Chapter 7
Gridshell, Ribbed–Shell Domes

7.1 Introduction

Shown in Fig. 7.1 is the schematic of the development of the covering geometry, starting with a cylindrical shell up to a polygonal shell formed from the series of intermingling cylindrical surfaces.

A spherical open covering (Fig. 7.1b) or a spherical closed covering (Fig. 7.1c) on the projection of a square can be built from two intermingling cylinders. An open shell covering (Fig. 7.1d) or a closed shell covering (Fig. 7.1e) from several intermingling cylinders on a polygonal projection can also be formed.

In stone structures it was customary to call domes axially symmetrical, masonry closed coverings. In wooden structures there occurs an enormous number of differentiated two-curvature coverings, both open and closed. In this work the scope of considerations covers the closed shell coverings in form of domes.

The covering shown in Fig. 7.1e is made up of sections of a cylindrical vault supported on meridional ribs as well as the lower and the upper ring. The intersection edges of the cylinders separate a section of the sphere limited by edge ribs called chords. The building of main ribs on the intermingling lines of the cylinders determined the essential stiffening and increasing of the load capacity of the covering at a small weight of the structure. Shown in Fig. 7.2 is the model of a sectoral dome from arch centres made on the projection of an octagon. The model is the visualization of a sectional dome having a span of 30.0 m at the scale of 1:50.

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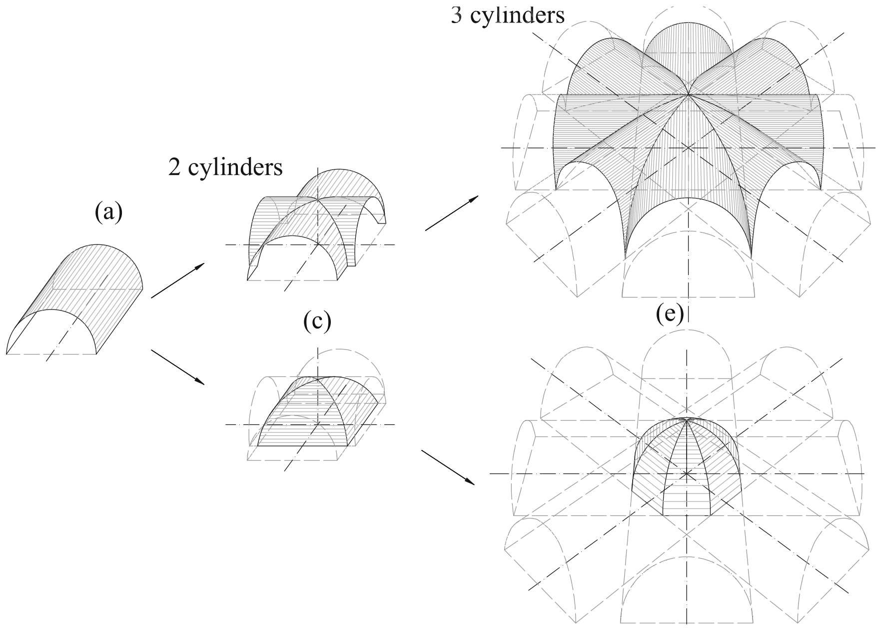

Fig. 7.1 Shaping of the geometry of open and closed shell coverings on polygonal projections

Fig. 7.2 Model of the gridshell covering, closed, on the projection of an octagon 1—arch centre, 2—chords, 3—lower supporting crown, 4—upper chord-connecting ring

7.2 Sectoral, Gridshell Domes

In coverings built on polygonal projections, in the intermingling lines of cylindrical surfaces there occur main ribs called chords, concurring in the upper ring of the dome [1]. The height of the load-bearing chords was adopted as $h \geq \frac{1}{150}L$, where L—the diameter of the circular projection of the dome. The fields between the chords was filled with a rhombic network of bar ribs using which the shell of the covering from planks was connected, transforming it into a complex construction (Fig. 7.3).

In such system it was recommended to determine the number of the sides of a polygonal projection of the closed vault (Fig. 7.1c, e) in accordance with the adopted angle ψ between the arch centres of the network so that the helices

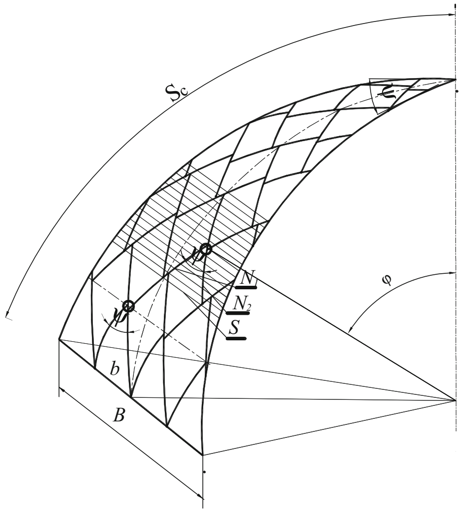

Fig. 7.3 "Spherical" triangle of a sectional gridshell 'dome' [1]

(from crossheads) reach as far as possible the side edges of the chords in one section (Fig. 7.2). This condition is met at the maintenance of the Eq. (7.1) according to [2]:

$$B/S_c = tg(\psi/2) \tag{7.1}$$

where

B—length of the side of the polygonal vault base, Fig. 7.4b.
S_c—length of the arch as shown in Fig. 7.4a.
ψ_m—360/m, where m—the number of the sides of the base's polygon. The Eq. (7.1) results from the similarity of the dome's section limited by ribs to the geometry of the arrangement of crossheads and can adopt the form:

$$s = b/2 \sin \frac{\psi_m}{2} \tag{7.2}$$

where: ψ_m—apex angle of the section. Attention should be drawn to the adjustment of the geometry of crossheads from their joining technology.

At the given number of the sides of the covering's polygonal projection, the angle between the arch centres of the rhombic shell was determined from the Eq. (7.1), whereby the angle should not go beyond the range of angles between the ribs recommended for cylindrical arch centre-gridshell vaults. This angle, designated in this work as **α**, should be included within the range of 30° up to 50° [1]. The variable thickness of chords in the projection was obtained using laps of various thicknesses nailed to the side edges of the chords at the place of the chord's connection with arch centres. While conferring a variable width on main ribs (chords) (Fig. 7.4b), a bilateral convergence of the helices from arch centres with the chord’s axis in one section was obtained, thus providing the full normalization of the length of arch centres. However, if at the pre-set number *n* of the sides of the polygonal plane of a closed vault it was impossible to adopt the angle ψ between the arch centres, chords of a constant width were built, and the arch centres differently joining them were connected with chords at various levels, or the length of the crossheads was changed. At an inaccurate accounting and the constant thickness of chords, the helices from crossheads, converging with chords of the constant length produce an unfavourable loading of the chords. The chords reinforced with side laps were also used at the assembly of the covering. An example of the gridshell sectoral covering from arch centres is shown on the projection of an octagon is shown in Fig. 7.4.

