# Chapter 16
# OR Models for Emergency Medical Service (EMS) Management

**S.M. Gholami-Zanjani, M.S. Pishvaee, and S. Ali Torabi**

## 16.1 Introduction to Emergency Medical Services

Emergency medical services (EMS) is a comprehensive procedure for prompt patient care in different settings such as the emergency departments (ED), urgent care clinics, pre-hospital settings, and also disaster situations (Soriya and Colwell 2012). EMS typically consists of call centers, dispatchers, ambulances and paramedics. Accordingly, process of EMS follows some steps such as reception of calls, vehicle dispatching, on-site treatment and release of patient or transportation to health care facilities. This process is presented in Fig. 16.1. Therefore, emergency medical service plays a critical role in any emergency care system. The growing demand for more efficient EMS has sparked off efforts to evaluate and improve the quality of many EMS systems in recent years. The key objective of many successful EMS systems can be operationally defined by the effective and consistent provision of immediate medical care to seriously ill or injured patients, and the expeditious conveyance of patients to advanced resuscitation and lifesaving care (I.O.M 2006). Medical conditions of patients who require immediate care imply that delay in treatment could result in the worsening of patients' conditions (Lam et al. 2015).

All abovementioned issues indicate the importance of EMS area. Among several relevant issues in this context, ambulance planning problem could help to improve public health level and social health care systems. Generally, the problem of

S.M. Gholami-Zanjani • M.S. Pishvaee
School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran
e-mail: se_Gholami@ind.iust.ac.ir; pishvaee@iust.ac.ir

S.A. Torabi (✉)
School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran
e-mail: satorabi@ut.ac.ir

 
C. Kahraman, Y.I. Topcu (eds.), *Operations Research Applications in Health Care Management*, International Series in Operations Research & Management Science 262, https://doi.org/10.1007/978-3-319-65455-3_16

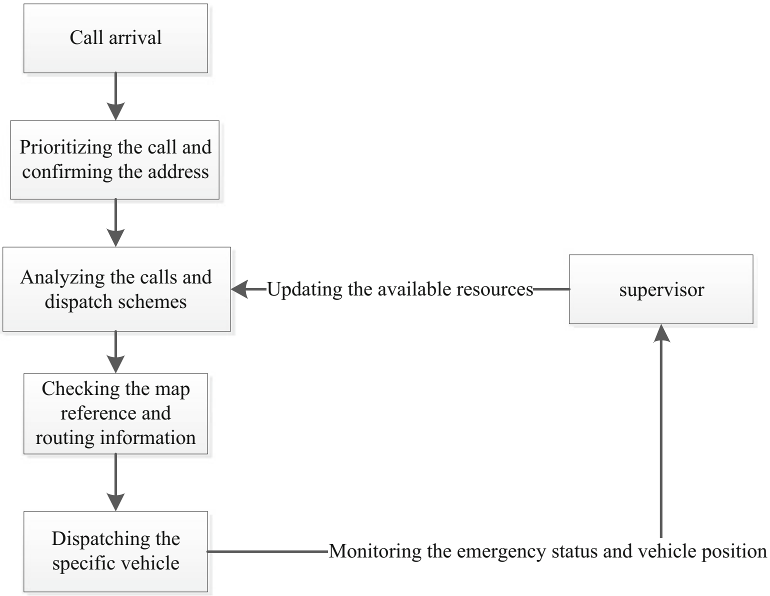


**Fig. 16.1** Emergency response process (Adopted from Haghani and Yang 2007)

locating ambulances and ambulance bases can be divided into strategic, tactical and operational levels. At the strategic level, the locations of the ambulance bases are determined while considering coverage constraints (Berman et al. 2011; Karasakal and Karasakal 2004; Brotcorne et al. 2003; Schmid 2012; Schmid and Doerner 2010). In the next step, the required number of ambulances to fulfill the demand of each base is specified over a multi-period mid-term decision horizon (Brotcorne et al. 2003; Schmid 2012; Essen et al. 2013; Knight et al. 2012; Galvão et al. 2005). Finally, at the operational level, the allocation of ambulances to emergency demands and relocation of ambulances must be carried out in the real-time in an on-going fashion. Therefore, the main decisions to be made involve: the number of required resources, ambulance/helicopter fleet management, location of ambulances, and ambulance routing. The important point is that all aforementioned decisions are highly interrelated and their joint optimization in a sole model could prevent sub-optimality of obtained solutions (Coppi et al. 2013; Jagtenberg et al. 2015).

Dynamic and complex nature of emergency medical systems imposes a high degree of uncertainty in ambulance planning decisions and significantly influences the overall performance of the achieved decisions. Some examples of uncertainties in this context are randomness in the demand of emergency services, location of incidents and variability of travel times. Thus, the significance of accounting for uncertainty has prompted the researchers to address uncertain parameters in EMS models. Stochastic programming approach is often employed by researchers to cope with data uncertainty in the proposed models (Essen et al. 2013; Galvão et al. 2005).

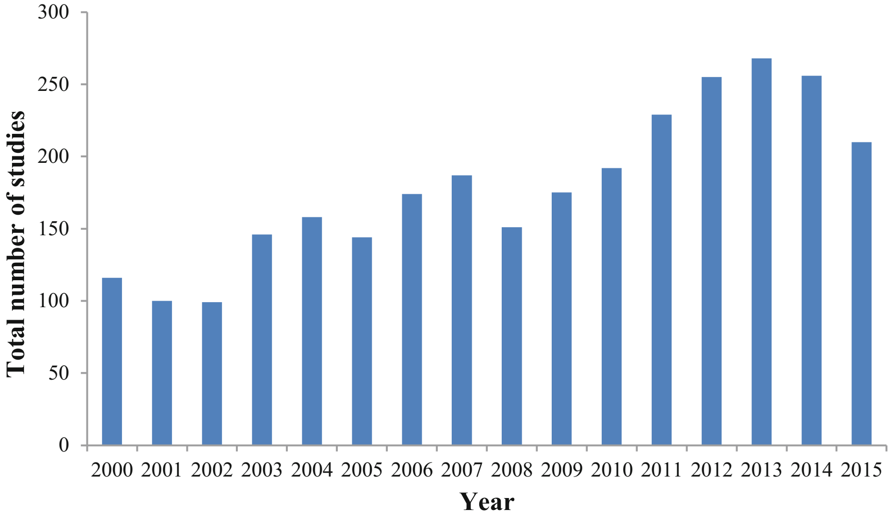


**Fig. 16.2** General trend of healthcare problems

Having reviewed the academic literature of EMS, the general trend of conducted studies, in terms of number of papers, is extracted. In this regard, we have investigated the research titles indexed in Google Scholar and Scopus. These research subjects include “location”, “emergency”, “ambulance”, “routing”, “hospital” and “medical” with various combinations in the titles and subjects. All relevant studies published since 2000 have been reviewed. Being mindful of the fact that we may miss some indirect relevant papers, overlapping some researches and considering papers that could not be directly involved in the investigated framework, the resulted papers could fairly mirror the general trend. All together, we have ended up with numerous papers in this field since 2000. Figure 16.2 shows the number of articles with respect to relevant years. It is obvious that there is a growing interest in the subject. It seems that this trend in drawing attentions is likely to continue due to rise in the elderly population of the societies.

As it is mentioned earlier, the EMS problems could be classified into two main categories: (1) strategic and (2) tactical/operational level problems. Strategic decisions are mainly concerned with location of ambulance stations (i.e. those sites where ambulances are waiting to be dispatched to incident locations), medical care facilities and staff hiring while tactical/operational decisions focus on short and mid-term decisions such as crew scheduling, crew pairing, number of needed vehicles, ambulance standby site location, ambulance relocation and dispatching rules. Dispatching decisions are related to ambulance routing problems. Generally, the main three types of decisions in the emergency medical service include: (1) ambulance location, (2) ambulance relocation, and (3) ambulance dispatching, which significantly influence the cost efficiency and responsiveness (e.g. response time) measures (Bruzzone and Signorile 1998). In this chapter, we focus on strategic and operational decisions as per their importance in this area.

