# Chapter 16
# Design of Statistics Learning Environments

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**Abstract** The goal of this chapter is to draw attention to the need to think about learning environments and their design as a way of considering how sustainable change in the learning and teaching of statistics can be supported. The goal is not to advocate one particular approach to the design of learning environments but rather to raise awareness of the need to consider this lens in statistics education. We first present the rationale for the importance of a focus on learning environments for statistics education. We provide several examples of learning environments that operationalize and integrate various design perspectives and are informed by two theoretical frameworks: social constructivist theory of learning and realistic mathematics education theory. We discuss these examples in a critical way by comparing and evaluating their designs, looking for common threads among them, and develop from them six design considerations for statistics learning environments. Finally, we discuss implications and emerging directions and goals for further implementation and research.

**Keywords** Learning environment • Statistics education • Learning and teaching of statistics

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## 16.1 Introduction

Many of the research studies in the learning and teaching of statistics (reviewed by Garfield & Ben-Zvi, 2007; see also chapters in part II of this handbook) suggest innovative approaches that differ from the traditional classroom practices through which most current statistics teachers learned this subject themselves. However, innovation which addresses only one aspect of the pedagogical context, for example, introducing technological tools in teaching when assessment practices remain unchanged, is likely to have only limited impact. This chapter offers starting points of theory and design for deep learning (Sawyer, 2014) of statistics to develop students' statistical reasoning. To do this, we use a *learning environment* perspective to provide a dynamic, holistic, integrated, and multidimensional framework for sustainable educational change in statistics.

The goal of this chapter is to draw attention to the need to think about learning environments and their design as a way of considering how sustainable change in the learning and teaching of statistics can be supported. We provide several examples of learning environments that operationalize and integrate various design perspectives (e.g., Hickey, Kindfield, Horwitz, & Christie, 2003) and are informed by various theoretical frameworks (social constructivist theory of learning and realistic mathematics education theory). We discuss these examples in a critical way by comparing and evaluating their designs, looking for common threads among them. We emphasize that the goal of this chapter is not to advocate one particular approach to the design of learning environments, but rather to raise awareness of the need to consider this lens in statistics education.

A learning environment perspective can guide statistics educators and researchers to view, design, and assess statistics teaching and learning in designed settings, such as classrooms and online courses, as a holistic entity. It can support the intentional transformation of an educational setting based upon conjectures about how the integration of features of the designed setting will support the learning of statistics. Such an entity is a complex and dynamic educational system, composed of multiple factors: key statistical ideas and competencies (content), engaging tasks, real or realistic data sets, technological tools, classroom culture including modes of discourse and argumentation among students and between students and teachers, norms and emotional aspects of engagement, and assessment methods. Integrating all these factors in order to reform the way statistics is learnt and taught is a challenging endeavor. In addition, the broader community (school-level policy makers, local and national authorities, etc.) plays a significant role in the constitution of the learning environment. For example, tensions may arise between required traditional examinations and alternative assessment methods employed in a learning environment or between national curricula and an emergent and dynamic learning trajectory in the learning environment.

New developments in mathematics, statistics, and science education, and more generally in the learning sciences, provide important ideas and practical implications about the design of learning environments (e.g., Bielaczyc, 2006; Collins, 1999; De Corte, Verschaffel, Entwistle, & van Merrienboer, 2003; Vosniadou, Ioannides, Dimitrakopoulou, & Papademetriou, 2001; Vosniadou & Vamvakoussi, 2006). These developments highlight the value of rethinking what is taught, how it is taught, and how it is assessed (e.g., Bransford, Brown, & Cocking, 2000). The focus in this chapter is on general characteristics of statistics learning environments that need to be examined and integrated in light of these new developments. Our specific objectives are to first present the rationale for the importance of a focus on learning environments for statistics education; we provide a potential framework for considering aspects of statistics learning environments building on social constructivist background theory and the domain-specific theory of realistic mathematics education (RME). We then present three examples of statistics learning environments used in diverse contexts (primary school, lower secondary school, and tertiary education) and develop from them six design considerations for statistics learning environments. Finally we discuss implications and emerging directions and goals for further implementation and research.

## 16.2 Learning Environments

Reform in statistics education is required and has been sought and evaluated in recent decades (see Chap. 2 of this volume; Cobb, 1992, 1993; Everson, Zieffler, & Garfield, 2008; Garfield, Hogg, Schau, & Whittinghill, 2002; Moore, 1998; Reston & Bersales, 2008). The core idea that underpins this reform is that learning statistics is not about passively acquiring a set of facts and procedures but rather about actively constructing meanings and understandings of big ideas, ways of reasoning, and articulating arguments, dispositions, and perspectives. Unidimensional changes, such as the redesign of particular tasks or aspects of the curriculum, are not sufficient to make extended and sustainable change (e.g., Cuban, 2003; Darling-Hammond, 1997; Kohn, 1999). We recognize that even comprehensive efforts to change several aspects of teaching and learning statistics are not necessarily a key to success (Savelsbergh et al., 2016).

The research literature in statistics education is filled with success stories, which are of importance to the advancement of the field but have not had a major impact on the way statistics is taught in all levels of education. We propose that one of the reasons for this is the lack of a systematic, comprehensive, and integrated approach to design for educational change. We suggest that what is needed is change in a combination of interrelated dimensions (content, pedagogy, space, time, tasks, tools, assessment, classroom culture, etc.) that can bring about significant and sustainable reform in the teaching and learning of statistics by providing a coherent framework in which each dimension operates synergistically with others. Moore (1997) similarly urged a reform of statistics instruction and curriculum based on strong synergies between content, pedagogy, and technology. A learning environment perspective can provide such a framework. One of the major goals of statistics education is to educate critical, independent, and statistically literate learners who are able to study topics of their own interest and become involved in data-based decisions. A learning environment perspective can provide fertile affordances to support learners' growth and development in this direction.

Design dimensions of statistics learning environments that will be considered and discussed in this chapter are based on a number of principles arising from recent research. In particular, we have drawn on research concerning the importance of prior knowledge and preference for depth over breadth (Bransford et al., 2000), the creation of failure-safe learning communities in which students can learn from their successes and mistakes (Bielaczyc & Collins, 1999; Kapur & Bielaczyc, 2012), the nurture and articulation of learners' diverse expertise, encouragement of reflection and feedback (Boud, Keogh, & Walker, 1985), formative assessment (Clark, 2012; Kingston & Nash, 2011), and enculturation into the statistics discipline (Edelson & Reiser, 2006).

Teacher education in statistics is not just about improving teachers' subject knowledge but also about challenging their thinking about the whole process of statistical inquiry as central to statistical thinking and learning (see Chaps. 10 and 14 of this volume; Pfannkuch & Ben-Zvi, 2011). A learning environment perspective can provide a guiding framework for teachers that can support their professional growth in statistics education.

