A New Type of Soft Subincline of Incline

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Abstract. Firstly, this paper presents the new concept of soft subincline. Then some equivalent conditions of it and two operations "RESTRICTED INTERSECT" and "AND" on it are discussed. After that the relationship between soft subincline and the dual of soft set based on the method of the dual of soft set are studied. In addition, the concepts and properties of maps between soft subincline are given. Finally, the chain condition of H which consists of all of the soft subinclines is introduced and obtain a necessary and sufficient condition for H is Artinian or Noetherian.

Keywords: Incline-algebra · Soft incline · Soft sets · Dual soft sets · Chain condition

1 Introduction

As the fuzzy set theory was proposed, mathematical tools dealing with incomplete and uncertain problems were also presented. In particular, Pawlak and Atanassov proposed the rough set theory [1] and intuitionistic fuzzy set theory [2]. However, these mathematical theories are lack of parameter tools. Therefore, in order to solve this problem, Molodtsov gave the concept of soft sets innovatively in 1999 [3]. After that, Aktas and Cagman propose the definition of soft group in 2007 [4] which created a new field of soft algebra. A few years afterwards, many scholars had done a series of researches in soft algebra [5–13].

The notion of incline algebra was proposed by Cao in 1981 [14]. He also published a monograph about incline algebra with other two scholars [15]. In 2001, Jun applied fuzzy sets to incline algebra and proposed the concept of fuzzy subincline [16].

Liao applied soft sets to incline algebra and proposed the concept of soft incline in 2012 [17]. The concept of fuzzy soft incline and $(\in, \in \vee q)-$ fuzzy soft incline were proposed by Alshehri in 2012 [18]. The study of inclines and incline matrices is significant both in theory and in practice, they have good foreground of applications in many areas including automation theory, decision theory, cybernetics, graph theory and nervous system [15]. At present, the theories of incline algebras and incline matrices are highly utilized by computer science applications [19–21].

In 2008, Yuan and Wen introduced algebraic structures in parameter set and obtained a new algebraic structure of soft sets [22]. They introduced a soft algebra

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structure which can be reduced to L-fuzzy algebra by using the concept of dual soft sets, where L = P(X) (P (X) is the power set of the common universe X) is a Boolean algebra. In general the element in lattice L has no structure. However, the elements over L = P(X) is a set which can also have many elements and algebraic structure. Therefore, more meaningful results can be obtained than general L-fuzzy algebra.

In this paper, by using the idea above, we give the new concept of soft subincline. The difference between our new concept of soft subincline and the concept of soft incline proposed in literature [17] is that: the parameter set of a soft subincline is a fixed incline in this paper, while it is a subincline of a certain incline in literature [17]. Furthermore, we investigate some algebraic properties of our new type of soft subincline and introduced some properties of the new type of soft subincline of incline under the chain condition. These results enrich the theory of soft algebra.

2 Preliminary Notes

Definition 2.1 [14]. An inline (algebra) is a set K with two binary operations denoted by "·" and "·" Satisfying the following axioms: for all $x, y, z \in K$,

(1) $x + y = y + x$;
(2) $(x + y) + z = x + (y + z)$;
(3) $(xy)z = x(yz)$;
(4) $x(y + z) = xy + xz$;
(5) $(y + z)x = yx + zx$;
(6) $x + x = x$;
(7) $x + xy = x$;
(8) $y + xy = y$.

For convenience, we pronounce " + " (resp."·") as addition (resp. multiplication).

Every distributive lattice is an incline. An incline is a distributive lattice if and only if $xx = x$ for all $x \in K$.

Note that $x \le y \Leftrightarrow x + y = y$ for all $x, y \in K$.

A subincline of an incline K is a non-empty subset M of K which is closed under addition and multiplication. A subincline M is said to be an ideal of an incline K if $x \in M$ and $y \le x$ then $y \in M$. By a homomorphism of inclines we shall mean a mapping f from an incline K into an incline L such that $f(x + y) = f(x)f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in K$.

Definition 2.2 (Cartesian product). Let A and B be two non-empty set, then $A \times B = \{(x, y) | x \in A, y \in B\}$ is called a Cartesian product over A and B.

Theorem 2.1 [14]. Let K_1 and K_2 be incline algebras, then their Cartesian product is an incline algebra if for all $(x_1, x_2), (y_1, y_2) \in K_1 \times K_2$:

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2),$$
$$(x_1, x_2)(y_1, y_2) = (x_1 y_1, x_2 y_2).$$

Definition 2.3 [14]. A pair (F, A) is called a soft set (over X) if and only if F is a mapping of E into the set of all subsets of the set X.