The remainder of this chapter is continued in Sect. 2 by introducing and reviewing EMS strategic planning models. EMS operational planning models are described and investigated in Sect. 3. These sections include distinguished models from the related literature as well. Section 3.1 describes how different uncertainty programming approaches are applied to EMS models. An application of EMS models on a real case is illustrated in Sect. 5. Finally, Sect. 6 includes concluding remarks and some future research directions.

## 16.2 EMS Strategic Planning Models

In EMS systems, response time is considered as a critical issue that directly affects patients' health. Insufficient prompt services dramatically threaten people's lives. The public expects availability and accessibility of EMS facilities to provide sufficient services (Daskin and Dean 2004). However, because of limited allocated governmental budget and other resources, the quality and quantity of services could be vulnerable. By availability we mean adequate supply of services. The opportunity to obtain medical services and resources is defined as the access to healthcare facilities (Gulliford et al. 2002). In other words, accessibility of facilities refers to the capability of the patients to reach the health care facilities. In the case of prompt response services, it is vice versa; capability of health providers such as ambulances to reach patients is considered as the accessibility. In order to attain the desired service level, location of health care facilities such as ambulance bases should be efficiently determined. In addition, high cost resources are involved in maintaining EMS in terms of ambulances, qualified staff and so on (Farahani et al. 2012). Hence, effectively locating the EMS facilities becomes an impressive decision on the system configuration. There are three common service goals used in EMS which are as follows (Kerkkamp 2014).

- Minimizing the average response time to patients' needs
- Minimizing the maximum response time to patients' needs
- Maximizing the number of regions that are supportable (i.e. covered) within a predefined response time

These three goals are corresponding to three basic location models namely, the P-median, P-center and the covering models, respectively. Throughout this chapter our attention is restricted to discrete location models because they have been used considerably in health care location problems while they are practical to reasonable extent. Covering models are used abundantly in the related literature as they are suitable enough for emergency situations. There are several comprehensive reviews that provide deep insights and state-of-the-art in this field (Brotcorne et al. 2003; Farahani et al. 2012; Goldberg 2004; Li et al. 2011; Looije 2013; Van Buuren 2011).

The classical Location Set Covering Model (LSCM) and Maximal Covering Location Problem (MCLP) introduced by Toregas et al. (1971) and Church and ReVelle (1974) are two variants of covering models. These models assume that all the parameters are deterministic and a demand point is roughly covered or uncovered by each located facility. By covering we mean that a node or a demand point is covered by a facility if the distance or response time between the facility and the demand point is less or equal to a predefined value. The sets, inputs and variables used to define the LSCM are as follows.

| ***Set*** | ***Description*** |
|---|---|
| $I$ | Set of demand nodes |
| $J$ | Set of potential facility sites |
| $f_j$ | Fixed cost of locating a facility at site $j$ |
| $a_{ij}$ | Equals to 1 if demand point $j$ can be covered by base (i.e. location) $i$, otherwise it equals to 0 |

| ***Variable*** | ***Description*** |
|---|---|
| $X_j$ | Equals to 1 if a facility is located at potential site j, otherwise 0 |

With this notation, the LSCP is as follows.

$$Minimize \sum_{j \in J} f_j X_j \tag{16.1}$$

$$Subject\ to \sum_{j \in J} a_{ij} X_j \geq 1 \forall i \in I \tag{16.2}$$

$$X_j \in \{0, 1\}\ \forall j \in J \tag{16.3}$$

The decision variable $x_j$ indicates whether base $j$ is opened or not. The objective function (16.1) minimizes the total cost of establishing the selected facilities that has EMS vehicles stationed whilst covering all demand points. Constraints (16.2) states that each demand point must be covered by at least one opened facility. Constraints (16.3) ensure that there is no fractional part of healthcare locations. In addition to this model, another objective function which minimizes the number of located facilities is of interest. In this case, the objective function is as follows.

$$Minimize\ \sum_{j \in J} X_j \tag{16.4}$$

Such an objective function could be justifiable when the fixed EMS facility costs are almost equal. Daskin and Dean (2004) referred to the model minimizing the total cost, as the set covering problem and to one minimizing the total number of located facilities, as the location set covering problem.

The above-mentioned model contains some shortcomings which are discussed in below.

- LSCM seeks the minimum amount of EMS bases required to cover all the demand points by considering unlimited amount of available stations. However, the needed resources (the cost or number of required bases) are often prohibitive. In real world problems, there is certain amount of available ambulances.
- Another disadvantage is that this model ensures that each demand point is covered just once whilst if EMS vehicles are dispatched and busy, other emergency demand points covered by the corresponding vehicles could not be covered any more.
- Furthermore, the model does not distinguish between demand nodes that relatively generate lots of demand and those that generate little demand. It is clear that the regions with more requests are of high investigation in comparison to less generated demand points.

In this regard, these concerns motivated some models to compensate for above-mentioned shortcomings. One of these models is Maximal Covering Location Problem (MCLP), developed by Church and ReVelle (1974). To formulate this model, the following notations should be denoted.

| | |
|---|---|
| $h_i$ | Demand at demand point $i$ |
| $P$ | Number of available EMS resources |
| $Z_i$ | Binary variable, it equals to 1 if demand point $i$ is covered, otherwise 0 |

**Notations**

Demand of node $i$ indicates for example the number of incidents per hour or the number of residents in that area. Accordingly, the mathematical formulation is as follows:

$$Maximize \sum_{i \in I} h_i Z_i \tag{16.5}$$

$$Subject\ to\ Z_i - \sum_{j \in J} a_{ij} X_j \leq 0 \forall i \in I \tag{16.6}$$

$$\sum_{j \in J} X_j = P \tag{16.7}$$

$$X_j \in \{0, 1\} \quad \forall j \in J \tag{16.8}$$

$$Z_i \in \{0, 1\} \quad \forall i \in I \tag{16.9}$$

Objective function (16.5) maximizes the total amount of covered demands (not number of demand points). Constraints (16.6) stipulate that demand point $i$ cannot be covered unless an EMS vehicle has been located within the relevant. Constraints (16.7) ensure that $p$ facilities should be exactly located. Constraints (16.8) and (16.9) are standard integrality constraints. Another objective function for this model could be minimizing uncovered demand as follows.

$$Minimize \sum_{i \in I} h_i \left(1 - Z_i\right) \tag{16.10}$$

As it was mentioned before, a considerable disadvantageous is that these models do not allow multiple EMS vehicles at each base. Therefore, in real-life cases, this property results in uncovered demand points if EMS vehicles are busy for handling other calls. To deal with this shortcoming, some extended versions are developed such as Backup Coverage Problems (BACOP) (ReVelle and Hogan 1989) and Double Standard Model (DSM) (Gendreau et al. 1997).

### *16.2.1 Backup Coverage Problems (BACOP)*

Backup coverage problems (BACOP) firstly developed by (ReVelle and Hogan 1989). BACOP models try to maximize demand points that are covered twice. The first BACOP (i.e., BACOP 1) model is similar to the MCLP but it ensures that each point is covered by at least one facility. The model's objective function maximizes the demands which are covered twice while in the second BACOP model (i.e., BACOP 2) objective function is modified rewarding demand points that are covered only once as well as those covered twice. In other words, it is combination of the demands covered once and the ones covered twice.

