# Green Distributed Power Control Algorithm for Multi-user Cognitive Radio Networks

Yinmeng Wang, Jian Chen(✉), Chao Ren, and Huiya Chang

Xidian University, Xi'an, Shannxi Province, People's Republic of China
jianchen@mail.xidian.edu.cn

**Abstract.** Considering both system energy efficiency (EE) and the implementation of distributed power control algorithm in multi-user cognitive radio networks (CRNs), a multi-leader Stackelberg power control game algorithm is proposed to achieve continuous Pareto improvements in non-cooperative power control game (NPG) in this paper. By combining the advantages of cooperative and non-cooperative games with consideration of secondary users' quality of service (QoS) requirements, the problems of low system EE of non-cooperative game and limited Pareto improvement of single leader Stackelberg game are solved. Simple utility function and time back-off are utilized to facilitate the implementation of distributed algorithm. Simulations show that the proposed algorithm improves the system EE as Pareto improvement is reached. Meanwhile, primary user's QoS is guaranteed as secondary users transmit with lower power.

**Keywords:** Energy efficiency · Cognitive radio networks · Stackelberg game theory · Pareto improvement

## 1 Introduction

With the increasing energy consumption in wireless networks, green wireless communications arouse great attention, which aim at improving energy efficiency (EE). Power control [1] is an efficient radio management method to reduce mutual interference and improve the EE.

Power control schemes based on game theory are investigated. In [2], a non-cooperative power control game (NPG) was investigated to solve the power control issues in multi-secondary-user underlay cognitive radio networks (CRNs). The utility function was designed based on EE, which was easy to realize distributed computation and reduced the power consumption of the base stations [2]. In [3], the authors modified the utility function designed in [2] with a novel pricing function to pursue higher EE. However, Nash equilibrium (NE) in the non-cooperative game is inefficient since the users act selfishly [4]. In [5], hierarchy-based cooperative Stackelberg game was introduced to deal with the inefficient NE problem. In [6], Stackelberg game was used to pursue high EE for single user while total EE of multiple secondary users (SUs) was ignored. In [7], the


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authors focused on maximizing multiple SUs' total EE with Stackelberg game. However, only one-shot Pareto improvement was obtained in [7]. Thus, the EE can be further improved by continuous Pareto improvements.

In this paper, we focus on achieving high total EE of multiple users by continuous Pareto improvements in underlay CRNs. We proposed a distributed power control algorithm based on Stackelberg game to improve EE. In the proposed algorithm, the utility function is simply defined based on EE and time back-off [8] is used to implement the distributed algorithm. Continuous Pareto improvements are achieved with multiple leaders implementing power-decreasing strategy. Thus, high total EE of multiple users is achieved in the green communications. We also prove the existence of Stackelberg Equilibrium (SE) and investigate the computational complexity of the proposed algorithm.

## 2 System Model and Problem Formulation

In underlay CRNs, $N$ pairs of SUs simultaneously share the same band with a pair of primary users (PUs). To make the figure simple and clear, in Fig. 1, only three pairs of SU transceivers (SU-TX$i$ and SU-RX$i$, $i = \{1, 2, 3\}$ and a pair of PU transceiver (PU-T and PU-R) are shown and the interference links between SUs are not shown. Log-normal channel model is considered. The channel gains of links SUs$-$PU and secondary transmitter (ST) $j-$secondary receiver (SR) $j$ are denoted by $g_j$ and $h_j$, respectively. $h_{ij}$ denotes channel gain between ST $i$ and SR $j$. Local knowledge between two direct links about channel information can be acquired by each SU. In underlay CRNs, three constraints should be satisfied. First, the interference to PUs caused by SUs should not exceed the interference threshold $I_{th}$. Second, the maximum power budget of SU $j$, e.g. $p_j$, is $p_j^{\max}$. Last, each pair of SUs (e.g. the $j$ th) needs to meet a target signal-to-noise plus interference ratio (SINR) $\gamma_j^{tar}$ to guarantee the quality of service (QoS). In this paper, we aim to maximize the total EE of the system. For each SU, the utility $u_j$ is defined as [2,3,5]

