About Approach of the Transactions Flow to Poisson One in Robot Control Systems

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Abstract. Flows of transactions in digital control systems of robots are investigated. On the base of the fact, that uses the conception of Poisson character of transactions flow permits to simplify analytical simulation of robot control process, the problem of estimation the degree of approach of real flow to Poisson one is putted on. Proposed the criterion based on evaluation of expectation of waiting function. On the example of investigation of "competition" in the swarm of robots it is shown that flow of transactions, generated by swarm, when quantity of robots aspire to infinity approximately aspire to Poisson one.

Keywords: Robot digital control system · Transaction · Poisson flow · Non-poisson flow · Regression · Correlation · Parametric criterion · Pearson's criterion · Waiting function

1 Introduction

Functioning of mobile robots may be considered as sequence of switching from one state of onboard equipment to another under control of commands flow [1, 2]. States of robot may include interpretation of the programs by onboard computer [3, 4], receiving/generating transactions, execution the commands by mechanical or electronic units, service a queue of command [5, 6], support the dialogue with remote operator [7] etc. Below, all switches and interconnections will be called transactions. Transactions occur in the physical time. Time intervals between transactions for the external observer are random values [8]. So transactions form a flow. One of variety of flows is the stationary Poisson flow, which possesses an important property – absence of aftereffect [9, 10]. Use such a property permits substantially to simplify the modeling behavior of robots. So when working out robot's digital control systems there is always emerges a question about a degree of approximation the real flow to Poisson flow. Methods of estimation of properties the flows of transactions are developed insufficiently, that explains the necessity and relevance of the work.

A. Ronzhin et al. (Eds.): ICR 2017, LNAI 10459, pp. 113–122, 2017.
DOI: 10.1007/978-3-319-66471-2_13

2 Quantitative Estimations of Approximation Degree of Transaction Flows to Poisson One

It is well known, that time intervals between transactions in Poisson flow are characterized with exponential distribution law [9–11]:

$$f(t) = \frac{1}{T}\exp\left(-\frac{t}{T}\right), \tag{1}$$

where T – is the expectation of time interval; t – is the time.

Regression criterion is based on estimation of standard-mean-square error as follows [12, 13]:

$$\varepsilon_r = \int_0^\infty [g(t) - f(t)]^2 dt, \tag{2}$$

where $g(t)$ – is the distribution under estimation.

Let $g(t) = \delta(t - T)$, where $\delta(t - T)$ – is Dirac δ-function. Then:

$$\varepsilon_r = \int_0^\infty [\delta(t - T) - f(t)]^2 dt = \varepsilon_{r1} + \varepsilon_{r2} + \varepsilon_{r3}, \tag{3}$$

where

$$\varepsilon_{r1} = \int_0^\infty \delta^2(t - T)dt = \lim_{a \to 0} \int_{T-a}^{T+a} \left(\tfrac{1}{2a}\right)^2 dt = \infty;$$
$$\varepsilon_{r2} = -2\int_0^\infty \delta\left(t - T_g\right) \cdot \tfrac{1}{T_f}\exp\left(-\tfrac{t}{T}\right)dt = -\tfrac{2}{eT};$$
$$\varepsilon_{r3} = \int_0^\infty \tfrac{1}{T^2}\exp\left(-\tfrac{2t}{T}\right)dt = \tfrac{1}{2T}.$$

Thus criterion ε_r changes from 0 (flow without aftereffect) till ∞ (flow with deterministic link between transactions), and it has the dimension as [time−1].

Correlation criterion is as follows [14]:

$$\varepsilon_c = \int_0^\infty g(t) \cdot \frac{1}{T}\exp\left(-\frac{t}{T}\right)dt. \tag{4}$$

This criterion changes from $\frac{1}{2T}$ (flow without aftereffect) till $\frac{1}{eT}$, where $e = 2,718$ (deterministic flow). With use the function:

$$\tilde{\varepsilon}_c = \frac{e(1 - 2T\varepsilon_c)}{e - 2}. \tag{5}$$

Criterion may be done the non-dimensional one, and it fits the interval $0 \leq \tilde{\varepsilon}_c \leq 1$. Parametrical criterion is based on the next property of exponential low (1) [9–11]:

$$T = \sqrt{D}, \tag{6}$$

where $D = \int_0^\infty \frac{(t-T)^2}{T} \exp\left(-\frac{t}{T}\right) dt$ – is the dispersion of the law (1).