**Backup Covering Problem 1**

$$Max \sum_{i \in I} h_i \mu_i \tag{16.11}$$

$$S.t. \sum_{j \in J} X_j = P \tag{16.12}$$

$$\left( \sum_{j \in J} X_j \right) - \mu_i \geq 1 \tag{16.13}$$

$$X_j \in \{0, 1\} \quad \forall j \in J \tag{16.14}$$

$$\mu_i \in \{0, 1\} \quad \forall i \in I \tag{16.15}$$

**Backup Covering Problem 2**

$$Max\ \alpha \sum_{i \in I} h_i \mu_i + (1 - \alpha) \sum_{i \in I} h_i \lambda_i \tag{16.16}$$

$$S.t. \sum_{j \in J} X_j = P \tag{16.17}$$

$$\mu_i - \lambda_i \leq 0 \tag{16.18}$$

$$\left( \sum_{j \in J} X_j \right) - \mu_i - \lambda_i \geq 1 \tag{16.19}$$

$$X_j, \mu_i, \lambda_i \in \{0, 1\}\ \forall j \in J, \forall i \in I \tag{16.20}$$

In BACOP1 binary variable $\mu_i$ is defined such that it equals to 1 if and only if demand point $i$ is covered at least twice by EMS bases. Therefore, in BACOP1, Objective function (16.11) maximizes the population covered at least twice. Constraints (16.12) states that there is only $p$ number of EMS resources. Constraints (16.13) stipulate that each demand point is covered twice.

In BACOP2, in addition to $\mu_i$, $\lambda_i$ is defined as a binary variable for the demand points that are covered at least once. In this model, single covered demand points are taken into account by a weight factor ($\alpha \in [0, 1]$). Constraints (16.18) state that demand points that are covered twice are also covered once. Constraints (16.19) imply that if demand point $i$ is covered once then $\lambda_i = 1$; and, if the demand point is covered by more than one EMS resource then $\mu_i$ is equal to 1.

Although LSCP and MCLP are relatively simple in their formulation, they have resulted in considerable number of extensions and variants. An extension of MCLP model is the tandem equipment allocation model (TEAM) introduced by Schilling et al. (1979). In this model, two types of vehicles, primary and special vehicles, are considered. This model assumes that the coverage distance is higher for primary vehicles in comparison to secondary ones. Therefore, the maximal travel time is lower than the maximal travel time for the special ones. Daskin and Stern (1981) developed Hierarchical Objective Set Covering Model (HOSC) as a combination of LSCM and MCLP models to locate EMS vehicles in Austin (Texas). This model simultaneously maximizes the extent of multiple coverage of demand and minimizes number of the locations. However, in real world problems, the likelihood of having two vehicles busy is low. A Double Standard Model (DSM) is a more recent model, developed by Gendreau et al. (1997). This model makes distinction between two coverage parameters (response times). It looks for solutions through which all demand points are covered at least once within response time $r_1$ and $\alpha$ percent of them must be handled within $r_2$. This model can be considered as an extension for BACOP1. Doerner et al. (2005) applied DSM to locate ambulances in Australia. The corresponding results demonstrate that a large proportion of population is covered in the limited amount of time and some vehicles are used more than once. Laporte et al. (2009) also applied DSM model to tackle real world problems in Belgium, Australia and Canada. Su et al. (2015) modified the DSM model by changing the objective function into minimization of operational costs and the cost of delayed services. They applied the resulted model to ambulance deployment in Shanghai, China. Backup Double Coverage model is a different backup coverage model developed by Basar et al. (2009). In this model, a demand point is covered if at least two opened bases are near to that demand zone. Church and Gerrard (2003) considered a generalization of LSCM in which multi-level coverage is needed.

### *16.2.2 Nonlinear Integer Programming Model for Real-Time EMS Vehicle Dispatching Model*

In this subsection, the model presented by Majzoubi et al. (2012) is reviewed as a sample while rich model involving vehicle dispatching and relocating decisions. This model assigns a vehicle to a patient based on vehicle's current location and availability status of all other vehicles. Further, patients are assigned to hospitals again based on the available resources and travel times. In addition to these assignments, vehicles are relocated after completing their services to serve the next demand points. This model incorporates assumptions for some medical emergencies such as influenza outbreak. The following assumptions are made to present this model.

- The vehicle fleet is heterogeneous and can serve every patient
- All vehicles can be tracked by global positioning equipment
- Information about hospital resources are known in advance as a priori
- At each state the model is invoked for running, only the available resources are considered
- The priority of requests are known by EMS call takers

The following notations are introduced to formulate the proposed model.

## Sets

| | |
|---|---|
| $I$ | Set of vehicles $i = 1 \ldots I$ |
| $J$ | Set of patients $j = 1 \ldots J$ |
| $K$ | Set of hospitals $k = 1 \ldots K$ |
| $S$ | Set of EMS stations $s = 1 \ldots S$ |
| $R$ | Set of census tracts $r = 1 \ldots R$ |
| $P$ | Set of patients' priority $p = 1, 2$ |
| $V$ | Set of available vehicles $V = V^P \cup V_1^H \cup V_2^H \cup V^E$ |
| $V^P$ | Set of vehicles en route to serve a patient |
| $V_1^H$ | Set of vehicles that are able to transport high priority patients or more than one patient to the hospitals |
| $V_2^H$ | Set of vehicles that are able to transport low-priority patients to the hospitals |
| $V^E$ | Set of vehicles that are en route to an EMS station or they are idle |
| $W$ | Set of patients or demand points $W = W^1 \cup W^2$ |
| $W^1$ | Set of high-priority patients |
| $W^2$ | Set of low-priority patients |

## Parameters

| | |
|---|---|
| $T_{ij}$ | The estimated travel time for vehicle $i$ to arrive at patient $j$'s location when departing at the current time |
| $T_{ik}$ | The estimated travel time for vehicle $i$ to arrive at hospital $k$ when departing at the current time |
| $T_{is}$ | The estimated travel time for vehicle $i$ to arrive at EMS base $s$ when departing at the current time |
| $T_{jk}$ | The estimated travel time from patient $j$ to hospital $k$ |
| $\mu$ | The average service time for serving each patient at her/his location |
| $PP_{pr(j)}$ | The travel cost per time unit to reach patient $j$ with priority $j$ |
| $IP$ | The travel cost per time unit of idle vehicles |
| $A_{ij}$ | The travel cost for vehicle $i$ to reach patient $j$; $A_{ij} = PP_{pr(j)} \times T_{ij}$ |
| $B_{ik}$ | The travel cost for vehicle $i$ to arrive at hospital $k$; $B_{ik} = PP_{pr(j)} \times T_{ik}$ |
| $B_{ij}$ | The travel cost for vehicle $i$ to reach patient $j$; $B_{ij} = PP_{pr(j)} \times T_{ij}$ |
| $B_{w(i)}$ | The cost of waiting for patient in a vehicle that is serving another patient; $B_{w(i)} = PP_{pr(j)} \times \mu$ |
| $B_{jk}$ | The cost of traveling from patient $j$ to hospital $k$; $B_{jk} = PP_{pr(j)} \times T_{jk}$ |
| $C_{is}$ | The cost of traveling to EMS base $s$ by vehicle $i$; $C_{is} = IP \times T_{is}$ |
| $D$ | The penalty cost incurred by serving more than one patient by one vehicle |
| $E$ | The penalty cost of inefficient routing |

| | |
|---|---|
| $\alpha_p$ | The penalty cost of violating time window for a customer with priority $p$ |
| $\beta_r$ | The penalty cost of not covering census tract $r$ by an ambulance |
| $\rho_j$ | The required response time for patient $j$ |
| $\xi$ | Threshold time for reroute acceptance |
| $\omega$ | Threshold value for serving another patient |

## Variables

| | |
|---|---|
| $x_{ij}$ | Equals to 1 if vehicle $i$ is dispatched to serve patient $j$; 0 otherwise |
| $y_{ik}$ | Equals to 1 if vehicle $i$ is dispatched to hospital $k$; 0 otherwise |
| $z_{is}$ | Equals to 1 if vehicle $i$ is dispatched to EMS station; 0 otherwise |
| $\pi_i$ | Equals to 1 if ambulance $i$ serves more than one patient; 0 otherwise |
| $\theta_i$ | Equals to 1 if vehicle $i$ is not efficiently rerouted to serve more than one patient; 0 otherwise |
| $u_j$ | Equals to 1 if vehicle if patient $j$ is not served in its time window; 0 otherwise |
| $\upsilon_r$ | Equals to 1 if vehicle $i$ is not covered by at least one ambulance; otherwise |
| $w_i$ | Equals to 1 if vehicle $i$ is not rerouted; 0 otherwise |
| $\tau_{ik}$ | $\tau_{ik} = T_{ik} \times y_{ik}$ if $\pi_i = 0$ and $\tau_{ik} = \sum_j T_{ik} \times y_{ik} \times x_{ij}$ if $\pi_i = 1$ |
| $\zeta_{ijk}$ | $\zeta_{ijk} = y_{ik} \times x_{ij}$, equals to 1 if vehicle $i$ is dispatched to serve patient $j$ and dispatched to hospital $k$ |

