Analysis for the Presence of Quantum Noise on the Teleportation

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Abstract. Quantum teleportation has provided us with an interesting way to transmit an arbitrary quantum state using one maximal entangled state and two classical bits of information. A variety of theoretical suggestions and experimental efforts have been made in this realm. In practical implementations of the teleportation protocol, quantum noise is an unavoidable factor. In this paper, we investigate the probabilistic quantum teleportation of two-particle. The fidelity of quantum state is calculated in detail, after suffering from the quantum noise. The relationship between quantum noise, quantum channel and quantum state fidelity is obtained. The effect of noise on the teleportation is analyzed.

Keywords: Quantum teleportation · Quantum noise · Fidelity · Quantum channel

1 Introduction

Quantum teleportation is a technique for the direct transmission of quantum states between the correspondents. In 1993, Bennett et al. [1] proposed the concept of quantum teleportation. At present, the technology has made numerous achievements in both theory [2–15] and experiment [16–18]. It has been studied from the original single particle quantum teleportation to many particle quantum teleportation and continuous variable quantum teleportation; from accurate quantum teleportation (the value of fidelity is 1) to the precise quantum teleportation (the value of fidelity is less than 1). All quantum teleportation protocols need establish a quantum channel by the entangled state. In this process, the particles of the entangled channel are susceptible to the interference by the quantum noise.

In this paper, we take the probability teleportation of two particle entangled state [19] as an example. and analyze the effects of several common quantum noises [20,21] on the quantum channel. We have obtained the relationship between the quantum fidelity, the parameters of noise and the state parameters of channel. It has an important reference value for the practical application of the quantum teleportation.

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2 Preparation

2.1 Fidelity and Quantum Noise

To quantify the efficiency of the probabilistic teleportation protocol we use the fidelity. The fidelity is

$$F = Tr[\rho_{Target}\rho_{out}] = \langle\psi|\rho_{out}|\psi\rangle.$$

The action of the noise on the qubit, described by the density matrix ρ, is (Table 1)

$$\rho \to \rho_{out} = \sum_{j=1}^{n} E_j \rho E_j^{\dagger}.\ \sum_{j=1}^{n} E_j E_j^{\dagger} = I.$$

Table 1. Four types of noise

Types of noise	Kraus operators	
Bit flip	$E_1 = \sqrt{1-p}I$	$E_2 = \sqrt{p}\sigma_x$
Phase flip	$E_1 = \sqrt{1-p}I$	$E_2 = \sqrt{p}\sigma_z$
Depolarizing	$E_1 = \sqrt{1-3p/4}I$	$E_2 = \sqrt{p/4}\sigma_x$
	$E_3 = \sqrt{p/4}\sigma_y$	$E_4 = \sqrt{p/4}\sigma_z$
Amplitude damping	$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$	$E_1 = \begin{pmatrix} 1 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$

2.2 Examples of Quantum Teleportation

The quantum teleportation of the participant are Alice and Bob. The quantum state that Alice wants to transmit is $|\varphi\rangle_{12}$. And the quantum entanglement channel is $|\phi\rangle_{34}$.

$$|\varphi\rangle_{12} = x|00\rangle + y|11\rangle, \tag{1}$$

$$|\phi\rangle_{34} = a|00\rangle + b|11\rangle, \tag{2}$$

where x and y are non negative real numbers, and $x^2 + y^2 = 1$; also a and b are non negative real numbers, and $a^2 + b^2 = 1$. We can suppose $0 < b \leq a < 1$.

Suppose Alice prepared entangled state $|\phi\rangle_{34}$, then sends Bob the qubit 4 by optical fiber to set up communication channel. In this process, the qubit 4 is easier affected by noise. So when noise act on the qubit 4, we study the influence of noise for the quantum teleportation.