To obtain from the property (6) non-dimensional criterion, fitting the interval $0 \leq \tilde{\varepsilon}_c \leq 1$, one should calculate the next function:

$$\varepsilon_p = \frac{\left(T_g - \sqrt{D_g}\right)^2}{T_g^2}, \tag{7}$$

where $T_g = \int_0^\infty t g(t) dt$ and $D_g = \int_0^\infty \left(t - T_g\right) g(t) dt$ – are expectation and dispersion of density $g(t)$.

In the case of experimental determining $g(t)$ as a histogram:

$$g(t) = \begin{pmatrix} t_0 \leq t < t_1 & \ldots & t_{i-1} \leq t < t_k & \ldots & t_{J-1} \leq t < t_J \\ n_1 & & n_i & & n_J \end{pmatrix}, \tag{8}$$

where n_i – is quantity of results fitting the interval $t_{i-1} \leq t < t_i$, then estimation of proximity of $f(t)$ and $g(t)$ one should use Pearson's criterion [15–17].

3 Criterion, Based on "Competition" Analysis

Let us consider transactions generation process as the "competition" of two subjects: external observer and transaction generator. Model of "competition" is the 2-parallel semi-Markov process, shown on the Fig. 1a.:

$$\mathbf{M} = [A, \mathbf{h}(t)], \tag{9}$$

where $A = \{a_{w1}, a_{w2}, a_{g1}, a_{g2}, \}$ – is the set of states; a_{w1}, a_{g1} – are the starting states; a_{w2}, a_{g2} – are the absorbing states; $\mathbf{h}(t)$ – is the semi-Markov matrix:

$$\mathbf{h}(t) = \begin{bmatrix} \begin{bmatrix} 0 & w(t) \\ 0 & 0 \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} 0 & g(t) \\ 0 & 0 \end{bmatrix} \end{bmatrix}; \quad \mathbf{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{10}$$

Let us consider the situation, when observer "wins" and at from the moment τ waits the event, when occurs the next transaction. For determine the waiting time let us construct on 2-parallel process ***M*** the ordinary semi-Markov process ***M′*** (Fig. 1b):

$$\mathbf{M}' = [\mathrm{A}', \ \mathbf{h}'(t)], \tag{11}$$

where $\mathrm{A}' = \mathrm{A} \cup \mathrm{B}$ – is the set of states; $\mathrm{A} = \{\alpha_1, \ \alpha_2, \ \alpha_3\}$ – is the subset of states, which simulate beginning and ending of wandering through semi-Markov process; α_1 – is the starting state; α_2 – is the absorbing state, which simulate "winning" of transaction generator; α_3 – is the absorbing state, which simulates end of waiting by the observer the event of generation transaction; $\mathrm{B} = \{\beta_1, \ \ldots, \ \beta_i, \ \ldots\}$ – is the infinite set of states, which define time intervals for various situation of finishing of transaction generator; $\mathbf{h}'(t) = \left\{h'_{m,n}(t)\right\}$ – semi-Markov matrix, which define time intervals.

