Chapter 10
Research on Statistics Teachers' Cognitive and Affective Characteristics

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Abstract Research about statistics teachers faces a unique challenge. It is not sufficient to account only for teachers' cognition and affect in regard to the subject matter of statistics. We also need to understand the personal characteristics teachers have related to developing the statistics-related cognitive and affective traits of students. Toward this end, researchers have supplemented studies of teachers' subject matter knowledge with studies of their pedagogical content knowledge, technological pedagogical statistical knowledge (TPSK), beliefs, and attitudes relevant to teaching statistics. We describe existing models and empirical research concerning each of these characteristics. Written assessments, interview techniques, and observation methods for assessing teachers' development of the characteristics are described as well. Strengths and limitations of existing models and assessments are discussed. We conclude by summarizing statistics teacher education research in the specific areas of data, uncertainty, and statistical inference. We close with recommendations about how statistics teachers' cognitive and affective characteristics may be developed by learning from teaching practice, immersion in statistical content, and use of technological environments. Opportunities and directions for future research appear throughout the chapter. Some specific research needs include progressive development of improved models for statistics teachers' cognition and affect along with robust qualitative and quantitative assessment tools.

Keywords Affect • Assessment techniques • Cognition • Pedagogical content knowledge • Subject matter knowledge • Teaching statistics • Technological pedagogical statistical knowledge (TPSK)

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10.1 Introduction

The previous chapters in this handbook have dealt with the nature of statistics and students' knowledge of the discipline. We now turn our attention to important mediators between students and the discipline: statistics teachers (primary, secondary, and tertiary). Teachers can be considered the third vertex in a didactic triangle, with students and content as the other two vertices (Goodchild & Sriraman, 2012). The mediating role of the teacher has motivated researchers to study teachers' cognitive and affective characteristics. Certainly, not all teacher and student interactions can be fully explained by these characteristics alone, since contextual constraints such as supervisor expectations, institutional policies, and instructional resources available are influential (Forgasz & Leder, 2008; Sullivan & Mousley, 2001). Diversity among students and equity concerns also come into play (Cobb, 1999). Nonetheless, research suggests that understanding teachers' individual characteristics is an essential part of studying teachers' impact on students' learning. For example, student achievement in statistics has been found to be positively associated with teachers' knowledge (Callingham, Carmichael, & Watson, 2016).

In this chapter, we focus specifically on research findings related to the cognitive and affective characteristics of statistics teachers. Precise definitions of "cognition" and "affect" are elusive in the literature, but we conceive of cognitive characteristics as being related to the knowledge and reasoning processes needed for teaching statistics and affective characteristics as being related to dispositions, emotions, attitudes, and beliefs about teaching statistics (McLeod, 1992). In many cases, it is difficult to separate cognition from affect. Beliefs, for example, though often discussed in connection with affect, are widely acknowledged to have cognitive components as well (Philipp, 2007). Hence, in this chapter, our primary goal is not to separate characteristics neatly into "cognitive" and "affective" bins, but rather to identify characteristics that may help shed light on the nature of teachers' mediating role between students and statistics.

We begin the chapter with descriptions of salient theoretical constructs related to statistics teachers' cognition (Sect. 10.2) and affect (Sect. 10.3). We then turn to methods for assessing attainment of these constructs (Sect. 10.4). Finally, we summarize findings from research in regard to the constructs (Sect. 10.5). In doing so, we seek to portray the current state of the art and identify fruitful directions for further research (Sects. 10.6 and 10.7).

10.2 Constructs for Describing Teachers' Cognitive Characteristics

Researchers employ various theoretical models to study cognition related to teaching statistics. These models generally acknowledge that knowing statistics is a necessary, but not sufficient, condition for teaching it. This resonates with Shulman's

(1987) influential assertion that teachers need pedagogical content knowledge, which is a "special amalgam of content and pedagogy that is uniquely the province of teachers" (p. 8). Building on Shulman's work, the Learning Mathematics for Teaching (LMT) Project conceptualized content knowledge for teaching as consisting of both subject matter knowledge and pedagogical content knowledge (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008). Several studies of statistical knowledge for teaching (SKT) have profitably used adaptations of the LMT model (e.g., Burgess, 2011; González, 2014; Groth, 2013; Leavy, 2015; Noll, 2011; Wassong & Biehler, 2010). Hence, we describe possible elements of subject matter knowledge (Sect. 10.2.1) and pedagogical content knowledge (Sect. 10.2.2) related to the LMT model next.

Although the LMT model appears frequently in statistics teacher education research, it would be inaccurate to portray it as the only model employed. We will also describe work that challenges the field to continue to think critically about the precise nature of the elements of SKT, their relationships with one another, and how they develop (Sect. 10.2.3). We close with a discussion of technological pedagogical content knowledge and how it relates to other research on teachers' cognitive characteristics (Sect. 10.2.4).

10.2.1 Subject Matter Knowledge

Subject matter knowledge can be conceptualized as having three sub-domains: common content knowledge, specialized content knowledge, and horizon knowledge (Ball et al., 2008; Hill et al., 2008).

Hill et al. (2008) described common content knowledge as "knowledge that is used in the work of teaching in ways in common with how it is used in many other professions or occupations that also use mathematics" (p. 377). At the university level, prospective teachers often study aspects of common statistical knowledge alongside those preparing for other professions. For example, knowing how to compute and interpret descriptive statistics such as mean, median, and interquartile range is valuable both to teachers and to other professionals (Groth, 2007).

Specialized content knowledge can be described as "the mathematical knowledge that allows teachers to engage in particular *teaching* tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (Hill et al., 2008, p. 378). Specialized knowledge of statistics might involve knowing how to represent the mean as a typical value, a fair share, and a signal amid noise (Wassong & Biehler, 2010). It might also entail the ability to analyze students' statistically naïve interpretations of data (Burgess, 2011). Similarly, appraising novel student-invented graphical representations may be done by drawing upon specialized knowledge (Groth, 2013).

Ball and Bass (2009) spoke of horizon knowledge as that which allows teachers to see connections between content studied at a particular grade level and major disciplinary structures, ideas, practices, and sensibilities. Consider the case of standard deviation. Seventh-grade US teachers using the Common Core State Standards (National Governors Association for Best Practices & Council of Chief State School Officers, 2010) would not teach this idea directly to their students, but they would be responsible for teaching the related idea of mean absolute deviation (MAD). Teachers might conceive of the MAD in at least two different ways: (1) as an algorithm for students to compute and master or (2) as a precursor for the study of standard deviation (Groth, 2014). Teachers holding the latter view would seem more likely to select tasks and ask questions that lead toward the concept of standard deviation that is on the statistical horizon. Teachers holding the former view may not be able to imbue their instruction with this perspective, and they may reduce the study of the MAD to rote learning of a procedure. Many questions about horizon knowledge remain for exploration by researchers, such as: (1) What specific aspects does horizon knowledge entail? (2) How is horizon knowledge best developed? (3) What can teachers with well-developed horizon knowledge do for students that others cannot?

Research involving teachers frequently focuses on their subject matter knowledge. Specific findings regarding the nature of different elements of teachers' subject matter knowledge appear in Sect. 10.5.

10.2.2 Pedagogical Content Knowledge

Hill et al. (2008) hypothesized that pedagogical content knowledge has three subdomains: knowledge of content and students, knowledge of content and teaching, and curriculum knowledge.

The first sub-domain, knowledge of content and students, pertains to teachers' knowledge of students' thinking patterns and problem-solving strategies (Hill et al., 2008). The importance of this type of knowledge is well established in the literature. Teachers participating in professional development about how students think about content tend to be more effective in facilitating students' learning (Franke, Kazemi, & Battey, 2007). Statistics education researchers have hypothesized that knowledge of content and students for statistics may consist of elements such as understanding students' difficulty learning the mean conceptually (Wassong & Biehler, 2010), comprehending student difficulties sorting data (Burgess, 2011), and knowing differences between how students tend to read dot plots and box plots (Groth, 2013). Comprehensively conceptualizing the nature of knowledge of content and students for statistics and its impact on student learning constitute important tasks for future research.

Knowledge of content and teaching is a combination of knowing about teaching and knowing about subject matter. It can help teachers with tasks such as choosing models and examples that bring out important aspects of content (Ball et al., 2008). It appears that knowledge of content and students contributes to knowledge of content and teaching. Consider, for example, a teacher who knows that transitioning from dot plot displays to box plots and histograms is a difficult task for students. Such a teacher is in position to select tasks to help students gradually transition from one display to the next (Groth, 2013).

Ball and Bass (2009) spoke of curriculum knowledge as including knowledge of educational goals, standards, and grade levels where specific concepts appear. This type of knowledge may help teachers appropriately sequence the introduction of statistical ideas in a curriculum (Godino, Ortiz, Roa, & Wilhelmi, 2011). However, there is considerable variability in how teachers interpret curriculum materials. When given a curriculum, some implement it with a high amount of fidelity to the curriculum authors' intentions, and others do not (Tarr et al., 2008). Sometimes this degrades the quality of instruction, but other times may help improve it (Brown, Pitvorec, Ditto, & Kelso, 2009). Hence, carefully examining statistics teachers' curriculum knowledge has the potential to help explain underlying reasons for instructional dynamics observed in the classroom.