Without noise, composite system composed of qubits 1, 2, 3, 4 is:

$$|\Phi\rangle_{1234} = |\varphi\rangle_{12} \otimes |\phi\rangle_{34}; \tag{3}$$

the quantum state of Eq. (3) can be written as

$$\begin{aligned}|\Phi\rangle_{1234} = \frac{1}{\sqrt{2}}[&|\phi^+\rangle_{23}(ax|00\rangle_{14} + by|11\rangle_{14}) + |\phi^-\rangle_{23}(ax|00\rangle_{14} - by|11\rangle_{14}) \\ &+|\psi^+\rangle_{23}(bx|01\rangle_{14} + ay|10\rangle_{14}) + |\psi^-\rangle_{23}(bx|01\rangle_{14} - ay|10\rangle_{14})];\end{aligned} \tag{4}$$

where $|\phi^{\pm}\rangle$ and $|\psi^{\pm}\rangle$ are Bell states.

Without noise, the quantum channel is $|\phi\rangle_{34}$. And the density matrix is

$$\rho_{in} = \begin{pmatrix} a^2 & 0 & 0 & ab \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ ab & 0 & 0 & b^2 \end{pmatrix}.$$

3 The Presence of Quantum Noise on the Teleportation

3.1 Bit Flip

After suffering from the noise named bit flip, the density matrix of $|\phi\rangle_{34}$ is

$$\begin{aligned}\rho_{out} &= (I \otimes E_1^{\dagger})\rho_{in}(I \otimes E_1) + (I \otimes E_2^{\dagger})\rho_{in}(I \otimes E_2) \\ &= (1-p)\begin{pmatrix} a^2 & 0 & 0 & ab \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ ab & 0 & 0 & b^2 \end{pmatrix} + p\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a^2 & ab & 0 \\ 0 & ab & b^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};\end{aligned}$$

so the quantum channel has changed as

$$|\phi\rangle'_{34} = \sqrt{1-p}(a|00\rangle_{34} + b|11\rangle_{34}) + \sqrt{p}(a|01\rangle_{34} + b|10\rangle_{34}); \tag{5}$$

and the composite system has changed as

$$\begin{aligned}|\Phi\rangle'_{1234} =& |\varphi\rangle_{12} \otimes |\phi\rangle'_{34} \\ =& \frac{1}{\sqrt{2}}\sqrt{1-p}[|\phi^+\rangle_{23}(ax|00\rangle_{14} + by|11\rangle_{14}) + |\phi^-\rangle_{23}(ax|00\rangle_{14} - by|11\rangle_{14}) \\ &+|\psi^+\rangle_{23}(bx|01\rangle_{14} + ay|10\rangle_{14}) + |\psi^-\rangle_{23}(bx|01\rangle_{14} - ay|10\rangle_{14})] \\ &+\frac{1}{\sqrt{2}}\sqrt{p}[|\phi^+\rangle_{23}(ax|01\rangle_{14} + by|10\rangle_{14}) + |\phi^-\rangle_{23}(ax|01\rangle_{14} - by|10\rangle_{14}) \\ &+|\psi^+\rangle_{23}(bx|00\rangle_{14} + ay|11\rangle_{14}) + |\psi^-\rangle_{23}(bx|00\rangle_{14} - ay|11\rangle_{14})];\end{aligned} \tag{6}$$

obviously, the fidelity of the composite system is $F_1 = (1-p) \times 1 + p \times 0 = 1-p$.

3.2 Phase Flip

After suffering from the noise named phase flip, the density matrix of $|\phi\rangle_{34}$ is

$$\begin{aligned}\rho_{out} &= (I\otimes E_1^{\dagger})\rho_{in}(I\otimes E_1) + (I\otimes E_2^{\dagger})\rho_{in}(I\otimes E_2)\\ &= (1-p)\begin{pmatrix} a^2 & 0 & 0 & ab\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ ab & 0 & 0 & b^2\end{pmatrix} + p\begin{pmatrix} a^2 & 0 & 0 & -ab\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -ab & 0 & 0 & b^2\end{pmatrix};\end{aligned}$$

