A New Decomposition Approach for the Home Health Care Problem

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Abstract Home Health Care (HHC) is a relatively new service that plays an important role to reduce hospitalization costs and improve the life quality for patients. Human resource planning is one of the most important processes in HHC systems, for which service providers have to deal with several operational problems, e.g., the assignment of operators to patients together with their routing process. In the literature, either these problems have been simultaneously solved, or decomposed by first solving the assignment problem and then the routing problem. In this work, we propose an alternative approach, where the decomposition is based on the *First Route and Second Assign* (FRSA) approach. An instance generation mechanism is developed as well, which generates instances inspired from real HHC providers, to test the proposed FRSA approach under different circumstances. Preliminary experiments show the effectiveness of the approach.

Keywords Home health care · Human resource planning · Matheuristic decomposition · First route second assign

1 Introduction

Home Health Care (HHC) is a relatively new service that plays an important role to reduce hospitalization costs and improve the life quality for patients, who receive service at their homes. Due to population ageing and high hospitalization costs, the

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demand for HHC service is increasing all around the world. In 2011, there were about 4.7 million patients in the U.S. and 1 million patients in Canada who were served by several HHC providers [7].

To balance the trade-off between cost and quality of service in HHC, providers need to deal with several optimization problems. Among them, we focus in this work on the patient assignment problem and the nurse (operator) routing problem. The first consists of matching patients with operators, while the latter determines the sequence of visits assigned to each nurse. These problems may be solved for a single period or multiple periods; in this work, we focus on the weekly problem. Both the assignment and the routing problems are handled by taking into account several features, such as patient requirements (frequency of visits), expected duration of each visit, possible visiting schedules (patterns) for patients, continuity of care, and nurse capacities.

We propose a new two-stage approach for the assignment and routing problem, which exploits a new concept for the HHC services, where routing decisions anticipate assignment decisions. In other words, we decompose the problem by deciding the routing at the first stage and the assignments at the second stage. Although this might seem counterintuitive, the variety of contexts in which HHC is provided (e.g., urban vs rural, dense vs sparse) legitimate to investigate the trade-offs for which focusing on the routing at the first stage can be beneficial. To validate the approach, as different HHC providers have different structures and cover different areas, we create a data generation mechanism to generate test instances which are able to mimic several situations of real HHC providers from different countries.

The reminder of this paper is organized as follows. A brief literature review and the problem description are presented in Sect. 2. The proposed methodology is described in Sect. 3. The data generation mechanism with some preliminary results are discussed in Sect. 4. Then, concluding remarks with future perspectives are presented in Sect. 5.

2 Problem Statement and Related Work

As discussed briefly in the previous section, the HHC assignment problem refers to the decision of matching nurses with patients, while the routing problem specifies visiting sequences of patients associated with each nurse. Several works related to these problems have been classified and discussed in a recent literature review [7]. Here, we present a short list of these works and classify them according to the length of the planning period (i.e., Single Period or Multiple Periods) and how the assignment and routing decision are held (i.e., Simultaneously or Sequentially).

The literature is mainly devoted to the simultaneous approach, where assignment and routing decision are obtained together in a single model (Vehicle Routing Problem) both for a single period [1, 6] and multiple periods [3, 9].

Moreover, due to computational complexity and operational flexibility, recent models based on two-stage *First Assign and Second Route* (FASR) approaches have been developed, where the output of the patient assignment problem is integrated as an input to the routing problem of each nurse (Traveling Salesman Problem). Both single planning period [11] and multiple planning period cases [10] have been investigated.

In this work, different from the HHC literature, we develop a new multiple periods two-stage *First Route and Second Assign* (FRSA) approach. Since different providers cover different areas (rural vs urban, different densities, etc.), such a model could be beneficial where travels are the key issue. More generally, with the development of the FRSA approach, a complete analysis can be conducted and the most appropriate approach can be selected depending on the trade-offs and the corresponding key issues of a given provider.

3 Methodology

We use the following notation:

- N: set of nurses, $N = \{1...n\}$;
- P: set of patients, $P = \{1...m\}$;
- D: set of days.
- N_d: subset of nurses available on day d;
- a_i: capacity of nurse i (duration of a workday including service time to provide visits and travel times);
- r_j: total number of visits required by patient j in horizon D;
- H: set of patterns (patterns are defined as the days in which the patient may be visited, e.g., for a frequency of two visits, patterns are: Monday and Thursday, Tuesday and Friday, …);
- H_j: subset of patterns for patient j;
- t_j: service time for each visit to patient j;
- c_{jk}: cost (time, distance, etc.) between patient j and patient k.