Definition 2.4 [23] (Restricted intersection operation of two soft sets). Let (F, A) and (G, B) be two soft sets over a common universe X. If the soft set (H, C) satisfy $C = A \cap B$ and for any $e \in C, H(e) = F(e) \cap G(e)$. We call (H, C) is the restricted intersection of (F, A) and (G, B), and denote $(H, C) = (F, A) \cap (G, B)$.

Definition 2.5 [24] (AND operation on two soft sets). Let (F, A) and (G, B) be two soft sets, then "(F, A) and (G, B)" denoted by $(F, A) \wedge (G, B)$ is defined to be $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall(\alpha, \beta) \in A \times B$.

Definition 2.6 [22] (The duality of soft sets)

$A_H : X \to P(E), x \mapsto A_H = \{g | x \in H(g)\}$ is called the duality soft set of H if $H : E \to P(X), g \mapsto H(g)$ is a soft set over K.

$H_A : E \to P(X), g \mapsto H_{(g)} = \{x | g \in A(x)\}$ is called the duality soft set of A if $A : X \to P(E)$ is a soft set over X.

Definition 2.7 [22] (The Extension Principle) let X be the common universe. Let f be defined by $f : K_1 \to K_2$ and let $H_1 : K_1 \to P(X)$ and $H_2 : K_2 \to P(X)$ are soft sets over K_1 and K_2 respectively. Define soft sets $f(H_1)$ over K_1 and $f^{-1}(H_2)$ over K_2 by $\forall g_2 \in K_2,\ f(H_1)(g_2) = \begin{cases} \cup_{f(g_1)=g_2} H_1(g_1)\ f^{-1}(g_2) \neq \varnothing \\ \varnothing \quad f^{-1}(g_2) = \varnothing \end{cases}$ and $\forall g_1 \in K_1, f^{-1}(H_2)(g_1) = H_2$. Then $f(H_1)$ is said to be the image of H_1 and $f^{-1}(H_2)$ is said to be the preimage of K_2.

Definition 2.8 [17]. Let K be an incline algebra. A pair (F, A) is called a soft incline over K if $F(x)$ is a subincline of K for all $x \in A$.

3 A New Type of Soft Subincline of Incline

Definition 3.1. Let K be an incline algebra and X be the common universe, $H : K \to P(X)$ is a soft set. H is called a new type of soft subincline of incline if it satisfies the following conditions: for all $g_1, g_2 \in K$,

(1) $H(g_1 + g_2) \supseteq H(g_1) \cap H(g_2)$;

(2) $H(g_1 g_2) \supseteq H(g_1) \cap H(g_2)$.

Example 3.1. Let $K = \{0, a, b, 1\}$ be an incline with the operation tables given in Table. Let $X = \{0, a, b, 1\}$ and $H : K \to P(X)$ be a soft set defined $H(0) = \{0, a\}$ $H(a) = \{0, 1\}, H(b) = \{a, b\}, H(1) = \{b\}$. Clearly, H is a new type of soft subincline of incline over K and it could be verified by Definition 2.1. Because $H(b) = \{a, b\}$ is not a subincline of K, so H is not a soft incline over K, then the new type of soft subincline is a new algebraic structure.

The operation tables of the incline

$+$	0	a	b	1
0	0	a	b	1
a	a	a	1	1
b	b	1	b	1
1	1	1	1	1

$\cdot$	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Theorem 3.1. Let K_1 and K_2 be two subinclines of K and let H_1 and H_2 be new type of soft subinclines of incline over K_1 and K_2 respectively, if $K_1 \cap K_2 \neq \varnothing$ and $(H_1, K_1 \cap K_2) = (H, K_1) \cap (H, K_2)$, then H is a new type of soft incline of incline over $K_1 \cap K_2$.

Proof. Because of K_1 and K_2 be two subinclines of K and $K_1 \cap K_2 \neq \varnothing$, it is easy to say that $K_1 \cap K_2$ is also a subincline of K. For any $g_1, g_2 \in K_1 \cap K_2$, we have

$$\begin{aligned} H(g_1 g_2) &= H_1(g_1 g_2) \cap H_2(g_1 g_2) \supseteq [H_1(g_1) \cap H_1(g_2)] \cap [H_2(g_1) \cap H_2(g_2)] \\ &= [H_1(g_1) \cap H_2(g_1)] \cap [H_1(g_2) \cap H_2(g_2)] = H(g_1) \cap H(g_2). \end{aligned} Z$$

Similarly, $H(g_1 + g_2) \supseteq H(g_1) \cap H(g_2), \forall g_1, g_2 \in K_1 \cap K_2$.