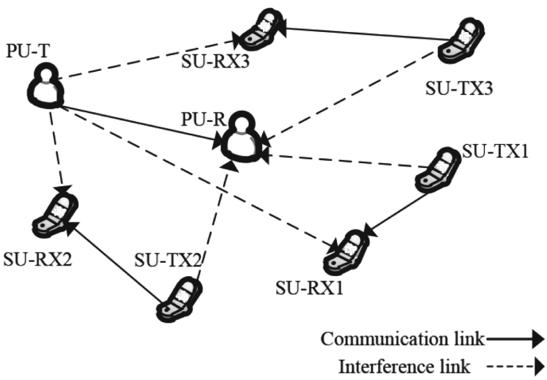


**Fig. 1.** System model

$$u_j\left(p_j, P_{-j}\right) = \frac{LR}{Mp_j} f_j(\gamma_j), \tag{1}$$

where $R$, $M$ and $L$ represent transmit rate, data length and information length of each packet, respectively. $P_{-j}$ is a set $\{p_1, \ldots, p_{j-1}, p_{j+1}, \ldots, p_N\}$. $f_j(\gamma_j)$ is a monotonic deceasing function to measure the probability of correct reception:

$$f_j(\gamma_j) = (1 - 2P_{ej})^M. \tag{2}$$

Here, $P_{ej} = 0.5 \exp(\frac{-\gamma_j}{2})$, which represents the binary error rate of a noncoherent frequency shift keying modem. The utility function can be interpreted as the number of information bits received per Joule of energy expended.

The optimization problem in this paper is written as:

$$\begin{aligned} \max \quad & w = \sum_{j \in N} u_j \\ s.t. \quad & \sum_{j \in N} p_j |g_j|^2 \le I_{th}, \\ & p_j \in (0, {p_j}^{\max}], \\ & \gamma_j = \frac{p_j |h_j|^2}{\sum\limits_{i=1,\, i \ne j}^{N} p_i |h_{ij}|^2 + \delta_j^2} \ge {\gamma_j}^{\text{tar}}. \end{aligned} \tag{3}$$

Here, $w$ is social welfare [5] as defined in [7]. $\delta_j^2$ contains the interference to SU $j$ caused by primary transmitter and additive white Gaussian noise.

## 3 Review of NPG and Stackelberg Game

### 3.1 NPG Algorithm and NE

An appropriate model for power control problem is given by NPG [4]. In [4], the optimization problem is written as

$$\max \quad u_j(p_j, P_{-j}), j \in \{1, \ldots, N\}. \tag{4}$$

We denote the utility function alternatively as $u_j(p_j,P_{-j})$, where $u_j$ is the same as (1) and $P_{-j}$ represents the power of players excluding j.$p_j$ represents the power of user $j$. Non-cooperative game is described as $G = [N, \{p_j\}, \{u_j\}]$, where $N = \{1, \ldots, N\}$ is the SU player set. $\{p_j\}$ is the policy set and $\{u_j\}$ is the utility function set.

The works [4,9] on NPG algorithm show that the unique NE exists when users choose their policies selfishly and rationally. The NE can be described as $p^{\text{NE}} = (p_1^{\text{NE}}, \ldots, p_N^{\text{NE}})$, where $p_j^{\text{NE}} = \min(p_j^{\max}, p_j^{\sim})$ and $p_j^{\sim}$ is

$$p_j^{\sim} = \frac{\gamma_j^{\sim} \left( \sum\limits_{k \ne j} h_{kj} {p_k}^{\text{NE}} + \delta_j^2 \right)}{h_j}, \tag{5}$$

which represents the transmit power of SU $j$. The $\gamma_j^{\sim}$ is the SINR, which is the solution to (6)