It should be noted that $T_{ij}$, $T_{ik}$ and $T_{is}$ are updated over the time. Using the abovementioned notations, the proposed non-linear mathematical model for the real-time vehicle dispatching (RTEMSVD) is as follows:

$$\begin{aligned} Min \sum_i \sum_j A_{ij}\, x_{ij} &+ \sum_{i \in V_1^H} \sum_k B_{ik} y_{ik} + \sum_{i \in V_2^H} \sum_k B_{ik} y_{ik} \left(1 - \sum_j x_{ij}\right) \\ &+ \sum_{i \in V_2^H} \sum_j B_{ij} x_{ij} + \sum_{i \in V_2^H} \sum_k y_{ik} \left(\sum_j x_{ij}\right) \left(\sum_j B_{jk} x_{ij} + B_{w(2)}\right) \\ &+ \sum_i \sum_s C_{is} z_{is} + \sum_r \beta_r \upsilon_r + \sum_p \alpha_p \sum_{j \in W^p} u_j \end{aligned} \tag{16.21}$$

$$\sum_i x_{ij} = 1 \forall j \tag{16.22}$$

$$\sum_i x_{ij} = 1 \forall i \in V^p \tag{16.23}$$

$$\sum_k y_{ik} + \sum_s z_{is} = 0 \forall i \in V^p \tag{16.24}$$

$$\sum_{i'=1} \sum_{j \in W^p} A_{ij} x_{i'j}^0 - \sum_{i'=1} \sum_{j \in W^p} A_{ij} x_{ij} - \xi \geq -M w_i \forall i \in V^p \tag{16.25}$$

$$\sum_j \left(1 - x_{ij}\right)\ x_{ij}^0 \leq M\left(1 - w_i\right) \forall i \in V^p \tag{16.26}$$

$$\sum_k y_{ik} = 1 \forall i \in V_1^H \tag{16.27}$$

$$\sum_j x_{ij} + \sum_s z_{is} = 0 \forall i \in V_1^H \tag{16.28}$$

$$\sum_k y_{ik} = 1 \forall i \in V_2^H \tag{16.29}$$

$$\sum_j x_{ij} \leq 1 \forall i \in V_2^H \tag{16.30}$$

$$\sum_s z_{is} = 0 \forall i \in V_2^H \tag{16.31}$$

$$\sum_j x_{ij} + \sum_s z_{is} = 0 \forall i \in V^E \tag{16.32}$$

$$\sum_k y_{ik} = 0 \forall i \in V^E \tag{16.33}$$

$$\sum_i x_{ij} T_{ij} - \rho_j \leq M u_j \forall j \tag{16.34}$$

$$1 - \sum_s \sum_i z_{is} \psi_{sr} \leq M \upsilon_r \forall r \tag{16.35}$$

It is clear that all the variables are binary in this model. Equation 16.21 represents the objective function that consists of eight terms including vehicles' cost to reach a patient, transporting high-priority patients, and the third through fifth terms are for low-priority patients. The sixth term is the cost of returning vehicles to EMS bases. Seventh term is a penalty for not serving patients in the defined time window and finally, the last term considers the penalty of not serving a census tract by ambulances.

Constraints (16.22) stipulate that each patient should be served by an ambulance. Constraints (16.23) through (16.26) are related to the vehicles on the way to serve patients. Constraints (16.23) state that any dispatched vehicle is assigned to a patient. Vehicles on their way can neither go to a hospital nor EMS bases; this is ensured by constraints (16.24). Constraints (16.25) and (16.26) consider route manipulation for a vehicle if there is a reduction in travel time. Moreover, Constraint (16.27) states that vehicles, transporting high-priority patients, are assigned to a hospital and also they are not allowed to serve other patient and return to the base, presented by constraints (16.28). As Constraints (16.29) assign the low-priority patients' vehicles to hospitals and in contradiction with serving high-priority vehicles, Constraints (16.30) allow them to serve other patients. Constraints (16.31) ensure that these vehicles are not returned to EMS bases. Constraints (16.32) and (16.33) are related to idle vehicles. Constraints (16.32) allow these vehicles to remain at the same (or other) stations or serve a patient while Constraints (16.33) states that they cannot be assigned to hospitals. Response times are satisfied by Constraints (16.34) and (16.35).

## 16.3 EMS Operational Planning Models

As mentioned before, this chapter has focused on strategic and operational levels. In this section operational problems are investigated. Response time to reach the demand points is a critical issue for locating and relocating EMS bases and especially ambulances. Emergency medical aspects are often combined with routing and dispatching problems. Over $2.5 billion out of $5 billion of EMS expenditure is spent on transportation of patients (Sayre et al. 2001). Apart from this potential benefit, there are some other objectives that are desired to be optimized. These criteria include improving the information utilization and maximizing the productivity of services. In other words, at the operational level planning, it has to be decided which ambulance should be assigned to which request. Usually, when an emergency request occurs, the ambulance with the shortest path is dispatched (Golden et al. 2008). The contributions in ambulance operational planning are sparse and there is a little OR-based research works conducted on dispatching of vehicle fleets. Two problems, however, as ambulance dispatch and relocation are of great interest (Andersson and Värbrand 2007).

In the real world conditions, the regular transportations and emergency ones are scheduled and controlled with the same fleet of ambulances. Form another perspective, emergency transportations are required to service the emergency demand points. This kind of transportation includes dynamic elements since the parameters such as demand points, driving times and service times are uncertain and unknown at the decision making moment (Créput et al. 2011). Generally, service providers face with two different types of services based on their fleet of vehicles. First, they should cover their assigned region, as an output of location problem, in case of emergency requirements. Secondly, some regular requirements should be satisfied in terms of periodic pick-ups, delivery to patients, etc. Considering these two issues, regular transportations beside emergency ones, in one problem requires a highly robust and dynamic plan. These two problems seek different objectives. From the perspective of regular transportation, the total travel cost in terms of time or distance is minimized whilst in the case of emergency services, providing services within a predetermined amount of time, is desired (Bruzzone and Signorile 1998). Taking these two issues into account simultaneously, the arisen problem becomes a very challenging problem. For the sake of simplicity, these two types of demands (emergency and regular transfers) can be managed independently by dividing the vehicle fleet and managing them separately. However, this attitude leads to less efficient results in comparison to the integrated approach (Kergosien et al. 2014).

Studying approaches applied to Dynamic Vehicle Routing (VRP) can provide notable insights for decision makers deciding on ambulance dispatching problems. In those VRP addressing ambulance dispatching problem, vehicles can dynamically change their routes to serve the demand points within a time window. The regular transportations are also known as "Dial a Ride Problem (DARP)" (Cordeau and Laporte 2003). This problem consists of designing vehicle routes and schedules for a set of clients to satisfy their transportation requests. In this regard, some considerations are inevitable such as vehicle capacity, maximum rid time constraint, meeting time windows, etc. Moreover, there could be different criteria like mean user ride time or total distance traveled. This problem can be employed for both static and dynamic contexts (Cordeau and Laporte 2007; Parragh et al. 2010; Attanasio et al. 2004). Beaudry et al. (2010) applied this approach into medical context. Also, Parragh (2011) studied DARP with different types of vehicles (i.e., seated, wheelchair and stretcher).