While any setting in which learning takes place can be viewed from a learning environment perspective, we focus now on statistics teaching and learning that occur in the context of designed[1] learning environments[2] (mostly in classrooms and online settings, but sometimes also at home or in the workplace). The use of the metaphor of an *environment* emphasizes that classrooms are interacting social, cultural, physical, psychological, and pedagogical systems rather than a collection of resources, tasks, and activities or a list of separate factors that influence learning. Because of the complex nature of learning environments, successful design requires a working model of how components of the design that help frame forms of student participation and responsibility are collectively constituted and orchestrated (Lehrer, 2009).

To achieve this kind of balance and orchestration, we argue that learning environments must be designed on the basis of learning theories, which can guide the design, help choose between the options, and lead to better achievement of the pedagogical goals. In the next section, we describe two theoretical frameworks that have been commonly used to guide the construction of learning environments.

[1] Learning occurs in a wide continuum of settings from the "designed" to the "ambient" (Kali, Tabak, Ben-Zvi, et al., 2015). On this continuum, this chapter focuses on designed learning environments rather than informal and ambient ways of learning.

[2] Others use the term *learning ecology* instead of learning environment to emphasize that the educational system is always dynamic and emerging rather than a static entity (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Lehrer & Pfaff, 2011).

## 16.3 Theories that Can Guide the Design of Learning Environments

The roles of theory in design research and in design of learning environments are complex and dynamic (Jonassen & Land, 2012). These vary in levels of generality. From the most general level to the most specific, these include (1) orienting frameworks or background theories; (2) domain-specific instruction theories as frameworks for action; and (3) local instruction theories/humble theories/hypothetical learning trajectories (Prediger, Gravemeijer, & Confrey, 2015). Theories do not provide straightforward recipes for designing effective learning environments. However, they (1) provide a rationale and motivation to use a learning environment approach rather than merely focusing on content, tasks, or what the teachers are doing and (2) can provide considerations, guidelines, and constraints to the practical task of learning environment design (see more on the nature and use of theories in statistics education in Chap. 11 of this volume).

We take social constructivist theory, which is a well-accepted theory in the education community, as our background theory on teaching and learning. This theory requires instructional designers to think through how students construct new knowledge and how the classroom community might interactively constitute instructional tasks. In addition to this general educational theory, one needs a theory that is specific for mathematics education. For reasons we explain below, we choose RME as our domain-specific instruction theory.

### *16.3.1 Social Constructivist Theory*

According to constructivist theory, people learn by *actively constructing knowledge*, rather than by passively receiving knowledge: new knowledge and understandings are based on the *existing knowledge* and *beliefs* one already has and are grounded in our experiences, understandings, and cultural practices (e.g., Cobb, 1994b; Piaget, 1978; Vygotsky, 1978). The thesis that students construct their own knowledge leads to the following questions (Cobb, 1994a): "What do we as educators/instructional designers want the students to construct?" or "What do we want mathematics/statistics to be for them?" and "How do we create a situation in which students construct what we want them to construct?"

When trying to answer the last question, a learning environment perspective suggests that it is not sufficient to design instructional tasks or instructional activities; rather the whole learning environment needs to be considered. Drawing on social constructivism, we may argue that what counts in the learning environment are not just the tasks as such, but the tasks as they are interactively constituted in the classroom. How the tasks are construed depends largely on the classroom social norms, the forms of interaction, and the pedagogical agenda of the teacher. Those in turn are closely related to the learning goals, including a wider goal for how the students understand the nature of statistics.

From our perspective, prioritizing the investigative nature of statistics (see Chap. 4 of this volume), the latter requires specific *classroom social norms*, expecting students to come up with their own questions and solutions and to explain and justify their thinking. Further, it requires appropriate *socio-statistical norms* or beliefs about what it means to do statistics, which concern ways of reasoning and articulation, dispositions, and perspectives. Thus, a social constructivist belief that students construct their own knowledge and our beliefs about what statistics should entail for the students both create the need to think in terms of learning environments and define how we want to shape those environments. (See the Sect. 16.3.2 for an example of a learning environment that uses these social constructivist theory tenets.)

Social constructivism further determines how one thinks about *symbolic representations*. The key here is the distinction that can be made between *inscriptions*—such as marks on paper—and what these inscriptions signify. From a social constructivist point of view, what inscriptions signify is determined by the social practice in which they are used. For example, circles on paper may signify countable objects for students who are participating in a counting activity, while similar circles may signify characters for students participating in a natural language lesson. Thus, from social constructivist stance, establishing social practices in which such inscriptions are produced and used will be a central issue in the design of learning environments.

Several social constructivist theoretical frameworks have been developed to describe learning as *active participation in a community*. Communities of practice (Wenger, 1998), communities of learners (Rogoff, 1994), and knowledge-building communities (Scardamalia & Bereiter, 2014) are three frameworks that have had considerable influence on educational research and practice. Though they may have some nuanced distinctions, they share three fundamental tenets: *activity*, *participation*, and *enculturation*. The active nature of learning is embodied in students' participation in negotiating meanings, developing understanding, evaluating, and orchestrating their own learning in collaborative environments, all with the guidance of an expert teacher (Barron et al., 1998; Ben-Zvi, 2007; Brown & Campione, 1994; Sfard & Cobb, 2014). These forms of participation are, in turn, viewed as processes of enculturation: students assume increasingly central roles in the classroom community and immerse themselves within a culture of learning through which they acquire competence in language, social practices, rituals, and values of the discipline (Barry, 2007). For the classroom community to function effectively, the students and the teacher must negotiate and agree upon standard values and norms that guide and constrain social behavior (Cialdini & Trost, 1998; Hod & Ben-Zvi, 2015). The participation in a classroom community yields not only valued and shared products but contributes to the ongoing development and growth of all members, as they take up and build on each other's knowledge and actions (Rogoff, Turkanis, & Bartlett, 2001).

The implication of a social constructivist stance is that good pedagogical practice consists of designing learning environments that stimulate students to construct knowledge in learning communities. *Statistical inquiry* is one such approach that provides students with many opportunities to participate, think, reason, and reflect on their learning, as well as to discuss and reflect with their peers. A social constructivist perspective on inquiry does not mean that teachers should never tell students anything directly. Rather it means that learning is enhanced when teachers recognize that "teacher telling" does not automatically result in "student knowing" and pay attention to ways in which learners construct knowledge. Monitoring students' changing conceptions as instruction proceeds can provide insights as a starting point for new instruction.

Research does not provide a recipe for designing effective learning environments, but it does support the value of asking certain kind of questions about the design of learning environments and shows their value and success in certain contexts. We argue that the main reason to adopt a learning environment approach is that it appears to be more effective in helping students build a deeper understanding of statistics (e.g., Baeten, Kyndt, Struyven, & Dochy, 2010; Bransford et al., 2000; Cognition and Technology Group at Vanderbilt, 1998; Sawyer, 2014; Sfard & Cobb, 2014).

