Insurance Portfolio Containing a Catastrophe Bond and an External Help with Imprecise Level—A Numerical Analysis

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Abstract. In this paper, an integrated insurer's portfolio, which consists of a few layers of insurance and financial instruments, is numerically analysed. A future behaviour of such a portfolio is related to stochastic processes (like a random interest rate yield and uncertain catastrophic losses), therefore the Monte Carlo (MC) approach is applied. A special attention is paid to a problem of a share of catastrophe bonds in such a portfolio and to an analysis of an influence of an additional layer—an external (e.g. governmental) help. Some important measures of an insurer's risk (like a probability of his bankruptcy) are then numerically analysed. In considered examples, apart from strictly crisp sets of parameters, also fuzzy numbers are used to model an imprecise information concerning the possible external help.

Keywords: Risk process · Insurance portfolio · Catastrophe bond · Monte Carlo simulations · Probability of ruin · Governmental help · Fuzzy numbers

1 Introduction

Nowadays, the insurers face the problem of catastrophic losses, which are caused by earthquakes, tsunamis and other natural catastrophes. Therefore, a problem of an estimation and an analysis of a probability of an insurer's ruin is even more significant and urgent. Moreover, the insurers apply new, financial (or, simultaneously, financial and insurance) instruments, which are intended to lower this probability. A catastrophe bond (or a cat bond in short) is an example of such an instrument (see, e.g., [8,10,11]). However, an issuance of additional instruments changes a whole structure of an insurer's portfolio. Then, a classical risk process, which describes the cash flows of an insurer, should be also generalized to take into account these additional layers of the portfolio. This new formula of the risk process requires more complex approaches and supplementary numerical simulations in order to estimate the probability of an insurer's ruin and other statistics, which are important for an insurer.

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In this paper, we continue a work, which was started in [13,14]. Then, the generalized form of the classical risk process for the insurer's portfolio is considered. A cat bond, which is issued by an insurer, and an external help are examples of the layers in such a portfolio. Contrary to the classical approach, we also assume that there is dependency between time and money, i.e., one unit of money, which is paid now, has other value than the same unit, which will be paid in the future. In the following, cash flows for the insurer's portfolio are analysed using Monte Carlo (MC) simulations.

A contribution of this paper is fourfold. Firstly, a special attention is paid to a problem of a share of catastrophe bonds in the portfolio. An optimum level of the issued bonds is an important factor for the insurer. A larger share minimizes a probability of his bankruptcy, but it also minimizes expected profits of the insurer. Therefore a relevant numerical analysis is conducted. Secondly, a structure of the portfolio is further developed and an additional layer—an external (e.g., governmental or foreign) help—is incorporated. This next layer changes the mentioned generalized form of the classical risk process in a new way. Thirdly, we consider both a probabilistic and an imprecise approach to a value of such a help. In this second case, fuzzy triangular numbers are used to model this external help. Fourthly, in order to directly compare some important risk factors for the insurer, a method of a reduction of an estimation error is applied.

This paper is organized as follows. In Sect. 2, the generalized version of the classical risk process is introduced. Also an applied model of an interest rate (the one–factor Vasicek model) is recalled there. Some notes about a possible optimization procedure, which maximizes the cash flow for an insurer and minimizes his probability of a ruin, are included in Sect. 3. Section 4 is devoted to a numerical analysis of some examples, which are close to practical situations. Section 5 concludes the paper with some final remarks.

2 Risk Reserve Process and Its Generalization

Traditionally, in the insurance industry, a risk reserve process R_t is defined as a model of the financial reserves of an insurer depending on time t, i.e.

$$R_t = u + pt - C_t^* \tag{1}$$

where u is an initial reserve of the insurer, p is a rate of premiums paid by the insureds per unit time and C_t^* is a claim process, which is given by

$$C_t^* = \sum_{i=1}^{N_t} C_i \tag{2}$$

where $C_1, C_2, \ldots$ are iid random values of the claims. These claims are traditionally identified with the losses U_i, which are caused by the natural catastrophes, so we have $C_i = U_i$. There are also models, where the claims are only some part of the losses, e.g.,