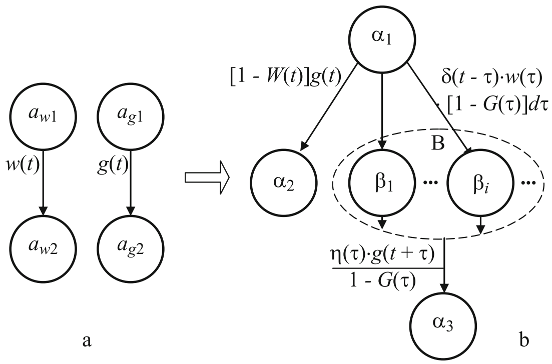

Fig. 1. To waiting time calculation

Elements $h'_{m,n}(t)$ are as follows: $h'_{m,n}(t)$ is the weighted density of time of finishing transaction generator if it "wins" the "competition":

$$h'_{12}(t) = g(t)[1 - W(t)], \tag{12}$$

where $\mathrm{W(t)} = \int_0^t \mathrm{w}(\theta)\mathrm{d}\theta$ – is the distribution function; θ – is an auxiliary variable;

$h'_{1,2+i}(t), \ i = 1, 2, \ldots,$ are defined as weighted densities of time of finishing of observer exactly at the time τ, if he "wins" the "competition" and waits the transaction:

$$h'_{1,2+i}(t) = \delta(t - \tau) \cdot w(\tau)[1 - G(\tau)]d\tau, \tag{13}$$

where $\delta(t - \tau)$ – Dirac function; $G(t) = \int_0^t g(\theta)d\theta; \ w(\tau)[1 - G(\tau)]d\tau$ – probability of finishing the observer exactly at the time τ, if he "wins" the "competition";

$\frac{\eta(t)\cdot g(t+\tau)}{1-G(\tau)}$ – is the density of time of residence semi-Markov process $\mathbf{h}'(t)$ in state B, where η(*t*) – is the Heaviside function.

Thus, probability of hitting the process $\mathbf{h}'(t)$ to the state B is as follows $p_{\alpha_0\beta} = \int\limits_0^\infty [1-G(\tau)]w(\tau)d\tau = \int\limits_0^\infty W(t)g(t)dt$. Weighted density of time of waiting by observer the next transaction is equal to $h_{w\to g}(t) = \eta(t)\int\limits_0^\infty w(\tau)g(t+\tau)d\tau$. Pure density is as follows:

$$f_{w\to g}(t) = \frac{\eta(t)\int\limits_0^\infty w(\tau)g(t+\tau)d\tau}{\int\limits_0^\infty W(t)dG(t)}. \tag{14}$$

Let us consider function $f_{w\to g}(t)$ behavior when $g(t) = \frac{1}{T_g}\exp\left(-\frac{t}{T_g}\right)$ (flow of transactions without aftereffect) and when $g(t) = \delta\left(t - T_g\right)$ (flow with deterministic link between transactions).

Formula (14) for the first case is as follows:

$$f_{w\to g}(t) = \frac{\eta(t)\int\limits_0^\infty w(\tau)\frac{1}{T_g}\exp\left[-\frac{t+\tau}{T_g}\right]d\tau}{1-\int\limits_{t=0}^\infty\left[1-\exp\left(-\frac{t}{T_g}\right)\right]dW(t)} = \frac{1}{T_g}\exp\left(-\frac{t}{T_g}\right). \tag{15}$$

Formula (14) for the second case is as follows:

$$f_{w\to g}(t) = \frac{\eta(t)w\left(T_g - t\right)}{W\left(T_g\right)}. \tag{16}$$

Suppose that $w(t)$ have range of definition $T_{w\min} \le arg\, w(t) \le T_{w\max}$ and expectation $T_{w\min} \le T_w \le T_{w\max}$. In dependence of location $w(t)$ and $g(t)$ onto time axis, it is possible next cases:

(a) $T_g < T_{w\min}$. In this case (14) is senseless.
(b) $T_{w\min} \le T_g \le T_{w\max}$. In this case $f_{w\to g}(t)$ is defined as (16), range of definition is $0 \le \arg\left[f_{w\to g}(t)\right] \le T_g - T_{w\min}$, and $\int\limits_0^\infty t f_{w\to g}(t)dt \le T_g$.
(c) $T_g > T_{w\max}$. In this case $f_{w\to g}(t) = w(T_g - t)$, $T_g - T_{w\max} \le \arg\left[f_{w\to g}(t)\right] \le T_g - T_{w\min}$, and $\int\limits_0^\infty t f_{w\to g}(t)dt \le T_g$.