We structure our two-stage FRSA approach as follows:

- **Stage 1: Routing problem** modeled as a Periodic Vehicle Routing Problem (PVRP):
  - total travel time minimization with respect to the overall capacity of nurses is pursued;
  - no nurse-to-patient assignment information is considered;
  - patients are assigned a pattern.

Only the decisions regarding the pattern assignment to patients are kept for the next stage.

- **Stage 2: Nurse-to-patient assignment problem** consists of splitting the giant tours obtained in stage 1, with the objective of minimizing the maximum workload. At this stage, decisions regarding nurse-to-patient assignments are made.

3.1 *Stage 1: Route First*

We model the HHC as a PVRP in which one vehicle only services all patients. We use the typical route duration constraint for each day, adding up all nurses' capacities a_i. Since the number of nurses is not explicitly considered, no attention is paid to the fact that the nurses' individual travel distances must be minimized after the assignments. To account for this (and to ensure that the territory is covered every day), we add a step before solving the PVRP to create *seeds* that will tie each nurse to a physical territory.

(a) Creation of seeds

The objective is to select n seeds (as many as there are nurses) with a large distance between them. They are selected using the existing Basic Territory Units (BTUs). Let $x_s = 1$ if BTU s is chosen to host a seed (and 0 otherwise) and $y_{s_1 s_2} = 1$ if both s_1 and s_2 are chosen to host a seed (and 0 otherwise). $c_{s_1 s_2}$ is the cost (time, distance, etc.) between the centers of BTUs s_1 and s_2. We solve the following problem:

$$\max \quad \sum_{s_1} \sum_{s_2} c_{s_1 s_2}\, y_{s_1 s_2} \tag{1}$$

s.t.

$$\sum_{s \in S} x_s = n \qquad \forall s \in S \tag{2}$$

$$\sum_{s_2 \in S} y_{s_1 s_2} \leq n x_{s_1} \qquad \forall s_1 \in S \tag{3}$$

$$y_{s_1 s_2} \leq x_{s_2} \qquad \forall s_1, s_2 \in S \tag{4}$$

$$x_s \in \{0, 1\} \qquad \forall s \in S$$

$$y_{s_1 s_2} \in \{0, 1\} \qquad \forall s_1, s_2 \in S$$

Constraints (2) force to choose n BTUs, while constraints (3) and (4) ensure that $y_{s_1 s_2} = 1$ only if both s_1 and s_2 are chosen. The objective (1) is to maximize the distance between the chosen BTUs.

Once the n BTUs are selected, we choose in each of them the patient with the highest number of visits; if there are several equivalent patients, we select the farthest from the depot. For this subset of patients $\tilde{P} \subset P$ chosen as seeds, we impose a frequency of visits equal to $|D|$ for solving the PVRP, while for all others we use the frequency provided for by parameter r_j.

(b) The PVRP algorithm

We use a tabu search algorithm (sketched in Algorithm 1 and based on [5]) that proves to perform very well on the PVRP. Two movements are used to explore the solutions space, i.e., *changing the day combination* or *reinserting on a different route*.

The algorithm allows visiting unfeasible solutions but penalizes the objective function whenever an unfeasible solution is met. These penalties are dynamically adapted during the search. Without loss of generality, the initial solution can be constructed randomly or based on constructive heuristics. Readers are referred to [5] for details.

Algorithm 1 Major lines of the PVRP algorithm

Generate initial solution: patients are assigned a valid pattern of visits and sorted in increasing order of the angle to the depot. Insertion into a route follows this order:
while Stopping criteria not reached **do**
  for each patient **do**
    for each day **do**
      Search for the best insertion using the defined movements;
      Implement best non tabu movement unless aspiration criteria is met;
      Adapt dynamically penalty parameters;
      Update tabu list and statistics.
    end for
  end for
end while

After solving the PVRP, each patient j is assigned to a pattern $p_j \in H_j$.