$$C_i = \alpha_{\text{claim}} Z_i U_i \ , \tag{3}$$

so $\alpha_{\text{claim}} \in [0,1]$, $Z_i \sim U[c_{\min}, c_{\max}]$, and Z_i, U_i are mutually independent variables. The parameter α_{claim} describes a deterministic share of the considered insurer in the whole insurance market (for the given region) and the random variable Z_i models a random part of the claim C_i in the loss U_i. In this case, a non-informative random distribution, i.e. a uniform distribution $U[c_{\min}, c_{\max}]$ (for $0 \leq c_{\min} \leq c_{\max} \leq 1$), is used. Then, we have a process of the losses, which is given by

$$N_t^* = \sum_{i=1}^{N_t} U_i \tag{4}$$

If the assumption (3) is applied, it can lead to a hedging problem (see, e.g., [13]).

A process of a number of the claims $N_t \geq 0$ is usually driven by a homogeneous Poisson process (HPP), or a non-homogeneous Poisson process (NHPP). In this paper, we assume that a cyclic intensity function

$$\lambda_{\text{NHPP}}(t) = a + b2\pi \sin(2\pi(t-c)) \tag{5}$$

is used to model NHPP of the number of the claims N_t. The parameters of (5), which are applied in the following part of the paper, were estimated in [2], based on the data from the United States, provided by the Property Claim Services (PCS) of the ISO (Insurance Service Office Inc.). Then we have $a = 30.875, b = 1.684, c = 0.3396$. Also, using a method described in [2], the value of the single loss U_i is further modelled by a lognormal distribution with parameters $\mu_{\text{LN}} = 17.357, \sigma_{\text{LN}} = 1.7643$.

Because a non-constant intensity function (5) is applied, then the premium in (1) is fixed as a constant function for some deterministic moment T (see also [14] for further details), so

$$p(T) = (1+\nu_p) E C_i \int_0^T \lambda_{\text{NHPP}}(s) ds \tag{6}$$

where ν_p is a safety loading (or security loading) of the insurer, which is usually, in practical situations, about 10%–20%.

In the following, we consider a more complex insurer's portfolio, which consists of an additional layer—a special financial instrument, which is known as a catastrophe bond (or a cat bond, see, e.g., [8,10,11,14]) Therefore, the classical risk process (1) has to be generalized into a more suitable form, so that the cash flows related to the cat bond can be taken into account.

In general, when a catastrophe bond is issued, the insurer pays an insurance premium p_{cb} in exchange for a coverage, when a triggering point (usually some catastrophic event, like an earthquake) occurs. The investors purchase an insurance–linked security for cash. The above mentioned premium and cash flows are usually managed by a SPV (Special Purpose Vehicle), which also issues the catastrophe bonds. The investors hold the issued assets, whose coupons and/or principal depend on the occurrence of the mentioned triggering point. If such a catastrophic event occurs during the specified period, then the SPV compensates the insurer and the cash flows for the investors are changed. Usually, these flows are lowered, i.e. there is full or partial forgiveness of the repayment of principal and/or interest. However, if the triggering point does not occur, the investors usually receive the full payment from a cat bond (see, e.g., [11,13,14]).

Taking into account the described cash flows of a catastrophe bond, the classical risk process (1) should be written as

$$R_T = \mathrm{FV}_T(u - p_{\mathrm{cb}}) + \mathrm{FV}_T(p(T)) - \mathrm{FV}_T(C_T^*) + n_{\mathrm{cb}} f_{\mathrm{cb}}^i(N_T^*) , \tag{7}$$

where $f_{\mathrm{cb}}^i(N_T^*)$ is a payment function of the single cat bond for the insurer and p_{cb} is an insurance premium. We assume, that p_{cb} is proportional to both a part α_{cb} of a whole price of the single catastrophic bond I_{cb}, and to a number of the issued bonds n_{cb}, so that $p_{\mathrm{cb}} = \alpha_{\mathrm{cb}} n_{\mathrm{cb}} I_{\mathrm{cb}}$.

Moreover, in our setting (which is contrary to the classical approach, see also [13,14]), a value of money depending on time is taken into account. Therefore, $\mathrm{FV}_T(.)$ denotes a future value of the cash flow in (7). In the following, to calculate this future value, the one–factor Vasicek model

$$dr_t = \kappa(\theta - r_t)dt + \sigma dW_t \tag{8}$$

is applied. The parameters for (8) are fitted in [1], so we get $\kappa = 0.1179, \theta = 0.086565, \sigma^2 = 0.0004$.