Visible in Fig. 7.4 are chords of a variable width in the intersection lines of cylindrical width. The assembly of the vault was conducted in the following

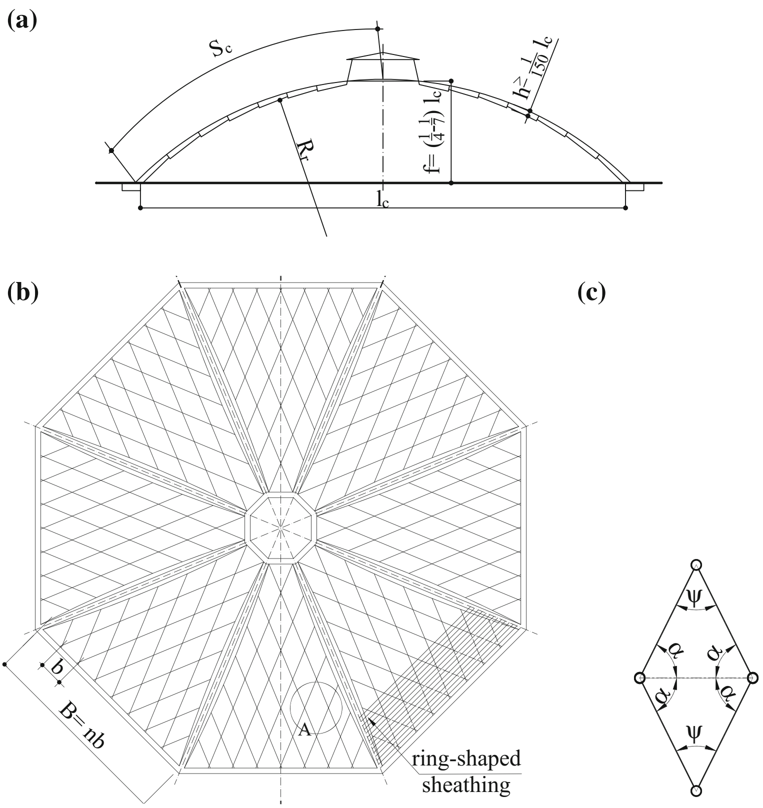

Fig. 7.4 Arch centre-gridshell dome on the projection of an octagon according to [3], with eight sectors; **a** section of a polygonal dome from crossheads, **b** projection at the level of the lower supporting ring, **c** angles of the rhombic network

sequence: first, the lower supporting ring and the central tower, on which the upper ring was laid, were placed and strengthened. Afterwards, the strengthened chords of the vault were disposed, and it was recommended to raise the chords and to place on the central tower of the scaffolding, pair-wise, in order to give the dome the spatial stiffness during the assembly. The chords were strengthened with mounting spacings from planks, afterwards a small scaffolding within one sector or its part was suspended to them.

The assembly of the vault was conducted sector-wise. Two oppositely lying sectors were mounted on a one-off basis.

7.3 Gridshell Domes from Arch Centres

At the realization of domes of large dimensions fabricated from wood, the load-bearing elements happen to be heavy and difficult in the assembly. Their transportation becomes uncomfortable and expensive. One of the most comfortable types of the structures of spherical domes are those made from separate crossheads. An essential stage of the development of wooden domes were domes built from a network of small ribs replacing meridional main ribs. Of particular importance was the introduction of crossheads called in the Russian references '**kosjaks**' [2], in the Polish references '**plant slants**' [1] (1959), which were used to model the surface between the main ribs of sectional-cylindrical domes built on a polygonal as well as circular projection (Figs. 7.1, 7.2, and 7.4). In this work the name '**crosshead**' was suggested for an element made from an element of the arch centre creating the grid of the shell.

Closed axially symmetrical coverings were built from wooden crossheads assembled into a rhombic spherical grid were called gridshell domes. This type of grid occurs in wooden structures only. The dimensions of such domes were selected, like gridshell coverings, on the projection of a polygon, that means that the ratio of the rise of arch f to the dome diameter $\boldsymbol{L}$ is included within the range of: $f/L = (1/7 \div 1/4)$.

Mischke D. in his paper [1] specifies that if in the sectional ribbed-gridshell coverings the condition $B/S_c \leq 1$ is met (where B—length of the polygon's base side, S_c—length of the arch of the dome's vertical section (Fig. 7.4a), then the sectoral dome may be treated as a rotary dome on the circular projection. Owing to the transformation of the polygonal projection into a circle, the edge of the intersections of cylindrical surfaces was eliminated, removing ribs off the construction. This required the strengthening of the lower supporting ring and the upper ring in the dome's key block. The lower supporting ring and the upper ring determined the stiffness of the gridshell dome's structure from arch centres on the circular plan. Shown in Fig. 7.5a, b are examples of domes built from arch centre gridshells on the circular plan.

The shaping of a shell made up from rhombuses, thus axially symmetrical gridshell domes from wood, followed through such calculation of the angles

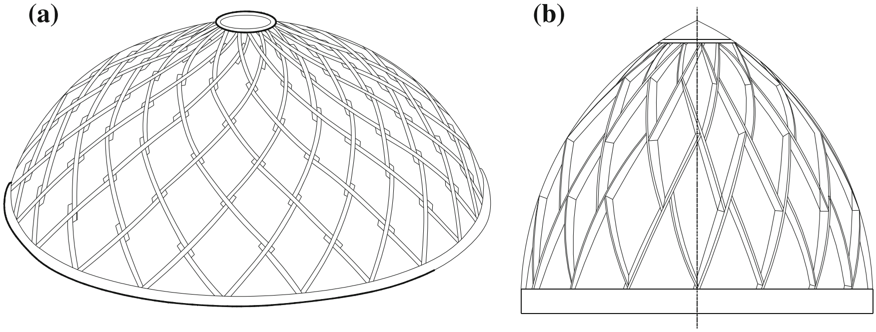

Fig. 7.5 Gridshell domes: **a** gridshell dome on a circular projection, **b** ogival dome on a circular projection

between ribs–arch centres "ψ", their inclination to the level of the dome's line of latitude "α", as well as the inclination angle to the dome's interior "φ" so as to obtain the closure of the last layer of crossheads in the central upper ring (Figs. 7.3 and 7.4c).

Shown in Fig. 7.6 is the axially symmetrical gridshell dome built at the beginning of the 20th century in Koblenz [4], Germany.

The dome on the side of its interior is demonstrated in Fig. 7.6. The ribs forming the rhombic gridshell of the dome are connected with a plank floor of a latitudinal arrangement of planks. This first layer of the shell was strengthened with successive inside layers of a spiral arrangement of planks. The connection of the ribs with the plank shell provided a spatial operation of the structure. The stiffness of the multilayer shell from planks joined with ribs allowed to make openings in the fields between crossheads in order to enhance the lighting of the interior.