### *16.3.2 Realistic Mathematics Education (RME) Theory*

According to social constructivism, everybody constructs his or her own knowledge. This puts teachers—and by extension instructional designers—in a difficult position. For how can you ensure that students construct what you want them to construct? Simon (1995) answered this question by proposing a *hypothetical learning trajectory* (HLT): try to anticipate the mental activities of the students when they engage in the instructional tasks under consideration, and relate those mental activities to the learning goal. By developing, enacting, analyzing, and revising HLTs, the teacher can guide the learning process of the students. Teachers can be supported in designing HLTs by being offered prototypical instructional sequences and the local instruction theories underpinning them. These can provide teachers with frameworks of reference for deciding what mental activities to aim for and choosing instructional tasks that may foster these mental activities.

The domain-specific instruction theory for RME offers a theoretical basis for the design of such local instruction theories. The founding father of RME, Freudenthal (1973), argued that students should experience mathematics as a human activity, similar to the activity of mathematicians. While engaging in mathematical activity, they could *reinvent* mathematics (or statistics) with the help of teachers and tasks. In relation to this, he speaks of *guided reinvention*, which, we may argue, is compatible with constructivism, as both Freudenthal and constructivists have in mind students who construct their own mathematics. Over time, those starting points were elaborated in a domain-specific instruction theory, initially formulated by Treffers (1987) and later worked out in the form of instructional design heuristics by Gravemeijer (2004). Those instructional design heuristics encompass *guided reinvention*, *didactical phenomenology*, and *emergent modeling*.

The *guided reinvention* design heuristic asks the designer to develop a potential reinvention route, of which the starting point should be experientially real to the students in that the students would know how to act and reason sensibly in those situations. Freudenthal (1973) pointed out that the designers could look at the history of mathematics/statistics as a source of inspiration (see, for instance, Bakker & Gravemeijer, 2006 who reviewed the historical phenomenology of mean and median). History could tell them which dead ends to avoid and how breakthroughs were achieved. Streefland (1991) added to this the idea of looking at students' informal solution strategies. Students may invent informal solution strategies that show the germs of the applicable mathematics, which could be used as starting points for a reinvention process. Building on Treffers (1987) and van Hiele (1986), we may argue that the learning goals should be framed in terms of networks of mathematical relations. In relation to this, we introduce the notion of *reification*, in which processes obtain an object-like character (Sfard & Linchevski, 1994). The conception of a distribution may, for instance, evolve from the process of organizing measurements within a space of possible outcomes to conceptualizing a distribution as an object with certain characteristics such as shape, center, spread, and skewedness (Bakker & Gravemeijer, 2004).

The *didactical phenomenology* heuristic, also originated by Freudenthal (1983), argues that mathematical thought things such as concepts, rules, and procedures were invented to organize certain phenomena. As examples of a thought thing, we may think of the conception of "the mean," summarizing a set of data in one number, or the conception of a distribution as a more sophisticated way of grasping a data set. The procedure of calculating the mean would offer an example of a procedure type of thought thing. Designers are advised to investigate how the mathematical thought things they are aiming for organize phenomena in applied situations. According to Freudenthal (1983), they can then use that information to create situations in which the need arises to organize phenomena by the very thought things that are to be invented. Related to this, the advice is to explore the variety of situations in which the thought thing is applied in order to create a broad phenomenological base.

The *emergent modeling* design heuristic refers to the roles that models and modeling can play in supporting the reinvention process. Of key importance here is the notion that symbolic representations do not come with an inherent meaning. In relation to this, Bereiter (1985) framed the learning paradox: to come to understand a new piece of mathematics, you have to understand the symbolic representations that derive their meaning from the very piece of mathematics you want to come to understand. The emergent modeling heuristic aims at circumventing this learning paradox by fostering a learning process in which symbolizations and meaning coevolve. Initially, the models come to the fore as context-specific models, referring to situations that are experientially real for the students. Initial models have to allow for informal solution strategies at the level of the contextual problem. Then, while the students gather more experience with similar problems, the teacher will support them in shifting their attention toward the mathematical relations and strategies. This will help them to further develop those mathematical relations, which enables them to see the model in a different light; the model starts to derive its meaning from the emerging network of mathematical relations and starts to become a base for more formal mathematical reasoning. Thus, a *model of* informal mathematical activity develops into a *model for* more formal mathematical reasoning, together with the development of a network of mathematical relations, which the students may experience as an expansion of their mathematical reality.[3]

Taken together, social constructivism and RME can provide a conceptual foundation to guide the design of learning environments. Although not the only relevant theories, they provide one example of how theory and practice are strongly linked and can enrich each other. We turn now to describe three examples of statistics learning environments which were designed based on one of these theories, each targeting a different age level: primary school, lower secondary school, and tertiary level.

## 16.4 Examples of Statistics Learning Environments

### *16.4.1 Example I: The Connections Learning Environment (Primary School)*

The connections learning environment is built upon the principles of social constructivist theory (Sect. 16.3.1 above) and is designed for young learners (ages 10–12). It is a design and research project which started in 2005 (Ben-Zvi, Gil, & Apel, 2007) to develop students' statistical reasoning in an inquiry and technology-enhanced learning environment in primary schools in Israel.

The project extends for 5 weeks (6 h per week) each year in grades 4–6 during which students actively experience some of the processes involved in data-based inquiry, mirroring the practice of statistics experts. Students conduct data and statistical modeling investigations through peer collaboration and classroom discussions using TinkerPlots (version 2; Konold & Miller, 2011), a computer tool for dynamic data and modeling explorations. By playing a role in helping students learn new ways of representing data and develop statistical reasoning, TinkerPlots gradually becomes a thinking tool for these students; it scaffolds their ongoing negotiations with data, statistical ideas, inferences and their meanings (Ben-Zvi & Ben-Arush, 2014).

The tasks undertaken by connections students are a series of open-ended real data investigations that provide students with rich and motivating experiences in posing statistical questions; collecting, representing, analyzing, and modeling data; and formulating inferences in authentic contexts, which result in meaningful use of statistical concepts (Ben-Zvi, Aridor, Makar, & Bakker, 2012; Makar & Ben-Zvi, 2011). The data are obtained from a questionnaire designed by the research team, teachers and students, and administered by students in their school. The connections classroom is conceptualized and organized as a learning community (Bielaczyc & Collins, 1999) that supports collaboration, argumentation, sharing, and reflection.

[3] In practice, "the model" in the emergent modeling heuristic is actually shaped as a series of consecutive sub-models that can be described as a cascade of inscriptions or a chain of signification.

This is done physically in the class and virtually in a website that includes all educational materials and supports, students' reflective diaries, and peer and teacher feedback.

Alternative methods of assessment (Garfield & Ben-Zvi, 2008, pp. 65-89) are used as an integral part of the connections learning environment. These assessment activities, including student projects, oral presentations, and peer and teacher feedback, are viewed as an important component of the learning processes rather than only as a means for "testing" of students' outcomes. For teachers, they provide opportunities to gain insights into students' developing constructions of meaning and so are a crucial part of the planning and design process. Students are usually highly motivated to present and discuss their work in short presentations during the project and at the statistical happening, a final festive event with their parents.