Although pedagogical content knowledge appears as a separate category from content knowledge in the LMT framework, in practice, it is difficult, and often not advisable, to separate the two. Hence, in the summary of research appearing in Sect. 10.5, we report findings about teachers' pedagogical content knowledge, for the most part, alongside subject matter knowledge findings.

10.2.3 Continuing the Work of Precisely Defining SKT Elements, Their Relationships to One Another, and Their Development

Given the preceding description of the elements of SKT, some may gain the impression that it is a static trait rather than one that evolves and changes continuously within classroom contexts. LMT-based models are sometimes perceived in this manner (Venkat & Adler, 2014). Some theoretical work serves to cast SKT in a more dynamic light.

Working from the LMT framework and empirical data, Groth (2013) theorized about processes involved in individuals' transformation of statistics subject matter knowledge into forms that are useful for teaching. Central to the analysis is the idea that teachers' key developmental understandings of subject matter knowledge (Simon, 2006) are, alone, not sufficient for teaching. Teachers who have key developmental understandings must also learn to view subject matter knowledge from students' perspectives in order to create pedagogically powerful ideas (Silverman & Thompson, 2008). In terms of the LMT framework, this suggests that knowledge of content and students is a precursor to developing knowledge of content and teaching. That is, teachers should understand students' learning needs in order to design and select teaching methods suitable for addressing them. The potential link between knowledge of content and students and knowledge of content and teaching deserves more research attention, as it is difficult to conceive of a teacher with robust knowledge of content and teaching but underdeveloped knowledge of content and students. Investigating whether one sort of knowledge is usually prerequisite to another could help effectively sequence learning experiences for teachers.

Studies in the traditions of design research (Bakker & van Eerde, 2015) and didactic engineering (Artigue, 2015) may help further reveal dynamic processes involved in SKT development. Such studies involve iterative cycles of research and development of instructional sequences in classroom settings. As the cycles occur, data yield contextually rich information about teachers' knowledge and its enactment in practice. Working from the perspective of teachers' knowledge situated in an institutional context, Godino et al. (2011) proposed facets of professional knowledge for teaching statistics that differ from those in the LMT model. One such facet was that teachers need knowledge of "students' attitudes, emotions, and motivations regarding the content" (p. 279). This type of knowledge is similar to knowledge of content and students in its focus on student characteristics, but different in that it deals with the importance of knowing children's affect in regard to statistics (and not just children's statistical cognition). It would be profitable for researchers to take advantage of different conceptualizations of the nature of SKT as starting points for comparing and contrasting viewpoints. Done systematically, such theoretical comparisons could lead to the incremental development of progressively more sophisticated models of SKT and how it develops.

In any model of SKT that is constructed, it is important for researchers to acknowledge that mathematics and statistics are distinct disciplines. Mathematics and statistics differ in their "origins, subject matter, foundational questions, and standards" (Moore, 1988, p. 3). Therefore, it is reasonable to assume that the knowledge needed for teaching statistics is not precisely equivalent to the knowledge needed for teaching mathematics (Groth, 2007). Hence, as theoretical work on the conceptualization of SKT continues, researchers must be careful to distinguish, as necessary, between professional knowledge needed for teaching mathematics and that needed for teaching statistics.

10.2.4 Technological Pedagogical Content Knowledge

Shulman's (1987) notion of pedagogical content knowledge is the basis for another related, yet somewhat distinct, body of research on teachers' knowledge. As digital technologies became more prevalent in classrooms, it was apparent that teachers needed *technological pedagogical content knowledge* (TPCK) to effectively use them for instruction (Koehler & Mishra, 2008). TPCK is a complex interaction among knowledge of content, pedagogy, and technology. Some theoretical work to conceptualize TPCK for statistics appears in this section, and some work to help develop teachers' TPCK appears in Sect. 10.6.3.

Lee and Hollebrands (2011) offered a framework to operationalize TPCK for statistics. Their framework posits statistical knowledge as the basis for technological statistical knowledge (TSK). TSK is a blend of technology knowledge and statistics knowledge. TSK includes knowledge of technologies that are both *amplifiers* and *reorganizers* (Ben-Zvi, 2000; Lee & Hollebrands, 2008). Amplifiers help automate processes that could be done by hand, such as computing least-squares regression lines (Lee & Hollebrands, 2008). Reorganizers "extend what teachers may be able to do without technology to help students reorganize and change their statistical conceptions" (Lee & Hollebrands, 2008, p. 329). For instance, TinkerPlots (Konold & Miller, 2011) affords the opportunity to generate and link multiple graphical representations. Using TinkerPlots to produce suitable representations for data is another activity that engages TSK (Lee et al., 2014).

TSK must ultimately merge with pedagogical knowledge if teachers are to develop technological pedagogical statistical knowledge (TPSK). An example of a task requiring TPSK was discussed by Wilson, Lee, and Hollebrands (2011). Teachers used pedagogical, statistical, and technological knowledge in analyzing a video case of children working with TinkerPlots. To analyze the case, teachers attended to how students thought about statistical tasks, how they used TinkerPlots in solving them, how the technology assisted or hindered students' learning, and the strengths and weaknesses of the task given to students. Statistical knowledge, or even TSK, would not be sufficient for analyzing these elements of the case.

At present, the literature provides a more comprehensive portrait of teachers' TSK than it does TPSK, though investigation of both types of knowledge is in its beginning stages. Also requiring research attention are the potential links among statistical knowledge, TSK, and TPSK. The Lee and Hollebrands (2011) framework provides a starting point for such investigations, but Lee et al. (2014) acknowledge that empirical work remains to be done to test the conjecture that teachers' TSK impacts their TPSK and teaching practices.

10.3 Constructs for Describing Teachers' Affective Characteristics

In the affective domain, beliefs and attitudes of statistics teachers have received research attention. However, the terms "beliefs" and "attitudes" are not used uniformly across studies. Philipp (2007) encountered the same dilemma in writing about beliefs and attitudes related to mathematics. To address the problem, he offered general descriptions that capture much of what authors often mean when using the two terms:

- Attitudes: "manners of acting, feeling, or thinking that show one's disposition or opinion. Attitudes change more slowly than emotions, but they change more quickly than beliefs" (p. 259).

- Beliefs: "Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes" (p. 259).

These characterizations provide starting points for our discussion of statistics teachers' beliefs (Sect. 10.3.1) and attitudes (Sect. 10.3.2).

10.3.1 Beliefs

Research describes several types of beliefs connected to teaching statistics. These include beliefs about the relationship between mathematics and statistics, goals and strategies for statistics instruction, and self-efficacy to teach statistics.

10.3.1.1 Beliefs About the Relationship Between Mathematics and Statistics

Statistics is often taught as part of a mathematics curriculum or in a mathematics department. This arrangement can support the belief that statistics is a branch of mathematics rather than a discipline in its own right (Burrill & Biehler, 2011). Rossman, Chance, and Medina (2006) argued that this is not a useful belief for teachers to hold, as it may lead to lack of instructional emphasis on the nature and role of context, measurement, data collection, and uncertainty in statistics. Similar concerns have been expressed by others (Gattuso, 2008; Scheaffer, 2006). Empirical data lend support to the validity of such concerns. Begg and Edwards (1999) found that teachers tended to acknowledge the cross-curricular nature of statistics yet still generally taught it as a unit of mathematics. Yang (2014) suggested that teacher beliefs about the differences between statistics and mathematics may be influenced by national curricula and assessments and that it would be worthwhile to explore the influence of these factors.

10.3.1.2 Beliefs About Goals and Strategies for Statistics Instruction

Eichler (2007) provided an empirically grounded framework for characterizing teachers' beliefs about the goals of statistics instruction. The framework included four categories of beliefs: traditionalist, application preparer, everyday life preparer, and structuralist. Traditionalists focus on the study of probability and algorithms in the abstract and not on applications. Application preparers value teaching students the interplay between theory and applications, focusing on the use of algorithms to solve real-world problems. Everyday life preparers take the focus on applications a step further, believing that the study of statistics should be driven by applications rather than theory. Structuralists focus heavily on probability theory, mathematical structure, and algorithms. Structuralists differ from traditionalists in that they believe in using applications as the basis for instruction. Structuralists differ from other groups in that their primary goal is to help students abstract mathematical structure from the applications rather than apply mathematical principles to make sense of situations students encounter outside of school.

Aspects of Eichler's framework resonate with other researchers' findings in regard to teachers' beliefs about strategies and goals of statistics instruction. Sedlmeier and Wassner (2008) found that teachers believed it to be valuable to relate statistics content to daily issues (similar to everyday life preparers), but did not believe in placing as much emphasis on student data gathering or student interests. Pierce and Chick (2011) found that some teachers believe in teaching procedures first and then using applications merely to try to make the procedures more interesting. Such a strategy may reflect application preparer and/or structuralist tendencies. Comprehensively mapping the relationships between observed teaching strategies and specific beliefs about the goals of statistics instruction is an interesting empirical task for which some infrastructure currently exists, and it awaits additional research attention.