The shape of a 'crosshead' used in the construction of gridshell domes from wood derives from an arch centre used for centuries (shown in Fig. 3.2 in Chap. 3), enriched at its end with tangs, and in the central section with an opening (key)—Fig. 7.5a. The proper cutting to size of the edges of the opening and tangs allowed to shape the spherical shell. The distinguishing feature of such shells are nodes in which three bars of a variable moment of inertia of the section converge. Such structure ensures the stiffness of the shell and makes possible the division of the wooden zone of the covering into rhombuses. **The rhombic division of the sphere distinguishes the gridshell domes from glued laminated timber from contemporary steel gridshell domes and those from glued laminated timber**.

Figure 7.7 exhibits a gridshell dome as described by the engineer Kleymienov B. in his paper [5] (1935). The dome was built in 1933–1934 over a gymnasium of the Palace of Physical Culture in Moscow. The sphere was built from crossheads made from factory-prefabricated planks and mounted on the construction yard. High quality wood, dried up to the moisture content of 17–18% was used, which ensured a higher stabilization of dimensions during the construction and operation of the facility. The nodes of the gridshell were fabricated according to the schematic in

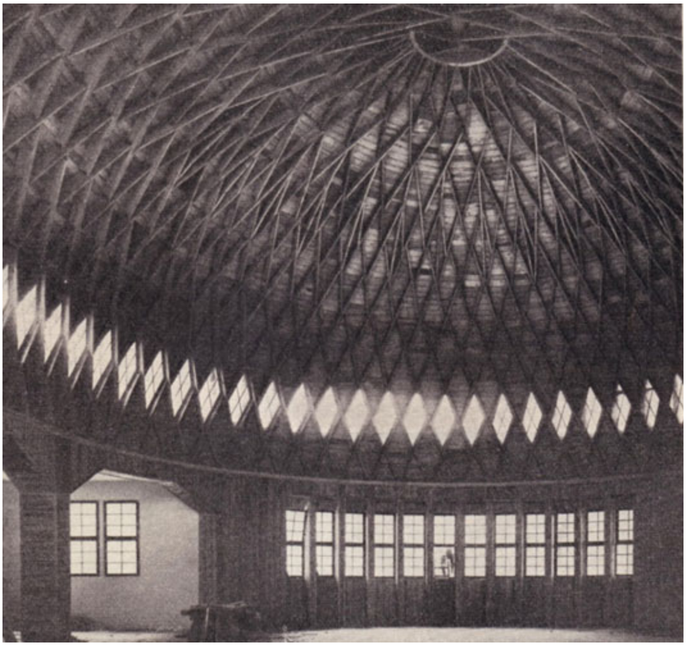

Fig. 7.6 Gridshell dome from cross braces of the Customs Office hall in Koblenz [4]

Fig. 7.7d. The connection of crossheads in the nodes were strengthened with steel bolts. The important condition of the durability of such structure is the exact adherence of crossheads in the nodes, as well as the securing of such adherence during the operation of the facility. This may be achieved owing to the meeting of three conditions: the wood has to be dry, the crossheads have to have an exact geometry, the connection in the nodes has to be careful and straightened with a fastener. Crossheads of thirteen various types were prefabricated. In each latitudinal chords crossheads of one type were used. Demonstrated in Fig. 7.7a is the mounted sphere from crossheads. Still visible on the sphere are horizontal assembly chords from planks to which the scaffolding platforms are screwed, allowing the cutting of crossheads to the shape of the sphere according to the template. Oval openings in planks required to make a node were cut on the construction yard—Fig. 7.7d, as well as the tang ends were cut off in two directions: α—inclination to the line of latitude and φ—inclination to the inside. The setting of crossheads followed in horizontal chords, forming closed circles, in accordance with the radius of the given line of latitude. Each circle was built by two working gangs going separate ways in opposite directions until their meeting. The setup of the first-level crossheads followed on the lower ring in nests designated with triangular laps determining the position of the lowest cross braces. Circa one hundred and twenty nodes were made on the lower ring. A need occurred to reduce the number of nodes at the height of the row 7 and 8 so that less bars of the gridshell reached the central upper metal

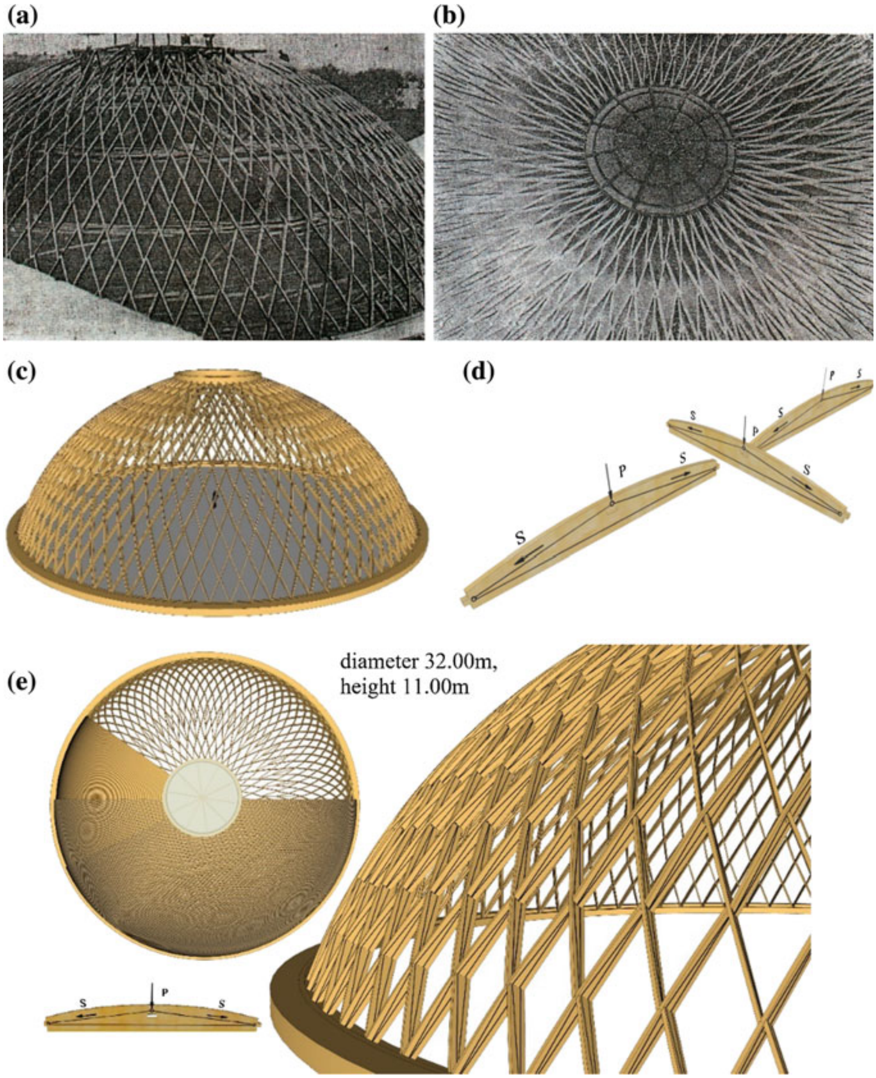

Fig. 7.7 Moscow, 1933/34, Gridshell dome from crossheads of a 32.0 m span, over the gymnasium of the Palace of Sports in Moscow [6], **a** archival photograph during the construction, **b** central, metallic upper node, **c** gridshell from crossheads reconstructed on a 3D model, **d** principle of the construction of a node of the shell from crossheads in which the bars of the Mises' truss were inscribed, **e** projection of the dome of a 32.0 m diameter and a magnification of the gridshell showing the Mises' trusses inscribed in arch centres

