Push–Pull Converter Transformer Maximum Efficiency Optimization

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Abstract. The contribution describes a maximum efficiency design of a push–pull (full-bridge and half-bridge) DC/DC converter transformer based on straightforward analytical calculations. The input parameters for the optimization are transformer core size, switching frequency and power. The output parameter is the core peak-to-peak flux density, from which all other essential parameters of the transformer (numbers of turns, core loss, winding loss etc.) can be derived. The optimization contains some simplifications which could cause a small error, but the result is an easy-to-use and simple to implement analytical formula, which can be used to quickly design a well-performing transformer or evaluate multiple designs.

Keywords: High-frequency transformer · Efficiency optimization · Push–pull converter · Full-bridge converter · Switch-mode power supply

1 Introduction

The leading trend in power conversion today is increasing of efficiency. The designer of an isolated DC/DC switching converter or power supply is faced with a problem of transformer parameter selection. With a given core (and given transformer cost), one can choose less winding turns for decreased copper losses, but this will result in increasing of core losses, and vice versa. With the high switching frequencies used today (often over 100 kHz), designing the transformer with peak flux density close to saturation is generally not optimal, as this will result in large core losses at the high frequency. The purpose of this contribution is to present a design tool, which can be easily applied using simple analytical equations and which outputs optimal transformer parameters, a balance between winding and core losses, which gives the lowest total loss and in turn the highest efficiency. With given transformer size, these design equations can be applied to maximize the efficiency of the transformer with little or no additional material cost.

The optimization can be applied to any core shape (toroidal, EE/EI, etc.) and for a half-bridge or full-bridge converter with a secondary-side bridge rectifier (diode or synchronous). For schematics, see Fig. 1.

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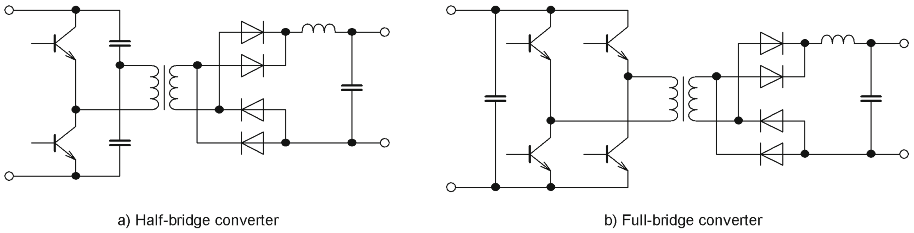

Fig. 1. (a) Half-bridge converter, (b) Full-bridge converter.

The size of the transformer core is given; if it was variable (subject of the optimization), then the core size would surely come out infinitely large for the lowest losses (approaching zero). The core size is based on the decision of the designer and acceptable costs. The optimization outputs losses of the transformer; if they come out too large or too small, another iteration of the optimization can be performed.

