Jan Łukasiewicz: A Creator of New Ideas in Logic and a Reinterpreter of Its History

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> *Poetic works do not differ from scientific in more imaginative fantasy. [. . .] However, scientists differ from poets in this respect, that the former* REASON *always and everywhere.*
>
> *About creativity in science* (Łukasiewicz [3, p. 32])

Abstract Jan Łukasiewicz was a leading figure of the Warsaw School of Logic—one of the branches of the Lvov-Warsaw School. The paper presents his personality, life and didactic activity, as well as the list of his main works and the greatest achievements in logic, its history and philosophy. In propositional logic, he invented the bracketless symbolism and constructed many systems of this logic. One of his greatest achievements was discovering three-valued logic. He also reconstruct the history of logic with the apparatus of modern logic. In ontology, Łukasiewicz made a logical analysis of the notion of causality and determinism. In epistemology, he established the precise formulation of the paradox of the liar.

Keywords Determinism · History of logic · Jan Łukasiewicz · Lvov-Warsaw School · Many-valued logic · Propositional logic

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1 Personality

People dabbling in science are eminent, if they see fundamental questions in their discipline and find original responses to these questions. If the theoretical construction proposed by an eminent scholar—in particular, an eminent logician—also has the value of simplicity and beauty, one can say about him that he is a genius. Łukasiewicz was certainly a genius in this sense.

It was said of him that he was shy, sensitive and irritable. He was sensitive to how others judged him—and whether he was appreciated by them. He could not hide the fact that he cared about his recognition.

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Sometimes, talents are born—as in the Polish phrase—*on the stone*; but a talent may not generally develop so as to become a genius: to do this, one needs the appropriate soil. This soil—in the case of Łukasiewicz—was the mental environment brought to life by Kazimierz Twardowski, that is, in short, the philosophical Lvov-Warsaw School. Łukasiewicz grew in this environment—and then he co-created this environment, not without mutual theoretical interaction also with his own colleagues and students.

What was the relationship of Łukasiewicz to that environment?

He identified himself with the Brentanian roots of the School—but not with all of them. Twardowski was highly estimated by him—but not for everything. He appreciated Stanisław Leśniewski—as a logician—at least initially, higher than himself.

Three women played a great role in the life of Łukasiewicz: firstly—his mother, Leopoldyna née Holtzer; then—princess Maria Józefina Sapieżanka, who was the object of his great love, but without reciprocity; in the end—his wife, Regina née Barwińska, who was his bedrock especially in the last years of his life (even though—let us add—a sometimes troublesome bedrock due to her antagonistic character).

He was a great Polish patriot, but his ancestors were Ruthenian (paternal) and Tyrolean (maternal). To strangers, with whom he came into contact, he referred differently. He had friends among the Germans—but he did not feel good in Germany. He treated Ukrainians with sympathy—but he was, i.a., an opponent of ukrainizing the Lvov University. He felt aversion to some Poles of Jewish origin—but he fiercely opposed anti-Jewish movements in the academic circles. In the last decade of the life, Ireland became his second homeland—but his attitude to the Irish people was rather (unfairly!) dry.

On the one hand he was a man of deep faith: in particular, a Catholic and a practicing Catholic. On the other hand, he avoided the public «confession of faith». He also avoided political declarations—which does not mean that he did not have an explicit (conservative) view on these issues.

2 Life

He was born on December 21, 1878, in Lvov; he died on February 13, 1956 in Dublin.

He studied law and later philosophy with Twardowski at the Lvov University—and at the universities of Berlin and Louvain. After his doctorate (1902) and habilitation (1909), he was a participant in the seminar of Alexius Meinong in Graz (1910). From the years 1911–1915, he was a professor of philosophy and logic at Lvov University; from the years 1915–1939 at the Warsaw University (with breaks from 1918–1920 and 1924–1929), where he served twice as the rector (1922/1923 and 1931/1932). In 1919 he was the Minister of Religious Denominations and Public Education. During one of the German raids at Warsaw in September 1939, his library collections and rich manuscript legacy burned totally. In 1944, as an avowed anti-communist—in the face of the approaching front of the Soviet troops, he decided to go to Switzerland, but because of the tense political situation after the assassination attempt on Adolf Hitler, he had to stop in Münster (under the supervision of Heinrich Scholz). After the war, he settled first in Brussels and then in Dublin, where he was professor of logic at the Royal Irish Academy.