3.2 Stage 2: Solving the Assignment Problem

After solving the PVRP, we obtain $|D|$ routes (one per day) that need to be *split* into n segments (one for each nurse). The splitting procedure must respect the continuity of care, i.e., each patient is assigned to exactly one nurse; for this purpose, the splitting is performed in parallel for all routes.

The procedure is summarized in Algorithm 2. We denote by $\boldsymbol{x}$ the solution obtained after the splitting procedure, and by $m_u(\boldsymbol{x})$ the maximum average utilization. $\boldsymbol{x}^*$ and $m_u(\boldsymbol{x}^*)$ refer to the best solution and its value, respectively. Initially, $m_u(\boldsymbol{x}^*)$ and $m_u(\boldsymbol{x})$ are set to a high value. The idea is inspired by the sweep algorithm where a random angle ω is generated, and patients are collected counter clockwise until a_i is reached. If the solution is infeasible, i.e. the last nurse is overloaded, we introduce parameter β_1 equal to this observed overload divided by n, ensuring to spread the "infeasibility" among all nurses. We repeat this procedure with $a_i += \beta_1$ until the solution is feasible.

We illustrate how this procedure can be visualized for a given day in Fig. 1. Full line represents the starting point for the collection of patients and the dotted line the point where the capacity of the nurse is reached.

Once the splitting procedure is complete, an assignment of patients to nurses is obtained (i_j denotes the nurse assigned to patient j). We add a step to revisit the assignment of a subset of patients either because the solution is not feasible after the

Algorithm 2 Description of the splitting procedure

$m_u(\boldsymbol{x}^*) = m_u(\boldsymbol{x}) = na_i$
for all $|D|$ routes resulting from PVRP: **do**
  Generate ω degrees (points);
  Select a random point;
  Order the nurses (according to: seniority, preferences or randomly);
  for $i = 1$ to n **do**
    for all routes corresponding to the days nurse i works; **do**
      while capacity of nurse is not reached **do**
        collect patients when pattern of visits and days of work coincide.
      end while
      if the problem is infeasible for at least one nurse: increase capacity by parameter β_1 and repeat the collection process starting from nurse 1.
    end for
    If $m_u(\boldsymbol{x}) < m_u(\boldsymbol{x}^*)$: $\boldsymbol{x}^* \leftarrow \boldsymbol{x}$ and $m_u(\boldsymbol{x}^*) = m_u(\boldsymbol{x})$.
  end for
end for

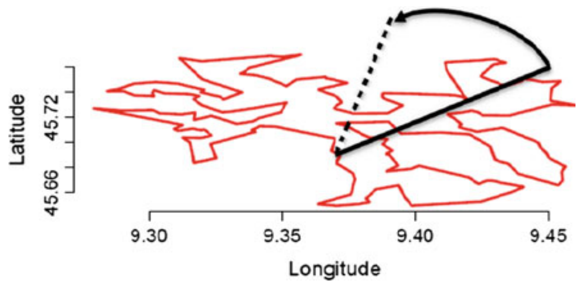

Fig. 1 Illustration of the splitting procedure

splitting procedure, or as a post-optimization step. We create the set of patients P' to be reassigned as follows:

1. Include in P' γ patients from the beginning and γ patients from the end of each nurse's route. The larger the value of γ is, the more we destroy the routing solution obtained in the previous stage.
2. Include in P' all patients close to the depot.
3. Include in P' all seeds in $\tilde{P}$.

Patients belonging to $P \backslash P'$ generate an initial fixed workload for each nurse.

To solve the assignment model, we use the formulation in [10]. For each patient j, we define τ_j as the average traveling time to reach him. Moreover, five decision variables are defined:

- $u_{ij} = 1$ if nurse i is assigned to patient j (and 0 otherwise);
- $u_{ij}^d = 1$ if nurse i visits patient j on day d (and 0 otherwise);
- $z_{jp} = 1$ if pattern p is assigned to patient j (and 0 otherwise);
- W_{id}: workload of nurse i in day d;
- m_u: maximum of the average utilization over D among the nurses.

