount of money the consumer $i \in \mathscr{N}$ is willing to pay for the energy $\mathbf{q}_i$. The consumers bid the monetary values or expenditures $\mathbf{k}_i$ to the control authority. In this scenario, the control authority obtains the value of the available net supply $v(t)$ for every $t \in \mathscr{T}$ and computes a system price $p(t)$ according to the following proportional allocation mechanism.

Definition 15.1 (*The proportional allocation mechanism*) The proportional allocation of the energy consumption at time slot $t \in \mathscr{T}$ is given by:

$$q_i(t) = \frac{k_i(t)}{p(t)}, \ i \in \mathscr{N}, \tag{15.4}$$

where $p(t) > 0$ is the price of electricity at time $t \in \mathscr{T}$, obtained by

$$p(t) = \frac{\sum_{i \in \mathscr{N}} k_i(t)}{v(t)}, \ t \in \mathscr{T}. \tag{15.5}$$

Since $v(t)$ is always positive, the system price $p(t)$ is well defined for every time slot $t \in \mathscr{T}$ and guarantees $v(t) = \sum_{i \in \mathscr{N}} q_i(t)$ for all t. Each consumer (flexible or fixed) is charged at the system price. Let the net utility of a consumer be defined as the total utility minus the expenditure. The flexible consumers maximize their own net utility function by a suitable selection of their consumptions $\mathbf{q}_i$. Let $\mathbf{p} \in \mathbb{R}^T$ denote the vector that collects the system prices for every time slot $t \in \mathscr{T}$. The distributed control problem for price takers is formulated as follows:

$$\max_{\mathbf{q}_i} \left\{ U_i(\mathbf{q}_i) - \mathbf{p}^\top \mathbf{q}_i : \mathbf{q}_i \in \mathscr{S}_i^{pt} \right\}, \ i \in \mathscr{N}, \tag{15.6}$$

where the set of feasible power consumptions is $\mathscr{S}_i^{pt} := \{\mathbf{q}_i : \mathbf{b}_i - \mathbf{H}_i \mathbf{q}_i \geq 0\}$.

The solution concept for the distributed control problem with price taking flexible consumers is the competitive equilibrium.

Definition 15.2 The set $\{(\mathbf{q}_i^E, \mathbf{p}^E) : i \in \mathscr{N}\}$ is a *competitive equilibrium* if each consumer selects its consumption vector $\mathbf{q}_i^E$ by solving the optimization problem (15.6) and the control authority obtains the price vector $\mathbf{p}^E$ using the proportional allocation mechanism (15.4)–(15.5).

The competitive equilibrium always exists if the feasibility set $\mathscr{S}$ is nonempty. Moreover, in such a case a competitive equilibrium is equivalent to a solution of the centralized control problem and maximizes the social welfare.

Theorem 15.1 *The set* $\{(\mathbf{q}_i^E, \mathbf{p}^E) : i \in \mathscr{N}\}$ *is a competitive equilibrium if and only if the set of consumptions* $\{\mathbf{q}_i^E : i \in \mathscr{N}\}$ *is a solution to the centralized control problem.*

15.2.4 Distributed Control with Price Anticipating Consumers

If the consumers can predict the mechanism that the control authority uses to set the price vector $\mathbf{p}$, they adjust their consumption decisions according to their impact on the price, and we say that they behave as price anticipators. By using as decision variables the monetary value vectors $\mathbf{k}_i$, where $k_i(t) = p(t)q(t)$ for $t \in \mathscr{T}$, the consumers can obtain the price vector $\mathbf{p}$ as a function of $\sum_{i\in\mathscr{N}} \mathbf{k}_i$, because we assume they know that $\mathbf{p}$ is decided by the formula $p(t) = \sum_{i\in\mathscr{N}} k_i(t)/v(t)$. Each consumer's monetary value depends on the sum of all the consumers' expenditures and the consumption assignment can be modeled as a non-cooperative game where the players are the flexible consumers.