In the connections learning environment, rather than first teaching students directly about statistical concepts and then asking them to apply them in investigations, the investigations themselves are designed to raise the need to attend to these concepts, hence deepening students' understanding of both their relevance and application. Additional strategies are used in the design of the educational materials such as growing samples (Bakker, 2004; Ben-Zvi, 2006; Konold & Pollatsek, 2002), which is a pedagogical heuristic in which students are gradually introduced to increasing sample sizes that are taken from the same population. For each sample, they are asked to make an informal inference (Chap. 8 of this volume) and then predict what would remain the same and what would change in the following larger sample. Thus, students are required to reason with stable features of distributions or variable processes and compare their hypotheses regarding larger samples with their observations in the data. They are also encouraged to think about how certain they are about their inferences. Beginning with small samples, students are expected to experience the limitations of what they can infer about this current sample. This is a useful pedagogical tool to sensitize and slowly introduce students to the decreasing variability of apparent signals in samples of increasing sizes.

Ben-Zvi (2006) found that the growing sample heuristic combined with "what if" questions not only helped connections students make sense of the data at hand but also supported their informal inferential reasoning by observing aggregate features of distributions, identifying signals out of noise, accounting for the constraints of their inferences, and providing persuasive data-based arguments. The growing awareness of students to uncertainty and variation in data enabled students to gain a sense of the middle ground of "knowing something" about the population with some level of uncertainty and helped them develop a language to talk about the gray areas of this middle ground (Makar, Bakker, & Ben-Zvi, 2011).

The growing sample heuristic does not stand alone but is part of the broader connections learning environment. For example, the students have a deep grasp of the sample data since they have collected them, and the technological tool allows them to present the growing samples easily and creatively in a supportive classroom culture.

The connections project was based initially on an exploratory data analysis (EDA) pedagogic approach (Shaughnessy, Garfield, & Greer, 1996). Students drew informal inferences from real samples following the statistical inquiry cycle. To foster students' appreciation of the power of their inferences, a model-based perspective has recently been added in which students build a model (a probability distribution) for an explored (hypothetical) population and produce data of generated random samples from their model using TinkerPlots. By analyzing generated random samples and comparing them with the suggested model, students learn about the relationships between samples and populations as well as about sampling variability and representativeness (Manor & Ben-Zvi, 2017).

Connections students gain considerable fluency in techniques common in exploratory data analysis: the use of statistical concepts, statistical habits of mind, inquiry-based reasoning skills, norms and habits of inquiry, and TinkerPlots as a tool to extend their reasoning about data (e.g., Ben-Zvi et al., 2012; Gil & Ben-Zvi, 2011). In a longitudinal mixed method study (Gil & Ben-Zvi, 2014), evidence of long-term impact of teaching and learning was sought among ninth graders, 3 years after their participation in the 3-year connections intervention. Students from two groups, who had/had not taken part in the program, were compared throughout three extended open-ended data inquiry tasks and took a statistical reasoning test. Connections students had significant gains in terms of their conceptual understanding of aggregate view of a distribution and informal statistical inference. They used statistical concepts in a more meaningful, integrated, and accurate manner in their explanations, were more fluent considering the uncertainty involved in generalizations from random samples, and supported their inferences with data-based evidence.

In sum, connections students learn by actively constructing knowledge of key statistical ideas and competencies; enjoy open, extended, and engaging tasks; investigate real data sets with sophisticated technology; and are assessed with alternative methods. The combination of these activities and entities, coupled with supportive and caring classroom learning culture, creates a learning environment that nurtures students' deep statistical learning (e.g., Aridor & Ben-Zvi, 2017; Manor & Ben-Zvi, 2017).

### *16.4.2 Example II: The Nashville Data Analysis Project (Lower Secondary School)*

In the late 1990s, two extended data analysis teaching experiments were carried out in a 7th and an 8th grade classroom. These design experiments (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) were part of a 10-year collaboration of Cobb, Gravemeijer, Yackel, and others, in which RME theory was elaborated while adopting the collectivist perspective on teaching and learning that is implied by a social constructivist view (Cobb, Gravemeijer, & Yackel, 2011). Similar to Freudenthal's (1973) adage of *mathematics as an activity*, the starting point for the design was that students would have to experience learning about data analysis by doing data analysis. This of course required a matching classroom culture in which the teacher and students could function as a community of practice/learning community. The structure of the lessons was tailored to this idea, which started with a whole-class discussion in which a problem or issue was explored, followed by small groups solving the problem with the help of a computer tool, and concluded with presentations and discussions.

This approach was worked out in the following manner (see also Gravemeijer & Cobb, 2013). The tasks were designed on the basis that the students would be doing data analysis "for a reason": to solve a problem or to answer question, preferably concerning a topic that the students considered relevant. To foster an effective reinvention process, a shift was made during the first teaching experiment from solving problems to considering and improving ways of data analysis and visualization, denoted as *cultivating statistical interest*. To achieve this, the students were given the role of data analysts working in the service of people who had to make decisions. The tasks usually involved comparing two data sets. Faithful to the idea of data analyses for a reason, the students were involved in the process of data creation. However, as assembling data was not feasible in most cases, this took the shape of *talking through the process of data creation*. In this manner, the researchers also tried to ensure that the starting points would become experientially real for the students.

Following the emergent modeling design heuristic, the researchers tried to provide for a process in which the ways of symbolizing/visualizing and the development of meaning coevolved. The backbone of the instructional sequence was formed by a series of visual representations that functioned as sub-models in an emergent modeling process (Fig. 16.1). We will briefly describe how this series of submodels evolves.

The starting point is the supposition that 7th grade students will be familiar with representing individual measurement values as lengths. When comparing data sets, the focus of the students will be on the end points of the individual value bars and the corresponding positions on the *x*-axis (Fig. 16.1a). So the bars can be left out, while the end points descend to the horizontal axis, resulting in a dot plot (Fig. 16.1b). Analyzing and comparing distributions represented by dot plots, students may start to reason about the shape of the distribution (Fig. 16.1b). In doing so, the vertical axis will come to signify the density of data points around a given *x*-value. While structuring data sets in various ways, structuring data in four equal groups may come to the fore as one of the powerful ways of structuring data (Fig. 16.1c, d). Here students may start using the partitioning in halves and quarters as means for comparing data sets while also starting to get a handle on distributions by realizing that the data density is the highest where the distance between the vertical bars is the smallest. Then the students may start to use four equal groups or boxplots as means to reason about distributions (Fig. 16.1e, f). Ideally the boxplots will come to signify the shape of the distribution for the students, thanks to the history of its emergence. In the process, distributions are expected to acquire an object-like quality for the students, objects with characteristics such as shape, spread, and skewness—which can be further defined with median, quartiles, and extreme values.

Building on this model, bivariate data sets may be sliced into a series of univariate distributions that can be represented as a series of (vertical) boxplots. Thinking of hill-type shapes corresponding with these boxplots, a ridge can be imagined running across the data set. This ridge may be interpreted in terms of a conjectured relationship of covariation between the two variables involved.