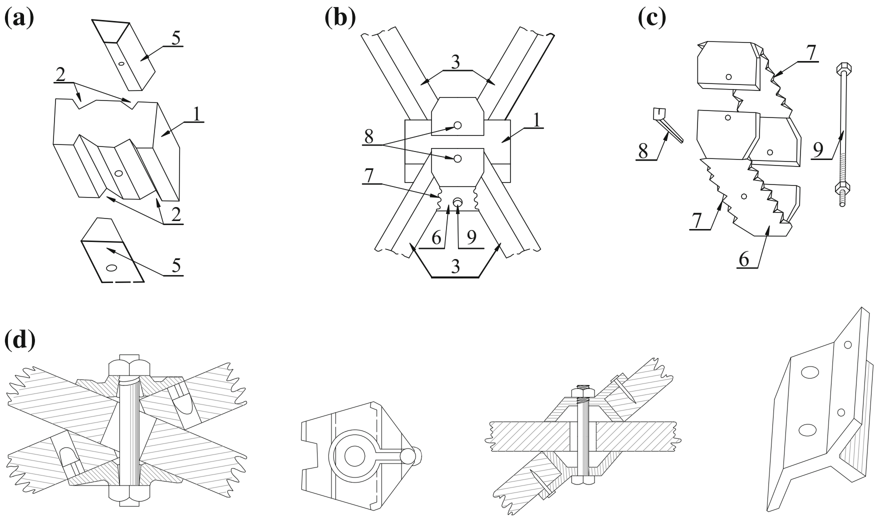

Fig. 7.8 Foreign fasteners used in the connections of arch centres according to [7], **a** wooden elements of the node **b** inclusion of the steel fasteners **c** steel elements of the node **d** iron castings connecting the crossheads of the gridshell

ring. After the reduction, circa forty nodes were made on the central metal ring. The planking was made upon prefabrication of the whole gridshell and the central upper ring, as well as the verification of all connections.

The latitudinal planking in the first-row sphere was made from quarterings of a 10–11 cm thickness. Starting with the second row until the sixth row, the planking from 5.0 cm thick planks, and higher, from 2.5 to 2.0 cm thick planks was applied. In order to make the latitudinal planking of the highest, ninth row, 2.0 cm thick planks were subjected to the interference of steam in order to achieve the required curvature. After the full planking of the dome with two layers of planks, its covering on the outside with metal sheet, the central scaffolding was removed. During the disassembly of the scaffolding, the deflection of the shell was monitored. The highest lowering of the central upper ring of a 1.50 cm value was recorded. The subsequent observations did not record an increase in the deflection.

The connections of arch centres in nodes used in the construction of axially symmetrical domes, like in closed domes, were strengthened using external foreign fasteners, e.g. pins from oak wood, (Fig. 7.8) (Pieselnik [7]), nails, iron castings and bolts.

A characteristic feature of the steel fasteners shown in Fig. 7.8 used in the wooden nodes of gridshell domes from solid wood is the strengthening of the ends of wooden bars through the clamping of the extreme sections of load-bearing elements from wood using iron castings.

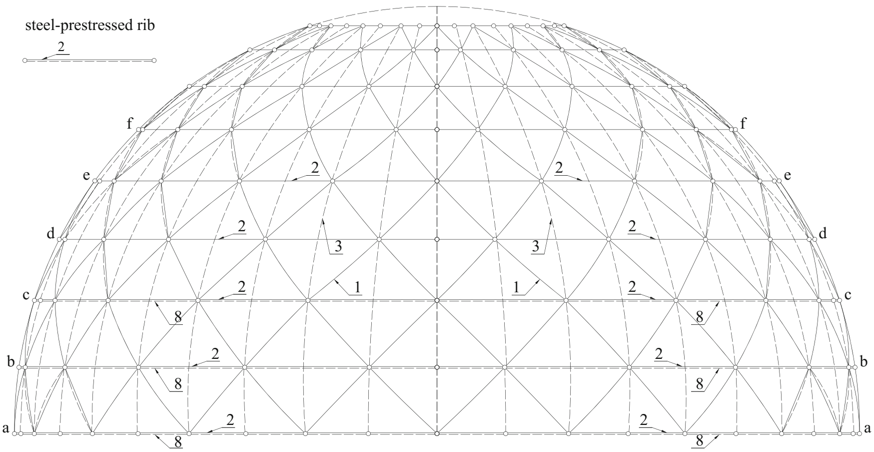

Fig. 7.9 Geometry of the gridshell dome designed by Sackur W. (1927), according to [7]

7.4 Steel-Prestressed Gridshell Domes

In the thirties of the 20th century steel-prestressed wooden domes were built. Examples of a steel-prestressed shell dome are described in Sect. 6.3 and shown in Fig. 6.5. Tshihatshiev H. A. described in his paper [7] (1947) a steel-prestressed gridshell dome as depicted in Fig. 7.9. The gridshell dome by Sackur W., protected by the German patent No. 415870 cl. 37b, 3_0 1 from 1927, features steel tie rods in lower latitudinal layers. The wooden load-bearing construction of the Sackur's dome constitute the ribs: **1**, **2** and **3** from arch centres set perpendicularly to the sphere. The elements: **1** are cross braces, **2** are latitudinal elements, prestressed at the level a, b, c, with steel bars, **3**—meridional ribs.

In each dome of the dome four diagonal bars **1** and two bars of the ring-shaped chords **2** converge. In each node a short plank **4** with angular cuts made from one piece of wood or from several planks were used, as shown in Fig. 7.10. At the top and at the bottom the short plank has two angular cut-outs each place according to the template. Diagonal bars **1** were inserted in those cut-outs. The short plank **4** has side cut-outs to insert latitudinal bars **2**. The strengthening of the node was achieved using side laps **6** and embracing clamps **7**.

Steel bars **8** strengthening the lowest positioned wooden ribs **2** against stretching were used in the lower tensile parts of the dome. An essential role in the maintaining the shape of the dome is played by meridional ribs **3** nailed on the outside of the nodes of the dome in a distance **d** $\sim$ circa 5 cm. (Fig. 7.10g) **A plank shell of the dome covering was nailed to meridional ribs 3, maintaining the distance d of the plank shell from the construction of the wooden dome. This is a next example of the care for air exchange and the provision of the ventilation for the wooden shell and ribs**.