Therefore, H is a new type of soft subincline of incline of $K_1 \cap K_2$.

Theorem 3.2. Let K_1 and K_2 be two inclines and let H_1 and H_2 be new type of soft subinclines of incline over K_1 and K_2 respectively, if $K = K_1 \times K_2$ and $(H, K) = (H_1, K_1) \wedge (H_2, K_2)$, then H is a new type of soft subincline of incline of K.

Proof. Since K_1 and K_2 are two inclines, by Theorem 2.1, it is sufficient to show that $K_1 \times K_2$ is also an incline. Then clearly $H[(x_1, y_1)(x_2, y_2)] = H(x_1 x_2, y_1 y_2) = H_1(x_1 x_2) \cap H_2(y_1 y_2) \supseteq [H_1(x_1) \cap H_1(x_2)] \cap [H_2(y_1) \cap H_2(y_2)] = [H_1(x_1) \cap H_2(y_1)] \cap [H_1(x_2) \cap H_2(y_2)] = H(x_1, y_1) \cap H(x_2, y_2)$ for all $(x_1, y_1), (x_2, y_2) \in K$.

Similarly, $H[(x_1, y_1) + (x_2, y_2)] \supseteq H(x_1, y_1) \cap H(x_2, y_2)$ for all (x_1, y_1), $(x_2, y_2) \in K$.

Therefore, H is a new type of soft subincline of incline of K.

Theorem 3.3. Let K be an incline, then the following are equivalent:

(i) H is a new type of soft subincline of incline of K.
(ii) For all $x \in X, A_H(x) \neq \emptyset$ is a subincline over K.

Proof. $(i) \Rightarrow (ii)$ For all $g_1, g_2(x) \in A_H(x)$, we have $x \in H(g_1)$ and $x \in H(g_2)$, therefore $x \in H(g_1) \cap H(g_2)$. Since H is a new type of soft subincline of incline of K, it follows that $H(g_1) \cap H(g_2) \subseteq H(g_1 g_2)$, then $x \in H(g_1 g_2)$. Hence $g_1 g_2 \in A_H(x)$.

Similarly, $g_1 + g_2 \in A_H(x)$ for all $g_1, g_2 \in A_H(x)$.

Therefore, $A_H(x)$ is a subincline over K for all $x \in X$.

$(ii) \Rightarrow (i)$: For all $g_1 + g_2 \in K$, if $H(g_1) \cap H(g_2) = \varnothing$, then $H(g_1) \cap H(g_2) = \varnothing \subseteq H(g_2)$; if $H(g_1) \cap H(g_2) \neq \varnothing$, assume that $x \in H(g_1) \cap H(g_2)$, then $x \in H(g_1)$ and $x \in H(g_2)$, hence $g_1, g_2 \in A_H(x)$ Note that $A_H(x)$ is a subincline over K, then $g_1 g_2 \in A_H(x)$ and so $x \in H(g_1 g_2)$. Thus $H(g_1) \cap H(g_2) \subseteq H(g_1 g_2)$.

Similarly, $H(g_1) \cap H(g_2) \subseteq H(g_1 + g_2)$ for all $x \in H(g_1) \cap H(g_2)$.

Therefore, H is a new type of soft subincline of incline of K.

Theorem 3.4. Let K be an incline, then the following are equivalent:

(i) Let A be defined by $A : X \to P(K)$, for all $x \in X, A_H(x) \neq \varnothing$ is a subincline over K.
(ii) H_A is a new type of soft subincline of incline over K.

Proof. $(i) \Rightarrow (ii)$: Assume that $x \in H_A(g_1) \cap H_A(g_2)$, then $g_1 \in A(x)$ and $g_2 \in A(x)$. Note that $A(x)$ is a subincline of K, then $g_1 g_2 \in A(x)$ and $g_1 + g_2 \in A(x)$. Clearly $x \in H_A(g_1 g_2)$ and $x \in H_A(g_1 + g_2)$. Thus $H_A(g_1) \cap H_A(g_2) \subseteq H_A(g_1 g_2)$ and $H_A(g_1) \cap H_A(g_2) \subseteq H_A(g_1 g_2)$.