The problem can be formulated in terms of the monetary expenditures by eliminating the price and the consumptions variables. Let $\mathbf{k}_{-i} = \{\mathbf{k}_j : j \in \mathscr{N} \setminus \{i\}\}$ denote the collection of monetary value vectors of all flexible consumers other than the consumer i. Note that $\mathbf{p}$ and $\mathbf{q}_i$ can be expressed as functions of $\mathbf{k}_i$ as $\mathbf{p}(\mathbf{k}_i; \mathbf{k}_{-i}) = \mathbf{D}^{-1}(\mathbf{v}) \sum_{j\in\mathscr{N}} \mathbf{k}_j$ and $\mathbf{q}_i(\mathbf{k}_i; \mathbf{k}_{-i}) = \mathbf{D}^{-1}(\mathbf{p}(\mathbf{k}_i; \mathbf{k}_{-i}))\mathbf{k}_i = \mathbf{D}^{-1}(\sum_{i\in\mathscr{N}} \mathbf{k}_i)\mathbf{D}(\mathbf{v})\mathbf{k}_i$ where $\mathbf{D}(\mathbf{x})$ denotes a diagonal square matrix whose main diagonal has the components of vector $\mathbf{x}$. Considering $\mathscr{S}_i^{pa}(\mathbf{k}_{-i}) := \{\mathbf{k}_i : \mathbf{b}_i - \mathbf{H}_i\mathbf{D}^{-1}\}$ $\{(\sum_{i\in\mathscr{N}} \mathbf{k}_i)\mathbf{D}(\mathbf{v})$ $\mathbf{k}_i \geq \mathbf{0}\}$, the distributed control problem for price anticipators is given by

$$\max_{k_i} \left\{ U_i(\mathbf{D}^{-1}(\sum_{j\in\mathscr{N}} \mathbf{k}_j)\mathbf{D}(\mathbf{v})\mathbf{k}_i) - \mathbf{1}^\top \mathbf{k}_i : \mathbf{k}_i \in \mathscr{S}_i^{pa}(\mathbf{k}_{-i}) \right\}, \; i \in \mathscr{N}. \quad (15.7)$$

Each consumer will try to maximize her own net utility, assuming that all other consumers' expenditures are fixed. This is called the *best response strategy* and the solution is called a Nash equilibrium. In a Nash equilibrium, no player has an incentive to deviate unilaterally of the equilibrium [20]. The Nash equilibrium for the distributed control problem with price anticipators is the set of expenditures $\{\mathbf{k}_i^G : i \in \mathscr{N}\}$ such that

$$U_i(\mathbf{q}_i(\mathbf{k}_i^G, \mathbf{k}_{-i}^G)) - \mathbf{1}^\top \mathbf{k}_i^G \geq U_i(\mathbf{q}_i(\mathbf{k}_i, \mathbf{k}_{-i}^G)) - \mathbf{1}^\top \mathbf{k}_i,\ \mathbf{k}_i \in \mathscr{S}_i^{pa}(\mathbf{k}_{-1}^G),\ i \in \mathscr{N}. \tag{15.8}$$

It can be proved that a Nash equilibrium always exists if the feasibility set $\mathscr{S}$ is nonempty.

Theorem 15.2 (Existence of Nash equilibrium) *The non-cooperative game described by Eq. (15.8) has a Nash equilibrium if the space $\mathscr{S}$ is nonempty.*

15.2.5 Price of Anarchy and Efficiency Improvement

The selfish behavior of agents in a non-cooperative game theoretic setting renders lower performance as compared to the optimal centralized control. Price of anarchy (PoA) is a measure to quantify the loss of efficiency in using game theoretic control over centralized control. PoA is defined as the worst-case ratio of the objective function value of a Nash equilibrium of a game and that of a centralized optimal solution [20]. The quantity 1 − PoA is a worst-case estimate of the loss of performance due to price anticipating behavior of agents.