He was a member of the Polish Academy of Sciences, the Polish Scientific Society in Exile, and received honorary doctorates from universities in Münster and Dublin. He was a leading figure in the Warsaw School of Logic, being an essential component of the philosophical Lvov-Warsaw School. The spirit of the School found in his philosophical and logical works its most perfect incarnation.

3 Works

Among the most important publications of Łukasiewicz, there are the following books: *On the principle of contradiction in Aristotle* [2], *Die Grundlagen der logischen Wahrscheinlichkeitsrechnung* [5] and *Aristotle's syllogistic from the standpoint of modern formal logic* [11].[1]

Most of his papers were collected in two Polish volumes: *Selected problems of logic and philosophy* [13] and *Logic and metaphysics. Miscellanea* [19].

Many of his works appeared also in translation into other languages, including English [13], French [16, 20, 22], Spanish [17], Japanese [18] and Russian [12, 21].

Łukasiewiczian *Elements of mathematical logic* [8] is the model of a manual. This work includes a lecture on the axiomatic system of classical propositional calculus, propositional calculus with quantifiers and a fragment of the calculus of names (Aristotelian syllogistic included). An unusual source for the history of Polish culture of the twentieth century is Łukasiewicz's *Diary* [23].

The most important scientific achievements were Łukasiewiczian logic, the philosophy and history—as well as ontology and methodology of philosophy.

4 Metaphysics

4.1 Ontology

Of ontological issues, Łukasiewicz was especially interested in two problems (though interrelated in many ways): what is a causal relationship and what are the reasons for determinism.

4.1.1 Causal Relationship

An attempt to define "causal relationship" was made by Łukasiewicz in his classic dissertation "Analysis and construction of the concept of cause" [1]. The analytical part of this dissertation has little equal in philosophical literature—and not only in Polish writing.

[1]This work is a continuation of the Polish monograph, prepared in 1939, which unfortunately burned during World War 2.

In the constructive part, Łukasiewicz proposed reducing the notion of a causal relationship to the notion of necessity. This reduction could be simplified in such a way: The fact that object P_1 has feature c_1 is the reason for the fact that object P_2 has feature c_2—when if object P_1 has feature c_1, then object P_2 must have feature c_2, where P_2 must have feature c_2, when if object P_2 did not have feature c_2, then object P_2 would be an internally contradictory object.

Łukasiewicz initially thought that he gave, in this way, an equivalent definition of "causal relationship". But it finally turned out, that the necessity of the relationship between the two states of affairs does not settle the fact that this is a causal relationship, since the impossibility of the occurring state of affairs S_2 without the occurring state of affairs S_1 can take place, i.a. in the case where the occurring state of affairs S_1 is later than the occurring state of affairs S_2, or when the two states of affairs are timeless (like, e.g., the fact that a certain figure is a square necessarily involves the fact that this figure is a rectangle)—whereas it is assumed that every state of affairs is later than its cause.

Consequently, the definition proposed by Łukasiewicz can be treated at most as an inclusive definition, indicating only a necessary condition for the occurrence of a causal connection.

4.1.2 Determinism

The ontological thesis of determinism (in one of its versions) holds that each state of affairs occurring in the real world is uniquely determined by causes preceding it.

Łukasiewicz pointed out that among the premises forming the basis of the thesis of determinism there are two principles: the principle of the excluded middle and the principle of causality [6]. The first—let us remember—states (in a certain version) that for any state of affairs, this state of affairs occurs or does not occur. The second one states (in a certain version) that every state of affairs has a cause in some previous state of affairs. Following the principle of the excluded middle applied to the future, we must recognize that the occurrence of any future state of affairs has been already determined; but—as Łukasiewicz says—there is no compulsion to accept the principle of the excluded middle (on the grounds of, e.g., trivalent logic, this rule does not apply). Supposing the principle of causality, we must accept the fact that causal-effectual chains are infinite «in the past» (i.e., they are eternal); but—as Łukasiewicz says—one can accept the hypothesis of infinite causal-effectual chains and at the same time recognize that in a certain—distant enough—moment, there is «already» no reason for a given state of affairs, if only one assumes that time intervals between successive links in the causal-effectual chain decrease «back» unlimitedly; under such a condition, adopting the principle of causality is compatible with indeterminism.