The ambulance dispatching problem is a kind of general assignment problem (Goldberg 2004). This problem is solved considering several criteria such as nearest origin, highest priority or first come first served (Haghani and Yang 2007). In other words, various dispatching rules such as Nearest Origin (NO), Highest Priority First Serve (HPFS) and First Come First Serve (FCFS) could be applied to different models. Dispatchers could have an ordered priority list for demand zones. Dispatching decisions can remarkably affect system outcome in terms of response time and survival rate. Hogan and ReVelle (1986) suggested dispatching the nearest ambulance to the demand with the highest priority. For the low-priority demands, they suggested dispatching ambulances in such a way that the demands are served in the predefined time windows. Green and Harries (1988) studied this problem and concluded that dispatching the nearest vehicle is efficient in the case of minimization of the average or total response time. Yang et al. (2004) developed a dispatching model that can be employed for EMS. Their model's objective is to minimize travel times in addition to the penalties for violating the desired time window and not covering demand points. Ibri et al. (2010) considered coverage and dispatching decisions as an integrated approach.

### 16.3.1 Joint Ground and Air Emergency Medical Services Coverage Model

This subsection presents the model introduced by Erdemir et al. (2010) as an operational sample model. They involved aeromedical services as well as ground ambulances in responding to trauma crashes. In this regard, they proposed a location-coverage model on the basis of both response time and total service time. This model addresses location of air and ground ambulances, and transfer points. Locating transfer points are justifiable since helicopters cannot always land at the crash sites. Therefore, it considers three options for transporting the demand requests as ground, air and joint ground-air. Two models were developed for this problem that one of them is presented here, which is a Set Covering with Backup Model (SCBM).

This model aims at covering all demand points using a combination of air and ground ambulances. As an extension to LSCP model, SCBM allows joint coverage through a combination of two facilities on a transfer point as well as exclusive coverage of two kinds of facilities. Further, other LSCP extensions consider single or backup coverage only in a direct way. To formulate the model the following notations are introduced.

**Sets**

| | |
|---|---|
| $M_h$ | Set of potential helicopter locations $h = 1 \ldots H$ |
| $M_a$ | Set of potential ground ambulance locations $a = 1 \ldots A$ |
| $M_r$ | Set of transfer point locations $r = 1 \ldots R$ |
| $N$ | Set of all crash paths |
| $P$ | Set of all crash nodes |

**Parameters**

| | |
|---|---|
| $C_h$ | Cost of locating an air ambulance |
| $C_a$ | Cost of locating a ground ambulance |
| $C_r$ | Cost of locating a transfer point |
| $A_{aj}(A_{ak})$ | Equals to 1 if a ground ambulance location covers node $j$ through path $k$; 0 otherwise |
| $A_{hj}(A_{hk})$ | Equals to 1 if potential helicopter location $h$ covers node $j$ through path $k$; 0 otherwise |
| $A_{ahrj}(A_{ahrk})$ | If potential air and ground ambulances ($a$ and $h$), and transfer point location $r$ covers node $j$ through path $k$; 0 otherwise |

**Variables**

| | |
|---|---|
| $x_a$ | Equals to 1 if ground ambulance is located at site $a$; 0 otherwise |
| $y_h$ | Equals to 1 if air ambulance is located at site $h$; 0 otherwise |
| $z_r$ | Equals to 1 if a transfer point is located at site $r$; 0 otherwise |
| $u_j$ | Equals to 1 if node or path $j$ is covered by at least one of the located helicopters; 0 if node or path $j$ is covered by at least two ground ambulances and/or combinations |
| $v_{ja}$ | Equals to 1 if node or path $j$ is covered by ground ambulance $a$; 0 otherwise |
| $l_{ahr}$ | $l_{ahr} = x_a y_h z_r$, Equals to 1 if a ground ambulance, helicopter and transfer points are located at $a$, $h$ and $r$, respectively; 0 otherwise |

Using the abovementioned notation, the proposed mathematical model is as follows:

$$Min\ Z = \sum_{a \in M_a} C_a x_a + \sum_{h \in M_h} C_h y_h + \sum_{r \in M_r} C_r z_r - \sum_{j \in N \cup P} u_j \varepsilon \tag{16.36}$$

$$s.t. \sum_{h \in M_h} A_{hj} y_h \geq u_j \forall j \in N \cup P \tag{16.37}$$

$$A_{aj} x_a + \sum_{h \in M_h} \sum_{r \in M_r} A_{ahrj} l_{ahr} \geq v_{ja} \forall j \in N \cup P, \forall a \in M_a \tag{16.38}$$

$$\sum_{a \in M_a} v_{ja} = 2\left(1 - u_j\right) \forall j \in N \cup P \tag{16.39}$$

$$x_a \geq l_{ahr} \forall a \in M_a, h \in M_h, r \in M_r \tag{16.40}$$

$$y_h \geq l_{ahr} \forall a \in M_a, h \in M_h, r \in M_r \tag{16.41}$$

$$z_r \geq l_{ahr} \forall a \in M_a, h \in M_h, r \in M_r \tag{16.42}$$

$$z_r + y_h + x_a - l_{ahr} \leq 2 \forall a \in M_a, h \in M_h, r \in M_r \tag{16.43}$$

$$z_r, y_h, x_a, l_{ahr}, u_j \& v_{ja} \in \{0, 1\} \tag{16.44}$$

Objective function (16.36) minimizes the total cost of locating ground ambulances, air ambulances and transfer points. Sum of variables $u_j$ multiplied by a very small number $\varepsilon$, is subtracted from the total cost because through this subtraction, assignment of two different ground ambulances to cover node $j$ is relaxed. Constraints (16.37–16.39) are the set covering and backup coverage constraints ensuring that all nodes are covered at least twice by ground ambulances, or once by an air ambulance and/or by a combination of air and ground ambulances. Constraints (16.40–16.43) linearized counterparts of the nonlinear terms stipulating that $l_{ahr}$ cannot be 1 unless at least one of $x_a$, $y_h$ or $z$ is 0. In other words, this term justifies combination of air and ground ambulances used for service. So, if all $x_a$, $y_h$ and $z_r$ are 1 then all the EMS servers that form the combination are located and available.

## 16.4 EMS Models Under Uncertainty

The complex nature and dynamic structure of EMS systems imposes a high degree of uncertainty that could remarkably affect the overall performance of the system. Since the emergency event may happen at anytime and anyplace, uncertainty is inevitable in emergency situations. On the other hand, there are some limitations in multiple coverage models since they cannot ensure a satisfying service level. Therefore, to be of any help, we must represent the inherent problem complexities. The deterministic models do not consider the busy times of vehicle fleets. However, in real world problems, resources in terms of the total number of vehicle fleets are limited. This assumption as well as busy fractions and reliability can be covered by probabilistic EMS models. In these models, distribution functions are extracted for the arrival calls. In EMS systems, usually arrival calls are assumed to follow the Poisson processes and service is done with respect to FCFS strategy.

Daskin (1983) extended the MCLP model by considering the ambulance busy fraction. The introduced model is the Maximum Expected Covering Location Model (MEXCLP) that includes a busy fraction $\rho \in (0, 1)$ for all ambulances. In this model, three simplifying assumptions are considered as follows:

- Servers operate independently
- Servers' busy probabilities are invariant regarding the locations
- Busy probabilities are the same for all servers.

The mathematical formulation of this model is as follows.

$$Maximize \sum_{j} d_j \sum_{k=1}^{q} (1-\rho)\, \rho^{k-1} z_j^k \tag{16.45}$$

$$S.t. \sum_{i} x_i \leq p \tag{16.46}$$

$$\sum_{i} y_i \leq q \tag{16.47}$$

$$y_i \leq q_i x_i \forall i \tag{16.48}$$

$$\sum_{i} a_{ij} y_i \geq \sum_{k=1}^{q} z_j^k \forall i \tag{16.49}$$

$$x_i \ \& \ z_j^k \in \{0, 1\} \tag{16.50}$$

$$y_i \in N \tag{16.51}$$

This model overestimates coverage since independent assumptions are made. In this model, $x_i$ is a binary variable that is 1 if a facility is set up at location $i$ and $y_i$ determines number of ambulances allocated to opened facility at location $i$. These two are limited by parameters $p$ and $q$. $z_j^k$ is a binary variable that is 1 if demand point $j$ is covered at least by $k$ number of ambulances. According to the base model, $a_{ij}$ shows that if base $i$ can cover demand point $j$. In this model, the objective function is to maximize the expected coverage by the term which is demand multiplied by the sum of marginal gains $(d_j(1-\rho)\rho^{k-1})$. As for the busy ambulance, $\rho^k$ shows the probability that k ambulances are busy. Constraints (16.46) indicate that up to $p$ emergency bases can be opened and Constraints (16.47) stipulate that total number of available ambulances is equal to $q$. Constraints (16.48) ensure that if a base is opened, only a limited number of ambulances can be assigned to it. Constraints (16.49) are covering limitations. Integrality constraints are shown by Constraints (16.50) and (16.51).