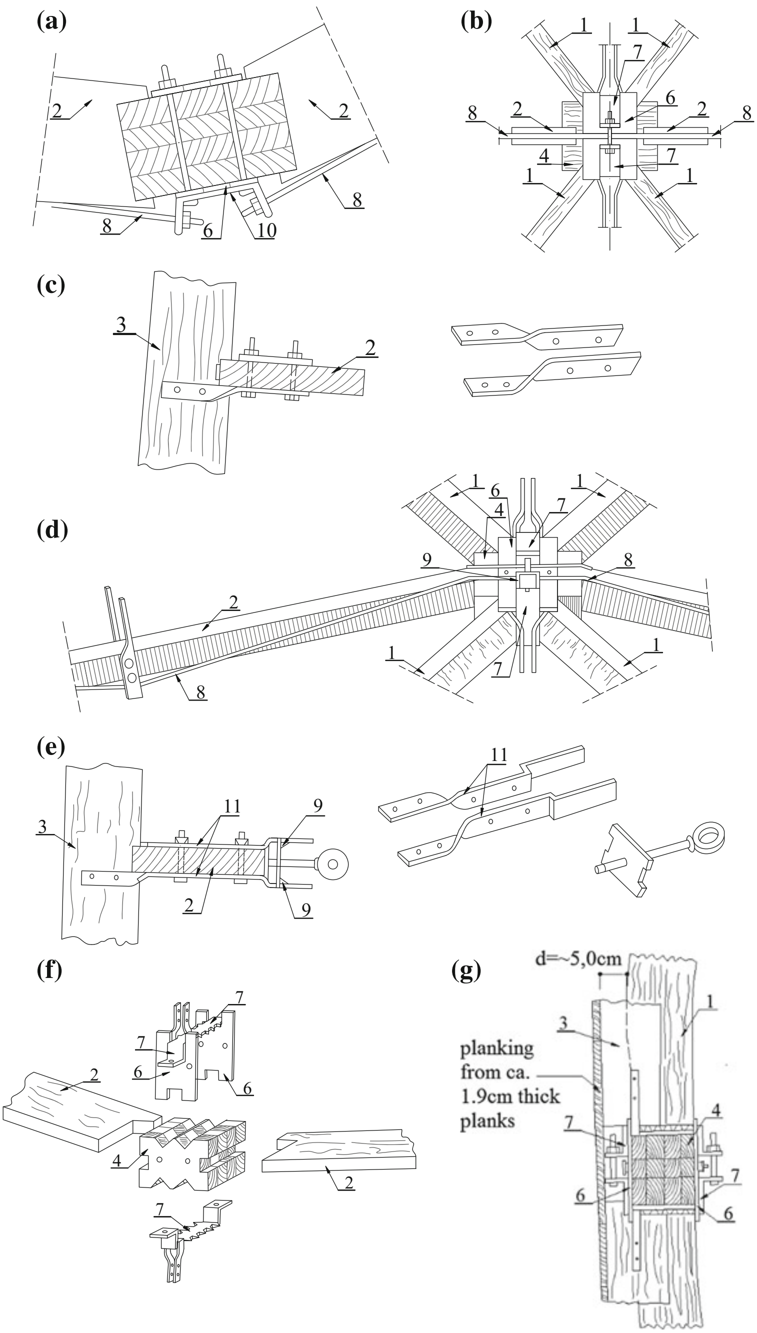

Fig. 7.10 Solutions of the nodes' structure in the Sackur's gridshell dome according to [7] (1927), **a** node from wood with mechanical clamp, **b** top view of a node, **c** connection of a rib with a steel tie rod, **d** prestressing of a latitudinal rib **2** with a steel bar 8, **e** connection of a rib with a steel tie rod, **f** construction of a dome's wooden-steel node, **g** d—distance of the covering from the structure of ribs

The meridional ribs **3**, bearing the planks of the sheathing, facilitate the air flow, exchange of humidity and drying of the wetted elements of the dome. Not flammable fire-proof boards can be mounted into the crossheads **1** on the interior inside.

The nodes of the dome are demonstrated in Fig. 7.10 compiled according to [7]. In each case, the steel elements strengthen the ends of wooden bars through the induction of a mechanical clamp. The stress of the bars **8** was achieved with threaded grip handles on ring-shaped bars **2** strengthened using laps **11** (Fig. 7.10).

Described in the paper [7] is the patent of the solution of the dome's structure, focusing on the building principle and the joining of elements. The details are not shown, including the dimensions of structural elements. It can be stated from the assessment made on the basis of Fig. 7.10 that the dimensions of load-bearing wooden elements are values calculated in several or a dozen or so centimetres.

The structure of the nodes of the Sackur's dome is complicated, however, its solution is conforming to the properties of wood. The steel elements applied strengthen through the mechanical clamp the most loaded, extreme sections nodal sections of wooden bars. Steel sheets are not introduced in the section from wood, but they embrace the most loaded extreme sections of wooden bars. Low-dimensional fasteners of a nail type are only driven into wood. The assembly of the Sackur's dome is described in the paper [7]. The building of the dome followed without an internal scaffolding. Halves of short planks **4** were laid on the lower thrust ring and the connection was made using halves of clamps **7** with the supporting ring. The lower ring from bars designated in Fig. 7.9a, b was mounted on the nodes thus built, afterwards the ring b and c was mounted, successively, up to the central upper ring.

7.5 Shell Domes on a Grid from Arch Centres Strengthened in the Node with a Flat Plank

A next example of gridshell domes is the system to cover tower water tanks used in Germany until World War II, called the GREIM system [8]. A model of such a dome from the paper [4] is shown in Fig. 7.11.

This system differs from the previously described gridshell domes from crossheads in the planks above the node being nailed on crossheads (perpendicularly to the sphere), along the helix resulting from the position of ribs. The shell of the covering of the dome shown on the left of the photograph were nailed on the planks at the angle of 60°. The planks nailed on the crossheads of the gridshell shell level the shape of the spherical coating, stiffening the nodes from arch centres, and facilitate the nailing of the second outside layer of the plank shell at the angle of 60°.