4.2 Epistemology

Łukasiewicz's attitude to epistemology was more than critical: he considered most epistemological problems to be apparent ones.

The problem of truth was one of the real problems to which he devoted a lot of attention; it was understandable: truth is one of the logical values, and Łukasiewicz worked on constructing logical systems in which more than two logical values could be admissible, and not only truth and falsehood.

By "truth"—or more precisely: a "true sentence"—Łukasiewicz understood a sentence that "admits this property to an object, which is really possessed by this object, or that refuses this property, which is not really possessed" [5, p. 55]. We use different criteria for the truth of sentences, but we are not able to justify that these or other criteria are valid; an attempt to give such a justification always ends in either a vicious circle or *regressus in infinitum*.

Łukasiewicz was probably the first to establish the precise formulation of the so-called paradox of the liar. As the source of the paradox, he indicated a sentence of the type "Sentence S is false": such that, if sentence Z, that is, the sentence "Sentence S if false", is true, it is just as it says, so it is false; if sentence S is false, then it is just not the case, so sentence S is not false—or it is true. This formulation became an inspiration for the semantic conception of truth, proposed by the student of Łukasiewicz, Alfred Tarski.

5 Logic

5.1 Propositional Logic

A particular subject of Łukasiewicz's interest was a classic propositional calculus, i.e. a logical theory which reports the meaning of conjunctions (functors) connecting sentences—such as "and", "or", "if . . . , then", "always and only if"—and the negation "it is not the case that".

Let us start with the fact that Łukasiewicz invented for this calculus a special symbolism, called "bracketless symbolism" and later "Polish symbolism". It consists in recording complex sentences in such a way that at first we give a functor, followed by sentences «bonded» by this functor. For example, the formula "If p, then (q and r)" is recorded in such a symbolism as: CpKqr (where 'C' signifies functor "if . . . , then", and 'K'—functor "and"). In the parenthetical symbolism, this formula has the form: $p \rightarrow (q \wedge r)$. Note that the bracketless formula, mentioned above, has two symbols less than its equivalent parenthetical formula.

Now: propositional calculus in the form of axiomatic theory is a set of statements about these conjunctions, each of which is either an axiom (and therefore the claim accepted without proof), or can be derived from axioms using determined rules called "rules of inference". Łukasiewicz constructed many versions of such propositional calculus—differing, among other, as to adopted axioms and their number, rules of inference and their number, as well as which functors are considered primary and which secondary, i.e. definable with the use of the former. Among those versions, one was considered a classic one: it is a system which operates one functor (i.e. the functor "always if . . . "—or the functor of implication), universal quantifier ("every . . . ") binding propositional variables, four rules of inference, symbols of truth and falsehood, and symbols of accepting and rejecting sentences.

Łukasiewicz's desire was to create a system which would contain as little as possible axioms, rules of inference etc. He succeeded in this respect, firstly, in inventing a system of implicational propositional calculus (or a calculus operating with only one functor: the functor of implication) based on one axiom numbering—in the bracketless symbolism—13 symbols (CCC*pqr*CC*rp*C*sp*), and then to prove that it is the shortest axiom of that implicational calculus. Secondly, Łukasiewicz honed the rules of inference. He invented, among others, a useful version of the rule of substitution, i.e. the rule according to which in a sentence containing variables we are free to insert in their place other variables or constants, as long as we are doing it consequently (i.e. to insert these variables or constants into the places of each appearance of a substituted variable). He developed a more precise reasoning called "generalizing deduction", i.e. the method of proving general statements on the basis of their particular cases. He codified the matrix characteristics of the functor of propositional calculus and the method examining the tautologicality of formulas of that calculus, involving the compilation of possible combinations of substituting propositional variables in these formulas by symbols of truth and falsehood.

5.2 Propositional Logic with Quantifiers

In turn, the system of propositional calculus with quantifiers was based by Łukasiewicz on: (a) the universal quantifier ($\prod$) and the functor of implication (C) as primary concepts; (b) the three axioms (Tarski and Paul Bernays's ones: C*q*C*pq*, CCC*pqpp* and CC*pq*QQ*qr*C*pr*); (c) the definition of negation ($\mathrm{N}p = \mathrm{C}p \prod pp$) and (d) five rules of inference: the rule of substitution (modified in comparison to the analogical rule in propositional calculus without quantifiers), the rule modus ponens, the rule of replacement, the rule of combining, and the rule of skipping quantifiers. In this system, Łukasiewicz presented proofs of 19 theorems (he left five theorems without proof), including the proofs of three axioms of the system of propositional logic without quantifiers, described above.

