

# A Tractable Traffic-Aware User Association Scheme in Heterogeneous Networks

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**Abstract.** In Heterogeneous networks (HetNets), the power difference between macro base stations (MBSs) and small base stations (SBSs) causes severe load unbalance. Therefore, cell range expansion (CRE) is proposed as an effective method to extend the coverage of SBSs and achieve balanced utilization of BSs. However, the downlink (DL) quality for offloaded user equipment (UE) cannot be guaranteed. In this paper, a traffic-aware user association scheme is proposed in HetNets. Distinct association biases are applied to different UEs according to their requirements. System performance of the proposed scheme is analyzed using the tool of stochastic geometry. The results show that the proposed scheme can improve DL throughput by enhancing the rate coverage of UEs, meanwhile signal-to-interference-plus-noise ratio (SINR) requirement with low data rate demand UEs is ensured. Moreover, the optimal association bias, which maximizes DL throughput, can be derived through particle swarm optimization (PSO), and it changes with different densities of BSs and UEs.

**Keywords:** Traffic demand · User association · Cell range expansion · System capacity · Heterogeneous networks

## 1 Introduction

With the rapid development of smart terminals, cellular networks face an overwhelming growth in data traffic [1, 2]. To satisfy the extremely high data demand, HetNets are proposed to improve the coverage and throughput of wireless communication systems [3, 4]. The SBSs in HetNets can provide network access for closer users and offload the data traffic from MBSs to earn traffic offloading gains. However, the transmit power of MBSs is always larger than that of SBSs, the conventional association policy of reference signal receiving power (RSRP) [5] leads to unbalanced load. Hence, CRE, which allows a UE to access in SBSs with lower DL SINR, was proposed as an effective method to extend the coverage of SBSs and achieve load balance.

Several studies have addressed the application of CRE. In [6], the authors focused on the joint transmission of DL and uplink (UL), and derived the transmission success probability and energy efficiency (EE) using stochastic geometry.

Authors of [7] concerned with the cell edge users and presented an adaptive CRE algorithm to improve the cell edge user throughput in HetNets. A scheme with rate-based CRE offset was proposed in [8], which adjusts the CRE offset according to the ratio of a user's UL and DL data demands. In [9], a Pico-specific upper bound CRE bias estimation algorithm for HetNets was proposed. However, these papers neglect the fact that the types and needs of UEs grow rapidly in fifth generation (5G) scenario, traffic demand for UEs should be considered.

In this paper, a tractable traffic-aware user association (TUA) scheme is proposed in HetNets, where distinct association biases are applied to UEs according to different user traffic demands. The main contributions are as follows:

- Proposed scheme is able to balance system loads by offloading some high traffic demand UEs from MBSs to SBSs for better rate coverage, while ensuring the SINR coverage of low traffic demand UEs. Furthermore, proposed scheme can improve rate performance of UEs and enhance system DL throughput compared to traditional RSRP association.
- The expression of SINR coverage, rate coverage, and system DL throughput are derived theoretically using the tool of stochastic geometry.
- The optimal association bias factor  $\theta_{S,u_H}^*$ , which can maximize DL throughput, can be derived through particle swarm optimization (PSO). Numerical results show that either UE density or BS density has an effect on  $\theta_{S,u_H}^*$ .

## 2 System Model

In this paper, a 2-tier HetNet consisting MBSs and SBSs is considered. The position of BSs are modeled according to an independent homogeneous Poisson Point Processes (PPP)  $\Phi_v$  with intensity  $\lambda_v$ , where  $v = M$  for MBS, and  $v = S$  for SBS. UEs are divided into high data rate UEs and low data rate UEs according to different traffic types. For example, UEs with voice demand are low data rate UEs, and UEs with video demand are high data rate UEs. It is also assumed that all kinds of UEs are located as independent PPP  $\Phi_u$  with intensity  $\lambda_u$ , where  $u = u_H$  for high data rate UEs, and  $u = u_L$  for low data rate UEs.

BSs in the same tier have the same transmit power  $P_v$  over bandwidth  $W$ . Assumed that downlink signals experience path loss with a path loss exponent  $\alpha$ , where  $\alpha > 2$ . The fading between a BS and a UE is assumed to be Rayleigh fading with unit average power, i.e.  $h \sim \exp(1)$ . Interference signals arise from all other BSs except the serving BS, and there is no intra-cell interference, e.g. orthogonal multiple access is applied within a BS area [5].

