Option-Implied Objective Measures of Market Risk with Leverage

Matthias Leiss and Heinrich H. Nax

Abstract Leverage has been shown to be procyclical and indicative of financial market risk. Here, we present a novel, inherently forward-looking way to estimate market leverage ratios based on derivative prices, option hedging, and the 'operational' riskiness measure by Foster and Hart (J Polit Econ 117(5):785–814, 2009). Furthermore, we report option-implied 'optimal' leverage levels inferred via the (Kelly, IRE Trans. Inf. Theory 2(3):185–189, 1956) criterion. The resulting measure of leverage exhibits strong procyclicality prior to the Global Financial Crisis of 2008. Finally, we find it to successfully predict large stock market downturns.

Keywords Objective risk • Foster-Hart • Leverage • Risk-neutral densities

1 Introduction

> With the benefit of hindsight, we clearly should have put even greater emphasis on the risks of excessive leverage.
> Hildebrand (2008)

The Global Financial Crisis of 2008 brought questions related to excessive leverage back on the table of risk regulation. Previous risk regulation frameworks (e.g., Basel I and II) posed capital requirements that were (at least partially) based on the relative riskiness of various types of assets (Hildebrand, 2008). While such risk-based capital measures signaled high stability of banks prior to the Global Financial Crisis, simple leverage ratio assessments exposed the largely undercapitalized situation of key financial actors which exacerbated the crisis. As a reaction to the crisis, the new regulatory framework (Basel III) contains a simple, non-risk-based leverage ratio requirement (Basel Committee on Banking Supervision, 2010).

Nevertheless, as Schularick and Taylor (2012) have noted, we have entered an age of unprecedented financial risk due to leverage. In particular, the vast expansion of credit and financial innovation, combined with implicit government insurance and

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the prospect of rescue operations, have resulted in massively increased leverage. As a result, the financial system has become more vulnerable to endogenously generated instabilities as manifested by recurring booms and busts (Von der Becke and Sornette, 2014).

A key issue inherent to leverage is procyclicality, which means that leverage ratios are only a partial remedy. In theory, standard portfolio rules would seem to imply anticyclical leverage; high leverage when the risk premium is high. Empirically, however, procyclicality of leverage has been documented extensively (Adrian and Shin, 2014). This empirical phenomenon has been explained through increased collateral requirements during downturns creating leverage cycles (Geanakoplos, 2010): increased uncertainty and volatility of asset returns lead lenders to require tighter margins, which, in turn, mechanically implies falling prices and consequently large losses for the most leveraged investors. Importantly, both of these elements feed back on each other, thus starting the leverage cycle. Any institution in the financial system where investors hold long-term, illiquid assets that are financed by short-term liabilities is particularly at risk of this, and falling leverage can consequently lead to 'runs' on such institutions (Adrian and Shin, 2014). Perhaps serving as the most famous example, the Global Financial Crisis of 2008 started as a run on the sale and repurchase (repo) market (Gorton and Metrick, 2012).

Generally, due to procyclicality, leveraged financial markets exhibit fat tails of the return distribution and clustered volatility (Thurner et al., 2012). This suggests the use of leverage ratios as indicators for the likelihood of future financial crashes and crises. Indeed, changes in dealer repos can be used to successfully forecast changes in financial market risk as measured by the Chicago Board Options Exchange Volatility Index (VIX) index (Adrian and Shin, 2010). Similarly, intermediary leverage has been shown to be negatively aligned with the banks' Value-at-Risk (VaR) (Adrian and Shin, 2014).

Our present paper pursues a similar goal, namely to use leverage procyclicality to predict market risk. Our contribution to the existing literature is the construction of leverage ratios from derivative markets. Prior work had either focused on leverage as the ratio of collateral values to the down payment (with data generally being inaccessible, Geanakoplos 2010), or as the ratio of total assets to book equity (Adrian and Shin, 2010, 2014). By contrast, our approach will be to construct forward-looking estimates of leverage ratios based on prices of financial options. Specifically, we will use risk-neutral probability distributions to evaluate the estimated, forward-looking performance of hedged portfolios as quantified by the recently proposed 'operational' riskiness measure of Foster and Hart (2009). In our generalization of the measure, allowing leverage, the measure indicates the level of leverage at which the estimated growth rate becomes negative. We note that this is fundamentally different from previous theoretical work on optimal trading with leverage. For example, the previous study by Grossman and Vila (1992) establishes optimal dynamic trading rules subject to a leverage constraint that is given. Here, our goal is to empirically determine such a constraint in the first place.