5.3 Reconstruction of Syllogisms

Łukasiewicz has reconstructed the most significant part of the logic of Aristotle, i.e. "assertoric" syllogistic (*scil*. non-modal) [11]. His intention was to make this reconstruction: on one hand—according to the intentions of the great philosopher; on the other hand—developed in the spirit of modern logic.

According Łukasiewicz—Aristotle's syllogistic is a part of the logic of names, namely a formal theory of three constants: "all ... are ...", "none ... are ...", "some ... are ..." and "some ... are not ...", where the values of the variables representing arguments of those functors of two arguments are adopted only in general terms (in particular, with the exclusion of empty and negative names). The so constructed syllogistic is superstructured over propositional logic; in particular, it contains the following constants of this logic: "if ..., then ...", " ... and ... " and (in some proofs), "it is not the case that ...."

Aristotle tried to axiomatize syllogistic, taking as its basis four modes of the first syllogistic figure, eventually reducing finally to two of them (*Barbara* and *Celarent*). It turned out, however, that we need to add to them two laws of conversion, and (in some cases) two laws of identity. The simplest axiomatic base contains as primary terms—the constants "all ... are ..." and " ... some ... are ..." (the rest can be defined with their help and the negation of sentences), and as axioms—two laws of identity and the moods *Barbara* and *Datani* (or: *Barbara* and *Dimaris*).

Reduction of imperfect syllogisms to perfect ones, postulated by Aristotle, was interpreted by Łuksiewicz as the proof of theorems of the system (*scil.* deriving them from axioms). According to Łukasiewicz, Aristotle gave not only proofs of the true syllogistic formulas, but he also tried to show that all the others formulas are wrong, and as such should be rejected. He rejected inconclusive formulas, usually using the method of exemplification by means of appropriate concrete terms (which satisfy "premises", but do not satisfy—"conclusions").

5.4 Many-Valued Logic

One of Łukasiewicz's most important achievements was discovering three-valued logic[2]—and more generally: logics more than two-valued; the philosophical importance of the three-valued logic was compared, by Łukasiewicz himself, to the importance of non-Euclidean geometry in mathematics.

Existing logical calculi based on the principle of bivalence—i.e. on the assumption that every sentence has exactly one logical value: it is either true or false: tertium non datur. Łukasiewicz generalized the concept of logical value in such a manner that he allowed the existence of "intermediate" values between truth and falsehood: one—in the case of the three-valued logic, two—in the case of the four-valued logic, or more—up to the infinitely-many-valued (in short: $\aleph_0$-valued) logic.

Behind the idea of the three-valued logic there were the following insights. There are sentences which at the given moment can not be principally (and not, e.g., because of someone's ignorance) determined to be true or false. These are sentences about future events, which at present are not determined (i.e. those that there are currently neither the cause of their occurrence, nor the cause of their non-occurrence—or sentences about past events, the effects of which have completely "expired" (i.e. those for which there are currently no effects of their occurrence). These are just sentences having a third logical value. So sentences can be not only true or false, but also undetermined.[3]

An example of intuitive interpretation of the four-valued logic is its interpretation in terms of modal logic, that is, one in which there are functors of the type "it is necessary

[2]The first outline of non-Chrisipian logic dates back to 1917, and the first system of it—to 1920.

[3]It may be worth noting that one of the consequences of adopting the so characterized indefiniteness as a third value in the three-valued logic is that we should to accept the view that sentences change their logical value in time: e.g. sentence undetermined at a certain moment may come to be true (or false) at some later moment, namely, one in which adequate causes have already occurred. Someone who would not want to agree with such a consequence, could not also agree with the sketched interpretation of the third value. Questioning the interpretation does not involve, of course, questioning the interpreted calculus.

that ... ” and “it is possible that ... .” Łukasiewicz made this interpretation when it turned out that the characteristics of propositional functors in the conceptual apparatus of the three-valued logic is unsatisfactory.

As regards the infinitely-many-valued logic, Łukasiewicz claims (at least initially) that logical values present in this logic can be identified with degrees of probability.