Simplifying assumptions of MEXCLP was later relaxed by Batta et al. (1989) through the use of Adjusted Maximum expected Covering Location Problem (AMEXCLP). As another extension, Erkut et al. (2008) incorporated busy fractions into EMS survival models entitled Maximum Expected Survival Location Problem (MEXSLP). The introduced model tends to maximize the survival probability of a patient rather than the expected coverage. In this respect, Galvao et al. (2005) developed a simulated annealing heuristic to cope with location hypercube models in order to calculate the expected coverage.

A valid strategy to incorporate the uncertainty is to determine several scenarios for uncertain parameters. For instance, uncertain demands can be estimated by a set of scenarios. In this respect, since the large number of scenarios makes the model more complex, scenario reduction methods could be useful. Nickel et al. (2016) used a sampling approach in case of large scenario numbers for ambulance location problem with stochastic demands. In this field, Bernaldi and Bruni (2009) developed the two-stage stochastic LSCM model with uncertain demands. In this model, the first stage finds the opened bases and allocated ambulances, and the second stage assigns the demand points to the EMS bases. Van Den Berg et al. (2015) presented the linear formulation of MEXCLP with fractional coverage. The computational results show that by their formulation, run time of the linear integer programming model is considerably shorter than the corresponding non-linear one.

From another perspective, research on stochastic EMS problem is limited due to the following reasons.

- Often, quantifying the uncertainty through the probability distribution functions is too difficult because of data scarcity
- In large-scale problems such as natural disasters, the queuing models could become too complex to tackle with
- Linearization of the non-linear terms is not an easy task

To cope with these challenges, robust optimization approach can be applied in order to provide appropriate solution. This approach uses uncertainty sets rather than probability density functions or scenarios. In this method, the obtained solution is optimal against all possible realizations of the uncertain parameters within their defined sets. Robust models have been rarely applied to EMS problems. Zhang and Jiang (2014) developed a robust counterpart model for a bi-objective EMS design problem. They considered two uncertainty sources as the number of emergency calls and the maximum number of concurrent calls. Among several robust models, they applied ellipsoidal uncertainty sets for both uncertainty sources.

Another application of operations research in EMS could be through quantifying and applying resiliency concepts. Today's EMSs need to be more agile and flexible in order to deal with demand fluctuations, variations in travel times, site disruptions and so on, to quickly respond to emergency demands. In other words, disruptions in EMS could be resolved by some recovery strategies such as: backup supplies, backup transportations modes, capacity buffers and expansions, and facility fortifications. As it was considered in this section, MEXCLP models are rather restrictive for two reasons (Ivanov et al. 2016):

- Failures or business disruption in the locations or networks are possible but not through describable probabilistic assumptions
- These models do not consider dynamics of the system that considerably impacts performance of the system.

In disruptive situations, disruption duration and recovery time considerably impact performance of the system; however it imposes dynamic dimensions to the system. In other words, the mathematical models should contain recovery functions such as differential equations to be able to describe unavailability time of the healthcare facilities and more importantly gradual recovery of capacities. Despite the rich literature on EMS models under uncertainty, most of the studies assume that the disrupted facilities are not returned to the operational mode during the planning horizon. So, regarding the healthcare models, dynamic functions embedded in mathematical models can be helpful to develop more realistic healthcare models.

## 16.5 Case Study

In this section, a real facility location model case study, presented by Jia et al. (2007) is reviewed. The case study is related to large scale emergency medical services (LEMS) in case of natural disasters, terrorist attacks, and etc. local emergency response providers are designed for dealing with regular small-scale emergencies such as vehicle accidents. Location problems, however, in large scales impose special assumptions to be taken into account in order to maximize coverage and services. For instance, these kinds of services follow low rate of frequencies and overwhelmed local emergency responders that often results in national assistances. In this regard, some modifications are inevitable for definition of facility coverage. Prior to present the elaborated models, we have to make some assumptions such as facility location objectives, facility quantity and service quantity. It is noteworthy to know that redundant and dispersed requirements in facility location problem helps to enhance reliability of facilities since some emergency situations such as earthquakes affect the availability of facilities (Jia et al. 2007). Moreover, in LEMS facility locations, there are different types of coverage since occurrence of an emergency at a specific location needs more than one facility to quell the impact of the emergency. Another point is that the demand areas in LEMS should be categorized with respect to attributes such as economic importance, weather pattern, and population density, and so on.

Being mindful of modifications, Jia et al. (2007) investigates locating medical supplies for large scale services. To be more specific, protective equipment and antidotes stocks against dirty bomb attacks are to be located. In this regard, three general facility location models namely covering model, P-median model and P-center model are developed with the analyzed assumptions to formulate the location problems in large scale disasters including anthrax, dirty bomb and smallpox terrorist attacks.

According to Fig. 16.3, the considered area for locating the facilities is Lose Angeles. This area is gridded into square zones and each zone's center is considered to be aggregated demand point. Downtown, West Hollywood, LAX airport, Port of LA, Port of Long Beach, Rowland Heights and Disneyland are considered as seven demand zones.

In this figure, a number of nodes are defined as potential nodes for locating facilities. It is assumed that resources are adequate to locate just four numbers of facilities in seven potential nodes.

Developed models are employed to solve the defined problem and afterwards a comparison is made by the obtained results between the classic and developed models. The notations of these models are presented as follows.

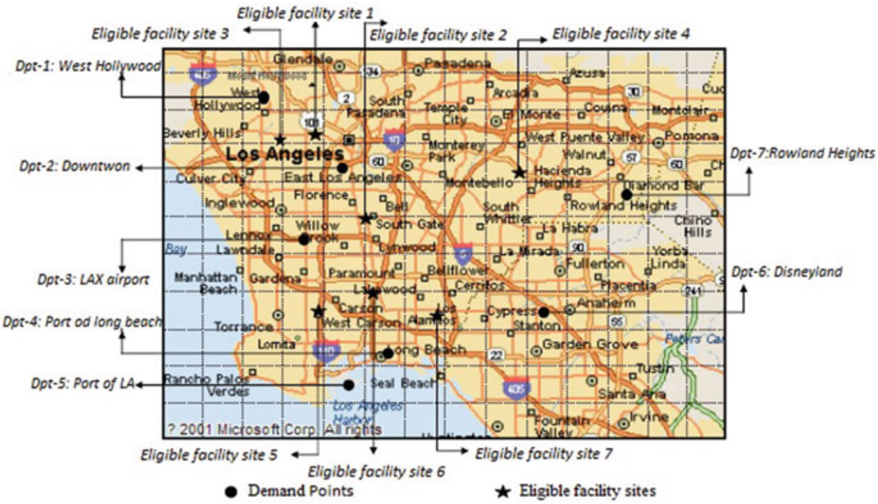


**Fig. 16.3** Los Angeles country gridded map (Adopted from Jia et al. 2007)

## Sets

| | |
|---|---|
| $I$ | Set of possible demand points $i = 1 \ldots I$ |
| $J$ | Set of possible facility locations $j = 1 \ldots J$ |
| $K$ | Set of emergency scenarios $k = 1 \ldots K$ |