### 2.1 Traffic-Aware User Association Scheme

In traditional RSRP association method [5], all UEs are associated with the serving BSs offering the maximum received power. However, in RSRP mode, MBSs have bigger coverage area than SBSs because of much higher transmit power, which causes unbalanced loads of BSs. Besides, different UEs have characteristic

requirements. For  $u_L$ , it is easy to achieve user demand, it is also important to ensure high signal-to-interference-plus-noise (SINR) quality and make data traffic reliable. For  $u_H$ , high data rate transmission is a key requirement. In proposed TUA scheme, different UEs use diverse association biases. Specifically, the SBS coverage area for  $u_L$  remains unchanged, while the SBS coverage area for  $u_H$  is expanded. That means  $u_L$  are connected to the serving BSs offering maximum DL received power for best SINR quality, while  $u_H$  are associated with BSs based on biased received power to have a probability of offloading from MBS to SBS. The TUA scheme can be expressed respectively as follows

$$\begin{cases} \text{if } P_M \theta_{M,u} \|x_M\|^{-\alpha} > P_S \theta_{S,u} \|x_S\|^{-\alpha} & \text{then to MBS} \\ \text{if } P_M \theta_{M,u} \|x_M\|^{-\alpha} < P_S \theta_{S,u} \|x_S\|^{-\alpha} & \text{then to SBS} \end{cases}, \quad (1)$$

where  $\|x_v\|$  is the distance between a UE to its nearest BS in  $v$ th tier, and  $\theta_{v,u}$  is the association bias factor of different UEs  $u$  in  $v$ th tier. In TUA scheme, we have

$$\theta_{M,u_L} = \theta_{S,u_L} = 1, \text{ and } \frac{\theta_{S,u_H}}{\theta_{M,u_H}} > 1. \quad (2)$$

Figure 1 illustrates the DL UE association with TUA scheme in HetNets. The biggest solid circle presents the MBS coverage area for higher transmit power of MBS. According to the association rule of TUA scheme, the SBS coverage area for UEs with different data rate demands differs. The smallest solid circle is the SBS coverage area for  $u_L$ , while the dotted circle is the expanded SBS coverage area for  $u_H$ . As shown in Fig. 1, some  $u_H$ , e.g. UE1, are offloaded from MBS to SBS for high achieved rate and also system balance.

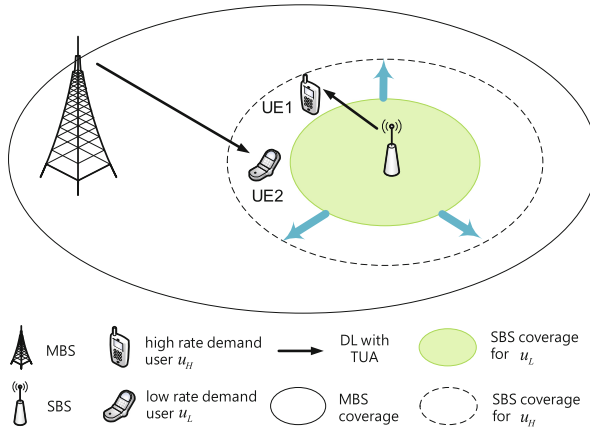


Fig. 1. An illustration of UE association with TUA scheme in HetNets.

## 2.2 SINR Model

According to Slivnyak's theorem [10], a typical UE located at origin is analyzed. Because all BSs are assumed to use the same bandwidth, the DL interference comes from all BSs except the serving BS, which can be modeled as a PPP  $\Phi_I$  with a density of  $\lambda_I = \sum_v \lambda_v$ . Hence, the SINR of a typical UE connected with  $v$ th tier is written as

$$SINR_v = \frac{P_v h_{x_v} \|x_v\|^{-\alpha}}{\sum_{y_i \in \Phi_I} P_i h_{y_i} \|y_i\|^{-\alpha} + \sigma^2}, \quad (3)$$

where  $\|y_i\|$  is the distance from interference BSs of  $i$ th tier to the typical UE, and  $\sigma^2$  is the constant additive noise power.