We can also enrich the considered portfolio and add some other layers (i.e. financial or insurance instruments), e.g. a reinsurance contract (see [14] for a more detailed discussion). But, in this paper, we focus only on a governmental (or, e.g., foreign), external help. We assume, that this help is supplied only if the losses surpass some given minimal limit A_{hlp}, and only with a fixed probability p_{hlp} (i.e. $\Pr(H = 1) = p_{\mathrm{hlp}}$ and $\Pr(H = 0) = 1 - p_{\mathrm{hlp}}$, where H is a binomial variable, which indicates, if this external fund is used or not). Then, a value of this help can be modelled by some function $f_{\mathrm{cb}}^i(N_T^*)$, e.g. by a constant value. If this external fund is incorporated into the generalized risk process (7), we get a new formula

$$\begin{aligned} R_T = \mathrm{FV}_T(u - p_{\mathrm{cb}}) + \mathrm{FV}_T(p(T)) - \mathrm{FV}_T(C_T^*) + n_{\mathrm{cb}} f_{\mathrm{cb}}^i(N_T^*) \\ + I(H = 1, N_T^* \geq A_{\mathrm{hlp}}) f_{\mathrm{hlp}}(N_T^*) , \end{aligned} \tag{9}$$

where $I(.)$ is an indicator function. Easily seen, such a help is treated as an additional source of funds by the insurer, because it lowers the overall losses and mitigates his expanses.

3 Optimization Goals

In a classical problem statement, an insurer is interested in a minimization of a probability of his ruin. For the given moment T, a probability of a ruin at the end of time interval T is given as

$$\phi(T) = \Pr(R_T < 0) . \tag{10}$$

Moreover, an insurer wants to maximize an overall cash flow for his portfolio, which is described by the generalized risk process (7) or (9). Therefore, in the following, we focus on an analysis of these two characteristics. Because of a stochastic and uncertain nature of (7) and (9), the MC approach is used to estimate an expected value of (7) or (9), namely ER_T. In practical situations, an insurer can be also interested in an overall optimization of his portfolio. Then, both the probability of the ruin and the expected value of the future cash flows can be combined in one optimization goal, e.g.,

$$\max(ER_T - \alpha_{\text{pen}} \Pr(R_T < 0)) , \tag{11}$$

where α_{pen} is some penalty factor, which is related to an occurrence of a ruin, and the maximum is taken for selected parameters of the portfolio (see, e.g., [3] for other approaches). In order to solve the problem (11), a stochastic optimization procedure can be necessary (see, e.g., [5]).

4 Numerical Analysis

As it was mentioned in Sect. 2, to model the trajectory of the process R_T, we apply NHPP with the intensity function (5) for the lognormal catastrophic losses. As for a payment function $f(N_T^*)$ for a holder of the considered cat bond, a piecewise linear function is applied (see [8,10,11,14] for a necessary introduction and an additional discussion), so

$$f(C_T^*) = \text{Fv} \left(1 - \sum_{i=1}^{n} \frac{\min(N_T^*, K_i) - \min(N_T^*, K_{i-1})}{K_i - K_{i-1}} w_i \right) \tag{12}$$

where Fv is a face value of the cat bond, $w_1, \ldots, w_n > 0$ are payoff decreases, and $0 \leq K_0 \leq K_1 \leq \ldots \leq K_n$ are the triggering points. We set $\text{Fv} = 1$ (i.e. one monetary unit assumption is used), and

$$K_0 = Q_{\text{NHPP-LN}}^{\text{loss}}(0.75), K_1 = Q_{\text{NHPP-LN}}^{\text{loss}}(0.9) , \tag{13}$$

where $Q_{\text{NHPP-LN}}^{\text{loss}}$ is x-th quantile of the cumulated value of the losses (for the considered NHPP and the lognormal distribution of the single loss). The payoff decrease is equal to $w_1 = 1$ and one year time horizon is applied, so $T = 1$. Then, if after one year, the cumulated value of losses surpasses K_1, the bond holder receives nothing. To find the price of such a catastrophe bond, we apply the method introduced in [8,10,11,14]. It requires analytical formulas and additional Monte Carlo simulations. Then, the mentioned price is estimated as $I_{\text{cb}} = 0.809896$ (see also [8,10,11,14] for a more detailed discussion), so such a value will be used further in this paper. We also assume, that $u = Q_{\text{NHPP-LN}}^{\text{loss}}(0.25)$, i.e. the initial reserve of the insurer is equal to 0.25-th quantile of the cumulated value of the claims, and that $\alpha_{\text{cb}} = 0.3$ (so 30% of the cat bond price is covered by the insurer) and $\nu_p = 0.1$ (i.e. the safety loading for the premiums is equal to 10%). For a better readability of results, the losses (hence, the claims also) are scaled in millions of money units.