In our energy assignment problem for flexible consumers, $\{\mathbf{q}_i^C : i \in \mathscr{N}\}$ denotes a solution of the centralized problem (15.3) and $\{\mathbf{q}_i^G : i \in \mathscr{N}\}$ denotes a Nash equilibrium for the distributed control problem with price anticipating consumers. The PoA is defined as follows:

$$\text{PoA} := \frac{\sum_{i \in \mathscr{N}} U_i(\mathbf{q}_i^G)}{\sum_{i \in \mathscr{N}} U_i(\mathbf{q}_i^C)}. \tag{15.9}$$

Theorem 15.3 *The tight lower bound of* PoA *of the Nash equilibrium solution for the distributed consumption assignment with flexible consumers that behave as price anticipators is* 0.75.

The worst-case loss of efficiency corresponds to the case where one agent consumes half of the total power consumed by all the agents at each time slot. Thus, the market power of a consumer plays a key role in the efficiency of the game. The theoretical worst case could only be attained under a particular setting. We are interested in developing strategies to improve efficiency. The following corollaries show two different ways to improve efficiency.

Corollary 15.1 *If all the consumers have same utility function, i.e., $U_i = U$, there is no efficiency loss at Nash equilibrium solution, i.e., PoA is 1.*

Corollary 15.2 *Suppose $\{\mathbf{q}_i = \mathbf{0} : i \in \mathscr{N}\}$ belongs to the set of load operational constraints, then the PoA approaches 1 as the number N of flexible consumers goes to infinity.*

Efficiency can be improved by recruiting consumers with similar utility functions, or by classifying them into groups of similar utility and designing specific programs for each group. The distributed control approach will have better efficiency if consumers share their utility functions with the central control authority. Another option is to reduce individual market power by increasing the number of consumers.

15.3 A Demand Response Program with Robust PoA Bound

In this section, we consider a different model for demand response. Unlike in Sect. 15.2, the price here is decided by desired energy consumption. Here, we formulate a decentralized control model assuming that the consumers are price anticipators and quantify that, in the worst case loss of efficiency of this problem is never greater than 50%.

We introduce some additional notation in this section. Let $\{q_i \in \mathbb{R}^T : i \in \mathscr{N}\}$ denote the set of power demand vectors for each consumer in the system. The vector of aggregated power demand in the system is $\mathbf{q}_{\mathscr{N}} = \sum_{i\in\mathscr{N}} \mathbf{q}_i$ where the entry t is denoted by $q_{\mathscr{N}}(t)$ and corresponds to the aggregated consumption at time slot $t \in \mathscr{T}$. The price of electricity in the system at time $t \in \mathscr{T}$ is a function of the aggregated consumption at that time and is denoted by $p(t) = p(q_{\mathscr{N}}(t))$. We assume that the price function is a convex, continuously differentiable and monotonically increasing function.

15.3.1 Centralized Control

We assume that the authority has full information about the supply function $p(q_{\mathscr{N}}(t))$ for that system. Let $\mathbf{p}(\mathbf{q}_{\mathscr{N}}) \in \mathbb{R}^T$ denote the vector of system prices for all time slots $t \in \mathscr{T}$. The authority aims to maximize the consumer's aggregated net utility subject to their operational constraints. For any feasible set of consumptions $\{\mathbf{q}_i \in \mathscr{Q}_i : i \in \mathscr{N}\}$, the objective is

$$\max \left\{ \sum_{i=1}^{N} U_i(\mathbf{q}_i) - \mathbf{p}^\top(\mathbf{q}_{\mathscr{N}})\mathbf{q}_{\mathscr{N}} \ : \mathbf{q}_i \in \mathscr{Q}_i, \ i \in \mathscr{N} \right\}. \tag{15.10}$$

Since the objective function is concave, the non-emptiness of the convex sets $\{\mathscr{Q}_i : i \in \mathscr{N}\}$ defined by the operational constraints ensure that a global maxima always exist [1]. But if the consumers want to control the power consumption of their loads on their own with the help of available information on their smart meters, then centralized control will not work. If consumers are price takers, like the earlier problem, it is easy to show that the distributed control has a competitive equilibrium where the solution is same as the centralized solution. We are more interested in the price anticipatory case. Thus we model this scenario as a game problem in the next subsection.