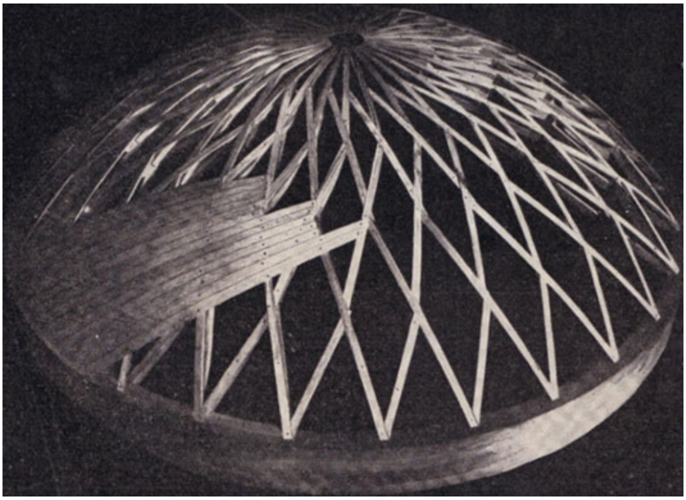

Fig. 7.11 Model of the gridshell dome of the Greim system from the paper [4]

7.6 Gridshell Dome on a Rhombic Grid from Planks

Gridshell domes made from flat planks were also built. An example of the dome of the Zollbau system, the German patent No. 456323 from 1928. Shown in Fig. 7.12a–c is the rule of building the network, as well as the geometry of the shell's grid from planks, according to [7] (1947). The dome was erected without a scaffolding. At the beginning, cross braces **1** and **1′** were set on the lower supporting ring, and fastened using wooden inserts. Afterwards, bars **2′** were positioned, being fastened to the ring and the ends of bars **1′**, and in the middle of bars **1**. In a similar way the bars of successive layers were connected according to the figure of the node shown in Fig. 7.12d.

The system shown in Fig. 7.12 is distinguished by the crossheads **2** and **3** connected in the middle of the length of the lower layer's crosshead. The planks **f** to strengthen the node and to level the gridshell shell of the dome were nailed along the helix (quasi-meridionally) on the ends of crossheads positioned on the outside of the shell (Fig. 7.12c). The planks of the covering's shell were nailed to the spiral planks at ca. 60°. The operational rule of the dome was verified on a model made from pine wood at the scale of 1:50 and shown in Fig. 7.12c. The outside planks **f**, nailed spirally over the nodes of the network protect (the bars of a constant section) the sphere against the node snap-through.

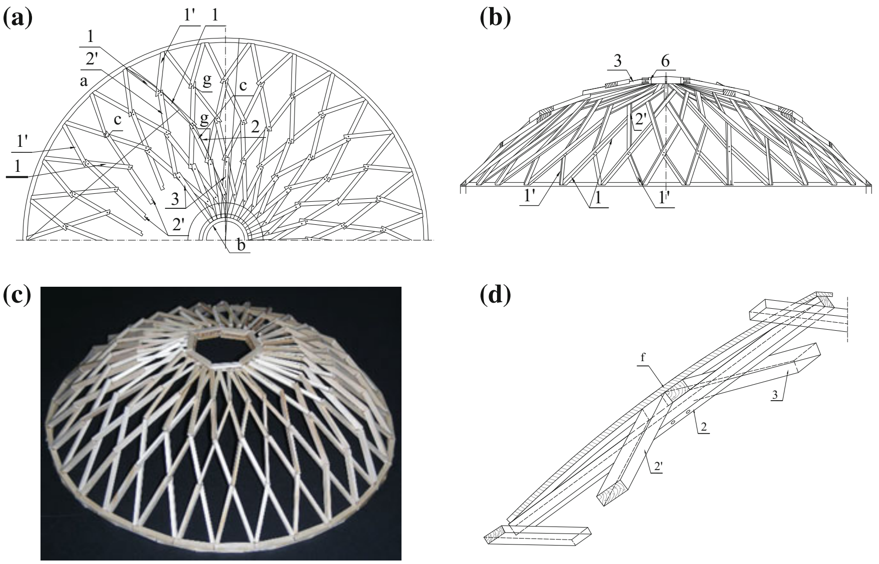

Fig. 7.12 Gridshell dome from arch centres fabricated following to the rules of the patent No. 456323 (1928) according to [7], **a** projection of the dome, **b** side view, **c** model of the dome made from pine wood, **d** rule of connecting the bars

7.7 Testing Gridshell Dome Models from Arch Centres

The author conducted the testing of shells and gridshell domes on models from various wood species and produced their computer visualizations. At each stage of building the model from wood the geometrical invariability and the spatial stiffness of the structure was verified. The models from wood were watched for many years, while checking random geometric deviations of nodes caused by the variable moisture of the environment. Attempts were also made to determine the standardization capacities of ribs-crossheads. To this aim, various models from crossheads of a similar length were built. The term 'of a similar length' was introduced for crossheads shown in Fig. 7.7d, f, having a similar geometry of the central part. A differentiated length of tangs made at the ends of the crosshead was assumed. This allowed to fit in a better way the ribs in the nodes of the gridshell. The models shown in Fig. 7.13 were made from arch centres 'of a similar length'. The differences between the models consist in the differentiation: (1) angle of contact ψ between crossheads, (2) angle α of the inclination of the small arch centre to the level of the equator, (3) angle φ of the inclination of small arch centre to the dome's inside.

The rhombic shell of the dome shown in Fig. 7.13a was built from ribs-crossheads of a similar length. The shell was shaped through changing the angle between the ribs "ψ" and their inclination to the level of the line of latitude

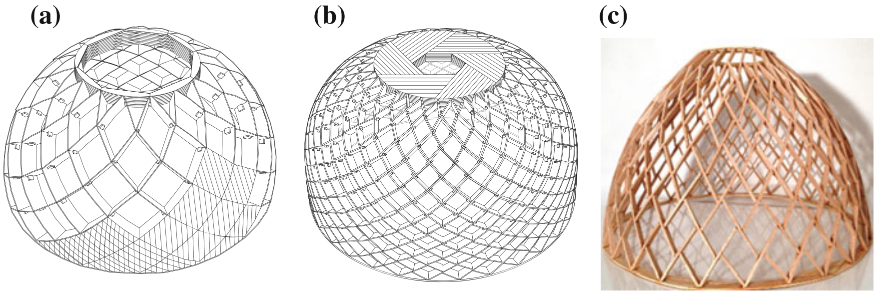

Fig. 7.13 Models of gridshell domes made from elements of a similar length, **a** hemi-spherical shell, **b** cylindrical shell combined with the hemi-spherical shell **c** ogival shell from pine wood

"α". In the first lower layer the crossheads were set up at an angle α = 36° to the level of the lower ring. The change of the angles "ψ" between the sides of the sphere's rhombuses and their inclination to the dome's inside "φ" leads to the closing of the vault. The closure of the last layer of crossheads in the upper ring made from horizontally laid layers of arch centres determines the shape of the dome's sphere. The stiffness of the upper ring of the dome determines the stiffness of the gridshell of the sphere made from ribs-crossheads.