Our findings are twofold. First, leverage ratios as constructed from derivative prices exhibit a pronounced and persistent peak prior to the Global Financial Crisis of 2008, thus quantifying the procyclical leverage regime of the market. Second, leverage ratios are found to be indicative of extreme future market-downturns. These findings complement our own investigation of option-implied operational market risks (Leiss and Nax, 2015), particularly during the build-up of the Global Financial Crisis of 2008, where our previous, leverage-free approach had only limited reach.

2 Operational Metrics of Disaster Risk

Well-known tail measures, like Value at Risk (VaR) and Expected Shortfall (ES), have become industry standards for assessing extreme market risks (Embrechts et al., 2005). By construction, they only characterize the risk of negative events while ignoring the potential upside. On the other hand, measures of dispersion such as volatility/variance or interquartile range account for up- and downturns, but are largely blind to rare extreme events on both sides of the spectrum. For example, the widely used Sharpe ratio (Sharpe, 1994) only accounts for the first two moments of the underlying return distribution, thus implicitly (and falsely) assuming that higher moments do not matter.

Two novel measures of riskiness (by Aumann and Serrano (2008) and Foster and Hart (2009)) promise to balance both, sensitivity to extreme risks and potential gains. Formally, these measures are defined for any gamble g in the set of gambles $\mathcal{G}$ characterized by random variables with positive expectation and positive probability of negative outcomes. For any gamble $g \in \mathcal{G}$, Foster and Hart (2009) uniquely define their risk measure, FH, as the zero of[1]

$$\mathbb{E}\left[\log(1 + FH(g)g)\right] = 0, \tag{1}$$

whereas Aumann and Serrano (2008) define their risk measure, AS, as the zero of

$$\mathbb{E}\left[\exp(-AS(g)g)\right] = 1. \tag{2}$$

One issue with expression (1), which will become extremely relevant for our leverage analysis, is that, for some continuous gambles $g \in \mathcal{G}$, FH thus defined may have no positive solution. In this case, Riedel and Hellmann (2015) extend the definition consistently by setting FH to the maximum possible loss incurred by that gamble. In particular, if g is a return distribution with maximum loss of 100%, FH is bound by 1.

Importantly, definitions (1) and (2) involve forming the expectation over the whole distribution of the gamble's outcomes. Thus, FH and AS are able to capture all moments of a gamble. This is formalized by Kadan and Liu (2014), who prove that higher moments do not necessarily have a weaker effect on FH and AS. In practice,

[1] The logarithmic growth rate had entered risk analysis already earlier. Examples involve the Kelly (1956) criterion (which aims to maximize growth rate), or, very similar to Foster and Hart (2009), Whitworth (1870, p. 217).

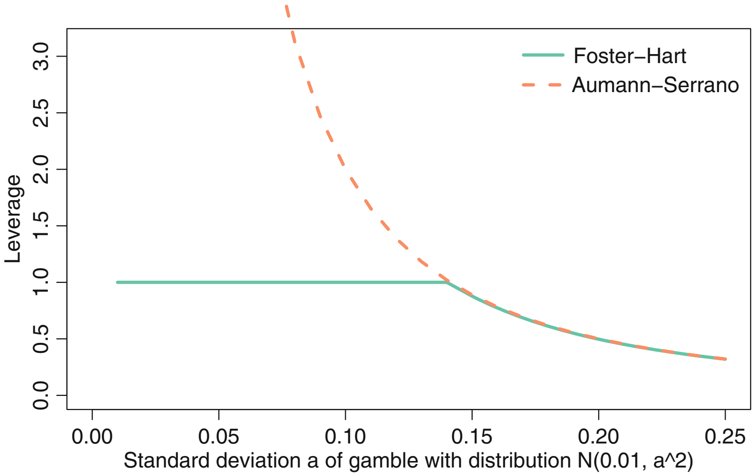

Fig. 1 Foster-Hart $FH(g)$ and Aumann-Serrano $AS(g)$ measures of riskiness vs. the standard deviation α of a normally distributed gamble $g \sim \mathcal{N}(0.01, \alpha^2)$. The implied leverage ratios coincide in the case of high risk ($\alpha \gg 0.01$). In the opposite case of vanishing risk ($\alpha \to 0$), AS diverges indicating zero risk and suggests infinite leverage, while the no-bankruptcy property of $FH(g)$ leads to an upper bound of 1