5.5 Metalogic

Among Łukasiewicz’s great achievements, there were the results of his metalogical analysis, and in particular: his results of research on consistency, independence and the completeness of axioms of the propositional calculus. The issue at stake was to determine whether or not the constructed sets of axioms contained axioms negative to each other such that some of them resulted in others, and finally, whether these axioms are sufficient to prove all the theses of the system, which appears to be true.

It was also demonstrated here that classical propositional calculus is a fragment of intuitionistic logic. If one considers that at the core of the latter belief there is the conviction that only proved theorems can be considered as theorems of a certain mathematical theory—and therefore among its theorems there is not, e.g., the law of excluded middle (“*p* or it is not the case that *p*”)—then Łukasiewicz’s result sounds surprising (removing this impression requires suitably enriching intuitionistic logic itself).

5.6 Philosophy of Logic

5.6.1 Anti-psychologism

In the second half of the nineteenth century—not without relation to the rapid development of psychological research—tendencies appeared to reformulate all philosophical issues, including problems of logic, in such a way that every philosophical (and logical) problem was replaced by its psychological paraphrase.

Łukasiewicz was one of those people who revolted earliest and most strongly against such a psychologization—especially the psychologization of logic. His anti-psychologism was reflected, among other things, in the fact that he precisely contrasted laws of logic with their alleged equivalents in the form of psychological laws, and—let us add—ontological ones. He stressed, e.g., the difference between logical, psychological and ontological principles of contradiction: the first principle (in one of its versions) is the law of propositional logic, according to which it is not the case that both *p* and not-*p*; the second principle (in one of its versions) claims that in reality is never so that a certain state of affairs occurs and does not occur at the same time; the third principle (in one of its versions) is the view that no one alive can hold a certain conviction and its negation at the same time.

Łukasiewicz—analyzing these versions of the principle of contradiction—noted that an experience is not able to confirm the ontological version of these principle. Here are excerpts of this argument—a beautiful example of Łukasiewicz’s philosophical prose:

> Any movement [...] takes place in such a way that the changing object loses some features that it possessed, and acquires new ones that it did not possess. In both cases, contradiction would arise, if there were not different temporal determinations.
>
> If the change is CONTINUOUS, e.g. the movement of an arrow released from a bow, [...] then in every smallest interval of time, the changing object loses in turn some features, and purchases second ones. The moving arrow is in any two moments of time in DIFFERENT places. [...] What [...] happens when this distance decreases to zero when we shall consider only ONE moment as unextended on timeline?
>
> Once, we heard the fable that when a princess pricked her little finger on a spindle, she fell immediately into a hundred-year's deep dream, and all life around her slept as well. In such a way, the legendary Popiel's court froze in the blink of an eye, enchanted by Rzepicha in the songs of *King of Spirit*. Suppose that what is only a poetic fantasy has become reality. [...] The arrow would [then] rest motionless in a certain place. But how do we know that it would be only in ONE place? Why, in an unextended moment, in a temporal point of the section, could it not be in at least two different places and thus be in a certain place and not be there at the same time? [...] Experience is silent on this issue. [...] All the perceived phenomena LAST shorter or longer and SHOULD last for a minimum period of time to be noticed by us at all. We do not know what happens in an unextended moment. However the principle of contradiction applies to such a moment; because if we say that the arrow cannot AT THE SAME TIME be and not be at the same place, the phrase "at the same time" refers to THE SAME, so the only ONE, unextended moment. [2, pp. 136–138]

So much for the ontological version of the principle of contradiction. As for the empirical justification of the psychological version, it would require painstaking research, which has not yet been undertaken and whose desirability at all is in question in light of the statements of some people (including philosophers and mystics) that they entertain contradictory beliefs. Finally—the only justification for accepting the logical version of the principle of contradiction is that it makes it possible to prove with a certainly unattainable in other cases that someone is mistaken or lying; because we recognize (assuming the principle of contradiction) a conviction entailing a contradiction as mistaken—and we have (assuming this principle) the basis to believe that someone is lying when that person has once said, that *p*, and some other time, that not-*p*; the inability to prove that someone is wrong or lying, would have far-reaching—negative—consequences for social life, because, among other things, it would make it difficult, and in some cases even prevent issuing of righteous judgments by the courts.