10.3.1.3 Self-Efficacy Beliefs About Teaching Statistics

Harrell-Williams, Sorto, Pierce, Lesser, and Murphy (2014) argued that it is important to measure self-efficacy to teach statistics. Teacher self-efficacy can be defined as a teacher's belief that he or she has the ability to bring about student learning (Ashton, 1985). Harrell-Williams et al. synthesized existing research to conclude that self-efficacy influences teachers' choices of instructional techniques and students' learning. They argued that it is particularly important to consider self-efficacy in regard to teaching statistical investigations. Such a domain-specific portrait of teacher self-efficacy is potentially more informative to teacher educators than more generic assessments.

10.3.2 Attitudes

There is a voluminous body of research on individuals' attitudes toward statistics (Nolan, Beran, & Hecker, 2012), but literature about *teachers'* attitudes toward statistics is more sparse (Estrada, Batanero, & Lancaster, 2011). Available research suggests that teachers tend to value statistics as a subject but find it difficult to enjoy, teach, and learn (Estrada, Batanero, Fortuny, & Díaz, 2005; Martins, Nascimento, & Estrada, 2012). Teachers' attitudes toward statistics are potentially important because they are hypothesized to relate to their persistence in gaining statistical knowledge (Estrada et al., 2005) and willingness to teach the subject (Leavy, Hannigan, & Fitzmaurice, 2013). Teachers' attitudes toward statistics are hypothesized to influence their knowledge of statistics, their teaching practices, and their students' attitudes

(Martins et al., 2012). Several attitude-related hypotheses, however, await strong empirical support. In studies of the impact of attitudes on teachers' content knowledge, for example, researchers have found moderate to low correlations (Hannigan, Gill, & Leavy, 2013; Nasser, 2004). Negative attitudes toward statistics appear to be clearly detrimental (Onwuegbuzie, 2000), but there seems be a limit on the extent to which positive attitudes relate to increased knowledge (Hannigan et al., 2013).

It appears that the field has not yet made a strong distinction between "teachers' attitudes toward statistics" and "attitudes toward teaching statistics." Although assessment items about attitudes toward teaching statistics have been included in some research studies (Martins et al., 2012; Pierce & Chick, 2011), many studies of teachers' attitudes have used instruments intended to measure the attitudes of the general population (Estrada et al., 2011; Hannigan et al., 2013). This might explain why empirical evidence about the impact of attitudes is elusive. If, for example, the field were to systematically conceptualize and investigate teachers' attitudes toward *pedagogical elements* such as statistics curriculum, children's statistical learning, and technology for teaching statistics, might we better understand the impact of teachers' attitudes on statistics teaching and learning?

10.4 Methods for Assessing Statistics Teachers' Cognition and Affect

Assessments of cognition and affect related to teaching statistics come in a variety of forms, spanning the spectrum of written assessments, interviews, and observations. Many studies make use of more than one type of assessment and may involve more than one aspect of teachers' cognition and affect. Below, a representative sample of assessments is discussed. Due to the scope of the chapter, we focus on assessments specifically designed for teachers rather than general standardized scales of cognition and affect that are sometimes used as part of research with teachers.

10.4.1 Written Assessments

Written assessments are often the most practical way to gather information from large groups of teachers. One such assessment, the Diagnostic Teacher Assessment of Mathematics and Science, includes a separate scale of multiple-choice and open-ended items for statistics (Saderholm, Ronau, Brown, & Collins, 2010). The LMT project also designed a scale of multiple-choice items specific to teaching statistics (G. Phelps, personal communication, June 11, 2010). An international comparison of teacher education, the Teacher Education and Development Study in Mathematics, included some items on pedagogical content knowledge for statistics among items pertaining to mathematics, though algebra, geometry, and number were more heavily assessed (Blömeke & Delaney, 2012).

Some studies of teachers' affect in regard to statistics and statistics teaching have used collections of questions from larger scales intended for a broad population. Estrada and Batanero (2008), for example, used a subset of items from the Survey of Attitudes Toward Statistics (Schau, Stevens, Dauphine, & del Vecchio, 1995) that had previously yielded lower scores for teachers. More recently, Harrell-Williams et al. (2014) designed a scale to assess teachers' self-efficacy to teach statistics. It measures teachers' feelings of preparedness to teach content from the *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) report for grades pre-K–12 (Franklin et al., 2007). This sort of assessment, which is specifically designed and psychometrically tested to measure affect in regard to *teaching* statistics, is relatively rare.

Some written assessments deal with both cognition and affect related to teaching statistics. Watson (2001) designed a survey to assess teachers' pedagogical content knowledge, self-efficacy to teach statistics, and beliefs about the value of statistics. Watson, Callingham, and Donne (2008) built on this survey to devise a 12-item scale of pedagogical content knowledge. The Statistics Teaching Inventory (Zieffler, Park, Garfield, del Mas, & Bjornsdottir, 2012) contains questions about statistics teachers' teaching practices, course characteristics, assessment practice, teaching beliefs, and assessment beliefs. González (2014) designed a written assessment of teachers' subject matter knowledge, pedagogical content knowledge, and beliefs and conceptions of variability. Instruments that assess aspects of both cognition and affect have the potential to help researchers understand complex relationships among teacher characteristics such as knowledge, beliefs, attitudes, goals, and teaching practices.

10.4.2 Interviews

Clinical interviews allow a high degree of interactivity between the researcher and study participants. They are more time intensive than written assessments. Interviews come in a variety of forms. They may be driven by a formal protocol, such as the StatSmart teacher interview protocol (Watson & Nathan, 2010), which probes the nature of teachers' subject matter knowledge and pedagogical content knowledge. More often, however, interview tasks and questions are designed to meet the specific objectives of a research study. For example, Noll (2011) interviewed graduate assistants to assess their statistical content knowledge of sampling. Participants were asked about written items they had completed and were given some new tasks to solve. Similarly, Browning, Goss, and Smith (2014) conducted interviews with preservice teachers to gain better understanding of the thinking they employed while solving written statistical tasks. Other studies incorporating interviews have probed subjects such as teachers' classroom practices (Casey, 2010), beliefs about the nature of statistics (Leavy et al., 2013), and perceptions of professional development sessions (Peters, Watkins, & Bennett, 2014).

10.4.3 Observations

Written assessments and interviews provide proxies of teachers' classroom practices and quality of instruction, but observation techniques allow researchers to see these firsthand. Using structured observations, researchers can infer the nature of teachers' SKT and knowledge of statistical investigations. For instance, Burgess (2011) illustrated how an SKT framework can guide such observations of teachers' practice. Casey (2010) described an observation-based process for assessing knowledge for teaching statistical association. Pfannkuch (2006) used observations to describe a teacher's knowledge of comparing distributions with box plots. Jacobbe and Horton (2010) used observations of teachers' lessons to detect their misconceptions related to data displays.

Another strategy is to observe teachers' interactions during professional development sessions. Although these are not always firsthand observations of teaching, they still provide data that may not be easily obtained through interviews or written tasks. Wilson et al. (2011) studied video of teacher education sessions to better understand teachers' pedagogical content knowledge in connection with the use of dynamic statistics software. Peters et al. (2014) observed how teachers' learning of measures of center developed as they interacted with one another. Leavy (2010, 2015) observed prospective teachers involved in lesson study. Lesson study engages a group of teachers in planning a lesson, carrying it out, observing students' reactions, and then debriefing on the lesson's effectiveness. Observations of these activities provide a window into teachers' thinking about planning and analyzing lessons. In general, teachers' discourse with one another during professional development can help explain the nature and origin of the knowledge and beliefs that guide their instruction.

Observations of teachers' lessons and professional development sessions typically yield a variety of artifacts. These may include written lesson plans (Garfield & Ben-Zvi, 2008), statistical tasks a teacher assigns to a class (Burgess, 2011), field notes (Casey, 2010), and teachers' responses to professional development tasks (Wilson et al., 2011). Artifacts of this nature can be synthesized with other data to help researchers develop portraits of teachers' cognitive and affective characteristics related to teaching statistics.

10.5 Research on Teachers' Statistical Knowledge

The research models, constructs, and techniques described up to this point in the chapter have been used in studies spanning various statistical content areas. In this section, we summarize findings from two broad, interrelated bodies of literature about teachers' knowledge related to data (Sect. 10.5.1) and uncertainty and statistical inference (Sect. 10.5.2).

10.5.1 Research on Data

We present a brief overview of research on teachers' subject matter knowledge and pedagogical content knowledge in regard to data displays, distribution and variability, associations between variables, and statistical literacy, reasoning, and thinking.