one often finds higher moments to have a strong impact on the risk measures (Kadan and Liu, 2014; Leiss and Nax, 2015; Anand et al., 2016). However, FH is significantly more sensitive to left-tail events than AS. Be g_α the composite gamble of $g_0 \in \mathcal{G}$ and an extreme loss $-L < 0$ with respective probabilities $1 - \alpha$ and $\alpha \in (0, 1)$ and $FH(g_0) > 1/L$. It is easy to show that (Kadan and Liu, 2014)

$$\lim_{\alpha \to 0} FH(g_\alpha) = 1/L, \tag{3}$$

whereas

$$\lim_{\alpha \to 0} AS(g_\alpha) = AS(g_0). \tag{4}$$

A variation of this is illustrated in Fig. 1. The gamble g is normally distributed with positive mean and standard deviation α, $g \sim \mathcal{N}(0.01, \alpha^2)$. In the high-risk scenario of large variance, $\alpha \gg 0.01$, FH and AS coincide almost perfectly. However, in the case of low risk, i.e. as $\alpha \to 0$, AS diverges indicating asymptotically zero risk and therefore infinite leverage, whereas FH is bounded by 1 to avoid bankruptcy with one shot.

Besides the above-mentioned practical appeal of taking into account the whole distribution of a gamble, both FH and AS also fill an important theoretical gap. It is known that risk-averse investors who choose their investments by maximizing expected utility may rank investments by second-order stochastic dominance

(SOSD) (Hadar and Russell, 1969; Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970). However, some pairs of investments cannot be ranked on the basis of SOSD. Kadan and Liu (2014) show that both FH and AS extend SOSD in a natural way as they induce a complete ranking on $\mathscr{G}$ that agrees with SOSD whenever applicable. The induced rankings differ, because loosely speaking FH and AH order independently of an investor's utility and wealth, respectively.

The theoretical reason for FH to be bounded is the no-bankruptcy theorem by Foster and Hart (2009). It states that when confronted with an infinite series of gambles $g_t \in \mathscr{G}$, the simple strategy of always investing a fraction of wealth smaller than $FH(g_t)$ guarantees no-bankruptcy, i.e.

$$\mathbb{P}\left[\lim_{t\to\infty} W_t = 0\right] = 0, \tag{5}$$

where W_t denotes wealth at time t. This bound is independent of the investor's risk attitudes, which is the sense in which FH is 'operational' according to Foster and Hart (2009). By contrast, following such a strategy leads to wealth divergence to infinity (a.s.).

3 Extending Operational Riskiness Measures to Leveraged Gambles

The hard bound of FH that is induced by the no-bankruptcy constraint poses a challenge for dynamic risk management, as in some scenarios there is no more variation in FH. Indeed, our empirical study of option-implied FH found FH to be at the upper bound on 27% of the business days during the decade 2003–2013, and on 45% of the business days during the 5 years leading up to the collapse of Lehman Brothers in September 2008 (Leiss and Nax, 2015). One might wonder, therefore, how much information is lost because of a lack of variation during those days.

Instead of focusing on other risk indicators, we would like to explore a different 'leverage route' in this paper. Since the hard bound of one inherent to the original FH measure is induced by the maximal loss, one could think of building a portfolio that is hedged against extreme events: let r_s be a gamble that describes the relative return distribution of buying at asset S at time $t = 0$ and holding it until time $t = T$. Accounting for dividends paid during that period Y and discounting

$$r_s = \frac{S_T + Y - S_0}{S_0}. \tag{6}$$

If the asset defaults and no dividends are being paid, the investor incurs a maximum loss of $\min(r_s) = -100\%$ such that $FH(r_s) \leq 1$. A simple way of hedging this portfolio is via a put option written on S with premium P_0 (at $t = 0$), strike price K, and maturity T. The return of a portfolio that consists of one unit of the stock and a put option is given by

$$r_h = \frac{\max(S_T, K) + Y - S_0 - P_0}{S_0 + P_0}, \tag{7}$$

with maximum loss of

$$\min(r_h) = \frac{K - S_0 - P_0}{S_0 + P_0} > -100\% \tag{8}$$

for $Y = 0$ and $K > 0$ (provided the seller of the option does not default). In other words, a gamble of the form (7) generally allows for $FH(r_h) > 1$, i.e. leverage.[2] Our definition (7) generalizes FH to allow for leverage.