4.1 Number of the Issued Cat Bonds

We start from an analysis of an influence of the number of the issued bonds n_{cb} on some key factors for the insurer, like a probability of his ruin. For a larger share of the cat bonds in the portfolio, the potentially catastrophic losses have lower impact on the insurer. This directly leads to the lower probability of his bankruptcy. On the other hand, the larger share also reduces the overall cash flows in the portfolio, even if the issued catastrophe bond will not be used afterwards (because a fixed triggering point of this cat bond is not even achieved). Hence, an issuance of the cat bonds works as an alternative way of a reinsurance (see also [14] for a comparison of these two approaches). Therefore, the insurer should choose an optimal level of the share of the catastrophe bonds in his portfolio. It should be not too large (because it does not maximize the expected insurer's profits) and not too low (because it leads to the higher probability of the bankruptcy at time T). In [13,14] there is no such an exact analysis.

In order to compare simulated outcomes for different values of n_{cb}, it is necessary to minimize other possible sources of variability. Therefore, to reduce a variance (and, furthermore, an estimation error), for an each value of n_{cb} the same set of $n = 1000000$ simulated trajectories is used. We also analyse three possible kinds of dependencies between the claims and the losses, namely $C_i = U_i$ (which is denoted further as Example I), $C_i = 0.5U_i$ (Example II, in this case each claim is always equal to 50% of the loss) and $C_i = Z_i U_i$, where $Z_i \sim U[0, 1]$ (Example III, the loss is transformed to the claim using a standard uniform distribution). Then, Example II reflects a situation, when the insurer has 50% of a whole insurance market, and Example III means, that there is no strict information about a level of such a share. Then, only a very general, non-informative statistical approach can be used (see [13] for a different approach to this problem).

Our analysis is done for a wide range of possible values of n_{cb}, which allows the insurer to directly compare his different possibilities in a construction of the portfolio. The estimated averages of the final value of the portfolio $\bar{R}_T$, as a function of n_{cb}, are plotted in Fig. 1 (outcomes for Example I are denoted by circles, for Example II—by squares, and for Example III—by rhombuses). They are almost linearly decreasing functions, which behave in a very similar way. However, an observed reduction of the estimated expected value is not very fast, e.g. in Example I for $n_{\mathrm{cb}} = 50$ we have $\bar{R}_T = 3487.55$, and for $n_{\mathrm{cb}} = 1600$ (i.e., the share of the cat bonds in such a portfolio is 32 times higher than in the previous case) we get $\bar{R}_T = 3349.06$ (only about 4% reduction).

It should be noted, that the averages $\bar{R}_T$ in Example II and Example III are very similar, but still they are not completely equal. It means, that even if an expected value of the loss in Example III is the same as a deterministic part of U_i in Example II, the outcomes are significantly different, which is rather in contrary to an "intuitive thinking".

Also the estimators of the ruin probabilities $\hat{\phi}(T)$ can be found in the similar way, using numerical simulations (see Fig. 2, the relevant plots are labelled in the same way, as previously). These probabilities are non-linearly decreasing

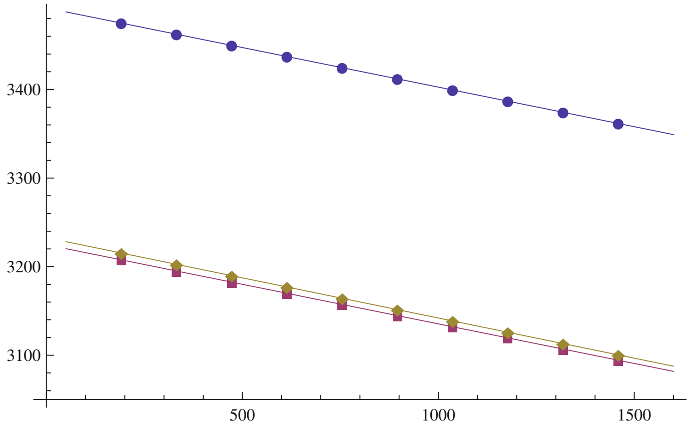

Fig. 1. Estimated averages of the final value of the portfolio (in Example I–Example III)