Therefore, H_A is a new type of soft subincline of incline over K.

$(ii) \Rightarrow (i)$: For any $g_1, g_2 \in A(x)$, we have $x \in H_A(g_1)$ and $x \in H_A(g_2)$, and so $x \in H_A(g_1) \cap H_A(g_2)$. Since H_A is a new type of soft subincline of incline over K, it follows that $H(g_1) \cap H(g_2) \subseteq H(g_1 g_2)$ and $H(g_1) \cap H(g_2) \subseteq H(g_1 + g_2)$, then $g_1 g_2 \in A(x)$ and $g_1 + g_2 \in A(x)$. Therefore,$A(x)$ is a subincline over K.

Theorem 3.5. Let K_1 and K_2 be two inclines and let X be the common universe. Let $f : K_1 \to K_2$ be a hemimorphic mapping. Let $H_1 : K_1 \to P(X)$ and $H_2 : K_2 \to P(X)$ be soft sets over K_1 and K_2 respectively. Then $f(H_1)$ is a new type of soft incline of incline over K_2 if H_1 is a new type of soft subincline of incline over K_1.

Proof. For all $g_2, g_2' \in K_2$.

Case1: Assume $f^{-1}(g_2) \neq \varnothing$ and $f^{-1}(g_2') \neq \varnothing$. if $f(H_1)(g_2) \cap f(H_1)(g_2') = \varnothing$, then $f(H_1)(g_2) \cap f(H_1)(g_2') \subseteq f(H_1)(g_2 + g_2')$ if $f(H_1)(g_2) \cap f(H_1)(g_2') \neq \varnothing$ then $\forall x \in f(H_1)(g_2) \cap f(H_1)(g_2')$ we have $x \in \cup_{f(g_1)=g_2} H_1(g_1)$ and $x \in \cup_{f(g_1')=g_2'} H_1(g_1')$. Thus there exists $g_1 \in K_1$ such that $x \in H_1(g_1)$ and $f(g_1) = g_2$. There also exists $g_1' \in K_1$ such that $x \in H_1(g_1')$ and $f(g_1') = g_2'$. Hence $x \in H_1(g_1) \cap H_1(g_1')$. Since H_1 is a new type of soft subincline of incline over K_1, we can get $H_1(g_1) \cap H_1(g_1') \subseteq H_1(g_1 + g_1')$, then $x \in H_1(g_1 + g_1')$. Note that f is a homomorphic mapping, so $f(g_1 + g_1') = f(g_1) + f(g_1') = g_2 + g_2'$ where $g_1 + g_1' \in K_1$. Therefore, $x \in H_1(g_1 + g_1') \subseteq \cup_{f(g)=g_2+g_2'} H_1(g) = f(H_1)(g_2 + g_2')$.

Case2: If $f^{-1}(g_2) = \varnothing$ or $f^{-1}(g_2') = \varnothing$, then $f(H_1)(g_2) = \varnothing$ or $f(H_1)(g_2') = \varnothing$, and so $f(H_1)(g_2) \cap f(H_1)(g_2') = \emptyset \subseteq f(H_1)(g_2 + g_2')$.

Similarly, $f(H_1)(g_2) \cap f(H_1)(g_2') \subseteq f(H_1)(g_2 g_2'), \forall g_2, g_2' \in K_2$.

Therefore, $f(H_1)$ is a new type of soft incline of incline over K_2 .

Theorem 3.6. Let K_1 and K_2 be two inclines and let X be the common universe. Let be $f : K_1 \to K_2$ be a homomorphic mapping. $H_1 : K_1 \to P(X)$ and $H_2 : K_2 \to P(X)$ are soft sets over K_1 and K_2 respectively. Then $f^{-1}(H_2)$ is a new type of soft subincline of incline of K_1, if H_2 is a new type of soft subincline of incline over K_2.

Proof. For any $g_1, g_1' \in K_1$, we have

$$f^{-1}(H_2)(g_1) \cap f^{-1}(H_2)(g_1') = H_2(f(g_1)) \cap H_2(f(g_1')) \subseteq H_2(f(g_1) + f(g_1')) \\ = H_2(f(g_1 + g_1')) = f^{-1}(H_2)(g_1 + g_1').$$

Similarly, $f^{-1}(H_2)(g_1) \cap f^{-1}(H_2)(g_1') \subseteq f^{-1}(H_2)(g_1 g_1'), \forall g_1, g_1' \in K_1$.