In later sections, we will compute and analyze our 'leverage Foster-Hart' $FH(r_h)$ for hedged portfolios based on risk-neutral probability distributions estimated from option prices. Thus, the forward-looking information contained in derivative prices enter $FH(r_h)$ twice: in P_0 via the return (7), and in the computation of the expectation via (1). Figure 2 illustrates this with an example showing the payoff for investment strategy (7) for buying the S&P 500 with the corresponding put option. Here, the values are t_0 = 2004-11-22, T = 2004-12-18, $S_0 = 1177.24$ USD, $K = 1190$, $P_0 = 21.50$ USD. Note that the strike of the put is higher than index price at time $t = 0$. Option pricing according to Black and Scholes (1973) suggests that the put option ask implies a volatility of only 11.9%. In this example, one finds $FH(r_h) = 10.7$, i.e. a leverage ratio of more than 10 (see Fig. 3).

Another sensible and closely related leverage ratio is the option-implied Kelly (1956) criterion K: instead of setting the expected logarithmic growth rate to zero as in (1), one asks for that multiple (or fraction) of wealth that maximizes it, thus defining

$$\alpha_K(g) = \arg\max_{\alpha} \mathbb{E}\left[\log(1 + \alpha g)\right]. \tag{9}$$

For gambles $g \in \mathscr{G}$, one has $\alpha_K(g) \leq FH(g)$. Continuing the example from above, we obtain a maximal growth rate at a leverage ratio of $\alpha_K(r_h) = 5.1$ (see Fig. 3). The leverage ratio implied by derivative prices is not meant to be identical to other definitions (Geanakoplos, 2010; Adrian and Shin, 2010, 2014), but should be seen as complementary.

[2] Sircar and Papanicolaou (1998) document that dynamic option hedging strategies imply feedback effects between the price of the asset and the price of the derivative, which results in increased volatility.

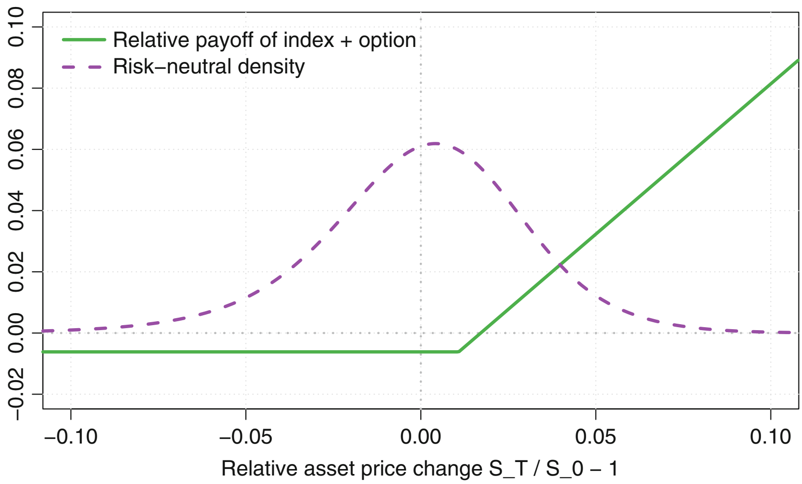

Fig. 2 Relative payoff r_h of an option-hedged portfolio example at maturity T and risk-neutral density of the underlying estimated at $t < T$ (scaled for visualization). The minimal loss of the hedged portfolio is $\min(r_h) = -0.6\%$