15.3.2 *Decentralized Control with Price Anticipating Consumers*

The consumers know the price function and they optimize their consumption schedules accordingly. As the price is a function of power consumption of all the consumers, we model the resulting situation as a non-cooperative game.

Definition 15.3 (*Demand response game*) The *demand response game* is defined by the triple $(\mathscr{N}, \mathscr{E}, V)$ where $\mathscr{N}$ is the set of players, $\mathscr{E} = \cup_{i \in \mathscr{N}} \mathscr{Q}_i$ is the set of feasible strategies, and $V : 2^{\mathscr{N}} \times \mathscr{E} \to \mathbb{R}$ is the welfare function for a subset of players $\mathscr{S} \in 2^{\mathscr{N}}$ and a strategy set $\{\mathbf{q}_i : i \in \mathscr{N}\} \in \mathscr{E}$.

Each consumer is individually rational and maximizes her individual welfare, assuming that the remaining players' strategies are fixed. Denoting the strategies of other players by $\mathbf{q}_{-i} = \{\mathbf{q}_j : j \in \mathscr{N} \setminus \{i\}\}$, the individual welfare of player $i \in \mathscr{N}$ is $V(\{i\}, \{\mathbf{q}_i, i \in \mathscr{N}\})$ and can be expressed as a function of the strategy of the player i and the strategies of the other players as follows:

$$L_i(\mathbf{q}_i, \mathbf{q}_{-i}) := V(\{i\}, \{\mathbf{q}_i, i \in \mathscr{N}\}) = U_i(\mathbf{q}_i) - \mathbf{p}^\top(\mathbf{q}_i, \mathbf{q}_{-i})\mathbf{q}_i \qquad (15.11)$$

The Nash equilibrium for the demand response game is the set of all players' strategies such that no player has an incentive to deviate unilaterally. Mathematically, Nash equilibrium is defined by the set of strategies $\{\mathbf{q}_i^* \in \mathscr{Q}_i : i \in \mathscr{N}\}$ such that $L_i(\mathbf{q}_i^*, \mathbf{q}_{-i}^*) \geq L_i(\mathbf{q}_i, \mathbf{q}_{-i}^*)$ for all $\mathbf{q}_i \in \mathscr{Q}_i,\ i \in \mathscr{N}$. Since each consumer's objective function is concave and the strategies set is convex and compact, a Nash equilibrium solution exists according to Rosen's theorem [19].

The demand response game as defined by Definition 15.3 belongs to the class of valid monotone utility games and this will allow us to bound its efficiency. Let us begin by characterizing this class of games. Consider a payoff maximization game given by the triple $(\mathscr{N}, \mathscr{E}, V)$ where $\mathscr{N}$ is the set of players, $\mathscr{E} = \cup_{i \in \mathscr{N}} \mathscr{E}_i$ is the set of feasible strategies for each player and $V : 2^{\mathscr{N}} \times \mathscr{E} \to \mathbb{R}$ is a function that provides the welfare associated with a subset of players for a given strategy.

Definition 15.4 (*Valid Utility Game* [21]) The payoff maximization game $(\mathscr{N}, \mathscr{E}, V)$ is a *valid utility game* if it satisfies the following three properties:

(i) V is submodular, i.e., for any $\mathscr{S} \subseteq \mathscr{S}' \subseteq \mathscr{N}$ and any player $i \in \mathscr{N} \setminus \mathscr{S}'$

$$V(\mathscr{S} \cup \{i\}, e) - V(\mathscr{S}, e) \geq V(\mathscr{S}' \cup \{i\}, e) - V(\mathscr{S}', e),\ \forall e \in \mathscr{E} \qquad (15.12)$$

(ii) The welfare of a player is never less than the value added to the social welfare, i.e., for any $\mathscr{S} \subseteq \mathscr{N}$ and any $i \in \mathscr{S}$,

$$V(\{i\}, e) \geq V(\mathscr{S}, e) - V(\mathscr{S} \setminus \{i\}, e), \ \forall e \in \mathscr{E} \tag{15.13}$$