Therefore, $f^{-1}(H_2)$ is a new type of soft subincline of incline of K_1.

Definition 3.2. Let K_1 and K_2 be two inclines and let H_1 be a new type of soft subincline of incline over K_1. Let $f : K_1 \to K_2$ be a map. For all $x, y \in K_1$, if $f(x) = f(y)$, we have $H_1(x) = H_1(y)$, then H_1 is said to be f-invariant.

Theorem 3.7. Let K_1 and K_2 be two inclines and let X be the common universe. If f is a homomorphic mapping from K_1 to K_2, $H_1 : K_1 \to P(X)$ is a soft set over K_1 and H_1 is f-invariant. Then the following are equivalent:

(i) H_1 is a new type of soft subincline of incline over K_1.
(ii) $f(H_1)$ is a new type of soft subincline of incline over K_2.

Proof. $(i) \Rightarrow (ii)$: Following Theorem 3.5, it is sufficient to show that the conclusion is correct.

$(ii) \Rightarrow (i)$: For any $g_1, g_1' \in K_1$ and $x \in H_1(g_1) \cap H_1(g_1')$, we have $x \in H_1(g_1)$ and $x \in H_1(g_1')$. Assume that $f(g_1) = g_2$ and $f(g_1') = g_2' \in K_2$, then $x \in \cup_{f(g)=g_2} H_1(g) = f(H_1)(g_2)$ and $x \in \cup_{f(g')=g_2'} H_1(g) = f(H_1)(g_2')$, and so $x \in f(H_1)(g_2) \cap f(H_1)(g_2')$. since $f(H_1)$ is a new type of soft subincline of incline over K_2, hence $f(H_1)(g_2) \cap f(H_1)(g_2') \subseteq f(H_1)(g_2 g_2')$, then $x \in f(H_1)(g_2 g_2') = \cup_{f(g)=g_2 g_2'} H_1(g)$

So clearly there exists $g \in K_1$ such that $f(g) = g_2 g_2'$ and $x \in H_1(g)$. Since f is a homomorphic mapping, then $f(g) = f(g_1) f(g_1') = f(g_1 g_1')$. Also note that H_1 is a f-invariant, then $H_1(g) = H_1(g_1 g_1')$, so $x \in H_1(g_1 g_1')$.

Hence $H_1(g_1) \cap H_1(g_1') \subseteq H_1(g_1 g_1')$.

Similarly, $H_1(g_1) \cap H_1(g_1') \subseteq H_1(g_1 + g_1'), \forall g_1, g_1' \in K$.

Therefore, H_1 is a new type of soft subincline of incline of K_1.

Theorem 3.8. Let K_1 and K_2 be two inclines and let X be the common universe. If f is a homomorphic mapping from K_1 to K_2 and $H_2 : K_2 \to P(X)$ is a soft set over K_2. Then the following are equivalent:

(i) H_2 is a new type of soft subincline of incline over K_2.
(ii) $f^{-1}(H_2)$ is a new type of soft subincline of incline over K_1.

Proof. $(i) \Rightarrow (ii)$: Following Theorem 3.6, it is sufficient to show that the conclusion is correct.

$(ii) \Rightarrow (i)$: For any $g_2, g_2' \in K_2$, note that f is a homomorphic mapping, so there exists $g_1, g_1' \in K_1$ such that $g_2 = f(g_1), g_2' = f(g_1')$ and $g_2 g_2' = f(g_1) f(g_1') = f(g_1 g_1')$, then $H_2(g_2) \cap H_2(g_2') = H_2(f(g_1)) \cap H_2(f(g_1')) = f^{-1}(H_2)(g_1) \cap f^{-1}(H_2)(g_1')$ since $f^{-1}(H_2)$ is a new type of soft subincline of incline over K_1, we get $f^{-1}(H_2)(g_1)$

$\cap f^{-1}(H_2)(g_1')\subseteq f^{-1}(H_2)(g_1g_1') = H_2(f(g_1g_1')) = H_2(g_1g_1')$. Hence $H_2(g_2)\cap H_2(g_2')$ $\subseteq H_2(g_2g_2')$.

Similarly, $H_1(g_2)\cap H_1(g_2')\subseteq H_1(g_2+g_2'), \forall g_2, g_2' \in K_2$.

Therefore, H_2 is a new type of soft subincline of incline over K_2.