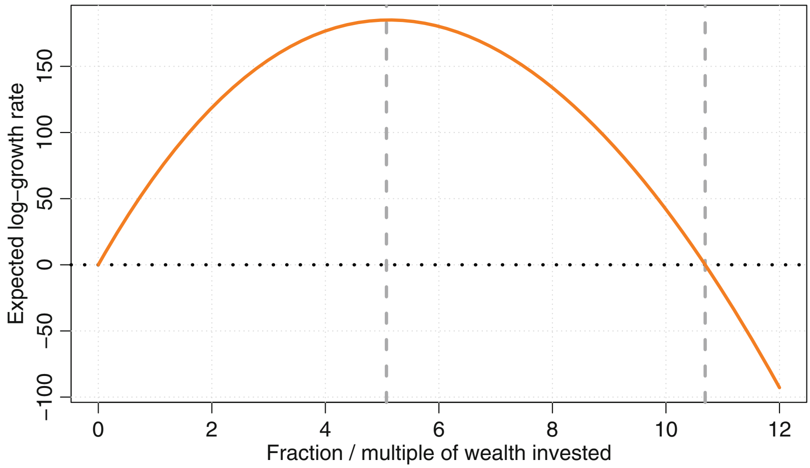

Fig. 3 Option-implied expected logarithmic growth rate of option-hedged portfolio example. The right zero crossing equals the Foster-Hart riskiness $FH(r_h) = 10.7$, the maximum growth rate the Kelly criterion $\alpha_K(r_h) = 5.1$

4 Data and Methods

In this section we discuss our data and the statistical methods employed in the empirical analysis.

4.1 Data

We obtain end-of-day bids, asks and open interest for standard European SPX call and put options on the S&P 500 stock market index for the period January 1st, 2003, to October 23rd, 2013, from Stricknet.[3] Throughout this decade the average daily market volume of SPX options grew from 150 to 890 K contracts and the open interest from 3840 to 11,883 K, respectively. In this study, we focus on monthly options, which are AM-settled and expire on the third Friday of a month. In addition, we use daily values for the S&P 500, its dividend yield, interest rates of 3-Month Treasury bills as a proxy of the risk-free rate, the (Chicago Board Options Exchange, 2009) Volatility Index (VIX) and the LIBOR from the Thomson Reuters Datastream.

4.2 Risk-Neutral Densities

Our first step is to extract risk-neutral densities from the option data as a market view on the probability distribution of the underlying gamble (which for our real-world finance application is of course unknown). There is a large literature on estimating risk-neutral probability distributions (Jackwerth, 2004). Here, we use our own method from Leiss et al. (2015), Leiss and Nax (2015) who generalize Figlewski (2010) for a modern, model-free method. We start with the fundamental theorem of asset pricing that states that in a complete market, the current price of an asset may be determined as the discounted expected value of the future payoff under the unique risk-neutral measure (e.g., Delbaen and Schachermayer, 1994). In particular, the price C_t of a standard European call option at time t with exercise price K and maturity T on a stock with price S is given as

$$C_t(K) = \mathrm{e}^{-r_f(T-t)}\,\mathbb{E}_t^{\mathbb{Q}}\left[\max(S_T - K, 0)\right] = \mathrm{e}^{-r_f(T-t)} \int_K^{\infty} (S_T - K) f_t(S_T)\mathrm{d}S_T, \tag{10}$$

where $\mathbb{Q}$ and f_t are the risk-neutral measure and the corresponding risk-neutral probability density, respectively. Since option prices C_t, the risk-free rate, r_f, and time to maturity, $T-t$ are observable, we can invert the pricing Eq. (10) to obtain an estimate for the risk-neutral density f_t. In practice, this involves numerical evaluation of derivatives (Breeden and Litzenberger, 1978) and fitting in implied volatility space (Shimko et al., 1993). Outside of the range of observable strike prices we fit tails of the family of generalized extreme value distributions, which are well-suited for the modeling extreme events (Embrechts et al., 1997). We refer the more interested reader to Figlewski (2010); Leiss et al. (2015); Leiss and Nax (2015) for details of the method.

[3]The data is available for purchase at http://www.stricknet.com/. More information on the SPX option contract specifications can be found at http://www.cboe.com/SPX.

4.3 Leverage Ratios

We will use the option-implied Foster-Hart riskiness of levered investments $FH^{\mathbb{Q}}(r_h)$ with r_h defined in (7) to estimate the prevailing leverage ratio. We compute $FH^{\mathbb{Q}}(r_h)$ for each business day and each put option available on that day. Be $\hat{P}_0$ the premium and $\hat{K}$ the exercise price with maximum $FH^{\mathbb{Q}}(r_h)$ on that business day. We report leverage ratios $FH^{\mathbb{Q}}(r_h(\hat{P}_0, \hat{K}))$ and, as a comparison, also the Kelly criterion $\alpha^{\mathbb{Q}}(r_h(\hat{P}_0, \hat{K}))$ as that quantity that numerically maximizes the option-implied logarithmic growth rate. Finally, we compute the future return $r_h(\hat{P}_0, \hat{K})$ with the realized value S_T of the underlying index at maturity.