For this case density of waiting time by Dirac function when transaction occurs is as follows:

$$f_{\delta\to g}(t) = \frac{\eta(t)\cdot g\left(t+T_g\right)}{\int\limits_T^\infty g(t)dt}. \qquad (17)$$

Expectation of $f_{\delta\to g}(t)$ is as follows:

$$T_{\delta\to g} = \int\limits_0^\infty t\frac{g\left(t+T_g\right)}{\int\limits_T^\infty g(t)dt}dt. \qquad (18)$$

So, the criterion, based on "competition" analysis, is as follows:

$$\varepsilon_w = \left(\frac{T_g - T_{\delta\to g}}{T_g}\right)^2, \qquad (19)$$

where T_g – is the expectation of density $g(t) = \delta\left(t - T_g\right)$; $T_{\delta\to g}$ – is the expectation of density $f_{\delta\to g}(t)$. For the exponential law:

$$\varepsilon_w = \left(\frac{T - T_{\delta\to g}}{T}\right)^2 = \left(\frac{T-T}{T}\right)^2 = 0. \qquad (20)$$

Let us investigate behavior of $T_{\delta\to g}$. For that let us expectation of $g(t)$ as (Fig. 2):

$$\begin{aligned}\int\limits_0^\infty tg(t)dt &= \int\limits_0^T tg(t)dt + \int\limits_0^\infty tg\left(t+T_g\right)dt + T_g\int\limits_0^\infty g\left(t+T_g\right)dt \\ &= p_{1g}T_{1g} + p_{2g}T_{\delta\to g} + p_{2g}T_g = T_g,\end{aligned} \qquad (21)$$

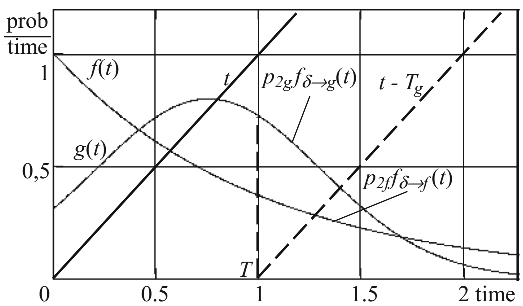

Fig. 2. To calculate of expectation

where $p_{1g} = \int_0^T g(t)dt; \quad p_{2g} = \int_T^\infty g(t)dt.$

If $g(t) = f(t)$, then $p_{1f} = \frac{e-1}{e}$; $p_{2f} = \frac{1}{e}$; $T_{1f} = T\frac{e-2}{e-1}$ and from equation:

$$p_{1f}T_{1f} + p_{2f}T_{\delta\to f} + p_{2f}T = T, \tag{22}$$

follows that:

$$T_{\delta\to f} = T. \tag{23}$$

If $g(t) \neq f(t)$ then from (21) follows that:

$$T_{\delta\to g} = \frac{p_{1g}\left(T - T_{1g}\right)}{1 - p_{1g}}. \tag{24}$$

4 Example

As an example let us consider the case, when transaction flow is formed as a result of competition inside the swarm of robots, when common quantity of robots is equal to K. Time from the start of observation and transaction, formed by k-th robot, $1 \le k \le K$, is defined with identical uniform lows. Forming the flow of transactions may be simulate with K-parallel semi-Markov process:

$$\mathbf{M}^K = \left[\mathrm{A}^K, \ \mathbf{h}^K(t)\right], \tag{25}$$

where $A^K = \{a_{11}, \ldots, a_{k1}, \ldots, a_{K1}, a_{12}, \ldots, a_{k2}, \ldots, a_{K2}\}$ – is the set of states; $a_{11}, \ldots, a_{k1}, \ldots, a_{K1}$ – is the subset of starting states; $a_{12}, \ldots, a_{k2}, \ldots, a_{K2}$ – is the subset of absorbing states; $\mathbf{h}^K(t)$ – semi-Markov matrix, *kk*-th elements of the main diagonal of which is equal to $\begin{bmatrix} 0 & v_k(t) \\ 0 & 0 \end{bmatrix}$, and other elements are equal to zeros (Fig. 3):