10.5.1.1 Data Displays

Studies of preservice or practicing teachers' graph reading and interpretation skills show a tendency to underestimate the complexities of learning and teaching graphical representations (Leavy, 2010). Teachers tend to express confidence in their understandings of graphical representations and to feel better equipped to teach this topic compared to other statistical ideas (González & Pinto, 2008; Watson, 2001). However, despite their positive attitudes and confidence toward teaching graphs, many educators have limited subject matter knowledge of graphical representations (González & Pinto, 2008; Jacobbe & Horton, 2010; Pierce & Chick, 2013; Sorto, 2004). They sometimes confuse histograms with bar diagrams (Bruno & Espinel, 2009), fail to integrate graphical knowledge with problem context (Burgess, 2002), and have trouble with graph selection and understandings of data type (Leavy, 2010). Monteiro and Ainley (2007) introduced the idea of "critical sense" as a key skill in the analysis and interpretation of graphical artifacts. They investigated critical sense in graphing among prospective primary school teachers from Brazil and England. They found that many preservice teachers did not have adequate mathematical knowledge to read graphs from the daily press. However, the majority displayed an ability to think critically and justify their ideas by combining statistical knowledge with personal experience and contextual knowledge.

A small number of studies have examined teachers' pedagogical content knowledge of graphs. González and Pinto (2008) concluded that teachers need more knowledge of the process of learning statistical graphs and the difficulties that students might have with them. Espinel, Bruno, and Plasencia (2008) observed lack of coherence between prospective primary teachers' graph building and their evaluation of tasks carried out by fictitious future students. Heaton and Mickelson (2002) observed that graph construction often became the endpoint of statistical investigation for preservice elementary teachers, who focused on the technical aspects of graph construction rather than on engaging children in reasoning with the data. However, some studies indicate that using dynamic data exploration tools (e.g., Finzer, 2002; Konold & Miller, 2011) can help teachers portray graph production as a means for understanding data rather than an end in itself (Meletiou-Mavrotheris, Mavrotheris, & Paparistodemou, 2011).

10.5.1.2 Distribution and Variability

Much of the research investigating teachers' reasoning with distributions has focused on their understanding of measures of central tendency, measures of variability, distributional thinking, and procedural aspects of statistics. Pedagogical content knowledge has also been explored.

Studies examining teachers' conceptions of measures of center have focused on the arithmetic mean (e.g., Batanero, Godino, & Navas, 1997; Gfeller, Niess, & Lederman, 1999); the mean, median, and mode (e.g., Groth & Bergner, 2006; Jacobbe, 2012); and the general concept of average (e.g., Begg & Edwards, 1999; Estrada, Batanero, & Fortuny, 2004; Leavy & O'Loughlin, 2006). Evidence from such studies illustrates that attaining deep understanding of these statistical concepts is nontrivial. Like students, many teachers struggle to view measures of central tendency as representative (or "typical") values. Although teachers can readily calculate the mean, they tend not to use it to compare groups (Canada, 2004; Hammerman & Rubin, 2004; Leavy & O'Loughlin, 2006; Makar, 2004; Makar & Confrey, 2002, 2004; Peters, 2009). Like students, teachers may rely upon procedural algorithms and need conceptual understanding (Gfeller et al., 1999; Leavy & O'Loughlin, 2006; Peters et al., 2014; Sorto, 2004).

As with measures of center, teachers' understandings of standard deviation and other formal measures of variation tend to be procedural (Leavy, 2006; Makar & Confrey, 2005; Sorto, 2004). Research indicates difficulties with the concept of variability for teachers of various grade levels (Mooney, Duni, van Meenen, & Langrall, 2014; Vermette, 2013) and similar misunderstandings to those seen in students (e.g., perceiving the normality shape of a distribution as an indication of low variability). Teachers often hold competing beliefs about random variation when the setting of a problem changes (Canada, 2004; Makar, 2004). Kuntze (2014) found that some secondary teachers did not consider learning about statistical variation to be an important instructional goal, though others did recognize the importance of teaching the concept.

Measures of center and measures of spread are inseparable. Conceptual understanding of standard deviation, for example, requires "a dynamic conception of distribution that coordinates changes to the relative density of values about the mean with their deviation from the mean" (Peters, 2009, p. 21). Teachers often have difficulty coordinating understandings of central tendency and dispersion (Dabos, 2014; Lee & Lee, 2011). Many teachers tend to focus either only on the center of the distribution, or on its range, or on small clusters or individual points, rather than integrating different aspects of data distributions (Canada, 2008; Makar & Confrey, 2005; Mooney et al., 2014). When beginning to reason about distributions, teachers can be encouraged to use informal terminology to describe spread and distribution, such as "clump," "bump," "bulk of this data," "scattered," and "bunched" (Canada, 2004; Makar, 2004; Makar & Confrey, 2005). Since children use similar language (e.g., Bakker & Gravemeijer, 2004), Makar and Confrey (2005) suggested recognizing and valuing this informal "variation talk" as a way to encourage intuitive statistical sensemaking.

Explorations with dynamic data software can also help improve teachers' distributional reasoning and pedagogical content knowledge (Canada, 2004; Hammerman & Rubin, 2004; Leavy, 2006; Lee & Lee, 2011; Makar, 2004; Meletiou-Mavrotheris, Paparistodemou, & Stylianou, 2009; Peters et al., 2014). For example, Meletiou-Mavrotheris and Serradó (2012) reported on EarlyStatistics, an intercultural professional development course in which teachers took part in authentic educational activities. The activities gave them the opportunity to reflect on the "big ideas" of statistics and their applications and to explore ways of improving statistics instruction through the adoption of a coherent technology-rich curriculum based on the statistical problem-solving process. Findings from the study indicated that EarlyStatistics met its objectives, improving teachers' knowledge of key statistical ideas including distributions. Data obtained from follow-up teaching interventions in some of the participants' classrooms suggested positive gains in student learning and attitudes toward statistics (Meletiou-Mavrotheris et al., 2011).

10.5.1.3 Associations Between Variables

Some researchers have designed instruction to help teachers confront their potential misunderstandings of association and those of their students. Batanero, Estepa, and Godino (1997) examined whether a computer-based teaching experiment would improve preservice teachers' understanding of association. They found improvement in covariational strategies and reduction in deterministic concepts of association. However, they also found that most teachers retained the belief that a strong association between two variables is adequate for drawing conclusions about cause and effect. Casey (2010) observed three experienced secondary teachers as they taught statistical association and interviewed them immediately following each observation. The research showed that to meet the demands of teaching, the teachers needed substantial knowledge of the concepts' underlying statistical association. For example, they needed to know not only how to compute the value of a correlation coefficient but also why it was computed as it was and the implications of the computation. Teachers also need to understand the nature of informal lines of best fit and criteria for placing them on graphs. Casey and Wasserman (2015) found that teachers hold a variety of conceptions of informal lines of best fit and how they should be placed. Despite the different conceptions, teachers positioned informal best fit lines in approximately the same place the least-squares regression line would appear in a scatterplot.

Along with subject matter knowledge of association, teachers need pedagogical content knowledge. Casey (2014) synthesized the results of three research studies centered on the teaching and learning of linear regression to describe the knowledge needed by teachers regarding learners' conceptions of linear regression. The synthesis illustrated that the knowledge needed to teach linear regression differs in significant ways from the knowledge needed to teach linear functions. Quintas, Ferreira, and Oliveira (2014) compared and contrasted the pedagogical content knowledge of two experienced secondary mathematics teachers as they taught bivariate data. The teachers had difficulty helping students reason about bivariate relationships. It was challenging for them to teach aspects of structure and strength, model fitting, and the role of the linear regression model in predicting events. Both teachers exhibited some of the common misunderstandings and errors with regard to bivariate relationships identified in the literature (e.g., Engel & Sedlmeier, 2011). Such findings indicate a need to design professional development that strengthens teachers' content knowledge and pedagogical content knowledge for teaching association in tandem.

10.5.1.4 Statistical Literacy, Reasoning, and Thinking

The development of students' statistical literacy has become an overarching goal of statistics education internationally. This broadening of the curriculum to encompass statistical literacy, reasoning, and thinking has put considerable demands on teachers (Hannigan et al., 2013). In particular, they must design lessons with engaging contexts (Chick & Pierce, 2008), focus on conceptual understanding (Watson, 2001), and pose critical questions (Reston, Jala, & Edullantes, 2006).

Research sheds light upon factors that influence the design and implementation of instruction that fosters students' statistical literacy, reasoning, and thinking. Burgess (2011) found that the students of a teacher with well-developed SKT were able to progress further with statistical investigations than students of a teacher whose knowledge was less developed. Callingham and Burgess (2014) conjectured that the national curriculum under which teachers operate may influence their approach to teaching statistics, since Australian teachers in their study tended to focus more on procedural aspects of instruction than did their counterparts from New Zealand. Makar and Fielding-Wells (2011) found that challenges in teaching statistical inquiry may stem from difficulties coping with the uncertainties of inquiry, managing classroom logistics, and developing the requisite content knowledge. Mickelson and Heaton (2004) found that the ability to translate content knowledge into effective teaching practices is complex and urged researchers to team with classroom teachers in order to help design meaningful experiences for students. Indeed, during the past decade, several researchers have been experimenting with new innovative models of preservice and in-service teacher training that are focused on inquiry-based instruction and on statistical problem-solving (e.g., Garfield & Ben-Zvi, 2008; Groth, Bergner, Burgess, Austin, & Holdai, 2016; Makar & Fielding-Wells, 2011; Meletiou-Mavrotheris & Serradó, 2012; Serradó, Meletiou-Mavrotheris, & Paparistodemou, 2014).