4.4 Return Downturn Regression

We will assess the predictive power of risk measures with respect to extreme losses in the form of logistic regressions. For this, we define a binary downturn variable Δr_t^{ρ} that equals 1 in the case of an extreme event, and 0 otherwise:

$$\Delta r_t^{\rho} = \begin{cases} 1, & \text{if } r_{t\to T} < \rho, \\ 0, & \text{if } r_{t\to T} \geq \rho, \end{cases} \tag{11}$$

where ρ is a quantile describing the 5%, 10%, or 20% worst return. We note that $r_{t\to T}$ is the future *realized* return from time t to the maturity of the option T, and corresponds to the capital gain of a non-levered r_s (6) or levered portfolio r_h (7). In this sense our analysis allows inference about the predictive power of risk measures. We will regress downturns on individual risk measures R

$$\Delta r_t^{\rho} = a_{0,t} + a_{R,t}\, R_t + \varepsilon_t, \tag{12}$$

and on sets of risk measures $\mathscr{R}$:

$$\Delta r_t^{\rho} = a_{0,t} + \sum_{R\in\mathscr{R}} a_{R,t} R_t + \varepsilon_t. \tag{13}$$

Specifically, we will include the option-implied Foster-Hart riskiness $FH^{\mathbb{Q}}(r_h)$ and 5% Value at Risk of levered portfolios $VaR^{\mathbb{Q}}(r_h)$.[4] Leiss and Nax (2015) performed rigorous variable selection using the least absolute shrinkage and selection operator and found three further risk measures to be indicative (Tibshirani, 1996): (1) option-implied 5% expected shortfall of non-levered portfolios $ES^{\mathbb{Q}}(r_s)$, (2) the Chicago Board Options Exchange (2009) Volatility Index (VIX), and (3) the difference between the 3-month LIBOR and 3-month T-Bill rates (TED), a measure of credit risk. We will consider those indicators as well.

[4] Our results are robust with respect to choosing a different VaR level.

Over successive business days the downturns (11) focus on the same maturity T, as option exercise dates are standardized. This may induce autocorrelation in the dependent variable, which we correct for by using the heteroskedasticity and autocorrelation consistent covariance matrix estimators by Newey and West (1987, 1994).

5 Empirical Results

Having established the leveraged Foster-Hart riskiness and methods used, we now study empirical applications. First, we discuss the time dynamics of the option-implied leverage ratios around the Global Financial Crisis of 2008. Next, we analyze the predictive power of various risk measures with respect to extreme losses of levered and non-levered portfolios.

5.1 Option-Implied Leverage Around the Global Financial Crisis

Geanakoplos (2010) reports dramatically increased leverage from 1999 to 2006. In 2006, a bank could borrow as much as 98.4% of the purchase price of a AAA-rated mortgage-backed security, which corresponds to an average ratio of about 60 to 1. However, these numbers should not be directly compared to our findings, as the leverage ratios are defined differently. We assess leverage in time periods before and after the onset of the Global Financial Crisis, which Leiss et al. (2015) identified as June 22, 2007. Table 1 summarizes the option-implied Foster-Hart riskiness for non-levered $FH(r_s)$ and levered investments $FH(r_h)$. Prior to the Global Financial Crisis of 2008 the non-levered $FH(r_s)$ on average recommends investments of about 78% of one's wealth. During and after the crisis this value drops to about half its previous level.

In terms of FH-recommended leverage, we find an average leverage ratio of 105 in the pre-crisis regime, albeit with a fairly large confidence interval of ± 40 (see Fig. 4). During and after the crash it shrinks drastically to about 3.4. Geanakoplos

Table 1 Average levels of option-implied Foster-Hart riskiness, levered Foster-Hart riskiness, and Kelly criterion with 95% confidence intervals prior and after the onset of the Global Financial Crisis identified as 22 June 2007

Pre-crisis			Crisis and post-crisis
Non-levered Foster-Hart riskiness	$FH^{\mathbb{Q}}(r_s)$	0.78 ± 0.03	0.40 ± 0.02
Levered Foster-Hart riskiness	$FH^{\mathbb{Q}}(r_h)$	105 ± 40	3.4 ± 0.8
Levered Kelly criterion	$\alpha_K^{\mathbb{Q}}(r_h)$	41.00 ± 0.03	1.57 ± 0.02