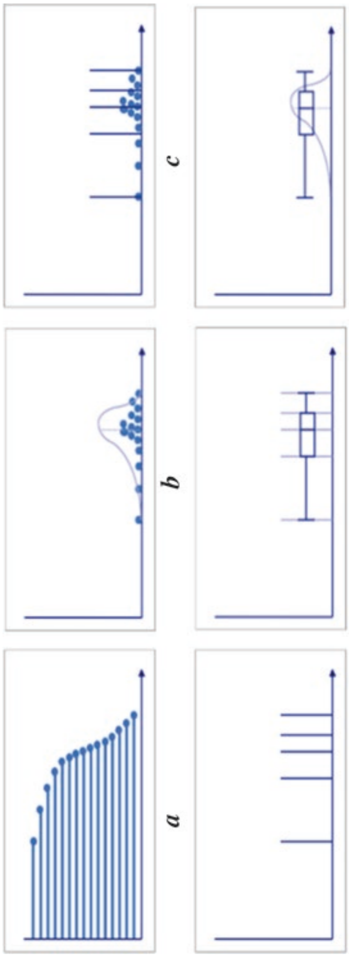


**Fig. 16.1** **(a–f)** Sub-models in an emergent modeling process

The visualizations are embedded in computer tools; and the computer tools, with the built-in tool options, were so designed that they would support the aforementioned reflexive process. The first tool shows data values as bars with a dot at the end. The students can structure data in various ways while comparing two or three data sets. The second tool shows data points in the form of dot plots, which can be structured in various ways, in particular in either two or four equal groups. The third tool shows bivariate data sets in a Cartesian graph and allows for slicing the data set vertically and structuring those slices in two or four equal groups.

There were several indicators that the students were in fact reinventing elementary statistics. At the end of the 7th grade, students used the tool options in original ways and invented idiosyncratic concepts such as "consistency" (small spread), "majority" (highest density), and a hill metaphor, which not just signified the visual shape for the students but also the way the data were distributed (Cobb, McClain, & Gravemeijer, 2003; Gravemeijer, 2002a, 2002b). They realized that a higher point on the hill corresponded with a higher density of data points.

At the end of the 7th grade teaching experiment, most students could readily interpret graphs of data sets in terms of characteristics of distributions while focusing on informative ways of organizing data. A limitation, however, was that they did not see the median as a characteristic of the data set, probably due to the fact that the median and the quartiles were often used to partition the data sets in order to compare them multiplicatively. However, the students did develop the notion of "hill" and "majority," which later on (in the 8th grade) could be further developed into the interpretation of the median as indicator of the location of a hill.[4]

### *16.4.3 Example III: Statistical Reasoning Learning Environment (Tertiary Level)*

Garfield and Ben-Zvi (2008, pp. 45-64; 2009) designed a learning environment model for an interactive, introductory secondary- or tertiary-level statistics course that is intended to develop students' statistical reasoning. This model is called a "Statistical Reasoning Learning Environment" (SRLE) and is built on the social constructivist theory of learning (Sect. 16.3.1 above). The model is also recommended for use in teacher education (Pfannkuch & Ben-Zvi, 2011).

The SRLE may be better understood through comparison with a "traditional" university class. In a "traditional" class, the students come to class with no anticipation of what they will learn, ready to copy down what the teacher has to say. The teacher presents a lecture that includes examples, some data analysis, and perhaps some demonstrations. The students listen, take notes, and perhaps ask questions. They leave the class with a homework assignment that uses information from the class they just attended. They go home and try to solve the problems by looking

[4] Note, however, that the students have to be made aware that not all distributions are unimodal.

back at their notes or looking up worked examples in the textbook, often getting frustrated if they do not find an exact match.

In an SRLE class, the students know that they have to prepare for class by reading a few pages in the textbook using study questions to guide their reading and note taking or by responding to a task, such as a data analysis task or an interview with a child. Students therefore come to class with a preliminary exposure to the topic, and sometimes with questions about it. Class begins with a short summary of, and reflection on, what was learned in the previous class, and students are asked if they have questions about the previous class or the assigned task. Students ask some questions that are answered by other students and/or the teacher. The teacher rarely answers a question directly but often asks students, “What do you think?,” and if another student gives an answer, the teacher asks, “Do you agree with this answer? Why?”

Now the class is ready to begin the first task. A question is given to the students such as “Do you think that female students spend more time on cell phones than male students?” Students form small groups to discuss these questions and sketch possible distributions and then share and compare their conjectures and reasoning with the class. The students move to computers and access a data set containing information that has previously been gathered about the students in the class using an online student survey. Working in pairs, students generate graphs and statistics to answer the questions on cell phone use. Students may discuss appropriate measures of center and spread for the data, revisiting those ideas from previous lessons. They may notice outliers in the data, and discussion may focus on how to find out if these are legitimate values or errors and on what happens to the graphs and statistics if those extreme values are removed?

The teacher’s role in the SRLE class is to present the problem, guide the discussion, anticipate misconceptions or difficulties in reasoning, and make sure students are engaged on task and not experiencing difficulties. The teacher has to know when to end discussions, how to learn from mistakes, and how to provide a good summary for the task using the work students have done, so students can appreciate what they learned from the task. At the end of class, after the wrap-up discussion and summary, students may be asked to complete a brief assessment task, providing the teacher with feedback on their learning for that class.

The contrast between the SRLE and traditional instructional approaches is large, and it is apparent that even an eager and enthusiastic teacher who wants to move from a more traditional approach to a more SRLE approach is faced with many challenges. These challenges include students, colleagues, and institution, as well as challenges to the teacher’s own expectations. These challenges are examined and addressed in Garfield and Ben-Zvi (2008, pp. 57-63).

The SRLE model integrates many previous research results and is based on current learning theories. It is hard to envision a way to empirically test it in its entirety since it is too complex and could translate differently in different courses and educational levels. Indeed, there is little empirical evidence as to what extent the entire SRLE improves students’ statistical reasoning and thinking (Baglin, 2013; Loveland, 2014). Conway (2015) studied the impact of conformity to SRLE principles on students' statistical reasoning in advanced placement statistics courses[5] in the USA. While the comparison between classrooms showing low and high conformity to SRLE principles revealed no statistically significant differences in students' statistical reasoning ability, results from this study suggest that beliefs and practices aligned with SRLE principles show potential to increase students' statistical reasoning at rates above national averages.

Several aspects of the SRLE were studied to assess learning outcomes. For example, both cognitive and affective/motivational factors were found associated with using real-life data to teach statistics in a first-year university statistics course (Neumann, Hood, & Neumann, 2013). Slootmaeckers, Kerremans, and Adriaensen (2014) used similar principles in the integration of quantitative material into non-methodological courses for political science students. Their results indicate that such an approach can not only foster interest in statistics but also retention of the acquired statistical skills.