4 The Chain Condition of Incline

Definition 4.1. Let K be an incline algebra and H is the set of all new type of soft subincline of incline over K. Suppose H_1 and H_2 are elements of H. Define a binary relation "$\leq$" over H as follows: $H_1 \leq H_2 \Leftrightarrow H_1(g)\subseteq H_2(g), \forall g \in K$.

Definition 4.2. Let K be an incline algebra, H is the set of all new type of soft subincline of incline over K, H_1 and H_2 are elements of H. Define a binary relation "=" over H as follows: $H_1 = H_2 \Leftrightarrow H_1(g) = H_2(g), \forall g \in K$.

Theorem 4.1. $(H, \leq)$ is a partially ordered set.

Proof. For any $H_1 \in H$, we have $H_1(g) \in H_1(g)$ for all $g \in K$, hence $H_1 \leq H_1$.
For any $H_1, H_2, H_3 \in H$, assume that $H_1 \leq H_2$ and $H_2 \leq H_3$. Clearly for any $g \in K$ we have $H_1(g)\subseteq H_2(g)$ and $H_2(g)\subseteq H_3(g)$, then $H_1(g)\subseteq H_3(g)$. Hence $H_1 \leq H_3$.

For any $H_1, H_2 \in H$, assume that $H_1 \leq H_2$ and $H_2 \leq H_1$. Clearly for any $g \in K$, we have $H_1(g)\subseteq H_2(g)$ and $H_2(g)\subseteq H_1(g)$, then $H_2(g) = H_1(g)$.

Hence $H_2 = H_1$.

Therefore, $(H, \leq)$ is a partially ordered set.

Theorem 4.2. K is an incline, H_1 and H_2 are new type of soft subinclines of incline over K. Then $H_1 \leq H_2$ if and only if $A_{H_1}(x)\subseteq A_{H_2}(x)$ for all $x \in X$.

Proof. Necessity: For any $g \in A_{H_1(x)}$, we have $H_1(g)\subseteq H_1(g)$. Since $H_1 \leq H_2$, then $H_1(g)\subseteq H_2(g)$. This implies that $x \in H_2(g)$, so that $g \in A_{H_2}(x)$. Therefore $A_{H_1}(x)\subseteq A_{H_2}(x)$.

Sufficiency: For any $g \in K$ and $x \in H_1(g)$, we have $g \in A_{H_1}(x)$. Since $A_{H_1}(x)\subseteq A_{H_2}(x)$ then $g \in A_{H_2}(x)$. This implies that $x \in H_2(g)$, so that $H_2(g)\subseteq H_2(g)$. Therefore $H_1 \leq H_2$.

Corollary 4.1. K is an incline, H_1 and H_2 are new type of soft subinclines of incline over K. Then $H_1 = H_2$ if and only if $A_{H_1}(x) = A_{H_2}(x)$ for all $x \in X$.

Definition 4.3. K is an incline, $\Omega(K)$ is a subincline family of K. For any ascending chain of $\Omega(K), K_1\subseteq K_2\subseteq \cdots \subseteq K_n\subseteq \cdots$, if there exists a positive integer n such that $K_n = K_m$ for all m > n, $\Omega(K)$ is called Noetherian. K is called a Noetherian incline if $\Omega(K)$ is the set of all subinclines over K. The number min $\{i|K_i = K_{i+1}, i = 1, 2, \cdots\}$ is called the stabilize index of Noetherian and denoted by $m_{\{K_i\}}$.

Definition 4.4. K is an incline, $\Omega(K)$ is a subincline family of K. For any descending chain of $\Omega(K), K_1 \supseteq K_2 \supseteq \cdots \supseteq K_n \supseteq \cdots$, if there exists a positive integer n such that $K_n = K_m$ *for all* $m > n$, $\Omega(K)$ is called Artinian. K is called a Artinian incline if $\Omega(K)$

is the set of all subinclines over K. The number min $\{i|K_i = K_{i+1}, i = 1, 2, \cdots\}$ is called the stabilize index of Artinian and denoted by $n_{\{K_i\}}$.

Definition 4.5. K is an incline, $\sum$ is the set of all subinclines over K. K is said to satisfy the maximal condition if every nonempty set over $\sum$ has a maximal element.

Definition 4.6. K is an incline, $\sum$ is the set of all subinclines over K. K is said to satisfy the minimal condition if every nonempty set over $\sum$ has a minimal element.

Theorem 4.3. K is an incline, K is a Noetherian incline if and only if K satisfies the maximal condition.