$$f'(\gamma_j)\gamma_j - f(\gamma_j) = 0. \tag{6}$$

At the NE, the corresponding SINR of $p_j^{\sim}$ depends on the function $f_j(\gamma_j)$ in (2). $\gamma_j^{\sim}$ is defined as 'the best SINR at the NE' and each SU transmits with the corresponding power of the best SINR. NPG algorithm provides a solution for power control when NE is achieved. However, the NE is inefficient from the sense of two perspectives: (1) if some users continue to decrease power to break the NE, the utility of each user will increase; (2) the SUs selfishly maximize their own utilities without considering the interference to PU and other SUs.

### 3.2 The Single-Leader Stackelberg Game for Power Control

Stackelberg game for power control is introduced to deal with the inefficient NE in NPG. In the game, one user is the leader while the other users are followers. If the leader chooses the power to maximize its utility, an equilibrium will be achieved among users, namely Stackelberg Equilibrium (SE).

In single-leader Stackelberg power game [5], both the leader and followers improve their EE with respect to non-cooperative setting. Two kinds of SUs exist when the followers reach a NE [4,9]. The first kind of SUs satisfies the equation $p_j^{\mathrm{NE}} = p_j^{\sim}$ and transmits with the corresponding 'best SINR at the NE' power. The second kind of SUs satisfies the equation $p_j^{\mathrm{NE}} = p_j^{\max}$ transmitting with maximum power. When leader decreases power, its interference to other SUs decreases. Then a new equilibrium is reached. The first kind of SUs keeps in the 'best SINR'. However, their power decreases and their EE increases according to (1). The power of the second kind of SUs is kept in $p_j^{\max}$. But the interference to them decreases as the leader and the first kind of SUs decrease their power. Hence, the SINR of the second kind of SUs increases and their EE increases according to (1). Last, if the leader continues to decrease power, the second kind of SUs will transform into the first kind of SUs. That means all the SUs transmit with the corresponding 'best SINR' power and their EE increases. No matter what power the leader chooses, followers enable to achieve a NE. With the decrease in leader's power, the EE of the two kinds of SUs increases. Thus, the new NE, namely SE, is a Pareto improvement.

However, in Stackelberg power control game, only one-shot Pareto improvement is achieved. What's more, some prerequisites are needed. Leader needs to acquire more knowledge than followers [6]. Followers are cognitive SUs able to know the leader's strategy [7]. However, in [6], distributed management was ignored. In underlay CRNs, SUs are unable to sense as [7].

## 4 Distributed Multi-leader Stackelberg Power Control Game

To achieve higher EE, a distributed multi-leader Stackelberg power control game algorithm (DMSPG) is proposed in this section. In DMSPG, firstly, single-leader game achieves SE and one-shot Pareto improvement is reached. Then, multiple leaders implement power-decreasing strategy to achieve continuous Pareto improvements. Since SUs interfere with each other, the change of the leader's power can affect followers. Hence, followers adjust their power according to their SINR to react to the leader. Finally, continuous Pareto improvements are achieved to improve the total EE.

### 4.1 The Criterion of Choosing Leader in DMSPG Algorithm

In DMSPG algorithm, SU $i^*$ is chosen as leader from the followers. The criterion is

$$i^* = \arg\min \left\{ Q_i \right\}, \gamma_i > \gamma_i^{\mathrm{tar}}, \tag{7}$$

where $Q_i$ is

$$Q_i = \frac{u_i \left( p_i, \mathbf{P}_{-i} \right)}{p_i |g_i|^2 + \sum\limits_{j \neq i} p_i \left| h_{ij} \right|^2}. \tag{8}$$

In (8), the numerator includes the EE of SU $i$ while the denominator consists of the interference to PU and other SUs caused by SU $i$. A SU is chosen as leader considering two aspects: (1) the SU induces more negative effects to the others; (2) the SU makes less contribution to the whole system. Leader implements Variable-step power-decreasing strategy to break the original NE. Then a SE is reached.