## 3 System Performance Analysis

In this section, we mainly analyze the SINR coverage, rate coverage, and DL throughput. Especially, DL throughput can be given as

$$DL \text{ Throughput} = \sum_{u \in \Phi_u} R_u, \quad (4)$$

where  $R_u$  is the achievable traffic rate of UEs.

### 3.1 SINR Coverage

SINR coverage is defined as the probability that the SINR of a typical UE is larger than SINR threshold  $\tau$ . Therefore, the SINR coverage can be expressed by definition as

$$\mathcal{S}(\tau) = \mathbb{P}(SINR > \tau). \quad (5)$$

In the HetNets, a typical UE has multiple BSs to connect with. Hence, let  $\mathcal{S}_v(\tau)$  defines the SINR coverage of a typical UE associated with  $v$ th tier, and the SINR coverage for a typical UE is

$$\mathcal{S}(\tau) = \sum_v A_v \mathcal{S}_v(\tau). \quad (6)$$

According to [11], the probability of UEs associated with  $v$ th tier in DL can be derived as bellows

$$A_v^u = \frac{\lambda_v}{\sum_{i \in \{M, S\}} \lambda_i \left( \frac{P_i}{P_v} \hat{\theta}_{i,u} \right)^{2/\alpha}}, \quad (7)$$

where  $\hat{\theta}_{i,u} = \frac{\theta_{i,u}}{\theta_{v,u}}$ .

Let  $D_v^u$  denotes the distance from a typical UE  $u$  to its DL serving BS in  $v$ th tier, and the probability distribution function (PDF) of  $D_v^u$  can be written as [11]

$$f_{D_v^u}(r) = \frac{2\pi\lambda_v}{A_v^u} r \exp \left\{ -\pi r^2 \sum_{i \in \{M, S\}} \lambda_i \left( \frac{P_i}{P_v} \hat{\theta}_{k,u} \right)^{2/\alpha} \right\}. \quad (8)$$

**Theorem 1.** *The DL SINR coverage of a typical UE  $u \in \{u_L, u_H\}$  can be given as*

$$\begin{aligned} \mathcal{S}^u(\tau) &= \sum_v 2\pi\lambda_v \int_{r>0} r \exp \left\{ -r^\alpha P_v^{-1} \tau \sigma^2 \right. \\ &\quad \left. -\pi r^2 \left( \sum_{i \in \{M, S\}} \lambda_i \left( \frac{P_i}{P_v} \right)^{\frac{2}{\alpha}} \left( \hat{\theta}_{i,u}^{2/\alpha} + \tau^{\frac{2}{\alpha}} \int_{\left(\frac{\hat{\theta}_{i,u}}{\tau}\right)^{2/\alpha}}^{\infty} \frac{1}{1+u^{\frac{\alpha}{2}}} du \right) \right) \right\} dr. \end{aligned} \quad (9)$$

*Proof.* Using (8) into the definition of SINR coverage in (5), and we can get the SINR coverage of a typical UE which is associated with  $v$ th tier BS as follows

$$\begin{aligned} \mathcal{S}_v(\tau) &= \mathbb{E}_r [\mathbb{P}(SINR_v > \tau) | r] \\ &= \int_{r>0} \mathbb{P} \left( \frac{P_v h r^{-\alpha}}{I + \sigma^2} > \tau \right) f_{D_v^u}(r) dr \\ &= \int_{r>0} \mathbb{P} \left( h > r^\alpha P_v^{-1} \tau (I + \sigma^2) \right) f_{D_v^u}(r) dr \\ &= \int_{r>0} \mathbb{E}_{I_r} \left( \exp(-r^\alpha P_v^{-1} \tau (I + \sigma^2)) \right) f_{D_v^u}(r) dr \\ &= \int_{r>0} \exp(-r^\alpha P_v^{-1} \tau \sigma^2) \prod_{i \in \{M, S\}} \mathcal{L}_{I_i}(r^\alpha P_v^{-1} \tau) f_{D_v^u}(r) dr, \end{aligned} \quad (10)$$

where  $I$  is the interference, and  $\mathcal{L}_{I_i}(r^\alpha P_v^{-1} \tau)$  is the Laplace transform of the interference from BSs in  $i$ th tier derived as

