Integrated AHP-TOPSIS Approach for Pareto Optimal Solution Selection in Multi-site Supply Chain Planning

Houssem Felfel and Faouzi Masmoudi

Abstract In this paper, a multi-objective, multi-period, multi-product stochastic model for a multi-site supply chain planning problem under demand uncertainty is proposed. The decisions to be made include the amounts of product to be produced, the amounts of products to be transported between the different sites and customers as well as the amounts of inventory of finished or semi-finished products. The developed model aims simultaneously to minimize the expected total cost, to maximize the customer demand satisfaction level and to minimize the downside risk. The e-constraint method is applied to solve the considered model and to generate the set of Pareto optimal solutions. This set of Pareto represents the trade-off between the different objective functions. Then, an integrated approach of the Analytic Hierarchy Process (AHP) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) methods is applied in order to select the best compromise Pareto solution. A numerical example is presented to illustrate the proposed approach.

Keywords Multi-supply chain planning · Multi-objective · Stochastic programming · AHP · TOPSIS

1 Introduction

In the face of today's highly competitive markets, manufacturing companies are required to expand their production capacity by adding more sites or outsourcing. Therefore, an integrated planning approach that coordinates the different entities of the multi-site supply chain should be established.

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M. Haddar et al. (eds.), *Design and Modeling of Mechanical Systems—III*,
Lecture Notes in Mechanical Engineering,
https://doi.org/10.1007/978-3-319-66697-6_30

A lot of attention has been made in the literature to treat multi-site supply chain planning problems. Many of these works are dealing with deterministic approaches (Felfel et al. 2014, 2015c, Ayadi et al. 2016). However, real multi-site supply chain planning problems are characterized by many sources of uncertainty such as customer demand. Thus, the assumption that these parameters are deterministic will lead to unrealistic results. So, it is crucial to develop an optimization planning model that takes into account existing uncertainties. Two-stage stochastic programming approach (Birge and Louveaux 1997) is widely used in the literature to deal with optimization problem under uncertainty. In this approach, the first-stage variables include the decisions to be made "here-and-now" before the revelation of the random events. The second-stage variables represent the variables that should be made in a "wait-and-see" mode after the revelation of uncertainty.

The minimization of the cost and the maximization of the profit are widely treated in multi-site supply chain planning problem. One can refer to Moon et al. (2002), Jackson et al. (2003), Lin and Chen (2006) and Felfel et al. (2015b). Nevertheless, other important criteria such as the customer demand satisfaction and the risk of having high total cost should be treated in multi-site supply chain planning problems. It is worthwhile mentioning that the objective functions usually conflict with each other in multi-objective optimization problem. Thus, the solution of this problem consists on a front of Pareto optimal solutions which represents the trade-off between the different objectives.

To solve the multi-objective optimization problem, the task of the decision maker consists on obtaining the front of Pareto optimal solutions and finding the most preferred compromise solution. The selection of a best solution from the front of Pareto can be considered as multiple criteria decision-making (MCDM) problem. A lot of methods have been developed in the literature for MCDM (Vincke 1992) such as AHP, ANP, ELCTRE, PROMETHEE, VIKOR and TOPSIS, etc. The technique for order performance by similarity to ideal solution (TOPSIS) and the analytic hierarchy process (AHP) were successfully applied in different areas such as supply chain management and logistics, manufacturing systems, design, engineering, and other many topics (Behzadian et al. 2012). To gain the benefits of these two methods, an integrated approach of the AHP and TOPSIS methods is applied to rank and to select the best compromise Pareto solution in a multi-objective supply chain planning problem.

The main objective of this paper is to develop a multi-objective, multi-product, multi-period, multi-site supply chain production and transportation model under demand uncertainty. The proposed model aims simultaneously to minimize the total cost, to maximize the customer demand satisfaction and to minimize the downside risk. In order to incorporate uncertainty in the considered model, a two-stage stochastic programming approach is adopted. An integrated approach of AHP and TOPSIS method is adopted to select the best Pareto optimal solution. Indeed, the AHP method is applied to determine the weights of the objectives and the TOPSIS method is used to rank the Pareto optimal solutions. A numerical example is presented to illustrate the proposed approach.

The remainder of the article is organized as follows. In Sect. 2, the problem statement is introduced. Section 3 describes the developed stochastic multi-objective planning model. In Sect. 4, the solution approach is presented. Section 5 details the numerical example and the computational results. Finally, Sect. 6 draws conclusions on this work and suggests future research.

2 Problem Statement

The supply network considered in this paper consists of a manufacturing system including many production sites that cooperate together in order to expand their capacities and competences. The end product is produced through different processes defined as multi-stage. Each production stage may involve more than one site, creating a multi-site supply chain structure. The considered supply chain is managed in a centralized way. The planning horizon contains several time period. Furthermore, finished products are characterized by unstable and uncertain demand. This uncertain demand could lead whether to excessive production and inventory costs or unsatisfied customer.