## 16.5 Design Dimensions for Statistics Learning Environment

In this section, we identify design dimensions that arise from theoretical and empirical sources we have discussed and the three learning environment examples described in the previous section. These dimensions are not meant to serve as a prescription for what teachers and designers should do but rather to provide a wide spectrum of factors, or starting points, that need to be considered, aligned, and balanced in designing statistics learning environments. The goal of designing effective and positive statistics learning environments is to support students to develop a deep and meaningful understanding of statistics and the ability to think and reason statistically. In considering the design of such learning environments, we discuss and expand on six dimensions of pedagogical design proposed by Cobb and McClain (2004), highlighting what we see as the important connections between them (Fig. 16.2).

### *16.5.1 Focus on Developing Central Statistical Ideas Rather than on Tools and Procedures*

There are several key statistical ideas that school and university students are expected to understand at a deep conceptual level (Burrill & Biehler, 2011; Garfield & Ben-Zvi, 2008). These ideas serve as overarching goals that direct teaching and

[5] Advanced placement is a US academic program with more than 30 courses in a wide variety of subject areas that provides secondary school students with the opportunity to study and learn at the college level.

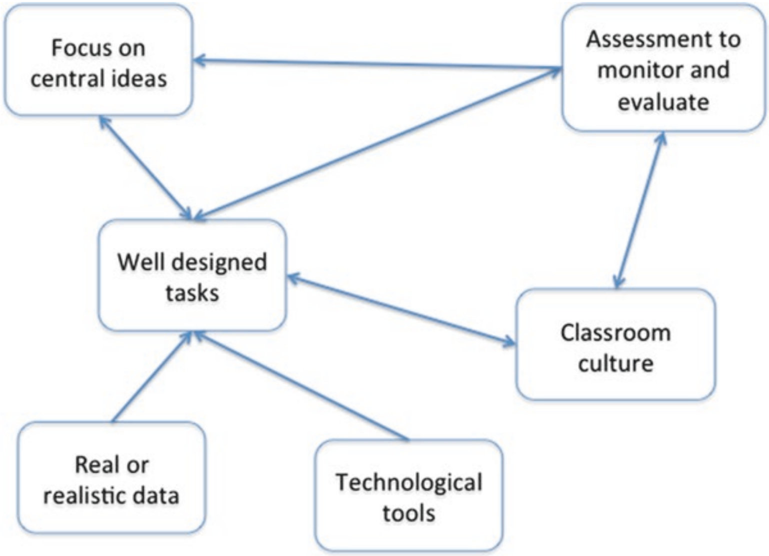


**Fig. 16.2** A web of interrelated dimensions of a learning environment

motivate and guide students' learning. They include data, distribution, center, variability, comparing groups, sampling, modeling, inference, and covariation.

Garfield and Ben-Zvi (2008; see Example III above) advocate a focus on key statistical ideas and the interrelations among them and suggest ways to present these ideas throughout a course, revisiting them in different contexts, illustrating their multiple representations and interrelationships, and helping students recognize how they form the supporting structure of statistical knowledge.

Following the RME approach, one would aim for reinventing statistical ideas and allowing procedures and definitions to emerge in the process of coming to terms with a key idea. As exemplified earlier with the example of the process of reinventing the conception of distribution as a mathematical object, measures of central tendency may be developed as means to get a handle on distributions.

### *16.5.2 Use Well-Designed Tasks to Support the Development of Statistical Reasoning*

An important part of a statistics learning environment is the use of carefully designed tasks, informed by research findings, that promote student learning through collaboration, interaction, discussion, and addressing interesting problems (e.g., Roseth, Garfield, & Ben-Zvi, 2008). It may be argued that such tasks should be part of a well-considered instructional sequence, informed by the aim of developing central statistical ideas, which is underpinned by a *local instruction theory*. A local instruction theory typically consists of a theory about a potential learning process and theories about the means of supporting that learning process (Gravemeijer & Cobb, 2013). The former offers teachers background information, on the basis of which they may decide, on a daily basis, what learning goals to aim for, while the latter offer them information on how potential tasks, tools, ways of interacting, and the classroom culture may support the intended learning process. This information will help teachers in choosing tasks and tools, anticipating the mental activities of the students, orchestrating classroom interaction, and evaluating the implied hypothetical learning trajectories.

Anticipating the notion of density, for instance, a step in the learning process, will concern the shift from measures represented as proportionally seized horizontal bars to measures represented by dots—where positions of the dots correspond with the end points of the bars (see Fig. 16.1a, b). The key here is to orient the students toward the end points of the bars and their position in respect to the horizontal axis. This asks for tasks in which the positions of the end points of the bars form a central issue. The battery life span task (Fig. 16.3) nicely fulfills this requirement (although the teacher may choose another task that can fulfill this function).

Designing high-quality tasks is demanding, not least because of some inherent tensions. One of these, which Ainley, Pratt, and Hansen (2006) call the “planning paradox,” is between engaging students’ interest by allowing freedom for them to develop their own ideas and ensuring that specific mathematical or statistical ideas are addressed. This is a tension between the design of appropriate tasks and the constraints of the institutional learning context.

Addressing this challenge, Ainley and Pratt (2014a) propose two linked principles for task design which are particularly appropriate in statistics education and potentially have wide application within the design of learning environments. The first is that tasks should have a clear *purpose* for the students within the context of the classroom. This might involve making a real or virtual object, such as a paper spinner or a model to generate data, or solving an intriguing problem. The purpose in this sense is not necessarily related to a real-world application: the purpose may arise within a fictional context, such as students advising on the movement of a new character in the “Angry Birds” computer game (Ainley & Pratt, 2014b). What is important is that the challenge of the task is engaging for students.

The second principle concerns the *utility* of statistical ideas, that is, the ways in which these ideas are useful. Engaging tasks should offer students opportunities to use statistical ideas in ways that enable them to see how and why they are powerful. For example, in modeling the movement of an “Angry Bird” which only moves horizontally (an “Angry Emu”), students can appreciate the need to express both signal and noise to describe the distance the Emu will travel relative to how far the sling is pulled back.