10.5.2 Research on Uncertainty and Statistical Inference

Uncertainty and statistical inference are challenging ideas for teachers, just as they are for the general population. Researchers have documented teachers' understanding of theoretical probability (Batanero & Díaz, 2012; Watson, 2001), empirical probability (Dollard, 2011; Groth, 2010; Theis & Savard, 2010), informal inference (Canada, 2008; Pfannkuch, 2006), samples and sampling distributions (Green, 2010; Green & Blankenship, 2014; Groth & Bergner, 2005; Maxara & Biehler, 2010; Noll, 2011), and formal inference (Liu & Thompson, 2009; Thompson & Liu, 2005).

Sound reasoning about uncertainty and inference require a departure from deterministic modes of thinking. Such modes of thinking influence teaching practices. Serradó, Azcárate, and Cardeñoso (2006), for example, found that deterministic beliefs about the nature of statistics prevented some teachers from embracing curricular goals related to probability and uncertainty. Liu and Thompson (2009) found that the majority of the high school teachers in their study tended to think deterministically. This made it difficult for the teachers to understand and portray hypothesis testing as a tool for drawing inferences.

Research suggests that enhancing teachers' subject matter knowledge about uncertainty and statistical inference must be given high priority. Building teachers' knowledge of pedagogical structures and tools by itself is not sufficient. Lee and Mojica (2008), for example, found that although a group of teachers involved their students in authentic statistical inquiry that included use of simulation tools, they missed the chance to develop students' understanding of the frequentist approach to probability because of limited subject matter knowledge. Deep understanding of probability is also needed for identifying student errors and implementing effective teaching practices (Maher & Muir, 2014; Paparistodemou, Potari, & Pitta, 2006). Such understanding can be developed through well-designed professional development. For example, Theis and Savard (2010) helped high school teachers design and implement a technology-based instructional intervention. They found that the use of simulation software within the intervention allowed teachers to adopt more inquiry-oriented strategies and begin to incorporate frequentist probability.

Although having subject matter knowledge is necessary for effective teaching of uncertainty, it is not sufficient. Leavy (2010) worked with a group of prospective teachers who demonstrated relatively strong subject matter knowledge about informal inference. However, they had trouble using this knowledge to develop pedagogical contexts for advancing children's learning. In particular, they had difficulty choosing sufficiently complex data, creating engaging contexts, handling unexpected student responses, and scaffolding children's thought processes. In other studies, gaps in pedagogical content knowledge have been framed as contributing factors to teachers' failure to emphasize important probability concepts when writing lesson plans (Chick & Pierce, 2008) and designing productive learning environments for students (Groth, 2010). Accordingly, researchers have begun to develop techniques capable of assessing both subject matter knowledge and pedagogical content knowledge (Meletiou-Mavrotheris, Kleanthous, & Paparistodemou, 2014) and to monitor and adjust their professional development efforts to ensure that they facilitate teachers' development of both of these aspects of SKT (Lee & Hollebrands, 2008; Serradó et al., 2014).

10.6 Teacher Education Frontiers

As illustrated in this chapter and in other reports on statistics teacher education (Franklin et al., 2015), developing cognitive and affective characteristics related to teaching statistics is a complex process deserving concentrated research attention. Such research attention is particularly important in light of the move away from traditional methods of teaching statistics (Batanero & Díaz, 2012). In order to better fulfill teachers' needs in a reform-oriented context, alternative approaches to teacher education have become prevalent. Some alternative approaches are situated within: the context of teachers' classroom practice, deep exploration of statistical content, and technological environments. To conclude the chapter, we summarize some of the work done using these approaches. We do so to encourage others to continue to develop and extend the approaches. We also foreshadow Chap. 12, which describes approaches to professional development in greater detail.

10.6.1 Learning from Teaching Practice

In practice-based approaches, teachers use real classrooms as sites for investigation. Lesson study is one example. Leavy (2010, 2015) used lesson study to develop prospective teachers' knowledge and ability to teach informal inferential reasoning and data handling. Roback, Chance, Legler, and Moore (2006) used lesson study to refine their own approaches to teaching inference. Other practice-based approaches involve researchers collaborating with teachers to design, implement, and analyze instruction. For example, Noll and Shaughnessy (2012) reported on a project in which teachers teamed with five university researchers to design and co-teach lessons to investigate secondary students' conceptions of variability. They found the project to be mutually beneficial; teachers and researchers learned from one another during collaboration. Groth et al. (2016) collaborated with prospective teachers to design instruction suitable for meeting students' learning needs in regard to measures of center and involved the prospective teachers in the process of disseminating the results (Groth, Kent, & Hitch, 2015). Under such approaches, the line between teachers and researchers is intentionally blurred in order to engage teachers in some of the same types of systematic classroom inquiry that are characteristic of formal research.

10.6.2 Immersion in Statistical Content

Examples of approaches that immerse teachers in deep exploration of statistics content can be found at the primary, secondary, and tertiary levels. Reston (2012) explored in-service elementary teachers' conceptions of probability, finding that problem-based learning, inquiry, and statistical investigations promoted stronger conceptual understandings of probability and enhanced pedagogical skills. Makar and Confrey (2002) immersed secondary teachers in focused investigations about student data and studied their statistical reasoning when comparing two groups. They concluded that such an immersion model can help improve teachers' conceptual understanding of inference, their instructional practices, and their disposition toward inquiry. Bargagliotti et al. (2014) developed materials capable of promoting secondary teachers' deep immersion in the study of variability and regression. At the tertiary level, Green and Blankenship (2014) designed a course to develop teaching assistants (TAs) as statistics educators. The course focused on how TAs can foster critical thinking and enhance learning in their classrooms. The TAs left the course with improved conceptual understanding of sampling distributions and strategies for teaching and assessing students.

10.6.3 Technological Environments

Numerous studies have investigated the use of dynamic statistical packages (Finzer, 2002; Konold & Miller, 2011) to develop teachers' knowledge for teaching concepts such as sampling distributions, the central limit theorem, confidence intervals, and hypothesis testing (e.g., Garfield & Everson, 2009; Maxara & Biehler, 2010; Meletiou-Mavrotheris et al., 2014). Such studies indicate that teachers' experimentation with statistical ideas through investigations of authentic and computer-simulated data can help them develop informal inferential reasoning, construct more sophisticated understandings of the logic of inferential statistics, and improve their repertoire of teaching strategies related to statistical inference. Those interested in exploring the potential of dynamic statistics software for supporting teachers' learning can take advantage of resources such as Lee and Hollebrands' (2008) teacher education curriculum that incorporates the software and Madden's (2011) framework describing the characteristics of statistically, contextually, and/or technologically provocative tasks.

Along with dynamic statistics software environments, there are many other technological frontiers to continue to explore for teacher education. These include online communities, mobile devices, and the use of big data in relation to assessment and instruction. Environments and tools of this nature help break traditional boundaries of time, location, and extent of teacher learning (Koehler & Mishra, 2008). As relatively new, emerging technologies, much of the story of their impact on statistics teachers' learning remains to be written.

10.7 Conclusion

Research on cognition and affect related to teaching statistics is a relatively new endeavor. Several opportunities for future research are identified in this chapter. Work remains to be done to more clearly define the elements of SKT, their relationships among one another, and the mechanisms through which they develop. As this work is carried out, it will be important to reconcile and refine different SKT models through systematic academic discourse among researchers working from diverse inquiry paradigms. We also need better understanding of interactions among affect, cognition, and teaching practices. We know that teachers' goals, beliefs, and attitudes influence teaching practices to an extent. The precise nature of the types of goals, beliefs, and attitudes that are most relevant and their degree of impact need further investigation. Additionally, we need better understanding of teachers' knowledge of the impact of social and environmental factors on students' achievement and interest in statistics and their levels of preparedness to help diverse populations of students learn statistics.

Qualitative and quantitative approaches each have roles to play as research on cognition and affect for teaching statistics continues to develop. Some psychometrically and theoretically sound quantitative instruments specific to teaching statistics exist, but many studies still have to rely on instruments developed for the general population. Qualitative research can help define the salient cognitive and affective characteristics to be assessed and can provide vivid portraits of how such characteristics may develop under different circumstances. As this work occurs, we can gain progressively better understanding of teachers' mediating role between statistical content and students.

References

Artigue, M. (2015). Perspectives on design research: The case of didactical engineering. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 467–496). Dordrecht, The Netherlands: Springer.

Ashton, P. T. (1985). Motivation and the teachers' sense of efficacy. In C. Ames & R. Ames (Eds.), *Research on motivation in education* (Vol. 2, pp. 141–171). Orlando, FL: Academic Press.

Bakker, A., & Gravemeijer, K. P. E. (2004). Learning to reason about distribution. In I. D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 147–168). Dordrecht, The Netherlands: Kluwer.

Bakker, A., & van Eerde, D. (2015). An introduction to design-based research with an example from statistics education. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 429–466). Dordrecht, The Netherlands: Springer.