5.6.2 Metaphysical Neutrality

Łukasiewicz argued not only for depsychologizing logic, but also for—so to speak—the metaphysical neutralizing of it. The idea was that—already in the twentieth century—there were tendencies to treat calculi of logical systems as systems implying one or another ontological or epistemological positions. Additionally, to show the groundlessness of such tendencies—it is enough to exactly distinguish the logical system from its permissible interpretation—and from metaphysical positions occupied by a logician who is the creator of this system or its interpreter. Logicians tend to be of defined metaphysical views—or of a defined worldview—but they feed these views as metaphysicians or members of a particular philosophical orientation, not as logicians: logical systems, as such, neither assume nor imply any metaphysical theses or any theses of faith or unbelief. Moreover, practicing formal logic does not require, e.g., accepting nominalism—or the view that logical systems are systems of unretrievable, «senseless» (i.e. having no reference to reality) symbols-strings. If it were so, logical systems would be always sets of a finite number of theses—because we can not «produce» (or build) an infinite number of symbols-strings. Meanwhile, these systems are infinite sets (it is enough to note that if the statement "If p, then p" is a thesis of the propositional calculus, then the statements "If *p*, then (*p* or *q*)", "If *p*, then [*p* or (*q* or *r*)]" etc. *ad infinitum* are such theses also. Similarly, e.g., the existence of many logical systems does not support relativism and conventionalism—or the view that our images of the world are determined by the freely accepted conceptual apparatus, so that none of them can reasonably be considered as a «true» image; because in this case, the issue is decided by experience, providing us knowledge about what the world «truly» is—not the mere fact of the existence (*scil.* constructability) of various its images.

5.7 *History of Logic*

Łukasiewicz's basic postulate in relation to historical research on logic (and more generally—philosophy) was the postulate to reconstruct history with the apparatus of contemporary logic; the first work realizing this postulate was the dissertation *On the principle of contradiction in Aristotle* [2], although the postulate itself was explicitly formulated much later [9].

According to this postulate—Łukasiewicz analyzed two great logical systems of antiquity: Aristotle's syllogistic and Stoic logic. Results of this analysis were surprising—and they questioned existing views on both systems.

It turned out, firstly, that Aristotle's syllogistic is not a theory of inference (as was previously thought), but a calculus of names: in particular, Aristotle's syllogisms are not rules of inference (of the type: If we accept the premise "Each *A* is *B*" and the premise "Each *B* is *C*", we must accept the conclusion "Each *A* is *C*"); they are theses of the calculus of names (of the type: If each *A* and each *B* is *C*, then each *A* is *C*). By the way, it turned out that Aristotle is the inventor of nominal variables and his syllogistic was the first (admittedly imperfect) axiomatized system in the history.

Secondly, Łukasiewicz showed that the Stoic logic is not (as previously thought) a calculus of names, but historically the first the system of propositional logic having its extension in the Middle Ages under the name of the theory consequences: in opposition to Aristotelian syllogisms—Stoic syllogisms are the rules of inference.

6 Methodology

In methodology—Łukasiewicz proposed an original classification of reasoning and reinterpreted the notion of inductive reasoning, the notion of probability and the notion of magnitude.

6.1 Reasoning

According Łukasiewicz—to reason (let us add: validly) is the same as to select for a certain sentence *S* a reason (or a sentence which is followed by sentence *S*) or a consequence (i.e., a sentence which follows sentence *S*); in the first case we are talking about reduction; in the second case—about deduction. Sentences—given and selected—may be accepted or not accepted (as true sentences) by the reasoner before beginning to reason. Depending on which of these two cases takes place—reduction is either an explanation (when we select an unaccepted as yet reason for an accepted sentence) or proof (when we select an accepted reason for an unaccepted as yet sentence); on the other hand deduction is inference (when we select an unaccepted as yet consequence to an accepted sentence) or confirmation (when we select an accepted consequence to an unaccepted as yet sentence).

The reasoning consisting in the fact that a certain law is accepted—therefore a general statement stating occurrence of certain general regularities—on the basis of earlier accepted individual sentences stating occurrence of a certain number of cases of this regularity is called "inductive reasoning" (more accurately: "incomplete enumerating induction"). Before Łukasiewicz induction was believed to be a kind of deduction. In light of Łukasiewiczian conception of reasoning, this is a misconception—inductive reasoning is a kind of explanation (in which a reason is selected to individual sentences), and thus it is a kind of reductive, and not deductive reasoning.