$$\begin{aligned} &\mathcal{L}_{I_i}(r^\alpha P_v^{-1} \tau) \\ &= \mathbb{E}_{I_i} \left( \exp(-r^\alpha P_v^{-1} \tau I_i) \right) \\ &= \mathbb{E}_{\Phi_i} \left( \exp \left( -r^\alpha P_v^{-1} \tau \sum_{y \in \Phi_i} P_i h_y \|y\|^{-\alpha} \right) \right) \\ &= \exp \left( -2\pi \lambda_i \int_{z_i}^{\infty} \left( 1 - \frac{1}{1+r^\alpha P_v^{-1} P_i \tau y^{-\alpha}} \right) y dy \right) \\ &= \exp \left( -\pi r^2 \lambda_i \left( \frac{P_i}{P_v} \tau \right)^{\frac{2}{\alpha}} \int_{\left(\frac{\hat{\theta}_i}{\tau}\right)^{2/\alpha}}^{\infty} \frac{1}{1+u^{\frac{\alpha}{2}}} du \right), \end{aligned} \quad (11)$$

where  $z_i$  is the distance of the nearest interference of  $i$ th tier expressed as

$$z_i = \left( \frac{P_i}{P_v} \hat{\theta}_k \right)^{1/\alpha} r.$$

Hence,  $\mathcal{S}_v(\tau)$  can be got by applying (10) and (11) in (5), and the SINR coverage of Theorem 1 can be easily derived from (7).

**Corollary 1.** *When  $\sigma^2 = 0$  and  $\alpha = 4$ , the expression of SINR coverage is*

$$\mathcal{S}(\tau) = \sum_{v \in \{M, S\}} \frac{\lambda_v}{\sum_{i \in \{M, S\}} \lambda_i \left( \sqrt{\frac{P_i}{P_v} \hat{\theta}_i} + \sqrt{\frac{P_i}{P_v} \tau} \arctan \sqrt{\tau / \hat{\theta}_i} \right)}. \quad (12)$$

### 3.2 Rate Coverage

According to Shannon theorem, the rate  $R$  of a typical UE is

$$R = \frac{W}{N} \log(1 + SINR), \quad (13)$$

where  $W$  denotes the bandwidth of the system, and  $N$  is the total number of UEs in coverage area of the serving BS.

The rate coverage is defined as the probability that the rate of a typical UE is larger than rate threshold  $\rho$ . Therefore, the rate coverage can be expressed by definition as

$$\mathcal{R}(\rho) = \mathbb{P}(R > \rho). \quad (14)$$

Denote demands of  $u$  as  $\rho_u$ . It is noticeable that  $\rho_{u_H} > \rho_{u_L}$  as definition. Similar to SINR coverage in last part, the rate coverage in HetNets is

$$\mathcal{R}(\rho) = \sum_v A_v \mathcal{R}_v(\rho). \quad (15)$$

**Theorem 2.** *The rate coverage of a typical UE  $u \in \{u_L, u_H\}$  is*

$$\begin{aligned} \mathcal{R}^u(\rho_u) = & \sum_v 2\pi\lambda_v \times \int_{r>0} r \exp \left\{ -r^\alpha P_v^{-1} \left( 2^{\frac{\rho_u \bar{N}_v}{W}} - 1 \right) \sigma^2 \right. \\ & \left. - \pi r^2 \left( \sum_{i \in \{M, S\}} \lambda_i \left( \frac{P_i}{P_v} \right)^{2/\alpha} \left( \hat{\theta}_i^{2/\alpha} + \left( 2^{\frac{\rho_u \bar{N}_v}{W}} - 1 \right)^{\frac{2}{\alpha}} \int_{\left( \frac{\hat{\theta}_i}{2^{\frac{\rho_u \bar{N}_v}{W}} - 1} \right)^{2/\alpha}}^{\infty} \frac{1}{1+u^{\frac{\alpha}{2}}} du \right) \right) \right\} dr, \end{aligned} \quad (16)$$

where  $\bar{N}_v$  is the average number of UEs connected to the serving BS in  $v$ th tier given as

$$\bar{N}_v = 1 + 1.28 \frac{\sum_{k \in \{u_H, u_L\}} \lambda_k A_v^k}{\lambda_v}.$$