Shown in Fig. 7.13b is the model of a gridshell dome of an outside diameter of 30.0 m, made at the scale of 1:50. Crossheads of a similar length were used to build the model, the angle "ψ" of the arch centres' connection and the inclination angle "α" to the line of latitude's level was changed. The lower layers of crossheads were set up at an angle of 36° to the level of the lower crown. Several lower latitudinal layers of the dome were made without using the inclination of the crossheads to the inside, i.e. the angle $\varphi = 90°$ was used. In the rhombuses of the lower part of the sphere just the angles "ψ" and "α" in successive layers were changed. The differentiation of the area of the rhombuses and the inclination of crossheads at an angle $\varphi < 90°$ leads to the closure of the shell, as shown in Fig. 7.13b. In order to stiffen the gridshell of the sphere in the dome's key block a multi-layer board from small planks was made. The use of the board stiffened the gridshell from crossheads better than the ring shown in Fig. 7.13c.

The differentiation of the values of the angles α, ψ, φ at a similar length of crossheads led to the building of an ogival dome, demonstrated in Fig. 3.13c.

The testing made on the models indicated difficulties in the maintaining of a fixed length of ribs-crossheads. The full standardization of the ribs of the rhombic gridshell can be maintained in single latitudinal chords at one level of the dome. The lower and the upper ring of the dome will ensure the spatial stiffness and the geometrical invariability of the gridshell structure. Many years' observations of the models of gridshell domes from wood demonstrated that the structure of a crosshead, provided with tangs on the ends, as well as the structure of the gridshell node allow to level the changes in the geometry of ribs-crossheads due to the impact of the moisture content and ambient temperature variations on wood.

7.8 Meaning of Ribs-Crossheads in the Development of Wooden Gridshell Domes

The possibility of using transformed into crossheads of a geometry adapted to the building of double-curvature, gridshell coverings seems to be extremely advantageous (Fig. 7.16a). Probably, this is why in the Polish and foreign bibliography no descriptions of failures of such type of ribbed-gridshell domes from wood were found. It may be presumed that they succumbed to fire and biological corrosion, however, no message is available on the occurrence and course of the failure.

The phenomena of the domes' disasters are known from the descriptions of the catastrophes of the steel structures of bar domes. They consisted mainly in the collapsing of spherical domes to the inside of the facility. Illustrated in Fig. 7.14 is the catastrophe of a steel, gridshell bar dome in the exhibition area in Chorzow as described by Augustyn J. in his paper [7] (1976). The sliding-down of snow due to wind, Fig. 7.14b led to the non-symmetrical load and non-symmetrical shifts of the bar dome's nodes, which produced a dynamic node snap-through to the dome's interior.

The collapse process of the steel dome of a 30.0 m diameter, of a curvature of $1/R = 1/25 = 0.04$,[1] built from bars of a length 1091 mm up to 1026 mm, lasted for 8.0 min. The failure created due to the action of loads as well as the following errors in the design project: a low curvature of the sphere, a minor angle of the intersection of bars $\alpha = \mathbf{2°32'}$, as well as clearances (ca. 1.3 mm) in the openings for the bolts in nodes [9].

In the designing of steel gridshell domes polyhedrons of an identical length inscribed into a sphere were looked for. Figure 7.15 exhibits the basis divisions of the sphere used in the designing and building of gridshell domes.

The number of regular polyhedrons of the identical lengths of edges, inscribed into the sphere, is limited. Practically, the following comes into consideration: a 12-hedron built from pentagons, a 20-hedron built from triangles and a 32-hedron, semi-regular, built from 12 pentagons and 20 hexagons. The tendency is visible on the examples shown in Fig. 7.15 to design steel domes to divide the ball-like sphere into identical polygons, being the bases of pyramids the apices of which are located on the sphere. In order to form a technically useful gridshell dome various polygons having edges of a similar length, adjustable during the assembly, are used. The edges of those pyramids, are arranged at slight angles, e.g. at an angle of 2°32′, as in the dome described in [9]. Such minimal inclination of bars, of a constant section and the moment of inertia, determines the sensibility of steel domes to the effect of the node snap-through.

As distinguished from steel structures, the effect of the node snap-through has not been noted up to the present in the bibliography related to the gridshell wooden coverings.

[1]Curvature—the inverse of a radius.

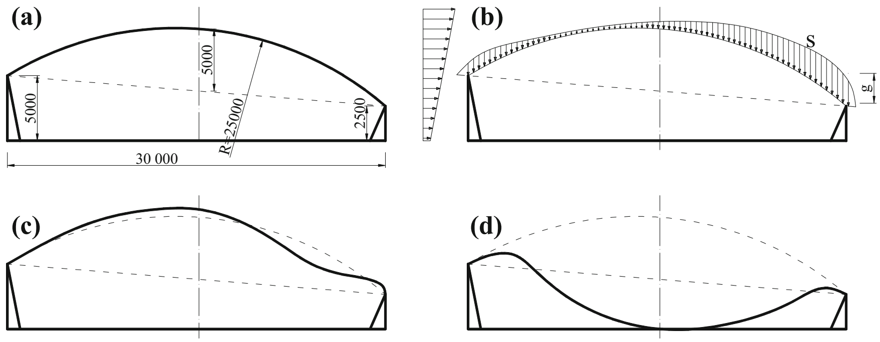

Fig. 7.14 Failure of the steel dome caused by the node snap-through, following the example of the steel dome in Chorzow (Poland), (Augustyn [9])

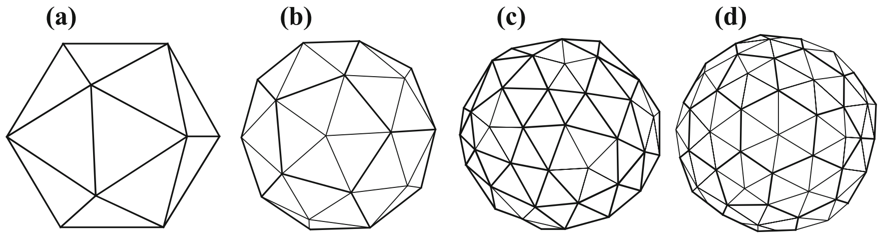

Fig. 7.15 The output polyhedrons to shape the optimum bar nets (*Poradnik projektanta konstrukcji metalowych*, Vol. 2, Part 2, Ch. 10, p. 203 Warszawa, Arkady 1982) **a** icosahedron **b** 60-hedron **c** 140-hendron **d** 180-hedron

The rib—crosshead, cut out from a beam having the geometry shown in Fig. 7.16a, has an advantageously variable moment of inertia of the section, impeding the node snap-through. The shape of the load-bearing element, the use of the variable moment of inertia of the crosshead's section, the largest one in the section of the potential node snap-through, the combination of ribs-crossheads with the plank shell of the dome, all this immunized the wooden gridshell domes to the node snap-through. The shell combined with the ribs became many times internally statically not determinable, and the structure of the node shown in Fig. 7.7d allowed slight local random deformations associated with the natural properties of wood.