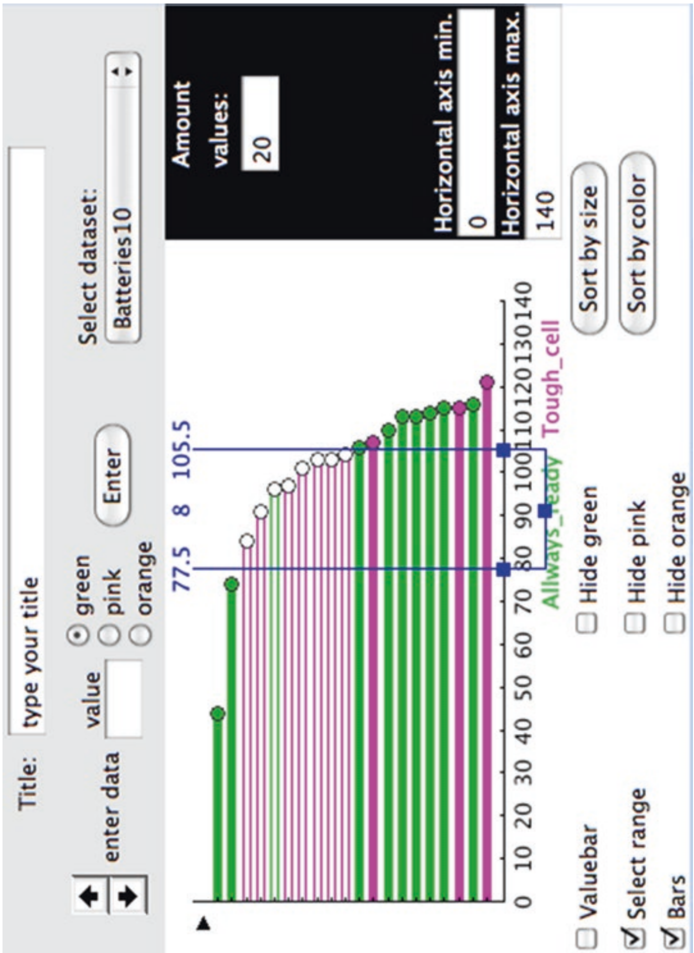


**Fig. 16.3** Comparing data on the life span of batteries with a computer tool

### 16.5.3 Use Real, or Realistic, and Motivating Data Sets

The design of pedagogic tasks in statistics must take account of the data that will be centrally involved. Data are at the heart of statistical work, and data should be the focus for statistical learning as well (Franklin & Garfield, 2006). Throughout their experience of learning statistics, students need to consider methods of data collection and production and how these methods affect the quality of the data and the types of analyses that are appropriate. One approach can be to look for interesting data sets to motivate students to engage in activities, especially ones that ask them to make conjectures about a data set before analyzing it (Ben-Zvi & Aridor, 2016). Another approach would be to start with a question and then discuss what data would be needed to answer it. However, the provision of real or "realistic" data is not always sufficient to engage students in tasks that develop statistical reasoning unless the task poses meaningful challenges and provides opportunities to use statistical ideas in realistic ways.

Consider two kinds of activities using real data which are relatively familiar within statistics education research. The first is exploratory data analysis based on a source of real data, such as CensusAtSchool (Connor, 2002). Although data about students like themselves may have intrinsic interest, posing meaningful questions about the data can be challenging for school students (e.g., Burgess, 2007). Open-ended exploration of relationships in the data without a clear goal may not lead them to use statistical ideas in realistic ways. The second is a sampling task, such as repeatedly drawing small samples to estimate the proportion of sweets of a particular color within a bowl. Here, the statistical idea of sampling is being used in a realistic way, to answer a specific question, but the task itself is not a meaningful challenge (Ainley, Gould, & Pratt, 2015). If you really wanted to know the numbers of sweets of different colors, it would be quicker and more reliable to empty the bowl and count them. What these tasks have in common is that, although based on real data, they do not emphasize opportunities for students to appreciate the utility of statistical ideas. As a result, they may appear contrived and fail to engage and motivate students.

There is a further tension concerning the role and nature of data in statistics tasks. Students, particularly younger students, need to experience collecting, recording, and cleaning their own data in order to develop their understandings of different forms of representation (e.g., Neumann et al., 2013). But data collection is time-consuming, often leaving relatively little time for analysis and discussion, and the features of the resulting data sets cannot be predicted. Providing real-world data sets (such as CensusAtSchool data, e.g., http://new.censusatschool.org.nz/), or devising data sets which are not authentic but embody the features that the teacher wants students to experience, will save time, but students may find such data sets harder to understand and engage with (Arnold, 2014).

In their teaching experiment on data analysis, Cobb et al. (2003) used an approach, which offers a resolution to this tension. They asked seventh grade students what data would be needed for a consumer report on batteries, providing an overall purpose for the task. The students came up with the variable "life span" and figured out how data on life span would have to be gathered, in an activity which the authors describe as "talking through the process of data creation." Subsequently, the students were offered life span data on two brands of batteries, which were not authentic but tailored to focus attention on statistical ideas involved in comparing two data sets, as part of instructional sequence. The data sets were constructed in such a way that one data set had a number of long-lasting batteries, while the other data set had a smaller spread. This allowed for a discussion of a small spread, for which the students invented the term "consistency" versus some high values. Students eventually linked this to the issue of what you would want to use the batteries for.

### *16.5.4 Integrate the Use of Technological Tools that Allow Students to Explore and Analyze Data*

The design of tasks (Watson & Ohtani, 2015) and the ways in which students may access and explore data are significantly influenced by the range of technological tools available to support the development of students' understanding and reasoning, such as computers, graphing calculators, Internet, statistical software, and web applets (e.g., Biehler, 2003). Students no longer have to spend time performing tedious calculations, or drawing graphs, and can focus instead on the more important task of learning how to choose appropriate analytic methods and how to interpret results. Technological tools are used not only to generate statistics, graph data, or analyze data but also to help students visualize concepts and develop an understanding of abstract ideas through simulations. For examples of innovative tools and ways to use these tools to help develop students' reasoning, see Ben-Zvi (2000); Chance, Ben-Zvi, Garfield, and Medina (2007); and Biehler, Ben-Zvi, Bakker, and Makar (2013).

A special category of technological tools is that of tools that are tailor made to instructional sequences, aiming to support "guided reinvention." As an example, we may refer to the data analysis experiment of Gravemeijer and Cobb (2013) described above that aimed to develop students' understanding of distribution as an object. Here an emergent modeling approach was applied in which the various sub-models instantiated the overarching idea of visualizing data sets. These visualizations were embedded in computer tools which enabled the students to structure the data in various ways. When comparing two data sets on the life span of batteries, for instance, the students used the tool options to compare the values of the AlwaysReady batteries with those of the Tough Cell batteries (Fig. 16.3).

Referring to the computer tool representation, they argued that they would prefer the "consistency" of the Tough Cell batteries over the many high values of the AlwaysReady batteries, when they needed a battery to really rely on; it would give you at least 80 h.

### *16.5.5 Establish a Classroom Culture that Fosters Statistical Arguments*

The design of tasks and technological and assessment tools has to take into account the expected forms of classroom *discourse*. In statistics learning environments, the use of activities and technology allows for a form of classroom discourse in which students learn to question each other and respond to such questions, as well as explaining their answers and arguments. Cobb and McClain (2004) describe the effective classroom discourse in which statistical arguments explain why the organization of data gives rise to insights about the phenomenon under investigation and students engage in sustained exchanges that focus on significant statistical ideas.