so the quantum channel has changed as

$$|\phi\rangle'_{34} = \sqrt{1-p}(a|00\rangle_{34} + b|11\rangle_{34}) + \sqrt{p}(a|00\rangle_{34} - b|11\rangle_{34}); \tag{7}$$

and the composite system has changed as

$$\begin{aligned}|\Phi\rangle'_{1234} =&|\varphi\rangle_{12}\otimes|\phi\rangle'_{34}\\ =&\frac{1}{\sqrt{2}}\sqrt{1-p}[|\phi^+\rangle_{23}(ax|00\rangle_{14} + by|11\rangle_{14}) + |\phi^-\rangle_{23}(ax|00\rangle_{14} - by|11\rangle_{14})\\ &+|\psi^+\rangle_{23}(bx|01\rangle_{14} + ay|10\rangle_{14}) + |\psi^-\rangle_{23}(bx|01\rangle_{14} - ay|10\rangle_{14})]\\ &+\frac{1}{\sqrt{2}}\sqrt{p}[|\phi^+\rangle_{23}(ax|00\rangle_{14} - by|11\rangle_{14}) + |\phi^-\rangle_{23}(ax|00\rangle_{14} + by|11\rangle_{14})\\ &+|\psi^+\rangle_{23}(-bx|01\rangle_{14} + ay|10\rangle_{14}) + |\psi^-\rangle_{23}(-bx|01\rangle_{14} - ay|10\rangle_{14})];\end{aligned} \tag{8}$$

obviously, the fidelity of the composite system is

$$\begin{aligned}F_2 =&(1-p)\times 1 + p\times\frac{1}{4}[|\ (a^2x^2-b^2y^2) + (a^2x^2-b^2x^2) + (-b^2x^2+a^2y^2) + (-b^2x^2+a^2y^2)\ |]\\ =&(1-p)\times 1 + \frac{1}{2}p\times\ |\ a^2-b^2\ |\\ =&1 + (a^2-\frac{3}{2})p.\end{aligned} \tag{9}$$

3.3 Depolarizing

After suffering from the noise named depolarizing, the density matrix of $|\phi\rangle_{34}$ is

$$\begin{aligned}\rho_{out} &= (I\otimes E_1^{\dagger})\rho_{in}(I\otimes E_1) + (I\otimes E_2^{\dagger})\rho_{in}(I\otimes E_2) + (I\otimes E_3^{\dagger})\rho_{in}(I\otimes E_3) + (I\otimes E_4^{\dagger})\rho_{in}(I\otimes E_4)\\ &= (1-\frac{3p}{4})\begin{pmatrix} a^2 & 0 & 0 & ab\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ ab & 0 & 0 & b^2\end{pmatrix} + \frac{p}{4}\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & a^2 & ab & 0\\ 0 & ab & b^2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} + \frac{p}{4}\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & -a^2 & -ab & 0\\ 0 & -ab & -b^2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} + \frac{p}{4}\begin{pmatrix} a^2 & 0 & 0 & -ab\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -ab & 0 & 0 & b^2\end{pmatrix};\end{aligned}$$

so the quantum channel has changed as

$$\begin{aligned}|\phi\rangle'_{34} =&\sqrt{1-\frac{3p}{4}}(a|00\rangle_{34} + b|11\rangle_{34}) + \sqrt{\frac{p}{4}}(a|01\rangle_{34} + b|10\rangle_{34})\\ &+\sqrt{\frac{p}{4}}(-ai|01\rangle_{34} - bi|10\rangle_{34}) + \sqrt{\frac{p}{4}}(a|00\rangle_{34} - b|11\rangle_{34});\end{aligned} \tag{10}$$