The objective of the considered multi-site supply chain planning problem is to minimize the total expected costs, to maximize the customer demand satisfaction level, and to minimize the downside risk (DRisk). The total costs include production costs, inventory costs, penalty costs of lost demand, and transportation costs. Model decision variables comprise the amount of product to be produced at each site in each period, the amount of inventory of each finished or semi-finished product that should be maintained on each site in each period, the amount of lost demand as well as the quantity of products to be transported between upstream and downstream sites and customers considering demand uncertainty.

3 Proposed Stochastic Mathematical Model

A multi-objective two-stage stochastic programming model is proposed in this section to deal the considered problem. The first-stage decisions include the quantity of products to be produced at each plant as well as the transportation quantity of products between the different plants. Decisions such as the quantity of products to be delivered to the customer, the inventory level and the lost demand quantity are considered as second-stage decisions. To formulate the mathematical model, we introduce the following indices parameters and decision variables:

Indices	
L_i	Set of direct successor plant of site i
ST_j	Set of stages (j = 1, 2, …, N)
i, i'	Production plant index (i, i' = 1, 2, …, I) where i belongs to stage n and i' belongs to stage $n + 1$
s	Scenario index (s = 1, 2, …, S)
k	Product index (k = 1, 2, …, K)
t	Period index (t = 1, 2, …, T)
Decision variables	
P_{ikt}	Production amounts of product k at plant i in period t
S_{ikt}^{s}	Quantity of end of period inventory of product k for scenario s at plant i in period t
JS_{ikt}^{s}	Quantity of end of period inventory of semi-finished product k for scenario s at plant i in period t
$TR_{i->i',kt}$	Quantity of product k transported from plant i to i' in period t
$TR_{i->CUS,kt}^{s}$	Quantity of product k transported from the last plant i to customer for scenario s in period t
$Dlost_{kt}^{s}$	Lost demand amounts of finished product k for scenario s in period t
$Q_{i,k}$	Amounts of product k received by plant i for scenario s in period t
Parameters	
cp_{ik}	Production unit cost for product k in at plant i
$ct_{i->i',k}$	Transportation unit cost of between plant i and i' for product k
$ct_{i->CUS,k}$	Transportation unit cost between the last plants i and the customer
cs_{ik}	Inventory unit cost of finished or semi-finished product k at plant i
D_{kt}^{s}	Demand of finished product k for scenario s in period t
DL	Distribution time of the finished products
pe_k	Penalty cost of product k
$capp_{it}$	Production capacity at plant i in period t
$caps_{it}$	Inventory capacity at plant i in period t
$capt_{i->i',t}$	Transportation capacity at plant i in period t
b_k	Time needed for the production of a product k [min]
π^s	The occurrence probability of scenario s where $\sum_{s=1}^{S} \pi^s = 1$

Problem formulation

$$\min E\,[Cost] = \sum_{s=1}^{S} \pi^s \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{i=1}^{I} cs_{ik}(S_{ikt}^{s} + JS_{ikt}^{s}) + pe_k Dlost_{k,t}^{s} + ct_{i->CUS,k} TR_{i->CUS,kt}^{s} + \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{i=1}^{I} cp_{ik} P_{ikt} + ct_{i->i',k} TR_{i->i',kt} \quad (1)$$

$$\max E[DS] = \sum_{s=1}^{S} \pi^s DS^s = \sum_{s=1}^{S} \pi^s \frac{\sum_{t=1}^{T} \sum_{k=1}^{K} D_{kt}^s - Dlost_{kt}^s}{\sum_{t=1}^{T} \sum_{k=1}^{K} D_{kt}^s} \tag{2}$$

$$DS^s \geq MDS \tag{3}$$

$$\min DRisk_{\Omega} = \sum_{s} \pi^s \psi^s \tag{4}$$

$$\psi^s \geq Cost_s - \Omega, \qquad \psi^s \geq 0, \ \forall s \tag{5}$$

$$S_{ik,t}^s = S_{ik,t-1}^s + P_{ikt} - \sum_{i' \in L_i} TR_{i->i',kt}, \quad \forall i \in ST_{j<N}, \ \forall k, t, s \tag{6}$$

$$\sum_{i=1}^{I} S_{ik,t}^s = \sum_{i=1}^{I} S_{ik,t-1}^s + P_{ikt} - TR_{i->CUS,kt}^s, \quad \forall i \in ST_{j=N}, k, t, s \tag{7}$$

$$JS_{ik,t}^s = JS_{ik,t-1}^s + Q_{ikt} - P_{ikt}, \quad \forall i, k, t, s \tag{8}$$