*Proof.* According to the definition of rate coverage in (14), the rate coverage of a typical UE  $u$  connected with  $v$ th tier with a rate coverage threshold  $\rho_u$  is

$$\begin{aligned} \mathcal{R}_v^u(\rho_u) &= \mathbb{P}(R^u > \rho_u) \\ &= \mathbb{E}_{N_v} \left[ \mathbb{P} \left( \frac{W}{N_v} \log(1 + SINR^u) > \rho_u \right) | N_v \right] \\ &\approx \mathbb{P} \left( SINR^u > 2^{\frac{\rho_u \bar{N}_v}{W}} - 1 \right) \\ &= \mathcal{S}^u \left( 2^{\frac{\rho_u \bar{N}_v}{W}} - 1 \right). \end{aligned} \quad (17)$$

In this paper, different UE clusters are deployed independently according to PPP in the system. Therefore, the average load of tagged serving BS in  $v$  tier can be derived referring to [11] as follows:

$$\begin{aligned} \bar{N}_v &= 1 + 1.28 \frac{\lambda_{u_H} A_v^{u_H}}{\lambda_v} + 1.28 \frac{\lambda_{u_L} A_v^{u_L}}{\lambda_v} \\ &= 1 + 1.28 \frac{\sum_{k \in \{u_H, u_L\}} \lambda_k A_v^k}{\lambda_v}. \end{aligned} \quad (18)$$

Hence, Theorem 2 can be derived by applying (7), (17) and (18) into (15).

**Corollary 2.** *When  $\sigma^2 = 0$  and  $\alpha = 4$ , the rate coverage can be expressed as*

$$\mathcal{R}^u(\rho_u) = \sum_{v \in \{M, S\}} \frac{\lambda_v}{\sum_{i \in \{M, S\}} \lambda_i \sqrt{\frac{P_i}{P_v}} \left( \sqrt{\hat{\theta}_{i,u}} + \sqrt{2 \frac{\rho_u N_v}{W} - 1} \arctan \sqrt{\frac{2 \frac{\rho_u N_v}{W} - 1}{\hat{\theta}_{i,u}}} \right)}. \quad (19)$$

### 3.3 DL Throughput

In this paper, two kinds of UEs classified by different data rate demands are served by HetNets. As definition of DL throughput in (4) previously, the total system DL throughput should consider achievable throughput of all UEs as below

$$DL \text{ Throughput} = \sum_{k \in \{u_H, u_L\}} \lambda_k \rho_k \mathcal{R}^k(\rho_k) \quad (Kbps/m^2). \quad (20)$$

## 4 Numerical Results

In this section, series of numerical results are presented to verify the accuracy of system model and analysis. Especially, the DL throughput in the HetNets is evaluated in detail. For clarity, the analysis in this section is limited to an interference-limited 2-tier HetNet. Table 1 shows the key simulation parameters of the system.

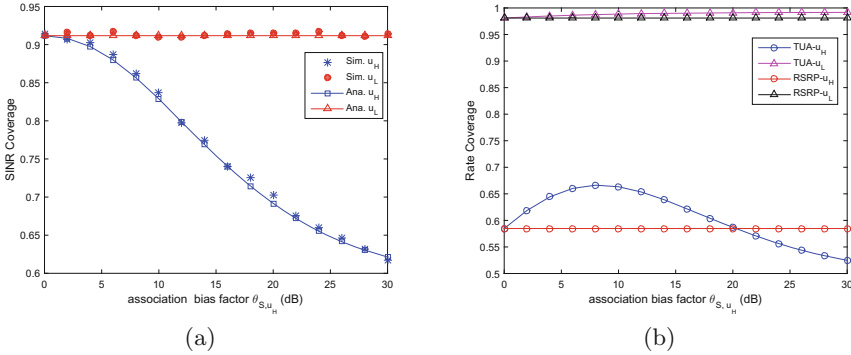
**Table 1.** Simulation parameters

Parameter	Value
MBS density ( $m^{-2}$ ), $\lambda_M$	$1 \times 10^{-6}$
SBSs density ( $m^{-2}$ ), $\lambda_S$	$1 \times 10^{-5}$
MBS transmit power (W), $P_M$	20
SBS transmit power (W), $P_S$	0.13
SINR coverage threshold (dB), $\tau$	-10
Rate coverage threshold of $u_H$ (Kbps), $\rho_{u_H}$	1024
Rate coverage threshold of $u_L$ (Kbps), $\rho_{u_L}$	12
System bandwidth (MHz), $W$	20
Path loss, $\alpha$	4