and the composite system has changed as

$$\begin{aligned}|\Phi\rangle'_{1234} =&|\varphi\rangle_{12}\otimes|\phi\rangle'_{34}\\ =&\frac{1}{\sqrt{2}}\sqrt{\frac{3p}{4}}[|\phi^+\rangle_{23}(ax|00\rangle_{14}+by|11\rangle_{14})+|\phi^-\rangle_{23}(ax|00\rangle_{14}-by|11\rangle_{14})\\ &+|\psi^+\rangle_{23}(bx|01\rangle_{14}+ay|10\rangle_{14})+|\psi^-\rangle_{23}(bx|01\rangle_{14}-ay|10\rangle_{14})]\\ &+\frac{1}{\sqrt{2}}\sqrt{\frac{p}{4}}[|\phi^+\rangle_{23}(ax|01\rangle_{14}+by|10\rangle_{14})+|\phi^-\rangle_{23}(ax|01\rangle_{14}-by|10\rangle_{14})\\ &+|\psi^+\rangle_{23}(bx|00\rangle_{14}+ay|11\rangle_{14})+|\psi^-\rangle_{23}(bx|00\rangle_{14}-ay|11\rangle_{14})]\\ &-\frac{1}{\sqrt{2}}i\sqrt{\frac{p}{4}}[|\phi^+\rangle_{23}(ax|01\rangle_{14}+by|10\rangle_{14})+|\phi^-\rangle_{23}(ax|01\rangle_{14}-by|10\rangle_{14})\\ &+|\psi^+\rangle_{23}(bx|00\rangle_{14}+ay|11\rangle_{14})+|\psi^-\rangle_{23}(bx|00\rangle_{14}-ay|11\rangle_{14})]\\ &+\frac{1}{\sqrt{2}}\sqrt{\frac{p}{4}}[|\phi^+\rangle_{23}(ax|00\rangle_{14}-by|11\rangle_{14})+|\phi^-\rangle_{23}(ax|00\rangle_{14}+by|11\rangle_{14})\\ &+|\psi^+\rangle_{23}(-bx|01\rangle_{14}+ay|10\rangle_{14})+|\psi^-\rangle_{23}(-bx|01\rangle_{14}-ay|10\rangle_{14})];\end{aligned} \tag{11}$$

obviously, the fidelity of the composite system is

$$\begin{aligned}F_3 =&(1-\frac{3p}{4})\times 1+\frac{p}{4}\times 0+\frac{p}{4}\times 0+\frac{p}{8}\times(a^2-b^2)\\ =&1-\frac{1}{8}(7-a^2)p.\end{aligned} \tag{12}$$

3.4 Amplitude Damping

After suffering from the noise named amplitude damping, the density matrix of $|\phi\rangle_{34}$ is

$$\begin{aligned}\rho_{out} &= (I\otimes E_1^\dagger)\rho_{in}(I\otimes E_1)+(I\otimes E_2^\dagger)\rho_{in}(I\otimes E_2)\\ &=\begin{pmatrix} a^2 & 0 & 0 & ab\sqrt{1-p}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ ab\sqrt{1-p} & 0 & 0 & b^2(1-p)\end{pmatrix}+\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&b^2p&0\\0&0&0&0\end{pmatrix};\end{aligned}$$

so the quantum channel has changed as

$$|\phi\rangle'_{34}=(a|00\rangle_{34}+b\sqrt{1-p}|11\rangle_{34})+b\sqrt{p}|10\rangle_{34}; \tag{13}$$