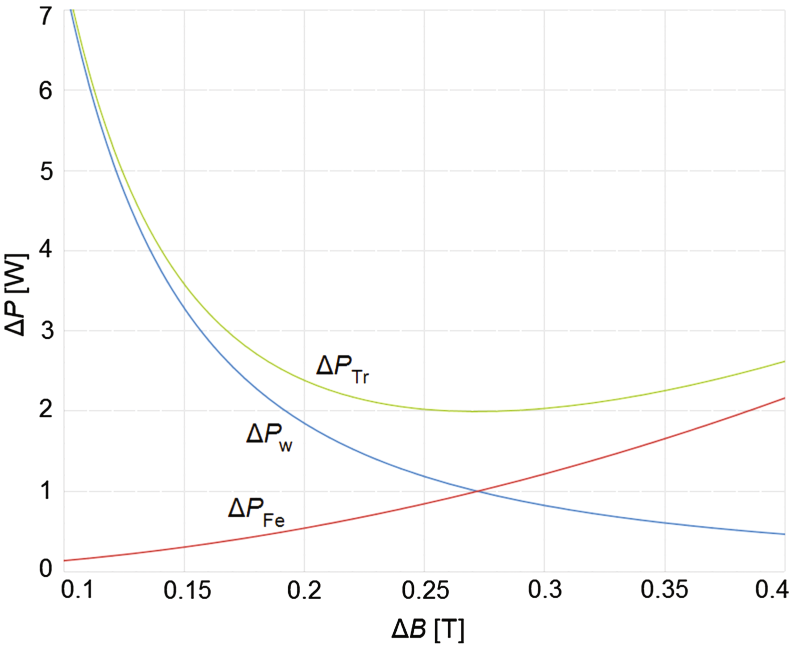

Fig. 2. ΔP_{w}, ΔP_{Fe} and ΔP_{Tr} versus ΔB

The common trend in power pulse converters nowadays is to use resonant and special topologies, see e.g. [1–3]. The classic full-bridge PWM topology described in this contribution has the advantage of seamless full controllability from zero to full load power and simple and robust control circuits. The disadvantage is slightly lower efficiency compared to modern topologies, but this disadvantage can be partlially compensated by optimal transformer design, as described in this contribution. Efficiency optimizations of the high-frequency transformer were performed in the past, see e.g. [1, 4, 5]. These optimizations however are based on different converter topologies, involve numerical methods, or are complex overall. In [6, 7], the design procedures are analytical, but the calculations encompass a much wider problem (including e.g. thermal design, eddy current calculations etc.), which result in more accurate, but much more complex equations. The purpose of this contribution is to present simple and easy to implement equations, however with some simplifications.

2 Definition of Parameters

2.1 Input Parameters

The input parameters for the optimization are following:

- Transformer core cross-section A_{Fe} [m^2];
- Transformer window area A_{w} [m^2];
- Winding copper fill factor k_{w} [-];
- Mean turn length l_{t} [m];
- Copper conductivity ρ_{Cu} [Ωm];
- Input power P [W];
- Switching frequency f [Hz];
- Switching duty cycle of one transistor d [-];
- reference core loss of the transformer $\Delta P_{\mathrm{Fe,ref}}$ [W] at a reference peak-to-peak flux density ΔB_{ref} [T] and reference frequency f_{ref} [Hz].

The “reference” values are provided in the datasheet of the core (core loss at a given frequency and flux density). If only datasheet of the material is available, the volume specific power loss must be recalculated for a volume of the given core.

Note that the input and output voltages and currents are not required for the optimization, only the power P.

2.2 Output Parameters

The optimization is based on finding an optimum value of peak-to-peak flux density ΔB_{opt} [T]. From this, core and winding loss can easily be calculated (from an intermediate step of the optimization) and by knowing the input and output voltages, the number of turns of the transformer can easily be calculated using standard formulas.

3 Simplifications

To keep the optimization straightforward, some simplifications were assumed during the optimization process. The designer should be aware of these simplifications as they might cause a substantial error in marginal cases. However, for most of the time, these simplifications should not degrade the result substantially.

These simplifications are:

- Eddy current loss of the core is omitted, as it is very small in standard power ferrite materials.
- Hysteresis energy loss per cycle is taken as proportional to the square of peak-to-peak flux density ΔB [T]. This is true if the shape of the hysteresis curve remains unchanged. For values of peak flux density close to saturation, there can be some inaccuracy, but for common values of peak flux density at high frequencies, the error caused should be small.
- The skin and proximity effects are not taken into account. By using a HF Litz-wire, the influence of these effects can be decreased down to an acceptable level. If required, these effects can be included in the increase of copper conductivity.

4 Optimization Procedure

The total power losses of the transformer ΔP_{Tr} consist of core losses ΔP_{Fe} and winding losses ΔP_{w}:

$$\Delta P_{\mathrm{Tr}} = \Delta P_{\mathrm{Fe}} + \Delta P_{\mathrm{w}} \tag{1}$$

The goal of the optimization is to find a minimum value of ΔP_{Tr}.

4.1 Winding Losses

The winding losses can be written as

$$\Delta P_{\mathrm{w}} = RI^2 = 2 \cdot \rho_{\mathrm{Cu}} \cdot \frac{l_{\mathrm{t}} \cdot N_1}{A_{\mathrm{Cu},1}} \cdot I^2_{\mathrm{rms},1}, \tag{2}$$

where N_1 is the number of turns of the primary winding, $A_{\mathrm{Cu},1}$ is the copper cross-section of primary conductor and $I_{\mathrm{rms},1}$ is the rms current of the primary winding.

Note that the just defined variables (N_1, $A_{\mathrm{Cu},1}$, $I_{\mathrm{rms},1}$) are not the input parameters of the optimization and they will be expressed using the input parameters.

The losses were calculated from the primary winding parameters, but the transformer also contains the secondary winding, this is why there is a double in the equation—optimal design assumes identical losses in the primary and secondary windings.