Proof. Necessity: Let $\sum$ be the set of all subinclines over K. Assume that nonempty subset $\sum'$ over $\sum$ without maximal element, then for any $K_i \in \sum'$, there exists K_{i+1} such that $K_i \subseteq K_{i+1}$. Hence there exists an infinite ascending chain $K_1 \subseteq K_2 \subseteq \cdots \subseteq K_i \subseteq K_{i+1} \subseteq \cdots$, a contradiction. Therefore, K satisfies the maximal condition.

Theorem 4.4. K is an incline, K is an Artinian incline if and only if K satisfy the minimal condition.

Proof. The proof of this theorem is similar to the proof of Theorem 4.3 and so is omitted.

Definition 4.7. K is an incline, let A be a subincline over K. A is said to be reducible if there exist B and C, which are subincline over K, properly including A, such that $A = B \cap C$. If $A = B \cap C$, there must have $A = B$ or $A = C$, then A is said to be irreducible.

Theorem 4.5. K is a Noetherian incline, then every subincline over K can be expressed as intersectiob of finite number of subinclines which are irreducible.

Proof. Let $\sum_1$ be the set of subinclines over K which are cannot be expressed as intersection of a finite number of irreducible subinclines. Assume that $\sum_1 \neq \varnothing$. Since K is a Noetherian incline, clearly we know K satisfies the maximal condition. Hence there exists a maximal element in $\sum_1$ and denoted by a. Because the elements in $\sum_1$ are reducible, there exist b and c, which are subinclines over K, such that $a = b \cap c$ where $a \subset b$ and $a \subset c$. For a is the maximal element, then $b, c \notin \sum_1$, and so b, c can be expressed as intersection of a finite number of irreducible subinclines, denoted by $b = b_1 \cap \cdots \cap b_m, c = c_1 \cap \cdots \cap c_n$ (the b_i, c_j are irreduclble). Clearly, $a = b \cap c = b_1 \cap \cdots \cap b_m \cap c_1 \cap \cdots \cap c_n$. a can be expressed as intersection of a finite number of irreducible subinclines, a contradiction. So we have $\sum_1 = \emptyset$, hence every subincline over K can be expressed as intersection of finite number of subinclines which are irreducible.

Definition 4.8. K is a Noetherian incline, H is the set of all new type of soft subinclines of incline over K. For any ascending chain of new type of soft subinclines of incline $H_1 \leq H_2 \leq H_3 \leq \cdots$, if there exists a positive integer n such that $H_m = H_n$ for all $m > n, H$ is said to have the ascending chain condition, or we say H is Noetherian.

Definition 4.9. K is an incline, H is the set of all new type of soft subinclines of incline over K. For any descending chain of new type of soft subinclines of incline $H_1 \geq H_2 \geq H_3 \geq \cdots$, if there exists a positive integer n such that $H_m = H_n$ for all $m > n, H$ is said to have the descending chain condition, or we say H is Artinian.

Theorem 4.6. K is an incline, H is the set of all new type of soft subinclines of incline over K. H is Noetherian if and only if $\Omega(K)(x) \triangleq \{A_{H_i}(x) | H_i \in H\}$ is Noetherian for all $x \in X$ and $\sup\{m_{\{A_{H_i(x)}\}} | x \in X\}$ is finited.

Proof. Necessity: Following Theorem 3.3, it is sufficient to show that $A_{H_i}(x)$ is a subincline of K. Then $\Omega(K)(x)$ is a subincline family. Let $A_{H_1}(x) \subseteq A_{H_2}(x) \subseteq \cdots A_{H_n}(x) \subseteq \cdots$ be an ascending chain over $\Omega(K)(x)$, according to Theorem 4.2, it follows that $H_1 \leq H_2 \leq \cdots \leq H_n \leq \cdots$ For H is Noetherian, so there exists a positive integer n such that $H_m = H_n$ for all $m > n$. According to Corollrry 4.1, it follows that $A_{H_m}(x) = A_{H_n}(x)$ for all $x \in X$.

Consequently, we infer that $\Omega(K)(x)$ is Noetherian for all $x \in X$ and $\sup\{m_{\{A_{H_i(x)}\}} | x \in X\}$ is finited.

Sufficiency: Let $H_1 \leq H_2 \leq \cdots \leq H_n \leq \cdots$ be an ascending chain over H, according to Theorem 4.2, we have that $A_{H_1}(x) \subseteq A_{H_2}(x) \subseteq \cdots A_{H_n}(x) \subseteq \cdots$ for all $x \in X$. Let $\sup\{m_{\{A_{H_i(x)}\}} | x \in X\} = n$, since $\Omega(K)(x)$ is Noetherian, so $A_{H_m}(x) = A_{H_n}(x)$ for all $x \in X$ if $m > n$. According to Corollary 4.1, it follows that $H_m{=}H_n$ if $m > n$. Therefore H is Noetherian.