and the composite system has changed as

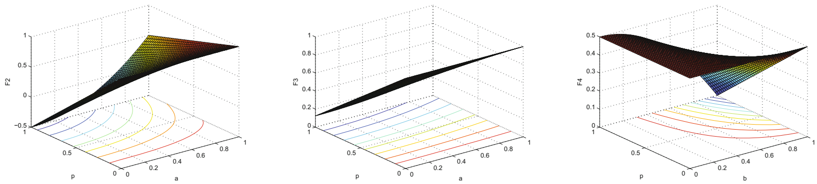
$$\begin{aligned}|\Phi\rangle'_{1234} =&|\varphi\rangle_{12}\otimes|\phi\rangle'_{34}\\ =&\frac{1}{\sqrt{2}}[|\phi^+\rangle_{23}(ax|00\rangle_{14}+by\sqrt{1-p}|11\rangle_{14})+|\phi^-\rangle_{23}(ax|00\rangle_{14}-by\sqrt{1-p}|11\rangle_{14})\\ &+|\psi^+\rangle_{23}(bx\sqrt{1-p}|01\rangle_{14}+ay|10\rangle_{14})+|\psi^-\rangle_{23}(bx\sqrt{1-p}|01\rangle_{14}-ay|10\rangle_{14})]\\ &+\frac{1}{\sqrt{2}}[|\phi^+\rangle_{23}(by\sqrt{p}|10\rangle_{14})+|\phi^-\rangle_{23}(-by\sqrt{p}|10\rangle_{14})\\ &+|\psi^+\rangle_{23}(bx\sqrt{p}|00\rangle_{14})+|\psi^-\rangle_{23}(bx\sqrt{p}|00\rangle_{14})];\end{aligned} \tag{14}$$

obviously, the fidelity of the composite system is

$$\begin{aligned} F_4 =& \frac{1}{4}[a^2x^2 + b^2y^2(1-p)] + [a^2x^2 + b^2y^2(1-p)] + [b^2x^2(1-p) + a^2y^2] + [b^2x^2(1-p) + a^2y^2] \\ =& \frac{1}{2}(1 - b^2p). \end{aligned} \tag{15}$$

4 Analysis

In the process of quantum teleportation, the fidelity of the composite system is $F_1 = 1 - p$, after the channel suffering from the noise named bit flip. So, we clearly known the fidelity F_1 decreases with the increase of the noise parameters p. If $p = 0$, means there is no noise, the value of fidelity is 1.



(a) Phase flip (b) Depolarizing (c) Amplitude damping

In the process of quantum teleportation, the fidelity of the composite system is $F_2 = 1 + (a^2 - \frac{3}{2})p$, after the channel suffering from the noise named phase flip. In this equation, we known $0 < b \leq a < 1$, and $a^2 + b^2 = 1$. So we get $\frac{\sqrt{2}}{2} \leq a < 1$. According to the figure (a), suppose a is constant, the fidelity F_2 decreases with the increase of the noise parameters p; suppose p is constant, the fidelity F_2 increases with the increase of a. However, if a close to 1, the entangled state will be unstable. similarly, if $p = 0$, the value of fidelity is 1.

In the process of quantum teleportation, the fidelity of the composite system is $F_3 = 1 - \frac{1}{8}(7 - a^2)p$, after the channel suffering from the noise named depolarizing. In this equation, we known $\frac{\sqrt{2}}{2} \leq a < 1$ and $0 \leq p \leq 1$. According to the figure (b), suppose a is constant, the fidelity F_3 decreases with the increase of the noise parameters p; suppose p is constant, the fidelity F_3 increases with the increase of a. Also a can not be close to 1, and if $p = 0$, the value of fidelity is 1.

In the process of quantum teleportation, the fidelity of the composite system is $F_4 = \frac{1}{2}(1 - b^2p)$, after the channel suffering from the noise named amplitude damping. In this equation, we known $0 < b \leq \frac{\sqrt{2}}{2}$ and $0 < p \leq 1$. In another case, if $p = 0$, the value of fidelity F_4 is 1. According to the figure (c), suppose b is constant, the fidelity F_4 decreases with the increase of the noise parameters p; suppose p is constant, the fidelity F_4 decreases with the increase of b. And b can not be close to 0.

5 Conclusion

We have calculated the fidelity of quantum state, and analysis of the effect of quantum noise on the quantum teleportation. It is useful for the practical application of the quantum teleportation. Next, we will study the situations that the quantum states suffer from the different noise at the same time.

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