$$Dlost_{kt}^s = D_{kt}^s - TR_{i->CUS,kt}^s, \quad \forall k, t, s \tag{9}$$

$$Q_{i'k,t+DL} = \sum_{i' \in L_i} TR_{i->i',kt}, \qquad \forall i, k, t, s \tag{10}$$

$$\sum_{k=1}^{K} b_k P_{ikt} \leq capp_{it}, \qquad \forall i, t \tag{11}$$

$$\sum_{k=1}^{K} S_{ikt}^s + JS_{ikt}^s \leq caps_{it}, \qquad \forall i, t, s \tag{12}$$

$$\sum_{k=1}^{K} TR_{i->i',kt} \leq captr_{it}, \qquad \forall i, t, s \tag{13}$$

$$P_{ikt}, S_{ikt}^s, JS_{ikt}^s, TR_{i->i',kt}, TR_{i->CUS,kt}^s, Q_{i,k}, Dlost_{kt}^s \geq 0, \qquad \forall i, k, t, s \tag{14}$$

The first objective function (1) aims to minimize the expected total cost (E [Cost]). The occurrence probability π^s of each scenario is considered in order to calculate the expected total cost. Equations (2) and (3) attempt to maximize the customer demand satisfaction level (MDS). The third objective function (4) aims to minimize the downside risk (DRisk) where ψ^s is a positive variable that measures deviation between the scenario cost value and a target Ω as shown in Eq. (8).

Constraints (6) and (7) provide the balance for the inventory level of products. Constraint (8) represents the inventory balance for the semi-finished products. Constraint (9) provides the balance equation for lost products demand. Constraint (10) gives the balance equations for the transportation between the different production stages. The set of constraints (11)–(13) denote the capacity constraints. Constraint (14) is the nonnegativity restriction on the decision variables.

4 Solution Approaches

4.1 Generation of the Front of Pareto Optimal Solutions

The obtained mathematical formulation can be finally expressed as follows:

$$\begin{aligned} \min\{E\,[Cost],\ \ -MDS, DRisk\,\} \\ s.t.\ Eqs.\ (1)-(14) \end{aligned} \tag{15}$$

The solution of the above problem consists of a set of Pareto optimal solutions. This set of Pareto represents the trade-offs that exist between the considered objective functions. In order generate this set of Pareto, we have applied the e-constraint method proposed by Haimes et al. (1971). This approach was widely used to solve multi-objective supply chain planning problems (Guillén et al. 2005; Franca et al. 2010; Felfel et al. 2015a). The main principle of this technique is to select one of the objective functions to be optimized whereas the other objectives are transformed into constraints with allowable bounds ε_i. In order to generate the entire set of Pareto optimal solutions, the level of ε_1 and ε_2 are changed as follows:

$$\begin{aligned} \min\,\{E\,[Cost]\} \\ s.t.\ Eqs.\ (1)-(14) \\ DRisk \le \varepsilon_1 \\ MDS \ge \varepsilon_2 \end{aligned} \tag{16}$$

4.2 Selection of the Best Pareto Optimal Solution

The selection of the most preferred Pareto optimal is based on a combination of the AHP method with the TOPSIS method. The analytic hierarchy process method (AHP) is a tool to evaluate and analyze multi-criteria decision-making problem first developed by Saaty (1980). In this paper, this method is used to determine the relative importance of each objective function. In order to obtain these weights, a pairwise comparison matrix is should be developed using Saaty preference scale detailed in Table 1.

Table 1 Preference scale (Saaty 1980)

Value (*bij*)	Description (*i* over *j*)
1	Equal importance
3	Weak importance
5	Strong importance
7	Very strong importance
9	Absolute importance
2, 4, 6, 8	Intermediate values

Then, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), developed by Hwang and Yoon (1981), is conducted to achieve the final ranking of the Pareto optimal solutions based on the weights obtained by the AHP method. In the TOPSIS approach, we calculate the best Pareto optimal solution that has simultaneously the farthest distance from the negative ideal solution and the shortest distance from the ideal solution. The positive ideal solution is a solution that attempts to minimize the expected total cost, to minimize the downside risk and to maximize the customer demand satisfaction level, whereas the negative ideal solution is the opposite of the previous one. For more details concerning each method, one can refer to Ayadi et al. (2016) and Felfel et al. (2017).

5 Illustrative Example

The considered example consists of a multi-site manufacturing network which contains five production stages with eight plants and two finished products. The planning horizon includes eight time periods where the length of each period is one week. It is assumed that the uncertain demand is considered as a set of discrete scenarios generated randomly and associated with known probability. The numerical example is solved using LINGO 14.0 and MS-Excel 2010 on a 32-bit with an INTEL(R) Core (TM) 2Duo CPU, T5670@1.8 GHZ, 1.8 GHZ, 2 GB RAM.