### 4.2 The Proposed DMSPG Algorithm

The DMSPG algorithm is described with four steps.

- Step 1: the initialization of NPG
  1. Execute NPG algorithm and the SUs reach a NE.
  2. Once SUs achieve the equilibrium, they broadcast flag information INIT-FINISHED + ID. INIT-FINISHED means initialization is finished and ID represents the identification.
  3. When those SUs succeed to hear the $N - 1$ INIT-FINISHED and equipment ID, they broadcast flag LEADER-START. It means the process of choosing leader starts.
- Step 2: choosing leader based on the criterion in (8)
  1. All SUs calculate their own $Q$ values. Then, $Q$ values are set as the start time of time back-off. SUs start to listen to the flag STOP-LEADER by virtual timer fashion.
  2. The SU whose timer is the first to become zero will broadcast flag STOP-LEADER.
  3. Once the other SUs listen to STOP-LEADER flag, the leader-choosing process will stop.

– Step 3: single leader performs power-decreasing game
  1. The leader decreases power level with an initial step size $\Delta p$. The SINR of the other SUs changes and the NE is broken. Then all the followers have to play non-cooperative game to achieve a SE.
  2. All SUs calculate their EE e.g. $u_j$. Then all SUs except the leader set their time back-off with $u_j$ as start time. These SUs start to listen to the flag STOP-EE by a time back-off fashion.
  3. Each SU sends a flag STOP-EE when the countdown finishes. The leader calculates $u_j$ based on the time all the STOP-EE heard. The total social welfare $w$ of the system is calculated according to (2). If $w$ is lower, it means the step size $\Delta p$ in (1) is big. SUs change to the original power and leader shortens step size to $0.5 * \Delta p$. Then go back to (1). This process keeps taking half the step size until $w$ increases and the QoS of SUs is satisfied. Then the transmit power of each SU remain unchanged. The step size of the leader is retained.
  4. Leader keeps repeating (3) with the step size retained in (3) till prescribed accuracy requirement is satisfied. It means no more utility could be attained even to decrease the leader's power and the SE is achieved.

– Step 4: multiple leaders perform power-decreasing game
  1. The leader chosen in Step 3 is put in leader set and does not take participate in leader-choosing any more. Then according to Step 2, another appropriate leader is chosen from followers.
  2. The new leader takes power-decreasing strategy in Step 3. Other leaders keep their original power.
  3. Estimate whether the algorithm is convergent according to practical application, for example, given terminal time. If it is not convergent, the process goes back to (1) until it is.

### 4.3 Analysis of Multi-leader Game and Continuous Pareto Improvements in DMSPG

**The Uniqueness and Existence of SE.** According to the analysis of single-leader game, no matter what power lever the leader sets, the followers will achieve a NE by non-cooperative game. If the leader chooses the power which maximizes its utility, the leader and the followers will achieve a SE. In this paper, the SE of multi-leader game exists as the single-leader Stackelberg game is played once more after a new leader is chosen. According to [5], the uniqueness of a SE is proved in single-leader Stackelberg game, the uniqueness of the SE in multi-leader Stackelberg game in this paper can be proved as [5].

**The Efficiency of DMSPG Analysis.** According to the analysis of single-leader game, the power of both leader and followers does not increase. In Step 4, the other leaders endure less interference and their SINR improves. Their EE increases according to (1). What's more, followers achieve one-shot Pareto improvement and their EE increases. The other leaders' power in leader set is invariable and followers' power does not increase. The partial derivative of $u_j(\cdot)$ with respect to $p_j$ is

$$\frac{\partial u_j(p_j, P_{-j})}{\partial p_j} = \frac{LR}{Mp_j^2}(f'(\gamma_j)\gamma_j - f(\gamma_j)). \tag{9}$$

The $u_j(\cdot)$ monotonously increases with respect to $p_j$ if $\gamma_j$ is no more than the solution $\gamma_j^{\sim}$ to (6). When leader1 in Step 3 decreases power, its utility decreases. According to Step 3, when the leader1 chooses the step, it should ensure that its EE doesn't decrease. If its SINR increases, its EE (utility) won't decrease. Thus, according to (9), this leader's power decreases and its interference caused by the other leaders and followers decreases much more. Hence, its SINR increases and its QoS is satisfied.