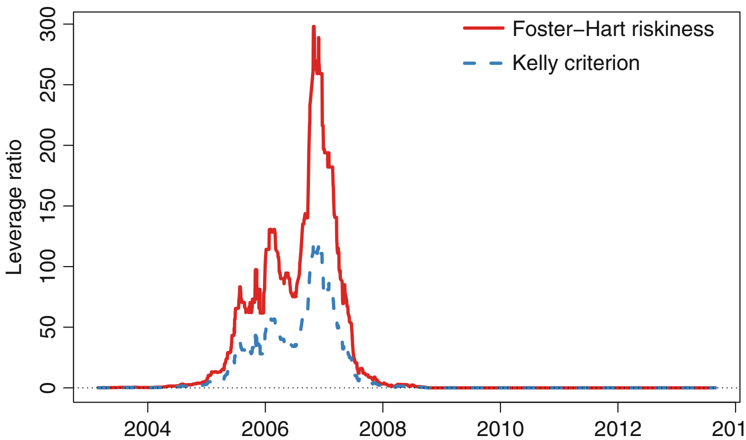

Fig. 4 Leverage according to option-implied Foster-Hart riskiness and Kelly criterion of leveraged gambles. Leverage ratios rise to drastically high values during the boom in mortgage-backed securities prior to 2008

(2010) explains the extraordinarily high leverage ratios during the pre-crisis years by financial innovation, namely the extensive use and abuse of credit default swaps (CDS). CDS are a vehicle for speculators to leverage their beliefs. Their standardization for mortgages led to enormous CDS trading prior at the peak of the housing bubble. Another reason for pronounced leverage before the crisis is the existence of two mutually reinforcing leverage cycles in mortgage-backed securities and housing (Geanakoplos, 2010). The option-implied Kelly criterion of hedged portfolios $\alpha_K^{\mathbb{Q}}(r_h)$recommends a leverage of 41 pre-crisis and 1.57 afterwards, with respective small confidence intervals of 0.03 and 0.02.

5.2 *Option-Implied Leveraged Foster-Hart Riskiness and Downturns*

We now assess the predictive power of various risk measures with respect to extreme future losses. Leiss and Nax (2015) empirically demonstrated that both Foster-Hart riskiness $FH(r_s)$ and the TED spread predict future downturns of non-hedged portfolios. Here, we will be specifically interested in the situation when the non-levered $FH(r_s)$ is stuck at the hard bound of 1 and therefore may only yield limited information. Thus, we subset our data to the 740 business days in our time period where $FH(r_s) = 1$.

Table 2 summarizes regression results for the 5%, 10%, 20% worst losses. We find that the option-implied Foster-Hart riskiness of levered portfolios helps predicting future downturns for very extreme events (at the 5% quantile and below). In the case of the 10% most negative performances, the option-implied value at risk

Table 2 Regressions of option-hedged portfolio downturns on various risk measures over 740 observations

Regression of the worst 5% downturns on risk measures (37 events)							
(Intercept)	−1.565*** (0.273)	−4.483*** (0.312)	−3.975*** (0.342)	−2.825*** (0.416)	−6.454*** (0.735)	−4.156*** (1.046)	−4.763*** (1.123)
$-FH^{\mathbb{Q}}(r_h)$	0.263*** (0.073)					0.218** (0.074)	0.138* (0.064)
$VaR^{\mathbb{Q}}(r_h)$		61.166*** (8.661)					20.326 (13.197)
$ES^{\mathbb{Q}}(r_s)$			0.329*** (0.081)			0.129 (0.131)	0.164 (0.156)
TED				−0.183 (0.548)		−0.452 (0.525)	−0.405 (0.519)
VIX					0.194*** (0.035)	0.119 (0.073)	0.098 (0.070)
Regression of the worst 10% downturns on risk measures (74 events)							
(Intercept)	−1.014*** (0.276)	−3.715*** (0.258)	−3.184*** (0.335)	−2.238*** (0.361)	−5.310*** (0.635)	−2.668*** (0.792)	−3.661*** (0.860)
$-FH^{\mathbb{Q}}(r_h)$	0.128 (0.066)					0.103 (0.070)	0.046 (0.034)
$VaR^{\mathbb{Q}}(r_h)$		64.658*** (7.962)					37.721*** (11.266)
$ES^{\mathbb{Q}}(r_s)$			0.351*** (0.105)			0.201 (0.138)	0.272 (0.166)
TED				0.110 (0.508)		−0.225 (0.440)	0.024 (0.414)
VIX					0.178*** (0.034)	0.057 (0.057)	0.017 (0.061)
Regression of the worst 20% downturns on risk measures (148 events)							
(Intercept)	−0.439* (0.214)	−2.328*** (0.240)	−2.287*** (0.292)	−1.831*** (0.337)	−4.889*** (0.738)	−2.605** (0.831)	−2.948*** (0.823)
$-FH^{\mathbb{Q}}(r_h)$	0.048** (0.018)					0.040* (0.018)	0.030 (0.017)
$VaR^{\mathbb{Q}}(r_h)$		49.538*** (8.005)					16.305 (10.774)
$ES^{\mathbb{Q}}(r_s)$			0.358** (0.113)			0.083 (0.099)	0.094 (0.105)
TED				0.778 (0.400)		0.084 (0.424)	0.229 (0.415)
VIX					0.207*** (0.044)	0.101 (0.054)	0.087 (0.053)