Ball, D. L., & Bass, H. (2009). *With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures*. Paper Presented at the 43rd Jahrestagung der Gesellschaft für Didaktik der Mathematik, Oldenburg, Germany.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Bargagliotti, A., Anderson, C., Casey, S., Everson, M., Franklin, C., Gould, R., et al. (2014). Project-SET materials for the teaching and learning of sampling variability and regression. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Batanero, C., & Díaz, D. (2012). Training school teachers to teach probability: Reflections and challenges. *Chilean Journal of Statistics, 3*(1), 3–13.

Batanero, C., Estepa, A., & Godino, J. D. (1997). Evolution of students' understanding of statistical association in a computer-based teaching environment. In J. B. Garfield & G. Burrill (Eds.), *Research on the role of technology in teaching and learning statistics: Proceedings of the 1996 IASE Round Table Conference* (pp. 191–205). Voorburg, The Netherlands: International Statistical Institute.

Batanero, C., Godino, J., & Navas, F. (1997). Concepciones de maestros de primaria en formción sobre promedios (Primary school teachers' conceptions on averages). In H. Salmerón (Ed.), *Actas de las VII Jornadas LOGSE: Evaluación Educativa* (pp. 310–340). Granada, Spain: University of Granada.

Begg, A., & Edwards, R. (1999). Teachers' ideas about teaching statistics. In *Proceedings of the 1999 combined Conference of the Australian Association for Research in Education and the New Zealand Association for Research in Education*. Melbourne: Australian Association for Research in Education & New Zealand Association for Research in Education. Retrieved from http://www.aare.edu.au/data/publications/1999/beg99082.pdf

Ben-Zvi, D. (2000). Toward understanding the role of technological tools in statistical learning. *Mathematical Thinking and Learning, 2*(1-2), 127–155.

Blömeke, S., & Delaney, S. (2012). Assessment of teacher knowledge across countries: A review of the state of research. *ZDM, 44*(3), 223–247.

Brown, S. A., Pitvorec, K., Ditto, C., & Kelso, C. R. (2009). Reconceiving fidelity of implementation: An investigation of elementary whole-number lessons. *Journal for Research in Mathematics Education, 40*(4), 363–395.

Browning, C., Goss, J., & Smith, D. (2014). Statistical knowledge for teaching: Elementary preservice teachers. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Bruno, A., & Espinel, M. C. (2009). Construction and evaluation of histograms in teacher training. *International Journal of Mathematical Education in Science and Technology, 40*(4), 473–493.

Burgess, T. (2002). Investigating the "data sense" of preservice teachers. In B. Phillips (Ed.), *Proceedings of the 6th International Conference on Teaching Statistics*. Cape Town, South Africa: International Association for Statistics Education. Retrieved from https://www.stat.auckland.ac.nz/~iase/publications/1/6e4_burg.pdf

Burgess, T. A. (2011). Teacher knowledge of and for statistical investigations. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education* (pp. 259–270). Dordrecht, The Netherlands: Springer.

Burrill, G., & Biehler, R. (2011). Fundamental statistical ideas in the school curriculum and in training teachers. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education* (pp. 57–69). Dordrecht, The Netherlands: Springer.

Callingham, R., & Burgess, T. (2014). How the curriculum shapes teachers' thinking: A comparison of New Zealand and Australian teachers' thinking about statistics. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Callingham, R., Carmichael, C., & Watson, J. M. (2016). Explaining student achievement: The influence of teachers' pedagogical content knowledge in statistics. *International Journal of Science and Mathematics Education, 14*(7), 1339–1357.

Canada, D. (2004). *Elementary preservice teachers' conceptions of variation*. Unpublished doctoral dissertation, Portland State University, Portland, OR.

Canada, D. L. (2008). Conceptions of distribution held by middle school students and preservice teachers. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Casey, S. A. (2010). Subject matter knowledge for teaching statistical association. *Statistics Education Research Journal, 9*(2), 50–68. Retrieved from http://iase-web.org/documents/SERJ/SERJ9%282%29_Casey.pdf

Casey, S. A. (2014). Teachers' knowledge of students' conceptions and their development when learning linear regression. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International statistical institute.

Casey, S. A., & Wasserman, N. H. (2015). Teachers' knowledge about informal line of best fit. *Statistics Education Research Journal, 14*(1), 8–35. Retrieved from http://iase-web.org/documents/SERJ/SERJ14(1)_Casey.pdf

Chick, H. L., & Pierce, R. U. (2008). Teaching statistics at the primary school level: Beliefs, affordances, and pedagogical content knowledge. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Cobb, P. (1999). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking and Learning, 1*, 5–43.

Dabos, M. (2014). A glimpse of two year college instructors' understanding of variation in histograms. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Dollard, C. (2011). Preservice elementary teachers and the fundamentals of probability. *Statistics Education Research Journal, 10*(2), 27–47. Retrieved from http://iase-web.org/documents/SERJ/SERJ10(2)_Dollard.pdf

Eichler, A. (2007). Individual curricula: Teachers' beliefs concerning stochastics instruction. *International Electronic Journal of Mathematics Education, 2*(3), 208–226. Retrieved from http://www.iejme.com/032007/d6.pdf

Engel, J., & Sedlmeier, P. (2011). Correlation and regression in the training of teachers. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (pp. 247–258). New York: Springer.

Espinel, M. C., Bruno, A., & Plasencia, I. (2008). Statistical graphs in the training of teachers. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Estrada, A., & Batanero, C. (2008). Explaining teachers' attitudes toward statistics. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Estrada, A., Batanero, C., & Fortuny, J. M. (2004). Un estudio sobre conocimientos de estadística elemental de profesores en formación (Prospective teachers' knowledge on elementary statistics). *Educación matemática, 16*(1), 89–111.

Estrada, A., Batanero, C., Fortuny, J. M., & Díaz, C. (2005). A structural study of future teachers' attitudes toward statistics. In M. Bosch (Ed.), *Proceedings of the 4th Congress of the European Society for research in Mathematics Education* (pp. 508–517). Barcelona: IQS Fundemi.

Estrada, A., Batanero, C., & Lancaster, S. (2011). Teachers' attitudes toward statistics. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (pp. 163–174). Dordrecht, The Netherlands: Springer.

Finzer, W. (2002). Fathom dynamic data software (Version 2.1) [Computer software]. Emeryville, CA: Key Curriculum Press.

Forgasz, H. J., & Leder, G. C. (2008). Beliefs about mathematics and mathematics teaching. In P. Sullivan & T. Wood (Eds.), *International handbook of mathematics teacher education, Knowledge and beliefs in mathematics teaching and teaching development* (Vol. 1, pp. 173–192). Rotterdam, Taipei: Sense Publishers.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Reston, VA: National Council of Teachers of Mathematics.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., et al. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-K–12 curriculum framework*. Alexandria, VA: American Statistical Association.

Franklin, C. A., Bargagliotti, A. E., Case, C. A., Kader, G. D., Scheaffer, R. L., & Spangler, D. A. (2015). *The statistical education of teachers*. Alexandria, VA: American Statistical Association.

Garfield, J., & Ben-Zvi, D. (2008). Preparing school teachers to develop students' statistical reasoning. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Garfield, J. B., & Everson, M. (2009). Preparing teachers of statistics: A graduate course for future teachers. *Journal of Statistics Education, 17*(2), 223–237. Retrieved from http://www.amstat.org/publications/jse/v17n2/garfield.pdf

Gattuso, L. (2008). Statistics in a mathematical context? In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Gfeller, M. K., Niess, M. L., & Lederman, N. G. (1999). Preservice teachers' use of multiple representations in solving arithmetic mean problems. *School Science and Mathematics, 99*(5), 250–257.

Godino, J. D., Ortiz, J. J., Roa, R., & Wilhelmi, M. R. (2011). Models for statistical pedagogical knowledge. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (pp. 271–282). Dordrecht, The Netherlands: Springer.

González, M. T., & Pinto, J. (2008). Conceptions of four pre-service teachers on graphical representation. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

González, O. (2014). Secondary mathematics teachers' professional competencies for effective teaching of variability-related ideas: A Japanese case study. *Statistique et Enseignment, 5*(1), 31–51.

Goodchild, S., & Sriraman, B. (2012). Revisiting the didactic triangle: From the particular to the general. *ZDM, 44*(5), 581–585.

Green, J. L. (2010). Teaching highs and lows: Exploring university teaching assistants' experiences. *Statistics Education Research Journal, 9*(2), 108–122. Retrieved from https://www.stat.auckland.ac.nz/~iase/serj/SERJ9(2)_Green.pdf

Green, J. L., & Blankenship, E. E. (2014). Beyond calculations: Fostering conceptual understanding in statistics graduate teaching assistants. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Groth, R. E. (2007). Toward a conceptualization of statistical knowledge for teaching. *Journal for Research in Mathematics Education, 38*(5), 427–437.