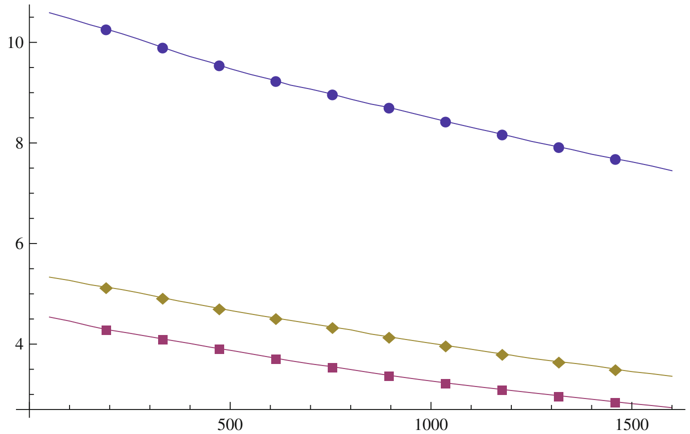

Fig. 2. Estimated probabilities of the final ruin (in Example I–Example III)

functions of n_{cb}. But now, the observed reduction for the increasing values of n_{cb} is more significant. In Example I, for $n_{\mathrm{cb}} = 50$ we have $\hat{\phi}(T) = 10.589\%$, and for $n_{\mathrm{cb}} = 1600$ we get $\hat{\phi}(T) = 7.448\%$ (almost 30% reduction of the ruin probability).

Because $\bar{R}_T$ and $\hat{\phi}(T)$ behave in a different way (linear vs. non-linear) as the functions of n_{cb}, then the outcomes, which are summarized in Figs. 1 and 2, can be directly merged using the optimization function (11) (or other one). Then, the optimal level of the issued cat bonds for the insurer can be directly found.

4.2 Influence of the External Help

We further develop our analysis of the insurer's portfolio and incorporate an additional layer—the external help (see (9)). As it was mentioned, the payment function for this help $f_{\text{hlp}}(N_T^*)$ can be modelled in various ways. In this paper, a function similar to a classical excess-of-loss policy is adopted. Then, we have

$$f_{\text{hlp}}(N_T^*) = \begin{cases} B_{\text{hlp}} - A_{\text{hlp}} & \text{if } \quad N_T^* \geq B_{\text{hlp}} \\ N_T^* - A_{\text{hlp}} & \text{if } B_{\text{hlp}} \geq N_T^* \geq A_{\text{hlp}} \end{cases} , \tag{14}$$

where B_{hlp} is a maximum limit for this help. Formulae (14) is, in some way, similar to commonly used reinsurance contracts (see, e.g., [14]), but without additional costs incurred by an insurer.

Let us suppose, that $n_{\text{cb}} = 1000$, the claims are equal to the losses, $p_{\text{hlp}} = 1$ (i.e., the help is always available, if the minimum limit of the losses is surpassed), and that $A_{\text{hlp}} = Q^{\text{loss}}_{\text{NHPP-LN}}(0.95), B_{\text{hlp}} = Q^{\text{loss}}_{\text{NHPP-LN}}(0.99)$, so the minimal limit for the external help is equal to 0.95-th quantile of the cumulated value of the losses and the maximal limit is given by 0.99-th quantile. Such a set of the parameters constitutes Example IV. Then, using simulations for the same set of trajectories as in Example I, the relevant outcomes can be easily compared. The average for the final value of the portfolio in Example IV is equal to 3598.21, comparing to 3402.67 in Example I (about 5.75% more in Example IV). However, a difference in the ruin probability is less visible—only about 0.01% (8.488% in Example IV vs. 8.498% in Example I).