The mathematical formulation of the problem is the following:

$$\min \quad m_u \tag{5}$$

s.t.

$$\sum_{i \in N} u_{ij} = 1 \qquad \forall j \in P \tag{6}$$

$$\sum_{p \in H_j} z_{jp} = 1 \qquad \forall j \in P \tag{7}$$

$$W_{id} = \sum_{j \in P} (t'_j + \tau_j) \cdot u^d_{ij} \le a_i \qquad \forall i \in N_d, \forall d \in D \tag{8}$$

$$\sum_{i \in N} u^d_{ij} = \sum_{p:p(d)=1} z_{jp} \qquad \forall j \in P, \forall d \in D \tag{9}$$

$$u^d_{ij} \le u_{ij} \qquad \forall i \in N_d, \forall j \in P, \forall d \in D \tag{10}$$

$$\sum_{i \in N} \sum_{d \in D} u^d_{ij} = r_j \qquad \forall j \in P \tag{11}$$

$$\frac{\sum_{d \in D} W_{id}}{|D| \cdot a_i} \le m_u \qquad \forall i \in N \tag{12}$$

$$u_{i_j j} = 1 \qquad \forall j \in P \backslash P' \tag{13}$$

$$z_{jp_j} = 1 \qquad \forall j \in P \backslash P' \tag{14}$$

$$u_{ij} \in \{0, 1\} \qquad \forall i \in N, j \in P \tag{15}$$

$$u^d_{ij} \in \{0, 1\} \qquad \forall i \in N, \forall j \in P, \forall d \in D \tag{16}$$

$$z_{jp} \in \{0, 1\} \qquad \forall j \in P, \forall p \in H \tag{17}$$

Constraints (6) decide the assignments. Constraints (7) are the scheduling constraints. Constraints (8) control the daily workload of nurses and use τ_j to estimate the travel time to reach patient j. Constraints (9) and (10) link together assignment and scheduling decisions; specifically, constraints (9) state that exactly one nurse per day must visit patient j only if a visit has been scheduled on that day for him/her ($p(d) = 1$ refers to patterns p having a visit on day d), and constraints (10) guarantee that a nurse can visit a patient only if she/he has been assigned to that patient. Constraints (12) link the maximum utilization m_u to the nurses' workloads.

As mentioned earlier, the usual 8 h per day may not allow to visit all patients; thus, parameter a_i in (8) should be carefully fixed. In our context, we solve an optimization problem to determine the smallest increase of this value that allows a feasible solution. We denote this value β_2, and $a_i = 8\,\text{h} + \beta_2$.

Two constraints are added to the general model when used for partial reassignment. Constraints (13) ensure that patients in $P \backslash P'$ are assigned to the nurse from the splitting procedure, and constraints (14) that they are assigned the pattern from the PVRP solution.

3.3 Obtaining the Operational Routes

Once the assignments to nurses and to patterns are obtained, a TSP is run for each day and each nurse to get the actual routes to perform and, thus, the actual daily workload for each nurse. We employ the genetic algorithm of [11], which provides equivalent performance to benchmark solvers, e.g., the *Concorde TSP Solver* [4], for the considered size of instances.

4 Data Generation and Preliminary Experiments

We create a mechanism to generate test instances, for the validation of our approach in a variety of situations inspired from real HHC providers. In particular, we use the characteristics of a large provider operating in the Northern Italy that has been already adopted for other analyses [2, 8].

Let us consider a test instance with m patients. We fix a geographical distribution of some BTUs, each one with a given center (latitude and longitude) and shape. Each patient j is randomly assigned to a BTU according to given probabilities to belong to each BTU; then, his/her coordinates are uniformly generated within the assigned BTU. A Care Profile (CP) is also assigned to each patient j, according to given probabilities to belong to each CP. Each CP is characterized by a range for the number of visits and an associated discrete distribution; once a patient is assigned to a CP, his/her demand is drawn from such distribution. Then, from the number of visits, the list H_j of all possible patterns is derived. The duration of each visit is finally generated from a uniform distribution within the interval [35, 45] minutes.

This mechanism is versatile and allows us to generate, for example, urban/rural instances by imposing high/low number of patients in a small/large territory. For example, we consider the average travel time $\overline{\tau}$ between patients as metrics to characterize an instance: $\overline{\tau} = 7$ min refers to an urban instance and $\overline{\tau} = 25$ min to a rural context.

An illustration of patient scattering in the territory is presented in Fig. 2.