6.2 Probability

In the traditional interpretation of probability, the probability of the occurrence of a certain events is discussed, e.g. about the probability that picking up a random a ball from a face-down box in which there are four balls: white, black, red and blue—I shall take up the white ball. Łukasiewicz proposed a «propositional» interpretation of probability. Certain probability is entitled not to the fact that (e.g.) I shall take up the white ball from the box, but to the sentence "I shall take up the white ball from the box". This sentence is a substitution of the formula "I shall take up ball *x* from the box". In this situation we can insert in place '*x*' a name of one of four colors: white, black, red or blue—but only in the case of one substitution, will the formula be transformed into a true sentence. Therefore, we can say that the probability of the sentence "I shall take up the white ball from the box" is equal to 1/4 in this case.

6.3 Magnitude

Łukasiewicz proposed—in the place of a very complicated «classic» (but logically flawed) definition of "magnitude" given by the mathematician Stanisław Zaremba—a very simple definition, according to which magnitude is the same as an element of a certain well-ordered set, or such a set, whose elements are «arranged» sequentially one after another, because it is determined that a suitable relationship for them is both asymmetrical, transitive and consistent (i.a. the relation of being-greater-than is such an ordering relation in the class of natural numbers).

6.4 Methodology of Philosophy

The methodology of science can be regarded as a description of how—and, in particular, by means of which research methods—science is actually done. We can also treat it as a set of rules defining how science should be done.

Łukasiewicz's contribution to the methodology of philosophy belongs mainly to methodology in this second sense. Łukasiewicz's expectations in this regard were very far-reaching, although he expressed them in a simple postulate:

> Metaphysical problems [have not been] resolved, but I do not think that [they are] unsolvable. We only need to approach them with the scientific method: with the same proven method which is used by a mathematician or a physicist. And above all, we need to learn to think: clearly, logically and strictly. [7, p. 368]

In practice, Łukasiewicz advocated practicing «axiomatic» philosophy, i.e. a philosophy with a structure similar to axiomatic logical systems.

The basis of such a philosophy should be its axioms. Łukasiewicz wrote:

> Every sentence, especially those that will be the basis of philosophy, [. . .] [should be] formulated as precisely as possible because only then can we duly justify or know the direct evidence of these sentences. [10, p. 372]

Evaluation of the current—especially so-called modern—philosophy was negative:

> The state of pre-Kantian philosophy: on the one hand, fantastic dreams, not withstanding scientific criticism; on the other hand, radical, dogmatic, unfounded skepticism. [But then also:] as we approach Kantian philosophy with the requirements of scientific criticism, its construction collapses like a house of cards. At every step, we have vague notions, incomprehensible sentences, unfounded statements, contradictions and logical errors. [7, p. 368]

The conclusion was crushing:

> Caused by a negligence of logic and by the ensuing mental dressage, the whole of modern philosophy was paralyzed by an impotence of strict and clear thinking. [10, p. 373]

7 Teacher

Even an approximate list of Łukasiewicz's students is not known; neither the audience of his lectures, nor the participants of his seminars. Even the list of masters and doctors promoted by him is incomplete and it raises doubts at various points.

Certainly, his postgraduate students were: Maria Ossowska, in the years 1923–1927, a senior assistant at the Seminary of Philosophy of Łukasiewicz; Mordchaj Wajsberg and Zygmunt Kobrzyński, both of whom died during World War 2—so they did not survive their promoter; finally, Stanisław Jaśkowski and Jerzy Słupecki. It seems that under the guidance of Łukasiewicz a doctorate on the Stoic logic was prepared by Czesław

Lejewski but there was no promotion because of the outbreak of the war. However, it is not clear, e.g., whether Bolesław Sobociński was a postgraduate student of Łukasiewicz or Leśniewski.

Łukasiewicz's assistants—though not postgraduate students—were: Tarski (from 1929) and Henryk Hiż (from 1940 to 1944).

Those who witnessed Łukasiewicz's lectures—colleagues and students—stressed that he was an excellent didactician. This was manifested in the fact that he could make contact with the participants of his lessons, and his lectures were strict and at the same time interesting and affordable, with a clear structure, delivered fluidly and in beautiful language.

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