Number of turns of the primary winding:

$$N_1 = \frac{V_1 \cdot d}{f \cdot \Delta B \cdot A_{\mathrm{Fe}}} \tag{3}$$

where V_1 is the primary voltage. This variable will also be expressed using the input parameters.

Expressing $A_{\mathrm{Cu},1}$ and $I_{\mathrm{rms},1}$ using basic mathematical formulas:

$$A_{\mathrm{Cu},1} = \frac{A_{\mathrm{w}} \cdot k_{\mathrm{w}}}{2N_1} = \frac{A_{\mathrm{w}} \cdot k_{\mathrm{w}} \cdot f \cdot \Delta B \cdot A_{\mathrm{Fe}}}{2V_1 \cdot d} \tag{4}$$

$$I_{\mathrm{rms},1} = \frac{P}{V_1 \cdot \sqrt{2d}} \tag{5}$$

Substituting for N_1, $A_{\mathrm{Cu},1}$ and $I_{\mathrm{ef},1}$ from (3), (4) and (5) in (1) we obtain:

$$\Delta P_{\mathrm{w}} = \frac{2 \cdot \rho_{\mathrm{Cu}} \cdot l_{\mathrm{t}} \cdot d \cdot P^2}{f^2 \cdot \Delta B^2 \cdot A_{\mathrm{Fe}}^2 \cdot A_{\mathrm{w}} \cdot k_{\mathrm{w}}} \tag{6}$$

For simplicity, constant C is introduced:

$$C = \frac{2 \cdot \rho_{\mathrm{Cu}} \cdot l_{\mathrm{t}} \cdot d \cdot P^2}{f^2 \cdot A_{\mathrm{Fe}}^2 \cdot A_{\mathrm{w}} \cdot k_{\mathrm{w}}} \tag{7}$$

Then (6) simplifies into:

$$\Delta P_{\mathrm{w}} = \frac{C}{\Delta B^2} \tag{8}$$

4.2 Core Losses

Knowing the basic relationships and considering the simplifications mentioned in Chap. 3, the following equation can be written:

$$\Delta P_{\mathrm{Fe}} = \Delta P_{\mathrm{Fe,ref}} \cdot \frac{f}{f_{\mathrm{ref}}} \cdot \left(\frac{\Delta B}{\Delta B_{\mathrm{ref}}} \right)^2 \tag{9}$$

For simplicity, constant D is introduced:

$$D = \frac{f \cdot \Delta P_{\mathrm{Fe,ref}}}{f_{\mathrm{ref}} \cdot \Delta B_{\mathrm{ref}}^2} \tag{10}$$

Then (9) simplifies into:

$$\Delta P_{\mathrm{Fe}} = D \cdot \Delta B^2 \tag{11}$$

4.3 Finding the Minimum Losses Point

Total transformer losses are the sum of (8) and (11):

$$\Delta P_{\mathrm{Tr}} = \frac{C}{\Delta B^2} + D \cdot \Delta B^2 \tag{12}$$

To find a minimum value of the function $\Delta P_{\mathrm{Tr}}(\Delta B)$, the function is differentiated with respect to ΔB and set equal to zero:

$$\frac{\mathrm{d}}{\mathrm{d}\Delta B}(P_{\mathrm{Tr}}) = \frac{-2C}{\Delta B^3} + 2D \cdot \Delta B \equiv 0 \tag{13}$$

From (12) the optimum value of ΔB can be expressed:

$$\Delta B_{\mathrm{opt}} = \sqrt[4]{\frac{C}{D}} \tag{14}$$

$$\Delta B_{\mathrm{opt}} = \sqrt[4]{\frac{2 \cdot \rho_{\mathrm{Cu}} \cdot l_{\mathrm{t}} \cdot d \cdot P^2 \cdot f_{\mathrm{ref}} \cdot \Delta B_{\mathrm{ref}}^2}{f^3 \cdot A_{\mathrm{Fe}}^2 \cdot A_{\mathrm{w}} \cdot k_{\mathrm{w}} \cdot \Delta P_{\mathrm{Fe,ref}}}} \tag{15}$$

5 Transformer Design Procedure

At first, an appropriate core size is selected based on the required converter power. The selection can be based on previous experience. Based on datasheet values and dimensions of the core, all of the core parameters are determined. The switching frequency is selected based on the speed of the switching transistors and their allowable power losses. Then, using (14), an optimum value of peak-to-peak flux density ΔB_{opt} is calculated. The transformer power losses are verified using (12). If the losses are too high or too low (not optimal), a different core size is selected and calculation of ΔB_{opt} is repeated.