## Variables

| | |
|---|---|
| $x_j$ | Equals to 1 if a facility is placed at $j$; 0 otherwise |
| $z_{ij}$ | Equals to 1 if facility $j$ covers demand point $i$; 0 otherwise |
| $u_i$ | Equals to 1 demand point $i$ is covered; 0 otherwise |

## Parameters

| | |
|---|---|
| $M_i$ | The population of demand point $i$ |
| $e_{ik}$ | The impact of demand point $i$ under emergency scenario $k$ |
| $\beta_{ik}$ | The probability of emergency scenario $k$ for demand point $i$ |
| $Q_i$ | Minimum number of facilities serving demand point $i$ to consider demand point $i$ as covered |
| $p_{jk}$ | Service level reduction in facility $j$ by emergency scenario $k$ |
| $P$ | The maximum available number of facilities to be located in possible locations |

The mathematical formulation for dirty bomb emergency situation is as follows:

$$Maximize \sum_{i \in I} \beta_{ik} e_{ik} M_i u_i \tag{16.52}$$

$$s.t. \sum_{j \in N_i} x_j p_{jk} \geq Q_i u_i \quad \forall i \tag{16.53}$$

$$\sum_{j \in J} x_j \leq P \tag{16.54}$$

$$u_i, x_j \in \{0, 1\} \quad \forall i, j \tag{16.55}$$

This model aims at maximizing coverage under occurrence of scenario $k$. Constraints (16.53) ensure coverage of each demand point by required number of facilities. Constraint (16.54) enforces that the maximum number of facilities does not exceed $P$. Now, the $P$-median model in presented for Anthrax emergency problem under occurrence of scenario $k$. As it was mentioned earlier, in this model the objective function minimizes the total distances between demand points and their serving facilities.

$$Minimize \sum_{j \in J} \sum_{i \in I} \beta_{ik} e_{ik} M_i z_{ij} d_{ij} \tag{16.56}$$

$$s.t. \sum_{j \in J} z_{ij} p_{jk} = Q_i \quad \forall i \tag{16.57}$$

$$\sum_{j \in J} x_j \leq P \tag{16.58}$$

$$z_{ij} \leq x_j \quad \forall i, j \tag{16.59}$$

$$z_{ij}, x_j \in \{0, 1\} \quad \forall i, j \tag{16.60}$$

In this model, constraints (16.57) indicate that each demand point is covered if there are $Q_i$ number of facilities serving it and constraints (16.59) ensures that a demand point $i$ can be served by a facility $j$ if the facility is already established.

In the following, the $P$-center model, corresponding to Small-pox emergency is presented. This model aims at minimizing the maximum distance for all demand points. As it can be seen, the only difference between the P-center and P-median model is about their objective functions, presented as objective function (16.61) and constraints (16.62).

$$Minimize \;\; L \tag{16.61}$$

*s.t.* (16.57–16.60)

$$L \geq \frac{\sum_{j \in J} \beta_{ik} e_{ik} M_i z_{ij} d_{ij}}{Q_i} \tag{16.62}$$

Each of the proposed large-scale examples is compared with its corresponding traditional model. According to Table 16.1, in case of dirty bombs, the optimal solution of classical coverage model locates the facilities at sites 1, 4, 6 and 7. This solution ensures that 100% of population is covered by one facility while in the multiple-coverage case only 21% of the population is covered. On the other hand, the proposed model can cover 97.5% of population as single coverage and 88% as for multiple-coverage. This result implies that the developed model provides an acceptable efficiency for multiple-coverage.

For the anthrax emergency, the optimal solution, on the basis of P-center model, suggests to locate facilities at sites 1, 2, 5 and 6. As it can be seen, although for the single-coverage both models, the same distances are resulted, for multiple-coverage the objective function value of classical model is greater than the developed model. Therefore, classical model provides unbalanced solutions that could result in more economic cost value and life losses.

In the third case, the classical model suggests to locate facilities at sites 1, 2, 5 and 7 on the basis of P-median for smallpox emergency In case of single coverage models, the developed model results in larger distances (2040 versus 1740). But, if the multiple coverage is considered, the weighted total distance equals to 11,018 for traditional model while in the proposed model the weighted total distance decreases to 7,528 miles.

## 16.6 Future Research Directions

In this chapter, an extensive review of emergency medical services mathematical models is presented. Specifically, these models are categorized into strategic and operational level problems. Then, two rich models are studied from the recent literature and a case study is presented and investigated in order to highlight the practical value of this area. Given the current state-of-the-art literature in EMS areas, there are various avenues for further research. Among these areas we refer to the following ones:

- The proposed models can be extended to include hospital evacuation problem. When an evacuation is inevitable, emergency medical services may assist hospitals with coordinating placement and transport of patients.
- According to relevant literature, considering some real-world features such as the varying travelling speeds of vehicles, through different scenarios, could provide

**Table 16.1** Comparison of the obtained results (Adopted from Jia et al. 2007)

| Models | Developed models | | | Traditional models | | |
|---|---|---|---|---|---|---|
| Emergency cases | Objective function | | Selected sites | Objective function | | |
| | Single coverage | Multiple coverage | | Single coverage | Multiple coverage | Selected sites |
| Dirty bomb emergency (covering model) | Covered population: 97.5% | Covered population: 88% | 1,2,3,6 | Covered population: 100% | Covered population: 21% | 1,4,6,7 |
| Anthrax emergency (P-center) | Maximal weighted distance: 4*48 | Maximal weighted distance: 7.5*31.4 | 1,2,3,7 | Maximal weighted distance: 4*48 | Maximal weighted distance: 7*48 | 1,2,5,6 |
| Smallpox emergency (P-median) | Weighted total distance: 2040 miles | Weighted total distance: 7528 miles | 1,2,3,6 | Weighted total distance: 1740 miles | Weighted total distance: 11,018 miles | 1,2,5,7 |

valuable insight for decision makers. However, EMS vehicles often do not follow all the rules and could use some exemptions.

- Following the real world problems, the current active location sites have not been included in the existed models. Therefore, it is advisable to include the current facilities in the model while making their decisions such as closing or keeping active the current facilities.
- Using bi-level programming approach including the location decisions and routing strategies, as embedded problems within another, could be another attractive subject to deal with EMS problems. In this way, global solutions rather than local ones can be obtained.
- Due to nature of inevitable uncertainties in this area, considering worst-cases could provide a remarkable vision for decision makers. Robust optimization models are helpful to reach such an understanding.

## References

Andersson T, Värbrand P (2007) Decision support tools for ambulance dispatch and relocation. J Oper Res Soc 58(2):195–201

Attanasio A, Cordeau JF, Ghiani G, Laporte G (2004) Parallel tabu search heuristics for the dynamic multi-vehicle dial-a-ride problem. Parallel Comput 30(3):377–387

Başar A, Catay B, Ünlüyurt T (2009) A new model and tabu search approach for planning the emergency service stations. In: Operations Research Proceedings 2008. Berlin, Springer, pp 41–46

Batta R, Dolan JM, Krishnamurthy NN (1989) The maximal expected covering location problem: revisited. Transp Sci 23(4):277–287

Beaudry A, Laporte G, Melo T, Nickel S (2010) Dynamic transportation of patients in hospitals. OR Spectr 32(1):77–107

Beraldi P, Bruni ME (2009) A probabilistic model applied to emergency service vehicle location. Eur J Oper Res 196(1):323–331

Berman O, Drezner Z, Krass D (2011) Discrete cooperative covering problems. J Oper Res Soc 62(11):2002–2012

Brotcorne L, Laporte G, Semet F (2003) Ambulance location and relocation models. Eur J Oper Res 147(3):451–463

Bruzzone A, Signorile R (1998) Simulation and genetic algorithms for ship planning and shipyard layout. Simulation 71(2):74–83

Church RL, Gerrard RA (2003) The multi-level location set covering model. Geogr Anal 35(4):277–289

Church R, Velle CR (1974) The maximal covering location problem. Pap Reg Sci 32(1):101–118

Coppi A, Detti P, Raffaelli J (2013) A planning and routing model for patient transportation in health care. Electron Notes Discrete Math 41:125–132