Theorem 4.7. K is an incline, H is the set of all new type of soft subinclines of incline over K. H is Artinian if and only if $\Omega(K)(x) \triangleq \{A_{H_i}(x) | H_i \in H\}$ is Artinian for all $x \in X$ and $\sup\{n_{\{A_{H_i(x)}\}} | x \in X\}$ is finited.

Proof. The proof of this theorem is similar to the proof of Theorem 4.6 and so is omitted.

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References

1. Pawlak, Z.: Rough sets. J. Inform. Comput. Sci. **11**(5), 341–356 (1982)
2. Atanassov, K.: Intuitionistic fuzzy sets. Fuzzy Sets Syst. **20**(2), 87–96 (1986)
3. Molodtsov, D.: Soft set theory-first results. Comput. Math Appl. **37**(4–5), 19–31 (1991)
4. Aktas, H., Cagman, N.: Soft sets and soft groups. Inform. Sci. **177**(13), 2726–2735 (2007)
5. Acar, Z.U., Koyuncu, F., Tanay, B.: Soft sets and soft rings. Comput. Math Appl. **59**(11), 3458–3463 (2010)

6. Atagun, A.O., Sezgin, A.: Softgroups and normalistic soft groups. Comput. Math Appl. **62**(2), 685–698 (2011)
7. Aygunoglu, A., Aygun, H.: Introduction to fuzzy soft groups. Comput. Math Appl. **58**(6), 1279–1286 (2009)
8. Feng, F., Jun, Y.B., Zhao, X.: Soft semiring. Comput. Math Appl. **56**(10), 2621–2628 (2008)
9. Guan, B.B., Liao, Z.H., Zhu, X.Y.: Soft subsmigroups. Fuzzy Syst. Math. **28**(4), 39–44 (2004)
10. Jun, Y.B.: Soft BCK/BCI-algebras. Comput. Math Appl. **56**(5), 1408–1413 (2008)
11. Jun, Y.B.: Union-soft sets with applications in BCK/BCI-algebras. Bull. Korean Math. Soc. **50**(6), 1937–1956 (2013)
12. Jun, Y.B., Kang, M.S., Lee, K.J.: Intersectional soft sets and applications to BCK/BCI-algebras. Commum. Korean Math. Soc. **28**(1), 11–24 (2013)
13. Qin, K.Y., Shao, Y.C., Zhu, Z.G.: Soft sets and softlattices. Comput. Eng. Appl. **49**(3), 10–14 (2013)
14. Cao, Z.Q.: A new algebraic system-slope. BUSEFAL **6**, 22–27 (1981)
15. Cao, Z.Q., Kim, K.H., Roush, F.W.: Incline Algebras and Applications. Wiley, New York (1984)
16. Ahn, S.S., Jun, Y.B., Kim, H.S.: Fuzzy subinclines (ideals)of incline algebras. Fuzzy Sets Syst. **123**(2), 217–225 (2001)
17. Liao, Z.H., Rui, M.L.: Soft Inclines. Comput. Eng. Appl. **48**(2), 30–32 (2012)
18. Alshehri, N.O.: fuzzy soft incline algebras. Word. Appl. Sci. J. **17**(12), 1783–1788 (2012)
19. Gu, Y.D., Han, S.C., Li, H.X.: An application of incline matrices in dynamic analysis of generalized fuzzy bidirectional associative memories. Fuzzy Sets Syst. **158**(12), 1340–1347 (2007)
20. Lu, L.X., Zhang, B.: Graphs and matroids weighted in a bounded incline algebra. Sci. World. J. Article ID 912715 (2014)
21. Zhang, L.M.: Generalized inverses of incline matrices and application. M.S.Dissertation, Liaocheng University (2007)
22. Wen, Y.C.: The study on soft set. M.S.Dissertation, Liaoning Normal University (2008)
23. Ali, M.I., Feng, F., Liu, X.Y., et al.: On some new operations in soft set theory. Comput. Math Appl. **57**(9), 1547–1553 (2009)
24. Biswas, R., Maji, P.K., Roy, A.R.: Soft set theory. Comput. Math Appl. **45**(4–5), 555–562 (2003)