Based on the above analysis, the EE of both other leaders and followers increases while the leader1's EE doesn't decrease. Hence, the total EE improves and a Pareto improvement is achieved. In Step 4, multiple leaders continue to decrease power and continuous Pareto improvements are achieved. Given the convergent conditions according to practical conditions, the algorithm ends.

**Computational Complexity of DMSPG.** The DMSPG algorithm ends in finite time. In the first step of proposed algorithm, the initialization of NPG results in a computation of $O(N^2)$. In the second step, every SU calculate its own utility and the computational complexity is $O(N)$. In the third step, the SUs play NPG after the leader decreases its power, resulting a computation of $O(N^2)$. In the fourth step, since another leader is chosen from the followers and the Stackelberg game is played once more, the computational complexity is $O(N^3)$. Thus, the overall computational complexity of proposed algorithm is $O(N^3)$. On the other hand, for followers, the computational complexity is mainly decided by the NPG, thus, the computational complexity of each follower is $O(N^2)$. For each leader, the computational complexity is decided by the forth step, since there are $N$ SUs in total, the computational complexity is $O(N)$.

## 5 Numerical Results

In underlay CRNs, nine SUs in set $\mathrm{SU}_i, i \in [1, 9]$ are sorted based on communication distance. $\mathrm{SU}_1$'s communication distance is the nearest while $\mathrm{SU}_9$ is the furthest. The comparison of the ESIA algorithm [3], the OL algorithm [6] and the DMSPG algorithm is given. The ESIA algorithm is based on non-cooperative game while the OL algorithm is based on single-leader game. The total bits $M$ is 80 and information bits $L$ is 64. The bits rate $R$ is 10 kbps. What's more, the SINR threshold of SU $\gamma^{\mathrm{tar}}$ is 6 dB and SINR threshold of PU $\gamma_{\mathrm{pu}}^{\mathrm{tar}}$ is 8 dB. The noise power $\delta^2$ is $5 \times 10^{-15}$ and PU power $P_{\mathrm{pu}}$ is 0.03 W. In addition, the precision is 5 and original step size is 0.001 W. The path loss $A$ is 0.097.

Figure 2 shows the average SINR of PU. With the increase in the number of SUs, SINR of PU decreases. However, the SINR of PU in DMSPG algorithm is

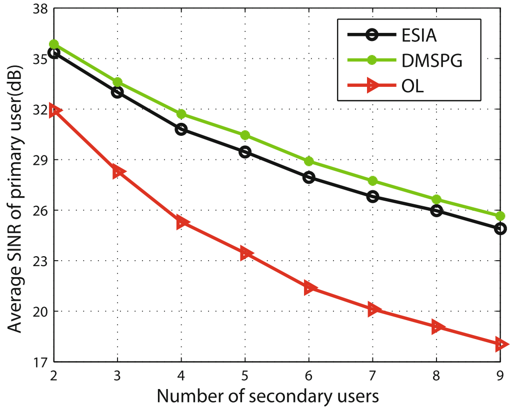


**Fig. 2.** Average SINR of PU of three algorithms (Color figure online)

higher than that in ESIA algorithm and OL algorithm. This can be explained as follows. In DMSPG, with the decrease in leader's power, the other SUs' power decreases. Hence, the interference to PU decreases and the SINR of PU improves. The ESIA is based on non-cooperative game and SUs cause more interference to PU. In the OL algorithm, only one leader is chosen and SUs cause more interference to PU.