In Fig. 2, the SINR coverage and rate coverage under different association bias factor  $\theta_{S,u_H}$  are shown in (a) and (b), respectively. As shown in Fig. 2(a), it is noticeable that  $u_L$  remains the same SINR coverage, while  $u_H$  has a decreasing trend. This is because different association biases are applied to  $u_L$  and  $u_H$ . For  $u_L$ , the association bias factor always has  $\theta_{u_L} = 1$ , which means that the

association for  $u_L$  is based on maximum received power, hence  $u_L$  can always get reliable SINR quality resulting from high SINR coverage. Moreover, for  $u_H$ , the bigger the association bias factor  $\theta_{S,u_H}$  is, the wider SBS coverage area is. Hence, more  $u_H$  are offloaded from MBSs to SBSs through expanding SBS coverage area, which causes declining SINR coverage performance due to both deterioration of the received signal power and enhanced interference.

Figure 2(b) presents the rate coverage of different UEs compared to traditional RSRP mode. Because  $u_H$  have higher data rate demands than  $u_L$ , the rate coverage of  $u_H$  is clearly interior to that of  $u_L$ . Moreover, proposed TUA scheme can obviously improve the rate coverage of  $u_H$ , and slightly enhance the rate coverage of  $u_L$ . That is due to the fact that some  $u_H$  are offloaded from MBSs to SBSs, which relieves the congestion of MBSs and balances the BSs load.

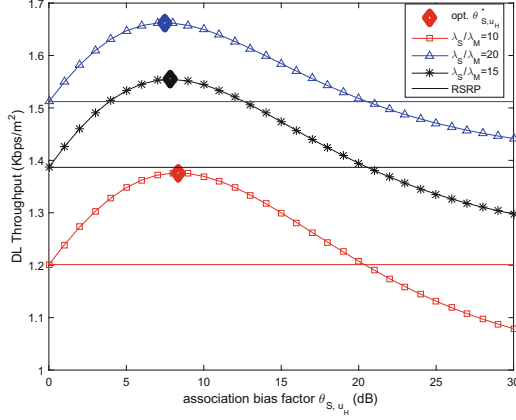


**Fig. 2.** Coverage performance of classified UEs under different association bias factor  $\theta_{S,u_H}$  ( $\lambda_{u_H} = 2e^{-5}m^{-2}$ ,  $\lambda_{u_L} = 4e^{-5}m^{-2}$ ): (a) SINR coverage with TUA scheme, (b) comparison of rate coverage with TUA and RSRP schemes.

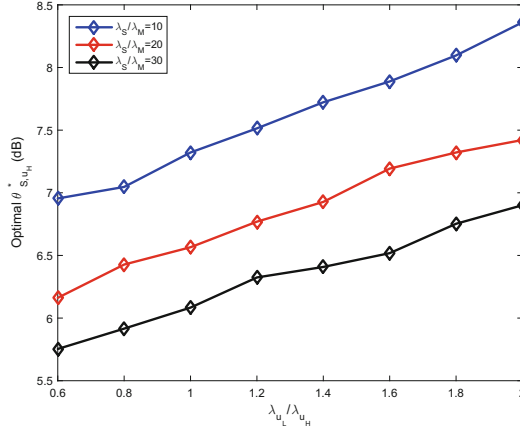
System DL throughput is demonstrated in Fig. 3. An optimal association bias factor  $\theta_{S,u_H}^*$ , which maximizes the DL throughput, is obtained using PSO. It is shown that  $\theta_{S,u_H}^*$  changes with different ratios of MBS density and SBS density. Compared to traditional RSRP mode, TUA scheme can enhance DL throughput by 16%, which mainly benefits from the improvement of rate performance. Furthermore, increasing density of SBSs can also yield significant DL throughput enhancement, because UEs are prone to be connected to SBSs as increase of SBSs, which also contributes to load balance in HetNets.