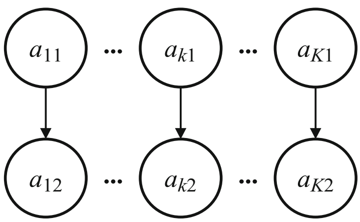

Fig. 3. Forming a flow of transactions in swarm of *K* robots

$$v_1(t) = \ldots = v_k(t) = \ldots v_K(t) = v(t) = \begin{cases} 1, \ when \ 0 \le t \le 1; \\ 0 \ in \ all \ other \ cases. \end{cases} \qquad (26)$$

K-parallel process starts from all states of subset $a_{11}, \ldots, a_{k1}, \ldots, a_{K1}$ contemporaneously. Transaction is generated when one of ordinary semi-Markov processes gets the state from the subset $a_{12}, \ldots, a_{k2}, \ldots, a_{K2}$. In accordance with theorem by Grigelionis [18] when $K \to \infty$ flow of transactions approaches to Poisson one.

Density of time when at least one of robot of swarm generates transaction is as follows:

$$g_K(t) = \frac{d\left\{1 - [1 - V(t)]^K\right\}}{dt}, \qquad (27)$$

where:

$$V(t) = \int_0^t v(\tau) d\tau = \begin{cases} 2t, \ when \ 0 \le t \le 1; \\ 0 \ in \ all \ other \ cases. \end{cases}$$

For this case:

$$g_K = \begin{cases} K(1-t)^{K-1}, \ when \ 0 \le t \le 1; \\ 0 \ in \ all \ other \ cases. \end{cases} \qquad (28)$$

Expectation of (27) is as follows:

$$T_K = \int_0^1 tK(1-t)^{K-1} dt = \frac{1}{K+1} [time]. \qquad (29)$$

Exponential law, which define Poisson flow of transactions is as follows:

$$f_K(t) = (K+1) \exp[-(K+1)t] \left[\frac{prob}{time}\right]. \qquad (30)$$

For waiting function:

$$\tilde{T}_K = \frac{K}{(K+1)^2} [time], \qquad (31)$$

$$\lim_{K \to \infty} \varepsilon_{lg}^K = \lim_{K \to \infty} \left(\frac{T_K - \tilde{T}_K}{T_K}\right)^2 = \lim_{K \to \infty} \frac{1}{(K+1)^2} = 0. \qquad (32)$$

I.e. with increasing K law approaches to exponential, in accordance with B. Grigelionis theorem. Densities are shown on the Fig. 4. Already at K = 0,012 what can be considered as a good approximation.

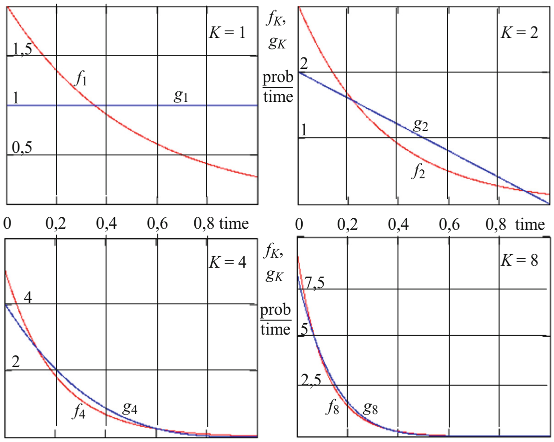

Fig. 4. Densities of time between transactions from swarm of robots

5 Conclusion

In such a way criteria with which may be estimated degree of approach of transaction flow in robotic system to Poisson flow were investigated. Criterion, based on calculation of the waiting time, is proposed, as one, which gives good representation about properties of flow of transactions, and have respectively low runtime when estimation.

Further investigation in the domain may be directed to description of method of calculations when processing statistics of time interval in a flow. Other research may concern the errors, to which leads the modeling of robot with non-Poisson character of transactions flows, when Poisson flows conception was laid into the base of simulation.

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