Figure 3 shows the total power of SUs. With the increase in number of SUs, the total power of SUs increases. However, SUs' power in DMSPG is lower than that in ESIA and OL. This is because SUs' power is non-increasing in DMSPG. ESIA is based on non-cooperative game and SUs' power decreases less. In OL, only one-shot Pareto improvement is achieved.

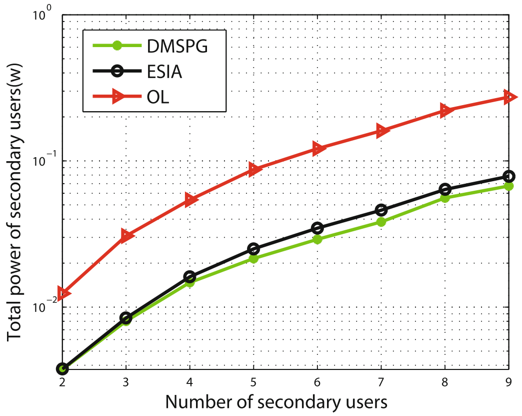


**Fig. 3.** Transmit power of SUs of three algorithms (Color figure online)

Figure 4 shows the total utilities of SUs. With the increase in number of SUs, the total utilities of SUs increase. The utilities of DMSPG are higher than ESIA and OL. The reason is that DMSPG is based on multi-leader Stackelberg game and enables to achieve continuous Pareto improvements. Thus, the proposed algorithm achieves higher total EE of multiple SUs.

The transmit power of the SUs (use $SU_3$, $SU_5$, $SU_8$ and $SU_9$ as example) with iteration times is shown in Fig. 5. We can see that the transmit power converges to the equilibrium within 30 iterations, which proves the convergence of proposed algorithm. The power of $SU_3$ is lowest while the power of $SU_9$ is the highest. The reason is that the power of SU increases With the distance of SU transceiver and the SUs are sorted based on communication distance as mentioned above.

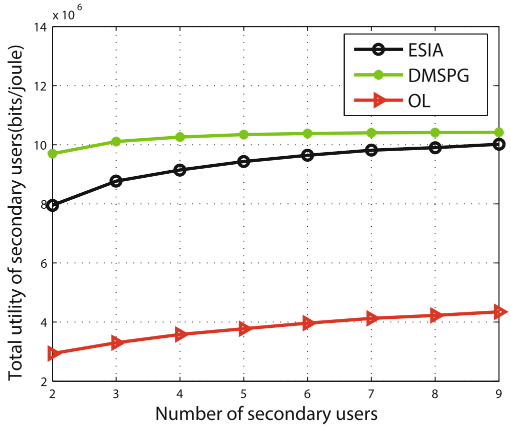


**Fig. 4.** Utilities of SUs of three algorithms (Color figure online)

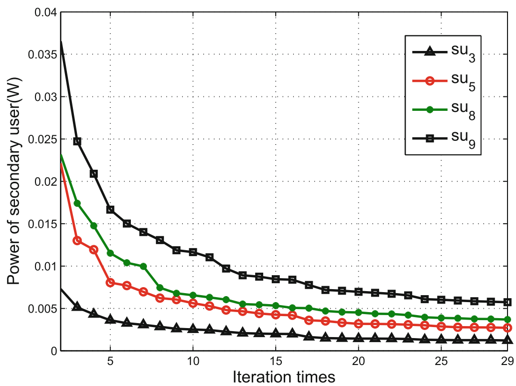


**Fig. 5.** Transmit power of SUs with iteration times (Color figure online)

## 6 Conclusion

In this paper, we aim at improving total EE of multiple users in green wireless communications. Considering distributed power control, we propose a distributed Stackelberg game power control algorithm to decrease SUs' power. Meanwhile, continuous Pareto improvements are achieved with multiple leaders implementing power-decreasing strategies. Simulations testify that the proposed DMSPG algorithm reduces SU's power and improves the total EE of the system.

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