Groth, R. E. (2010). Teachers' construction of learning environments for conditional probability and independence. *International Electronic Journal of Mathematics Education, 5*, 32–55. Retrieved from http://mathedujournal.com/dosyalar/IJEM_v5n1_2.pdf
Groth, R. E. (2013). Characterizing key developmental understandings and pedagogically powerful ideas within a statistical knowledge for teaching framework. *Mathematical Thinking and Learning, 15*(2), 121–145.
Groth, R. E. (2014). Prospective teachers' procedural and conceptual knowledge of mean absolute deviation. *Investigations in Mathematics Learning, 6*(3), 51–69.
Groth, R. E., & Bergner, J. A. (2005). Pre-service elementary school teachers' metaphors for the concept of statistical sample. *Statistics Education Research Journal, 4*(2), 27–42. Retrieved from https://www.stat.auckland.ac.nz/~iase/serj/SERJ4(2)_groth_bergner.pdf
Groth, R. E., & Bergner, J. A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning, 8*, 37–63.
Groth, R. E., Bergner, J. A., Burgess, C. R., Austin, J. W., & Holdai, V. (2016). Re-imagining education of mathematics teachers through undergraduate research. *Council on Undergraduate Research (CUR) Quarterly, 36*(3), 41–46.
Groth, R. E., Kent, K., & Hitch, E. (2015). Journey to centers in the core. *Mathematics Teaching in the Middle School, 21*, 295–302.
Hammerman, J. K., & Rubin, A. (2004). Strategies for managing statistical complexity with new software tools. *Statistics Education Research Journal, 3*(2), 17–41. Retrieved from https://www.stat.auckland.ac.nz/~iase/serj/SERJ3(2)_Hammerman_Rubin.pdf
Hannigan, A., Gill, O., & Leavy, A. M. (2013). An investigation of prospective secondary mathematics teachers' conceptual knowledge of and attitudes towards statistics. *Journal of Mathematics Teacher Education, 16*(6), 427–449.
Harrell-Williams, L., Sorto, M. A., Pierce, R. L., Lesser, L. M., & Murphy, T. J. (2014). Validation of scores from a new measure of pre-service teachers' self-efficacy to teach statistics in the middle grades. *Journal of Psychoeducational Assessment, 32*(1), 40–50.
Heaton, R. M., & Mickelson, W. T. (2002). The learning and teaching of statistical investigation in teaching and teacher education. *Journal of Mathematics Teacher Education, 5*(1), 35–59.
Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.
Jacobbe, T. (2012). Elementary school teachers' understanding of mean and median. *International Journal of Science and Mathematics Education, 10*, 1143–1161.
Jacobbe, T., & Horton, R. M. (2010). Elementary school teachers' comprehension of data displays. *Statistics Education Research Journal, 9*(1), 27–45. Retrieved from https://www.stat.auckland.ac.nz/~iase/serj/SERJ9%281%29_Jacobbe_Horton.pdf
Koehler, M. J., & Mishra, P. (2008). Introducing TPCK. In AACTE Committee on Innovation and Technology (Ed.), *Handbook of technological pedagogical content knowledge (TPCK) for educators* (pp. 3–29). New York: Routledge.
Konold, C., & Miller, C. (2011). Tinker plots (Version v2.0) [Computer software]. Emeryville, CA: Key Curriculum Press.
Kuntze, S. (2014). Teachers' views related to goals of the statistics classroom – from global to content-specific. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics (ICOTS 9, July, 2014), Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.
Leavy, A., & O'Loughlin, N. (2006). Preservice teachers' understanding of the mean: Moving beyond the arithmetic average. *Journal of Mathematics Teacher Education, 9*, 53–90.
Leavy, A. M. (2006). Using data comparison to support a focus on distribution: Examining preservice teachers' understandings of distribution when engaged in statistical inquiry. *Statistics Education Research Journal, 5*(2), 89–114. Retrieved from http://iase-web.org/documents/SERJ/SERJ5(2)_Leavy.pdf

Leavy, A. M. (2010). The challenge of preparing preservice teachers to teach informal inferential reasoning. *Statistics Education Research Journal, 9*(1), 46–67. Retrieved from http://iase-web.org/documents/SERJ/SERJ9(1)_Leavy.pdf

Leavy, A. M. (2015). Looking at practice: Revealing the knowledge demands of teaching data handling in the primary classroom. *Mathematics Education Research Journal, 27*(3), 283–309.

Leavy, A. M., Hannigan, A., & Fitzmaurice, O. (2013). If you're doubting yourself, what's the fun in that? An exploration of why prospective secondary mathematics teachers perceive statistics as difficult. *Journal of Statistics Education, 21*(3), n3. Retrieved from www.amstat.org/publications/jse/v21n3/leavy.pdf

Lee, H., & Hollebrands, K. (2008). Preparing to teach mathematics with technology: An integrated approach to developing technological pedagogical content knowledge. *Contemporary Issues in Technology and Teacher Education, 8*(4), 326–341. Retrieved from http://www.citejournal.org/vol8/iss4/mathematics/article1.cfm

Lee, H. S., & Hollebrands, K. F. (2011). Characterising and developing teachers' knowledge for teaching statistics with technology. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (pp. 359–369). Dordrecht, The Netherlands: Springer.

Lee, H. S., Kersaint, G., Harper, S. R., Driskell, S. O., Jones, D. L., Leatham, K. R., et al. (2014). Teachers' use of transnumeration in solving statistical tasks with dynamic statistical software. *Statistics Education Research Journal, 13*(1), 25–52. Retrieved from http://iase-web.org/documents/SERJ/SERJ13%281%29_Lee.pdf

Lee, H. S., & Lee, J. T. (2011). Enhancing prospective teachers' coordination of center and spread: A window into teacher education material development. *The Mathematics Educator, 21*(1), 33–47.

Lee, H. S., & Mojica, G. S. (2008). Examining how teachers' practices support statistical investigations. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Liu, Y., & Thompson, P. W. (2009). Mathematics teachers' understandings of proto-hypothesis testing. *Pedagogies, 4*(2), 126–138.

Madden, S. R. (2011). Statistically, technologically, and contextually provocative tasks: Supporting teachers' informal inferential reasoning. *Mathematical Thinking and Learning, 13*(1-2), 109–131.

Maher, N., & Muir, T. (2014). "I don't really understand probability at all": Final year pre-service teachers' understanding of probability. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 437–444). Sydney: MERGA.

Makar, K. (2004). *Developing statistical inquiry: Prospective secondary mathematics and science teachers' investigations of equity and fairness through analysis of accountability data*. Unpublished doctoral dissertation, University of Texas at Austin, Austin.

Makar, K., & Confrey, J. (2002). Comparing two distributions: Investigating secondary teachers' statistical thinking. In B. Phillips (Ed.), *Proceedings of the 6th International Conference on Teaching Statistics [CD-ROM]*. International Statistical Institute and International Commission on Mathematical Instruction: Voorburg, The Netherlands.

Makar, K., & Confrey, J. (2004). Secondary teachers' statistical reasoning in comparing two groups. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 353–374). Boston: Kluwer Academic.

Makar, K., & Confrey, J. (2005). "Variation-talk": Articulating meaning in statistics. *Statistics Education Research Journal, 4*(1), 27–54. Retrieved from http://iase-web.org/documents/SERJ/SERJ4(1)_Makar_Confrey.pdf

Makar, K., & Fielding-Wells, J. (2011). Teaching teachers to teach statistical investigations. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics:*

Challenges for teaching and teacher education (pp. 347–358). Dordrecht, The Netherlands: Springer.

Martins, J. A., Nascimento, M. M., & Estrada, A. (2012). Looking back over their shoulders: A qualitative analysis of Portuguese teachers' attitudes towards statistics. *Statistics Education Research Journal, 11*(2), 26–44. Retrieved from http://iase-web.org/documents/SERJ/SERJ11(2)_Martins.pdf

Maxara, C., & Biehler, R. (2010). Students' understanding and reasoning with sample size and the law of large numbers after a computer-intensive introductory course on stochastics. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society. Proceedings of the 8th International Conference on Teaching Statistics, Ljubljana, Slovenia*. Voorburg, The Netherlands: International Statistical Institute.

McLeod, D. B. (1992). Research on affect in mathematics: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: MacMillan.

Meletiou-Mavrotheris, M., Kleanthous, I., & Paparistodemou, E. (2014). Developing pre-service teachers' technological pedagogical content knowledge (TPACK) of sampling. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Meletiou-Mavrotheris, M., Mavrotheris, E., & Paparistodemou, E. (2011). Distance learning for teacher professional development in statistics education. *Teaching Statistics, 33*(1), 2–8.

Meletiou-Mavrotheris, M., Paparistodemou, E., & Stylianou, D. (2009). Enhancing statistics instruction in elementary schools: Integrating technology in professional development. *The Montana Mathematics Enthusiast, 16*(1&2), 57–78.

Meletiou-Mavrotheris, M., & Serradó, A. (2012). Distance training of mathematics teachers: The EarlyStatistics experience. *Universities and Knowledge Society Journal (RUSC), 9*(1), 340–353.

Mickelson, W., & Heaton, R. (2004). Primary teachers' statistical reasoning with data. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 327–352). Dordrecht, The Netherlands: Kluwer.

Monteiro, C., & Ainley, J. (2007). Investigating the interpretation of media graphs among student teachers. *International Electronic Journal of Mathematics Education, 2*(3), 188–207. Retrieved from http://www.mathedujournal.com/dosyalar/IJEM_v2n3_5.pdf

Mooney, E., Duni, D., van Meenen, E., & Langrall, C. (2014). Preservice teachers' awareness of variability. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Moore, D. S. (1988). Should mathematicians teach statistics? *The College Mathematics Journal, 19*(1), 3–7.