Cordeau JF, Laporte G (2003) A tabu search heuristic for the static multi-vehicle dial-a-ride problem. Transp Res B Methodol 37(6):579–594

Cordeau JF, Laporte G (2007) The dial-a-ride problem: models and algorithms. Ann Oper Res 153(1):29–46

Créput JC, Koukam A, Hajjam A, Kuhn O (2011) Dynamic vehicle routing problem for medical emergency management. INTECH Open Access Publisher

Cuninghame-Green RA, Harries G (1988) Nearest-neighbour rules for emergency services. Z Oper Res 32(5):299–306

Daskin MS (1983) A maximum expected covering location model: formulation, properties and heuristic solution. Transp Sci 17(1):48–70

Daskin MS, Dean LK (2004) Location of health care facilities. In: Operations research and health care. Springer, pp 43–76

Daskin MS, Stern EH (1981) A hierarchical objective set covering model for emergency medical service vehicle deployment. Transp Sci 15(2):137–152

Doerner KF, Gutjahr WJ, Hartl RF, Karall M, Reimann M (2005) Heuristic solution of an extended double-coverage ambulance location problem for Austria. CEJOR 13(4):325–340

Erdemir ET, Batta R, Rogerson PA, Blatt A, Flanigan M (2010) Joint ground and air emergency medical services coverage models: a greedy heuristic solution approach. Eur J Oper Res 207(2):736–749

Erkut E, Ingolfsson A, Erdoğan G (2008) Ambulance location for maximum survival. Naval Res Logist (NRL) 55(1):42–58

Essen JT, Hurink JL, Nickel S, Reuter M (2013) Models for ambulance planning on the strategic and the tactical level. Beta publishing. http://doc.utwente.nl/87377

Farahani RZ, Asgari N, Heidari N, Hosseininia M, Goh M (2012) Covering problems in facility location: a review. Comput Ind Eng 62(1):368–407

Galvão RD, Chiyoshi FY, Morabito R (2005) Towards unified formulations and extensions of two classical probabilistic location models. Comput Oper Res 32(1):15–33

Gendreau M, Laporte G, Semet F (1997) Solving an ambulance location model by tabu search. Locat Sci 5(2):75–88

Goldberg JB (2004) Operations research models for the deployment of emergency services vehicles. EMS Manag J 1(1):20–39

Golden BL, Raghavan S, Wasil EA (2008) The vehicle routing problem: latest advances and new challenges: latest advances and new challenges, vol 43. Springer Science & Business Media

Gulliford M, Figueroa-Munoz J, Morgan M, Hughes D, Gibson B, Beech R, Hudson M (2002) What does' access to health care' mean? J Health Serv Res Policy 7(3):186–188

Haghani A, Yang S (2007) Real-time emergency response fleet deployment: concepts, systems, simulation & case studies. In: Dynamic fleet management. Springer, Boston, pp 133–162

Hogan K, ReVelle C (1986) Concepts and applications of backup coverage. Manag Sci 32(11):1434–1444

Institute of Medicine Committee on the Future of Emergency Care in the US Health System (I.O.M) (2006) Emergency medical services: at the crossroads. National Academy Press, Washington, DC

Ibri S, Drias H, Nourelfath M (2010) A parallel hybrid ant-tabu algorithm for integrated emergency vehicle dispatching and covering problem. Int J Innov Comput Appl 2(4):226–236

Ivanov D, Pavlov A, Dolgui A, Pavlov D, Sokolov B (2016) Disruption-driven supply chain (re)-planning and performance impact assessment with consideration of pro-active and recovery policies. Transp Res Part E: Logist Transp Rev 90:7–24

Jagtenberg CJ, Bhulai S, van der Mei RD (2015) An efficient heuristic for real-time ambulance redeployment. Oper Res Health Care 4:27–35

Jia H, Ordóñez F, Dessouky M (2007) A modeling framework for facility location of medical services for large-scale emergencies. IIE Trans 39(1):41–55

Karasakal O, Karasakal EK (2004) A maximal covering location model in the presence of partial coverage. Comput Oper Res 31(9):1515–1526

Kergosien Y, Gendreau M, Ruiz A, Soriano P (2014) Managing a fleet of ambulances to respond to emergency and transfer patient transportation demands. In: Proceedings of the International Conference on Health Care Systems Engineering. Springer International Publishing, pp 303–315

Kerkkamp RBO (2014) Facility location models in emergency medical service: robustness and approximations (Doctoral dissertation, TU Delft, Delft University of Technology)

Knight VA, Harper PR, Smith L (2012) Ambulance allocation for maximal survival with heterogeneous outcome measures. Omega 40(6):918–926

Lam SSW, Nguyen FNHL, Ng YY, Lee VPX, Wong TH, Fook-Chong SMC, Ong MEH (2015) Factors affecting the ambulance response times of trauma incidents in Singapore. Accid Anal Prev 82:27–35
Laporte G, Louveaux FV, Semet F, Thirion A (2009) Application of the double standard model for ambulance location. In: Innovations in distribution logistics. Springer, Berlin/Heidelberg, pp 235–249
Li X, Zhao Z, Zhu X, Wyatt T (2011) Covering models and optimization techniques for emergency response facility location and planning: a review. Math Meth Oper Res 74(3):281–310
Looije D (2013) Optimal ambulance locations in the Netherlands (Doctoral dissertation, Delft University of Technology)
Majzoubi F, Bai L, Heragu SS (2012) An optimization approach for dispatching and relocating EMS vehicles. IIE Trans Healthc Syst Eng 2(3):211–223
Nickel S, Reuter-Oppermann M, Saldanha-da-Gama F (2016) Ambulance location under stochastic demand: a sampling approach. Oper Res Health Care 8:24–32
Parragh SN (2011) Introducing heterogeneous users and vehicles into models and algorithms for the dial-a-ride problem. Trans Res Part C: Emerg Technol 19(5):912–930
Parragh SN, Doerner KF, Hartl RF (2010) Variable neighborhood search for the dial-a-ride problem. Comput Oper Res 37(6):1129–1138
ReVelle C, Hogan K (1989) The maximum availability location problem. Transp Sci 23(3):192–200
Sayre MR, White LJ, Brown LH, McHenry SD (2001) National EMS research agenda. Prehospital Emerg Care: Off J Natl Assoc EMS Phys Natl Assoc State EMS Dir 6(3 Suppl):S1–43
Schilling D, Elzinga DJ, Cohon J, Church R, ReVelle C (1979) The TEAM/FLEET models for simultaneous facility and equipment siting. Transp Sci 13(2):163–175
Schmid V (2012) Solving the dynamic ambulance relocation and dispatching problem using approximate dynamic programming. Eur J Oper Res 219(3):611–621
Schmid V, Doerner KF (2010) Ambulance location and relocation problems with time-dependent travel times. Eur J Oper Res 207(3):1293–1303
Soriya G, Colwell CB (2012) Emergency medical services. In: Encyclopedia of intensive care medicine. Springer, Berlin Heidelberg, pp 839–840
Su Q, Luo Q, Huang SH (2015) Cost-effective analyses for emergency medical services deployment: a case study in Shanghai. Int J Prod Econ 163:112–123
Toregas C, Swain R, ReVelle C, Bergman L (1971) The location of emergency service facilities. Oper Res 19(6):1363–1373
Van Buuren M (2011) TIFAR modeling package for the evaluation of emergency medical services: with EMS modeling results for the Amsterdam area (Doctoral dissertation, TU Delft, Delft University of Technology)
Van den Berg PL, Kommer GJ, Zuzáková B (2015) Linear formulation for the maximum expected coverage location model with fractional coverage. Oper Res Health Care 8:33–41
Yang S, Hamedi M, Haghani A (2004) Integrated approach for emergency medical service location and assignment problem. Transp Res Rec: J Transp Res Board 1882:184–192
Zhang ZH, Jiang H (2014) A robust counterpart approach to the bi-objective emergency medical service design problem. Appl Math Model 38(3):1033–1040