It can be challenging to create a statistics learning environment with classroom discourse that enables students to engage in discussions in which significant statistical issues emerge and where arguments are presented and their meaning is openly negotiated. Creating a classroom climate where students feel safe expressing their views, even if they are tentative, is another challenging task and is related to classroom *culture*, in which the teacher and students have to develop the corresponding classroom social norms and socio-mathematical (or socio-statistical) *norms* (Yackel & Cobb, 1996). These norms encompass the obligation for the students to explain and justify their solutions, to try to understand the explanations and reasoning of the other students, to ask for clarification when needed, and eventually to challenge the ways of thinking with which they do not agree. The teacher is not expected to give explanations but to pose tasks and ask questions that may foster students' thinking. Socio-statistical norms would be tailored to what it means to do statistics, for example, what a statistical problem is, what a statistical argument is, and so forth.

As described in the three learning environment examples above, the shift in the classroom culture is related to a potential shift in the role of the students, from problem-solvers to statisticians who analyze and represent data to make them easily accessible for decision makers. When adopting the role of a data analyst, or data detective (Pfannkuch & Rubick, 2002), students can start reflecting on the adequacy and clarity of condensed descriptions and representations of data, which may foster the reinvention of more sophisticated representations and concepts.

### *16.5.6 Use Assessment to Monitor the Development of Students' Statistical Learning and to Evaluate Instructional Plans*

Assessment should be aligned to well-designed tasks that focus on central statistical ideas in a discourse-rich classroom. Much of the value of changes in the other design dimensions will be lost if assessment practices are not aligned in this way, since the attention of students and teachers will be shaped by the requirements of assessment. In recent years, many alternative forms of assessment have been used in statistics classes. In addition to quizzes, homework, and exams, many teachers use statistical projects as a form of assessment (MacGillivray & Pereira-Mendoza, 2011). Other forms of alternative assessment are also used to assess students' statistical literacy (e.g., critique a graph in a newspaper) and reasoning (e.g., write a meaningful short essay) or to provide feedback to the teacher (e.g., minute papers) (Bidgood, Hunt, & Jolliffe, 2010; Franklin & Garfield, 2006; Gal & Garfield, 1997).

Assessments need to be aligned with learning goals, focusing on understanding key ideas and not just on skills, procedures, and computed answers. This can be done with formative assessments used during a course (e.g., quizzes, small projects, or observing and listening to students in class) as well as with summative evaluations (course grades). Useful and timely feedback is essential for assessments to lead to learning. Types of assessment may be more or less practical in different types of courses. However, it is possible, even in large classes, to implement good assessment practices (Garfield & Ben-Zvi, 2008, pp. 65–89).

## 16.6 Discussion: Contemporary Issues and Emerging Directions

The goal of this chapter has been to draw attention to the need to think about learning environments and their design in statistics education as a way of considering how sustainable change in the learning and teaching of statistics can be supported. It is not to advocate one particular approach to the design of learning environments, but rather to raise awareness to the need to consider this lens in statistics education research and practice. We have provided several examples of statistics learning environments that were informed by the social constructivist and the realistic mathematics education theories. Drawing on these examples and theories, we have discussed six dimensions of statistics learning environments.

Designing for educational change to support the development of students' statistical reasoning is a challenging task. Using a lever to make a one-dimensional change (e.g., formulate new tasks, the use of a new pedagogical strategy) may make a difference that is not necessarily a sustainable change in students' understanding of statistical ideas. This chapter has argued for a holistic and integrated approach that advocates a learning environment where students are engaged in making and testing conjectures using data, discussing and explaining statistical reasoning, focusing on the important big ideas of statistics, using innovative tools in creative ways to assist their learning, and being assessed in appropriate ways.

We have discussed how the design of a statistics learning environment might take into consideration the following interrelated dimensions: a focus on central statistical ideas, the use of real or realistic data sets, well-designed tasks, integration of the use of appropriate technological tools, promoting classroom culture that nurtures discourse and socio-statistical norms, and the use of appropriate assessment methods (Cobb & McClain, 2004).

A key factor in this discussion is that these dimensions, which are interrelated (see Fig. 16.2), must be aligned and balanced. Issues of alignment are important for accelerating statistics learning both within and outside of schools. The meaning of these design principles being part of integrative whole is that using one of them separately is not enough to make deep and sustainable change in students' learning. The learning environment approach helps to interlink them. For example, the design of motivating tasks is linked to real data collection; these data can be used to build students' statistical understanding taking advantages of the innovative affordances of technological tools; productive classroom discourse is supported by the design of open-ended tasks that support argumentation and by appropriate responses by the teacher (Makar, Bakker, & Ben-Zvi, 2015); assessment methods need to align with the design of tasks; a provision of a new tool must consider the potential interactions with content and pedagogy (Moore, 1997).

Thus we argue that pedagogical and research efforts for change must consider the interactions among these dimensions. There are however other important dimensions of learning environments that were not included in this chapter. One example is the emotional aspects of engagement and identity to motivate all students to participate and reflect on their experiences (Heyd-Metzuyanim, 2013).

Learning environments should become part of the statistics education community discussion. Rather than the limited current focus on a specific tool or a set of innovative tasks, we hope to see more studies that report on integrated learning environments in statistics. The challenge is manifold. Planning a learning environment study is more complicated than a single-factor experiment, there are possibly greater tensions with local and national institutional constraints, and the design of assessment has to take into account multiple dimensions and use mixed methods.

If taken seriously, there are contemporary issues and future directions in this area of statistics learning environments. First, further research is crucially needed to provide more well-researched holistic examples in different contexts and age levels. Systematic studies are also needed about the effectiveness of statistics learning environments, learning environment design issues, the role of alignment between the various dimensions of statistics learning environments, new possibilities for teaching and learning in innovative designs, and opportunities in cutting-edge areas, such as model-based reasoning, visual representation to teach complex abstract concepts, learning in virtual worlds, net-based collaborative teams and communities, and big data (see Chaps. 1, 13, and 15 this volume).

The difficulty of demonstrating the effectiveness of the approach in Example III above raises profound methodological issues in researching learning environments. A traditional approach to research is one in which most variables are controlled as far as possible and the focus is on the unidimensional variable in question. The learning environment approach acknowledges a complex system or ecology in which such a methodology is not sustainable. Instead, a design research approach (Cobb et al., 2003; Gravemeijer & Cobb, 2013) is needed, where iterative design of the learning environment sensitizes the research team to the key mechanisms for learning within the design. Note, however, that in design research also, empirical data on what students gain from participating in the learning environment is indispensable. We recommend that more attention be given to methodological aspects of researching the design of learning environments.

Secondly, due to the proliferation of learning in online settings, there is an increase of designs for online learning communities such as MOOCs and virtual environments (e.g., Pratt, Griffiths, Jennings, & Schmoller, 2016; Wild, 2007). There is therefore a need to study designs for learning environments of the future (Jacobson & Reimann, 2010). We argue that taking a learning environment perspective can advance our understanding of the online learning arenas.

First steps in moving toward the learning environment perspective in the statistics education community are for researchers to consider the implications of this approach in their studies and for professional development to support teachers to consider how current curricula and materials align in the context of social, cultural, physical, psychological, and pedagogical components of a learning environment. Careful and steady change over a period of time, rather than a push for radical change, may lead to a successful implementation of a learning environment in the statistics education world, both among researchers and teachers.

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