If the value of ΔB_{opt} results so large that it would cause core saturation (close to 2 times the saturation flux density B_{sat}), then ΔB_{opt} must be decreased so that it does not cause core saturation. The core should not saturate for the maximum available duty cycle of the converter (usually close to 0.5).

From this value ΔB_{opt}, the number of turns of the primary winding N_1 and secondary winding N_2 can be calculated:

$$N_1 = \frac{V_1 \cdot d}{f \cdot \Delta B_{\mathrm{opt}} \cdot A_{\mathrm{Fe}}}; N_2 = \frac{V_2}{2f \cdot \Delta B_{\mathrm{opt}} \cdot A_{\mathrm{Fe}}}, \tag{16}$$

where V_2 is the output voltage of the converter (after the LC filter).

Copper cross-section of the primary winding conductor $A_{\mathrm{Cu,1}}$ and secondary winding conductor $A_{\mathrm{Cu,2}}$:

$$A_{\mathrm{Cu,1}} = \frac{A_{\mathrm{w}} k_{\mathrm{w}}}{2N_1}; A_{\mathrm{Cu,2}} = \frac{A_{\mathrm{w}} k_{\mathrm{w}}}{2N_2} \tag{17}$$

6 Design Example

A design example is provided to evaluate the objectivity of the optimization. Parameters usable for a practical converter are selected.

The selected input parameters are:

- Converter power: $P = 1000$ W
- Switching frequency: $f = 120$ kHz
- Duty cycle: $d = 0.35$
- Core: T 3813 (toroidal; 38 mm outer dia., 25 mm inner dia., 13 mm height)
- Core material: 3C94 (N87)
- Core reference losses: $P_{\mathrm{Fe,ref}} = 0.45$ W at $f_{\mathrm{ref}} = 100$ kHz and $\Delta B_{\mathrm{ref}} = 0.2$ T
- Core area: $A_{\mathrm{Fe}} = 77.5\ \mathrm{mm}^2$; winding area: $A_{\mathrm{w}} = 507\ \mathrm{mm}^2$
- Winding copper fill factor: $k_{\mathrm{w}} = 0.25$
- Mean turn length: $l_{\mathrm{t}} = 64$ mm
- Copper conductivity: $\rho_{\mathrm{Cu}} = 1.8\ 10^{-8}\ \Omega\mathrm{m}$
- Input voltage $V_1 = 300$ V; output voltage $V_2 = 48$ V

The C and D coefficients according to (7) and (10) are calculated at first. Then, (14) is used to obtain the optimum peak-to-peak flux density: $\Delta B_{\mathrm{opt}} = 0.27$ T. This value does not cause core saturation (peak flux density is one half of the peak-to-peak flux density) and so can be used.

Using (8), we obtain the winding losses: $\Delta P_{\mathrm{w}} \approx 1$ W.

Using (11), we obtain the core losses: $\Delta P_{\mathrm{Fe}} \approx 1$ W.

It can be seen that the core and winding losses are identical. This is not a coincidence; the equality can be explained from mathematical theory. The winding and core losses are equal regardless of the transformer and winding parameters, e.g. they are equal for different copper fill factors, for different reference core losses etc.

The total transformer loss is then about 2 W. This is an acceptable value for a transformer of this size, even with passive cooling.

The numbers of turns according to (16) are: $N_1 = 42$; $N_2 = 10$.

To validate the correctness of the optimization, Fig. 2 is presented. It shows the relationship of winding loss ΔP_{w}, core loss ΔP_{Fe} and total transformer loss ΔP_{Tr} with respect to peak-to-peak flux density ΔB. A local minimum at the calculated optimum peak-to-peak flux density ΔB_{opt} of ~0.27 T can be observed. At this point, it can be seen that the core and winding losses are identical.

7 Conclusion

The parameters of a HF-converter transformer (number of turns, peak-to-peak flux) are today often selected based on educated guess or experience. The contribution presented one method of calculating the transformer parameters for lowest total loss. Some simplifications were performed during the optimization process, however allowing the resulting equation to be analytical and relatively simple. For this reason, the described method can be implemented where calculation speed is the priority, e.g. in converter design tools, especially for evaluation of multiple designs. The accuracy of the result is moderate, however the possibility of false solutions is eliminated, as can sometimes be the case e.g. with numerical methods.

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