Of course, in practical situations, A_{hlp} and B_{hlp} can be given as imprecise values, not as strictly precise information. For example, the minimum limit can be stated as "about $Q^{\text{loss}}_{\text{NHPP-LN}}(0.95)$". Such inexact data can be modelled with, e.g., fuzzy sets, in contrary to an application of real numbers (i.e., "exact" information, see [6,7,9,11,13] for examples of applications of the fuzzy numbers in some areas). Fuzzy sets can be also combined with a probabilistic approach, and this leads to random fuzzy variables (see, e.g., [4] for a more detailed review). Therefore, in the next case—Example V—we use triangular fuzzy numbers to describe A_{hlp} and B_{hlp}, and analyse influence of such an assumption on the simulated output. We restrict ourselves to the triangular fuzzy numbers, but the presented further approach can be also used for other kinds of L–R fuzzy numbers.

Let $\tilde{a} = [a_L, a_C, a_R]$ denote a triangular fuzzy number, where a_L is its left end of a support, a_R—its right end of a support, and a_C—a core. Then, $\tilde{a}[\alpha] = [a_L[\alpha], a_R[\alpha]]$ is an α-cut of $\tilde{a}$, if $\alpha \in [0, 1]$.

We assume that

$$\tilde{A}_{\text{hlp}} = [Q^{\text{loss}}_{\text{NHPP-LN}}(0.95) - 200, Q^{\text{loss}}_{\text{NHPP-LN}}(0.95), Q^{\text{loss}}_{\text{NHPP-LN}}(0.95) + 200] \tag{15}$$

(so, the minimum limit of the external help is 0.95-th quantile ± 200), and, in the same way,

$$\tilde{B}_{\text{hlp}} = [Q^{\text{loss}}_{\text{NHPP-LN}}(0.99) - 200, Q^{\text{loss}}_{\text{NHPP-LN}}(0.99), Q^{\text{loss}}_{\text{NHPP-LN}}(0.99) + 200] . \tag{16}$$

Then, simulations for consecutive α-cuts of $\tilde{A}_{\text{hlp}}$ and $\tilde{B}_{\text{hlp}}$ can be performed to obtain an outcome (i.e. an approximation of a fuzzy number) for a desired function $f(x)$. During the MC simulations, α is changed from some starting value $\alpha_0 \geq 0$ up to an upper bound $\alpha_1 \in (\alpha_0, 1]$ with an increment $\Delta\alpha > 0$. After an evaluation of the left and right end points of the different α-level sets of the considered function of the output, i.e. $[\tilde{f}_L[\alpha](x), \tilde{f}_R[\alpha](x)]$, the obtained intervals are put on one another, so they form an approximation of a final fuzzy outcome $\tilde{f}(x)$. During this procedure, we should keep in mind, if $f(x)$ is an increasing or decreasing function of the fixed x, in order to select relevant left or right ends of the α-cuts for $\tilde{A}_{\text{hlp}}$ and $\tilde{B}_{\text{hlp}}$ (see [11–13] for further details of this approach).

The estimated average of the final value of the portfolio forms a L–R fuzzy number, which is almost a triangular fuzzy number (see Fig. 3, a plot labelled with circles). Its support is equal to $[3586.59, 3610.27]$ (respectively, 5.5% and 6.1% more than in Example I) and its core is given by the relevant value from Example IV. A supplier of the external help can be also interested in an evaluation of a probability of using such a help. This value can be directly estimated, if the introduced approach is applied (see Fig. 4, a plot labelled with circles), and it is also a L–R fuzzy number. Its support is equal to $[4.65\%, 5.141\%]$ and its core is given by 4.88%.

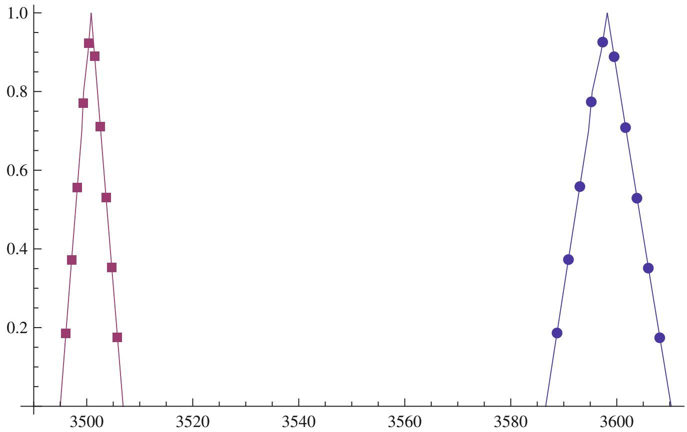

Fig. 3. Estimated averages of the final value of the portfolio (in Example V and Example VI)