Nasser, F. M. (2004). Structural model of the effects of cognitive and affective factors on the achievement of Arabic-speaking prospective teachers in introductory statistics. *Journal of Statistics Education, 12*(1), 1–28. Retrieved from http://www.amstat.org/publications/jse/v12n1/nasser.html

National Governors Association for Best Practices & Council of Chief State School Officers. (2010). Common core state standards for mathematics. Retrieved from http://www.corestandards.org/

Nolan, M. M., Beran, T., & Hecker, K. G. (2012). Surveys assessing students' attitudes toward statistics: A systematic review of validity and reliability. *Statistics Education Research Journal, 11*(2), 103–123. Retrieved from https://www.stat.auckland.ac.nz/~iase/serj/SERJ11(2)_Nolan.pdf

Noll, J., & Shaughnessy, J. M. (2012). Aspects of students' reasoning with variation in empirical sampling distributions. *Journal for Research in Mathematics Education, 43*(5), 509–556.

Noll, J. A. (2011). Graduate teaching assistants' statistical content knowledge of sampling. *Statistics Education Research Journal, 10*(2), 48–74. Retrieved from https://www.stat.auckland.ac.nz/~iase/serj/SERJ10(2)_Noll.pdf

Onwuegbuzie, A. J. (2000). Statistics anxiety and the role of self-perceptions. *Journal of Educational Research, 93*(5), 323–335.

Paparistodemou, E., Potari, D., & Pitta, D. (2006). Prospective teachers' awareness of young children's stochastic activities. In A. Rossman & B. Chance (Eds.), *Proceedings of the 7th International Conference on Teaching Statistics*. Salvador, Brazil: International Statistical Institute and International Association for Statistical Education.

Peters, S. A. (2009). *Developing an understanding of variation: AP statistics teachers' perceptions and recollections of critical moments*. Unpublished doctoral dissertation, The Pennsylvania State University, University Park, PA.

Peters, S. A., Watkins, J. D., & Bennett, V. M. (2014). Middle and high school teachers' transformative learning of center. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Pfannkuch, M. (2006). Comparing box plot distributions: A teacher's reasoning. *Statistics Education Research Journal, 5*(2), 27–45. Retrieved from http://iase-web.org/documents/SERJ/SERJ5%282%29_Pfannkuch.pdf

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Reston, VA: National Council of Teachers of Mathematics.

Pierce, R., & Chick, H. (2013). Workplace statistical literacy for teachers: Interpreting box plots. *Mathematics Education Research Journal, 25*(2), 189–205.

Pierce, R., & Chick, H. L. (2011). Teachers' beliefs about statistics education. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (pp. 151–162). Dordrecht, The Netherlands: Springer.

Quintas, S., Ferreira, R. T., & Oliveira, H. (2014). Attending to students' thinking on bivariate statistical data at secondary level: Two teachers' pedagogical content knowledge. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, Flagstaff, AZ, USA*. Voorburg, The Netherlands: International Statistical Institute.

Reston, E. (2012). *Exploring inservice elementary mathematics teachers' conceptions of probability through inductive teaching and learning methods*. Paper presented in the 12th International Congress on Mathematical Education, 8–15 July, 2012, COEX, Seoul, Korea.

Reston, E., Jala, L., & Edullantes, T. (2006). Probing college statistics teachers' instructional goals and classroom practices within a statistical literacy framework. In A. Rossman & B. Chance (Eds.), *Working cooperatively in statistics education: Proceedings of the 7th International Conference on Teaching Statistics, Salvador, Brazil*. International Association for Statistical Education and International Statistical Institute: Voorburg, The Netherlands.

Roback, P., Chance, B., Legler, J., & Moore, T. (2006). Applying Japanese lesson study principles to an upper-level undergraduate statistics course. *Journal of Statistics Education, 14*(2.) Retrieved from http://www.amstat.org/publications/jse/v14n2/roback.html

Rossman, A., Chance, B., & Medina, E. (2006). Some important comparisons between statistics and mathematics and why teachers should care. In G. F. Burrill & P. C. Elliot (Eds.), *Thinking and reasoning with data and chance: 68th yearbook* (pp. 323–333). Reston, VA: National Council of Teachers of Mathematics.

Saderholm, J., Ronau, R., Brown, E. T., & Collins, G. (2010). Validation of the diagnostic teacher assessment of mathematics and science (DTAMS) instrument. *School Science and Mathematics, 110*(4), 180–192.

Schau, C., Stevens, J., Dauphine, T., & del Vecchio, A. (1995). The development and validation of the survey of attitudes toward statistics. *Educational and Psychological Measurement, 55*(5), 868–875.

Scheaffer, R. L. (2006). Statistics and mathematics: On making a happy marriage. In G. F. Burrill & P. C. Elliot (Eds.), *Thinking and reasoning with data and chance: 68th yearbook* (pp. 309–321). Reston, VA: National Council of Teachers of Mathematics.

Sedlmeier, P., & Wassner, C. (2008). German mathematics teachers' views on statistics education. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Serradó, A., Azcárate, P., & Cardeñoso, J. M. (2006). Analyzing teacher resistance to teaching probability in compulsory education. In A. Rossman & B. Chance (Eds.), *Proceedings of the 7th International Conference on Teaching Statistics*. International Statistical Institute and International Association for Statistical Education: Salvador, Brazil. Retrieved from http://www.stat.auckland.ac.nz/~iase

Serradó, A., Meletiou-Mavrotheris, M., & Paparistodemou, E. (2014). Early statistics: A course for developing teachers' statistics technological and pedagogical content. *Statistique et Enseignement, 5*(1), 5–29.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–23.

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education, 11*(6), 499–511.

Simon, M. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. *Mathematical Thinking and Learning, 8*(4), 359–371.

Sorto, M. A. (2004). *Prospective middle school teachers' knowledge about data analysis and its application to teaching*. Unpublished doctoral dissertation, Michigan State University.

Sullivan, P., & Mousley, J. (2001). Thinking teaching: Seeing mathematics teachers as active decision makers. In F.-L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 147–163). Dordrecht, The Netherlands: Kluwer.

Tarr, J. E., Reys, R. E., Reys, B. J., Chávez, Ó., Shih, J., & Osterlind, S. J. (2008). The impact of middle-grades mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education, 39*(3), 247–280.

Theis, L., & Savard, A. (2010). Linking probability to real-world situations: How do teachers make use of the mathematical potential of simulation programs? In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society. Proceedings of the 8th International Conference on Teaching Statistics, Ljubljana, Slovenia*. Voorburg, The Netherlands: International Statistical Institute.

Thompson, P. W., & Liu, Y. (2005). Understandings of margin of error. In S. Wilson (Ed.), *Proceedings of the 27th Annual Meeting of the International Group for the Psychology of Mathematics Education*. Virginia Polytechnic and State Institute: Vicksburg, VA.

Venkat, H., & Adler, J. (2014). Pedagogical content knowledge in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 477–480). Dordrecht, The Netherlands: Springer.

Vermette, S. (2013). *Le concept de variabilité chez des enseignants de mathématiques au secondaire*. Unpublished doctoral dissertation, Université de Sherbrooke, Sherbrooke.

Wassong, T., & Biehler, R. (2010). A model for teacher knowledge as a basis for online courses for professional development of statistics teachers. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society. Proceedings of the 8th International Conference on Teaching Statistics. Ljubljana, Slovenia*.

Watson, J. M. (2001). Profiling teachers' competence and confidence to teach particular mathematics topics: The case of chance and data. *Journal of Mathematics Teacher Education, 4*(4), 305–337.

Watson, J. M., Callingham, R., & Donne, J. (2008). Establishing pedagogical content knowledge for teaching statistics. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and* *teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Monterrey: ICMI and IASE. Retrieved from http://www.ugr.es/~icmi/iase_study

Watson, J. M., & Nathan, E. L. (2010). Approaching the borderlands of statistics and mathematics in the classroom: Qualitative analysis engendering an unexpected journey. *Statistics Education Research Journal, 9*(2), 69–87. Retrieved from http://iase-web.org/documents/SERJ/SERJ9%282%29_Watson_Nathan.pdf

Wilson, P. H., Lee, H. S., & Hollebrands, K. F. (2011). Understanding prospective mathematics teachers' processes for making sense of students' work with technology. *Journal for Research in Mathematics Education, 42*(1), 42–67.

Yang, K.-L. (2014). An exploratory study of Taiwanese mathematics teachers' conceptions of school mathematics, school statistics, and their differences. *International Journal of Science and Mathematics Education, 12*(6), 1497–1518.

Zieffler, A., Park, J., Garfield, J., del Mas, R., & Bjornsdottir, A. (2012). The statistics teaching inventory: A survey on statistics teachers' classroom practices and beliefs. *Journal of Statistics Education, 20*(1), 1–29. Retrieved from http://www.amstat.org/publications/jse/v20n1/zieffler.pdf