In this paragraph, the e-constraint method is used to solve the multi-objective stochastic supply chain planning problem and to generate the set of Pareto optimal solutions. The obtained set of Pareto drawn in Fig. 1 contains 30 points. It should be noted that every point of the set of Pareto shown in Fig. 1 represents a particular set of supply chain planning decisions.

In order to apply the AHP method, each pair of objective functions is compared pairwise to determine their relative importance using the ratio scale shown in

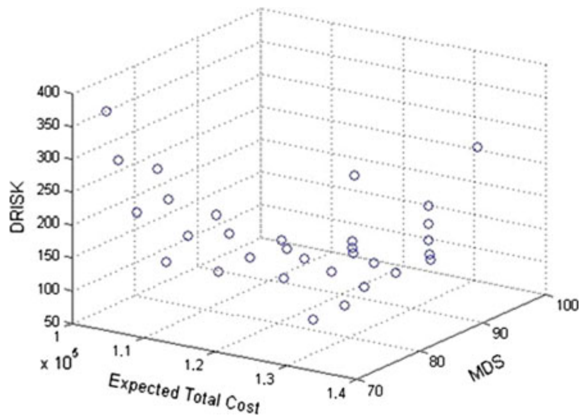

Fig. 1 Set of Pareto optimal solutions

Table 1. It is worth mentioning that expected the total cost is considered more important for the decision maker than the other objective functions. The least important objective is considered to be the DRisk. The relative weights of the different objectives are detailed in the last column of Table 2.

Table 2 Pairwise comparison of the objective functions and obtained weights

	E [Cost]	MDS	DRisk	Weight
E [Cost]	1	3	6	0.655
MDS	0.33	1	3	0.250
DRisk	0.17	0.33	1	0.095

Table 3 Separation measure from the ideal solution and negative ideal solution and relative closeness coefficient

Solution	D_j^+	D_j^-	CC_j^*	Rank
S1	0.019	0.0318	0.3739	30
S2	0.0242	0.0244	0.4975	28
S3	0.0264	0.0237	0.5266	24
S4	0.0284	0.0234	0.5482	22
S5	0.0302	0.0233	0.5641	19
S6	0.0309	0.0234	0.5691	17
S7	0.0283	0.0178	0.614	11
S8	0.0291	0.0176	0.6231	8
S9	0.0298	0.0176	0.6291	7
S10	0.0296	0.0203	0.5935	15
S11	0.0297	0.0233	0.5605	20
S12	0.029	0.0142	0.6714	2
S13	0.0296	0.0144	0.6726	1
S14	0.0292	0.0163	0.6418	6
S15	0.0288	0.0197	0.5938	14
S16	0.0288	0.024	0.5462	23
S17	0.028	0.0152	0.6481	5
S18	0.0283	0.0148	0.6562	3
S19	0.0288	0.0155	0.6496	4
S20	0.0284	0.0189	0.6001	12
S21	0.0284	0.0264	0.5183	25
S22	0.0266	0.0218	0.5498	21
S23	0.0272	0.0189	0.59	16
S24	0.0278	0.0168	0.6231	9
S25	0.0284	0.0174	0.6205	10
S26	0.028	0.0275	0.5043	27
S27	0.0272	0.0316	0.4628	29
S28	0.0265	0.0259	0.5062	26
S29	0.027	0.0209	0.5643	18
S30	0.0278	0.0189	0.5954	13

Subsequently, the TOPSIS method is applied to evaluate and select the compromise solution based on the weights obtained by AHP method. The separation distances of each Pareto solution from the positive and negative ideal solution D_j^+ and D_j^- respectively, the relative closeness measure CC_j^* to the ideal solution, as well as the rank of each Pareto solution are reported in Table 3. According to Table 3, the most preferred Pareto optimal solution is S13 since it has highest CC_j^* value of 0.6726.

6 Conclusion

In this paper, a two-stage stochastic, multi-objective, multi-site, multi-period, supply chain production and transportation model is developed. Three objective functions are considered which are the minimization of the expected total cost, the minimization of the DRisk, and the maximization of the customer demand satisfaction level. A front of Pareto optimal solutions is generated for the proposed model by means of the e-constraint method. Subsequently, an integrated approach of AHP and TOPSIS methods is applied in order to select the best Pareto solution. In the first step, the weights of each objective are calculated using the AHP approach. Then, in the second step, the Pareto optimal solutions are ranked by using TOPSIS to find the most preferred solution. As future work, other multi-criteria decision-making approach could be evaluated in order to compare their performance with the proposed integrated approach.

Acknowledgements We would like to acknowledge the financial support provided by the Mobility of researchers and research for the creation of value (MOBIDOC) as well as LINDO Systems, Inc for giving us a free educational research license of the extended version of LINGO 14.0 software package.

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