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

The dependent variable reflects if the realized ahead-return of an option-hedged portfolio belongs to the set of the worst 5% (top panel), 10% (middle), or 20% (bottom) downturns in that period. The risk measures involve the option-implied Foster-Hart riskiness $FH^{\mathbb{Q}}(r_h)$ and value at risk $VaR^{\mathbb{Q}}(r_h)$ of the hedged portfolio, the option-implied expected shortfall $ES^{\mathbb{Q}}(r_s)$ of the non-hedged portfolio, as well as the industry measures TED spread (credit risk) and the volatility index VIX

Table 3 Regressions of stock market downturns on various risk measures over 740 observations

Regression of the worst 5% index downturns on risk measures							
(Intercept)	-2.369^{***}	-3.460^{***}	-3.999^{***}	-3.820^{***}	-6.666^{***}	-4.450^{***}	-4.492^{**}
	(0.296)	(0.423)	(0.410)	(0.367)	(0.752)	(1.318)	(1.429)
$FH^{\mathbb{Q}}(r_h)$	0.022^{**}					0.016^{***}	0.015^{***}
	(0.008)					(0.005)	(0.004)
$VaR^{\mathbb{Q}}(r_h)$		26.867^{**}					2.595
		(10.186)					(11.657)
$ES^{\mathbb{Q}}(r_s)$			0.336^{***}			0.281^{*}	0.282^{*}
			(0.079)			(0.134)	(0.138)
TED				1.315^{**}		0.862	0.886
				(0.500)		(0.624)	(0.571)
VIX					0.204^{***}	0.022	0.020
					(0.038)	(0.102)	(0.101)

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$
The dependent variable reflects if the realized ahead-return of the S&P 500 stock market index belongs to the set of the worst 5% downturns in that period. The risk measures involve the option-implied Foster-Hart riskiness $FH^{\mathbb{Q}}(r_h)$ and value at risk $VaR^{\mathbb{Q}}(r_h)$ of the hedged portfolio, the option-implied expected shortfall $ES^{\mathbb{Q}}(r_s)$ of the non-hedged portfolio, as well as the industry measures TED spread (credit risk) and the volatility index VIX

of levered portfolios shows to be a significant predictor. Including even less extreme events, we find that while individually risk measures remain predictively successful, they lose significance in a joint regression.

Finally, we study if risk measures inferred from levered portfolios contain information about the future performance of non-levered investments. Table 3 summarizes our findings. The Foster-Hart riskiness estimated for hedged returns significantly explains future drops of simple returns both individually and in a joint regression. The same is true for the expected shortfall of non-levered investments as already documented in Leiss and Nax (2015).

6 Conclusion

In this paper we discussed a theoretical extension of the Foster-Hart measure of riskiness to study leverage. Option hedging prevents the value of portfolios from vanishing completely (provided the seller of the option does not default). In turn, this “frees” the Foster-Hart riskiness measure to values larger than 1, i.e. allows for leverage. Based on options data, we applied this new way of estimating prevailing leverage ratios to the decade 2003–2013 around the Global Financial Crisis. We found (1) a strong procyclicality of leverage during the bubble prior to the crash and (2) predictive power of risk measures computed for levered portfolios with respect to extreme losses.

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