(iii) The aggregated value of the individual welfare of a group of players is never greater than the social welfare of the group, i.e., for any $\mathscr{S} \subseteq \mathscr{N}$

$$\sum_{i \in \mathscr{S}} V(\{i\}, e) \leq V(\mathscr{S}, e), \forall e \in \mathscr{E} \tag{15.14}$$

Definition 15.5 (*Monotone Game* [21]) The payoff maximization game $(\mathscr{N}, \mathscr{E}, V)$ is a *monotone game* if for any $\mathscr{S} \subseteq \mathscr{S}' \subseteq \mathscr{N}$, $V(\mathscr{S}, e) \leq V(\mathscr{S}', e)$.

Definition 15.6 (*Valid Monotone Utility Game*) The payoff maximization game $(\mathscr{N}, \mathscr{E}, V)$ is a *valid monotone utility game* if it is simultaneously a valid utility and a monotone game.

For the demand response game of Definition 15.3, we assume that each consumer's utility function satisfies the following condition.

Assumption 15.1 The utility function of any consumer $i \in \mathscr{N}$ is such that

$$U_i(\mathbf{q}_i) \geq \sum_{t=1}^{T} p(\tilde{q}_{-i} + q_i(t))(\tilde{q}_{-i} + q_i(t)) - p(\tilde{q}_{-i})\tilde{q}_{-i}$$

where $\tilde{q}_{-i} = \sum_{j \in \mathscr{N} \setminus \{i\}} q_j^{\max}$.

The above condition implies that the value of the utility function of a consumer $i \in \mathscr{N}$ for any feasible demand $\mathbf{q}_i$ will be greater than the increase in the maximum cost of power consumption in the system due to the addition of that demand $\mathbf{q}_i$. The central authority can broadcast this requirement to all the consumers as a prerequisite for participation in the demand response program. We also assume that the authority have an estimate of the upper-bound of $\sum_{i \in \mathscr{N}} q_i^{\max}$ which it also broadcasts to all the consumers. Under Assumption 15.1, the demand response game is a valid monotone utility game.

Theorem 15.4 *The demand response game as defined by Definition 15.3 is a valid monotone utility game if Assumption 15.1 is satisfied.*

15.3.3 Price of Anarchy

A payoff maximization game which satisfies (15.14) for any solution set $e \in \mathscr{E}$ is said to be a (λ, μ) smooth game if it satisfy

$$\sum_{i \in \mathscr{N}} V(\{i\}, e^*) \geq \lambda V(\{\mathscr{N}\}, e') - \mu V(\{\mathscr{N}\}, e^*) \tag{15.15}$$

where $e^*, e' \in \mathscr{E}$ are any two solution strategies of the game.

The (1, 1)-smooth games have the property that a lower bound for its price of anarchy is 0.5. Since any valid monotone game is (1, 1)-smooth [20], we have the following result.

Corollary 15.3 *The demand response game as defined by Definition 15.3 is a (1, 1)-smooth game. Moreover, the lower bound of the price of anarchy of any pure Nash equilibrium is at least 0.5.*

We have shown that Nash equilibrium for our game exists, but there can be a number of reasons for which the players may not reach an equilibrium [20]. So, we can consider a weaker notion of equilibria i.e., coarse correlated equilibria for a game for which Nash equilibria does not exist or exists but can not be reached. Since the demand response game is a (1, 1) smooth game, the bound derived via smoothness argument extends automatically, with no quantitative degradation to other weaker equilibria notions [20]. This is called intrinsic robustness property of the price of anarchy. So, lower bound of price of anarchy is 0.5 even in case of coarse correlated equilibrium solution.

15.4 Conclusions

Variability of renewable resources can be accommodated by shaping demand. Effective solutions to this problem will necessarily require distributed control. In this paper, we have discussed our initial research on bounding loss of efficiency by using distributed control. These ideas can be applied to other distributed control problems like supply side market with deep renewable penetration, energy resource aggregation, and scheduling, energy trading between microgrids, etc.

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