In Fig. 4, the relations between optimal association bias factor  $\theta_{S,u_H}^*$  and distributions of both UEs and BSs are presented. When the density of  $u_L$  is increasing, the  $\theta_{S,u_H}^*$  is on the rise. Moreover, as the decrease of SBS density, the  $\theta_{S,u_H}^*$  also has a remarkable growth. It is obvious that both more UEs and less SBSs can cause more serious congestion of MBSs. Therefore, larger association bias factor  $\theta_{S,u_H}$  contributes to system load balance, which is beneficial to enhance DL throughput.





**Fig. 3.** Comparison of DL throughput performance under different association bias factor and SBSs deployments ( $\lambda_M = 1e^{-6}m^{-2}$ ,  $\lambda_{u_H} = 2e^{-5}m^{-2}$ ,  $\lambda_{u_L} = 4e^{-5}m^{-2}$ ).



**Fig. 4.** The relations between optimal association bias and both UEs and BSs densities ( $\lambda_{u_H} = 2e^{-5}m^{-2}$ ,  $\lambda_M = 1e^{-6}m^{-2}$ ).

### 5 Conclusion

In this paper, a tractable TUA scheme in HetNets is proposed, where different association biases are applied to satisfy various UEs traffic demands. The expression of SINR coverage, rate coverage, and DL throughput are derived theoretically. Specifically, TUA scheme is able to balance loads by offloading some high traffic demand UEs  $u_H$  from MBSS to SBSs in DL. Numerical results illustrate that TUA scheme can improve rate coverage of UEs and enhance system DL throughput by 16% compared to traditional RSRP association method, while the SINR quality of  $u_L$  is ensured. Moreover, there is an optimal association bias

factor  $\theta_{S,u_H}^*$  which maximizes DL throughput, and it reveals that  $\theta_{S,u_H}^*$  differs as either UE density or BS density changes.

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## References

1. Andrews, J., et al.: What Will 5G Be? IEEE J. Select. Areas Commun. 136–144 (2013). IEEE Press, New York
2. Zhang, X., Zhang, J., Wang, W.: Macro-assisted data-only carrier for 5G green cellular systems. IEEE Commun. Mag. **53**, 223–231 (2015). IEEE Press, New York
3. Zhang, X., Zhang, Y., Yu, R.: Enhancing spectral-energy efficiency for LTE-advanced heterogeneous networks: a users social pattern perspective. IEEE Wirel. Commun. **21**, 10–17 (2014). IEEE Press, New York
4. Zhang, X., Yu, R., Zhang, Y.: Energy-efficient multimedia transmissions through base station cooperation over heterogeneous cellular networks exploiting user behavior. IEEE Wirel. Commun. **21**, 54–61 (2014). IEEE Press, New York
5. Andrews, J., Baccelli, F., Ganti, R.: A tractable approach to coverage and rate in cellular networks. IEEE Trans. Commun. **59**, 3122–3134 (2011). IEEE Press, New York
6. Yang, K., Wang, P., Hong, X., Zhang, X.: Joint downlink and uplink network performance analysis with CRE in heterogeneous wireless network. In: 26th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC), pp. 1659–1663. IEEE Press, Hong Kong (2015)
7. Kikuchi, K., Otsuka, H.: Proposal of adaptive control CRE in heterogeneous networks. In: 23rd Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC), pp. 910–914. IEEE Press, Sydney (2012)
8. Sun, S., Liao, W., Chen, W.: Traffic offloading with rate-based cell range expansion offsets in heterogeneous networks. In: IEEE Wireless Communications and Networking Conference (WCNC), pp. 2833–2838. IEEE Press, Istanbul (2014)
9. Sun, Y., Deng, T., Fang, Y., et al.: A method for pico-specific upper bound CRE bias setting in HetNet. In: IEEE Wireless Communications and Networking Conference Workshops (WCNCW), pp. 80–84. IEEE Press, Shanghai (2013)
10. Chiu, S., Stoyan, D., Kendall, W., et al.: Stochastic Geometry and Its Applications. Wiley, Hoboken (2013)
11. Singh, S., Dhillon, H., Andrews, J.: Offloading in heterogeneous networks: modeling, analysis, and design insights. IEEE Trans. Wirel. Commun. **12**, 2484–2497 (2013). IEEE Press, New York