An average is an important measure, however, a practitioner can be also interested in a more detailed analysis of other characteristics of the portfolio, e.g., a statistical behaviour of its final value. An example of such a study can be seen in Fig. 5, where a quantile plot for the final value of the insurer's portfolio is plotted. In this case, the quantiles for $\alpha = 0$ of $\tilde{A}_{\text{hlp}}$ and $\tilde{B}_{\text{hlp}}$ are calculated, using the approach described previously. Main differences between the portfolios are seen in Fig. 5 only for lower ranks of the quantiles, e.g., for 0.01-th quantile

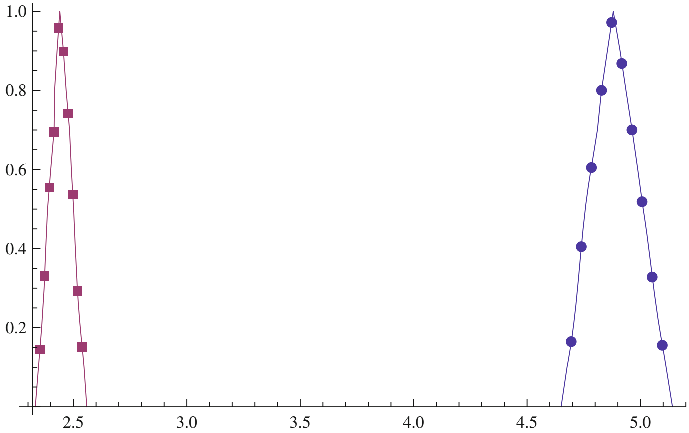

Fig. 4. Estimated probabilities of using the external help (in Example V and Example VI)

we have the final value of the portfolio −2722.21 versus −2247.15 (the difference is equal to 475.06), and for 0.99-th quantile we have the final value 7224.41 versus 7224.56 (so the difference is only 0.15). Then, a major effect of the external help is related rather to the "really catastrophic" events, which are statistically rare (only about 5% cases).

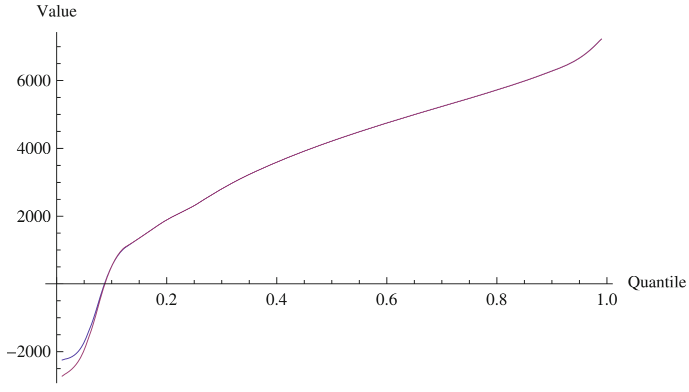

Fig. 5. Quantile plot of the final value of the portfolio (Example V)

In practical situations, we are not completely sure, if the external help will be supplied, i.e., we have $p_{\text{hlp}} \leq 1$. Therefore, we analyse a case (which is labelled further as Example VI), when $p_{\text{hlp}} = 0.5$ (so, there is 50% chance, that the external help can be used in a relevant situation) and all of the other parameters are the same as in Example V. Then, the estimated average of the final value of the portfolio and the probability, if the external help is used, can be seen in Figs. 3 and in 4 (plots labelled with squares). Easily seen, both of these fuzzy numbers are shifted to a left hand side, and their supports are narrower than in Example V.

5 Conclusions

In this paper, we focus on the analysis of the influence of the catastrophe bonds and the external help on the behaviour of the integrated insurer's portfolio. In order to evaluate the probability of the ruin and other important factors for the insurer, Monte Carlo simulations, together with the reduction of the estimation error, are applied. Then, various scenarios for the insurer's portfolio with different parameters are analysed. The outcomes from these examples are compared, using statistical measures. Apart from the crisp approach, the fuzzy numbers are also used to model an imprecise information, like the borderline limits of the external help.

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