We test the approach on the instances reported in Table 1. Details about their features are given in the first three columns. The following two columns show the solution obtained with the classical FASR decomposition in terms of maximum average utilization m_u and maximum workload W_{max} observed over the horizon. Finally, the last columns give the improvement (in %) when using the FRSA approach in two settings: only using the splitting, or using the splitting and the partial reassignment.

FRSA generally outperforms FASR, and in particular FRSA with partial reassignment outperforms FASR in all instances when comparing m_u. Moreover, results show that the partial reassignment step after the FRSA is very important when small distances are involved; however, the reassignment deteriorates the solutions when the distances are large. With partial reassignment, the improvement ranges between 0.37% and 1.61% when the distances are small ($\overline{\tau_j} = 7$), while the improvement is

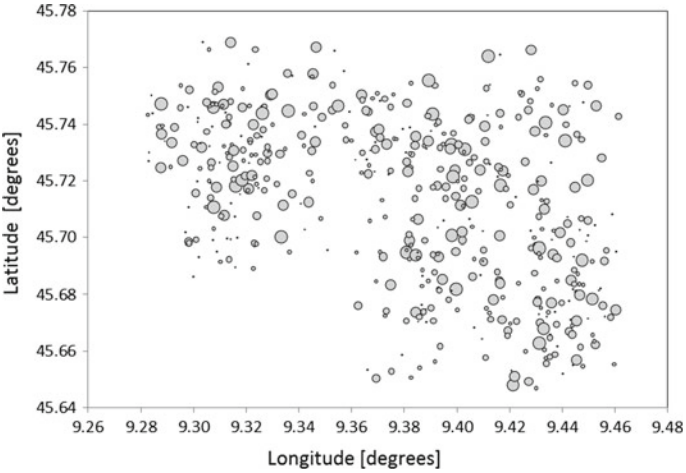

Fig. 2 Example of tested instance; each point represents a patient, whose size is proportional to the number of visits required in the time horizon

Table 1 Comparison of utilization rates and workloads (in %) obtained with FASR and FRSA

Instance			Classical FASR		FRSA: only splitting		FRSA: splitting & partial reassignment	
m	n	$\overline{\tau}$	m_u	W_{max}	m_u (%)	W_{max} (%)	m_u (%)	W_{max} (%)
300	18	7	90.51	95.26	+12.28	+9.14	**−1.25**	+6.60
			87.34	93.82	+7.53	+10.93	**−0.72**	+8.31
			94.71	96.03	+3.12	+10.39	**−1.61**	+6.04
300	20	7	82.63	94.91	+11.01	+9.87	**−0.37**	+8.64
			79.40	92.40	+22.85	+12.34	**−0.93**	+6.97
			85.60	94.64	+16.27	+12.06	**−0.76**	+6.35
300	18	25	113.60	117.44	**−4.64**	+1.61	−2.34	+1.00
			122.58	128.13	**−9.48**	**−7.51**	−2.95	+0.79
			118.27	121.07	**−11.54**	**−3.37**	−4.39	+0.91
300	20	25	106.26	111.04	**−4.41**	0.00	−3.05	+0.66
			114.31	117.35	**−9.04**	**−4.45**	−2.70	**−0.13**
			105.78	111.21	**−4.40**	+2.04	−2.15	**−0.27**

higher, up to the 11.54% without partial reassignments, when the distances are large ($\overline{\tau_j} = 25$). We remark that, in absolute terms, 1% improvement corresponds to 5 min approximately. As for W_{max}, we only observe an improvement with FRSA when the distances are large.

Finally, we may observe that m_u is higher than the 100% in some instances; in fact, for $\overline{\tau_j} = 25$, we considered cases in which the staff is overloaded to test both underloaded and overloaded situations.

5 Conclusion and Perspectives

In this work we propose an alternative decomposition for the HHC problem. While the literature usually proposes to first assign patients to nurses and then solve the routing problem, we first consider the routing problem and then solve the assignment problem. Moreover, to evaluate such approach in a variety of realistic cases, we define a versatile mechanism to generate instances. Preliminary experiments are promising, and seem to assess the appropriateness of our alternative decomposition.

In our future work, we will test additional instances considering in particular large settings. Finally, we will further compare our FRSA approach with the classical FASR decomposition available in the literature [10, 11], to deeply investigate for which types of instances our alternative decomposition performs better.

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