Shown in Fig. 7.16 is the shape of a 'crosshead' made from one plank, as well as the schematic diagramme of the distribution of internal forces at the impact of a concentrated load. Figure 7.16b illustrates the schematic diagramme of the node snap-through occurring in steel structures. It consists in the shortening of bars concurring in a node and the lowering of the node, which produces a strong rise of forces in bars, leading to the node snap-through from position "C" to position "C'". Figure 7.16b depicts the phenomenon occurring in the so-called Richard von

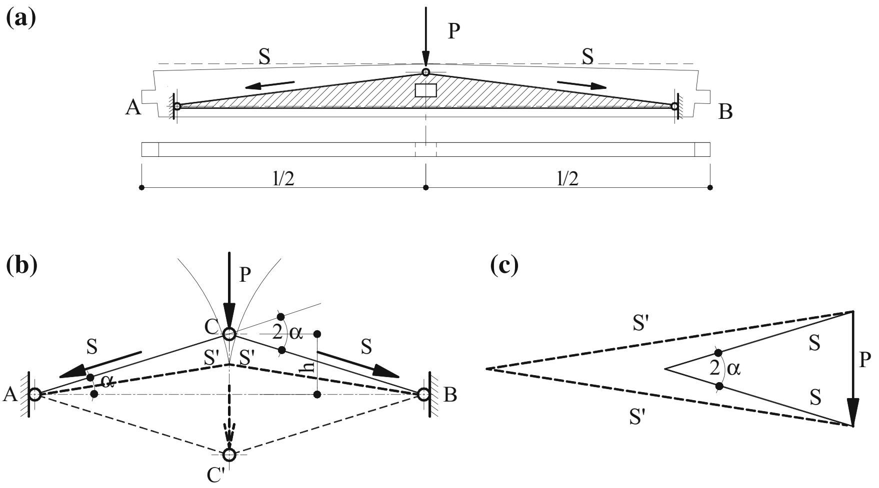

Fig. 7.16 Analysis of a node from 'crossheads'—illustration of the geometry and the effect of the node snap-through **a** the von Mises' truss inscribed into the shape of the arch centres used for the building of gridshell domes from wood, **b** the node snap-through effect in the von Mises' truss. **S′ ≫ S**, **c** rise of forces caused by the shortening of a bar from the von Mises's truss

Mises' truss, who noticed the problem of the node snap-through, however, did not explain the drastic increase of forces in nodes at the impact of the load on the node [10, 11].

The axial forces can locally change in wood due to shrink or swelling at the impact of moisture as well as deformations caused by creep. The shortening of a bar results in the movement of a node to the inside of the sphere and a non-linear increase of forces within the node. It may be presumed that the designers of gridshell domes from crossheads, aware of the impact of the moisture content and temperature and a change in the length of bars from wood rom resulting from it perfected the shape of 'ribs-crossheads' so as to ensure the natural shift of wooden bars in nodes and to provide, at the same time, the stability of the shell. **The rhombic gridshell, the structure of the crosshead and that of the node connecting them allows to match the bars from wood in the situation of load variations, including also the thermal-moisture adaptation**. In the von Mises' truss, inscribed into an arch centre, Fig. 7.16a, there follows a considerable exchange of bending for axial forces. **This mechanism fosters the decrease of wood creep since the smallest creep occurs at forces acting along the fibres when those forces are lower than the minimum axial critical load capacity**. The highest creep is caused by transverse forces, particularly unfavourable for wood.

The above-described phenomenon of the node snap-through was quoted due to the fact that it can appear in the structures of wooden domes built from other elements, as the diameter of the sphere's horizontal projection increases.

7.9 Conclusions

It follows from the performed review of the bibliography that historical, gridshell domes from solid wood were built on a rhombic network of ribs-crossheads. The gridshell of the sphere of such domes were built from a perfected arch centre, provided at its ends with tangs and an opening in the middle of its span. In each node three ribs of the shell were connected. After putting on and consolidating the shell with the rhombic gridshell structure, the multi-layer, ribbed shell of the dome constituted a geometrically invariable and internally statically undeterminable structure capable of transferring non-symmetrical external loads from wind and snow, as well as from own load and thermal and moisture-related impacts.

A next stage was the structure of the rhombic gridshell from flat laid planks. Very important for the development for the building technology of domes was the application of flat planks laid spirally on the nodes of rhombuses, preventing the node snap-through of the gridshell built from flat planks of a fixed section. A multi-layer planking was applied on the protected grid, creating a geometrically invariable and reliable construction of domes with the minimum wood consumption.

The building technology of wooden domes of a rhombic gridshell structure from ribs-crossheads or flat planks and a multi-layer plank shell worked well, which is evidenced by lack of reports of the catastrophes of such domes.

An essential disadvantage of the axially symmetrical gridshell domes is the non-standardness of crossheads whose dimensions and setting vary as they approach the upper ring of the dome. A conclusion was drawn from the testing made on actual and computer models that it is possible to standardize the shape of ribs of only one latitudinal chord of the dome's shell at one level. Probably, for this reason, it was preferred to build gridshell domes in form of closed sectional vaults, stiffened with chords, among which it was possible to standardize all ribs of the rhombic gridshell. The structures in which it was possible to reach the full standardization of arch centres are shown in Figs. 7.2, and 7.4.

Notice-worthy is the fact that the first gridshell steel domes imitated the rhombic structure of gridshell wooden domes. Kersten C. in his publication [4] quotes after the German Building Journal (1926, p. 366.) the description of a steel dome for the building of the Customs Office as demonstrated in Fig. 7.17. The gridshell structure of the steel dome of a 17.0 m diameter was developed according to the rules for gridshell domes from wooden ribs-crossheads. Undoubtedly, in order to hinder the node snap-through. The creators of the later accomplishments of wooden and steel domes, while building domes from bars of a fixed section, forgot the principal rules of the building of geometrically invariable gridshell structures.

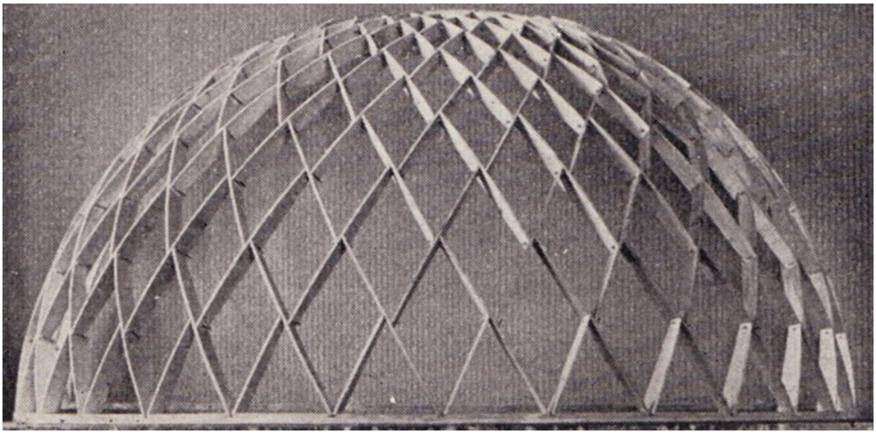

Fig. 7.17 Model of a steel dome of a 17.0 m diameter from 1926 [4]

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