Isotropy Analysis of a Stiffness Decoupling 8/4-4 Parallel Force Sensing Mechanism

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Abstract. A stiffness decoupling 8/4-4 parallel force sensing mechanism (PFSM) is presented. Its mathematic model is established with screw theory. The force mapping relation is studied and the stiffness matrix is found to be a diagonal matrix, which proves the stiffness decoupling characteristics of the mechanism. According to the concept of fully isotropy, the isotropy conditions are analyzed, the parameters which meet fully isotropy are given. The 8/4-4 PFSM's configuration under isotropy parameters is analyzed. Based on this configuration, an 8/4-4 mechanism cluster which meets the fully isotropy is presented. The cluster's configuration is classified and induced into four main configurations according to the different parameter conditions.

Keywords: Stiffness decoupling · Parallel force sensing mechanism · Isotropy analysis · 8/4-4 mechanism cluster

1 Introduction

Parallel mechanism possesses the distinguishing advantages including high rigidity, high accuracy and easy decoupling. Due to these advantages, parallel mechanism has been widely used in many fields of science and engineering, such as micro-manipulators [1, 2], machine tools [3], wind tunnel experiments [4, 5] and so on.

The force sensing mechanism based on generalized parallel structure has been widely studied by many scholars. Gaillet and Reboulet [6] firstly proposed the application of the Stewart platform to six-axis parallel force sensing mechanism (PFSM). Kerr [7] considered the axial stiffness of the branch in the studied of Stewart PFSM and enumerated some design criteria for the sensor structure. Dwarakanath and Venkatesh [8] presented a six-axis parallel mechanism based force/torque sensor with no mechanical joint. Yao et al. [9, 10] presented the theoretical analysis and experiment research of a novel statically indeterminate six-axis PFSM. Zhen Gao and Dan Zhang [11] designed a multidimensional acceleration sensor based on fully decoupled compliant parallel mechanism. Dwarakanath and Bhutani [12] proposed a beam type PFSM based on "joint less" connector configuration for the transmission of axial forces.

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Yao et al. [13] presented a novel six-component force sensor based on parallel and flexible mechanisms and manufactured the sensor prototype with 3-D printing technology.

As can be seen from the literature survey, isotropy is a very significant principle in the design of six-axis PFSM. Xiong [14] presented the concept "isotropy" for robot's force sensors on the basis of Fisher's information matrix. A Fattah and AMH Ghasemi [15] presented the concept of kinematic isotropy and used as a criterion in the design of various parallel manipulators with ideal kinematic and dynamic performance. JIN Zhenlin [16] defined the sensitivity isotropy evaluation criteria of the force sensing mechanism, investigated the relationships between the criteria and the parameters of all the transducers based on the Stewart platform within the geometric model of the solution space. Gogu G [17, 18] firstly defined fully isotropy and concluded the characteristics under the isotropy configuration. Yao et al. [19] defined the modified isotropy indices which considered stiffness coupling effect. [20] presented four kinds of redundant six-axis PFSM, analyzed the four mechanisms' isotropy performance based on modified isotropy indices.

It is known that stiffness decoupling is the precondition of isotropy, but the current stiffness decoupling idea is to get the numerical solution when coupled elements all equal to zero, its decoupled characteristics can only be achieved under certain parameter conditions, the coupled elements are not eliminated fundamentally. In this paper, a stiffness decoupling 8/4-4 PFMS without coupled element is proposed, fundamentally eliminates the coupled characteristics. Then its isotropy performance and derivative structure are analyzed. Finally, an 8/4-4 mechanism cluster based on 8/4-4 PFMS is presented.

2 Structure Model and Mathematic Model

2.1 The Structure Characteristics of the Generalized Six-Axis PFSM

Generalized six-axis PFSM is shown in Fig. 1, which is composed of a measuring platform, a fixed platform, and n flexible measuring branches connecting two platforms with spherical joints. The measuring coordinate system is in the center of the measuring platform. In order to ensure the integrity of the measurement model, the number of branches should be no less than 6. When external force applied on the measuring platform, ignoring the branches' weight and spherical joints' friction, the branches bear only the force acting along its axis.

Based on the screw theory, the force and moment applied on the measuring platform are distributed on all branches. For the equilibrium of the measuring platform, the Eq. (1) can be obtained:

$$\boldsymbol{F} + \in \boldsymbol{M} = \sum_{i=1}^{n} f_i \boldsymbol{\$}_i \quad (n \geq 6) \tag{1}$$

where $\boldsymbol{F}$ and $\boldsymbol{M}$ respectively represent the force vector and moment vector acted on the measuring platform, f_i represents the reacting force produced on the i-th flexible

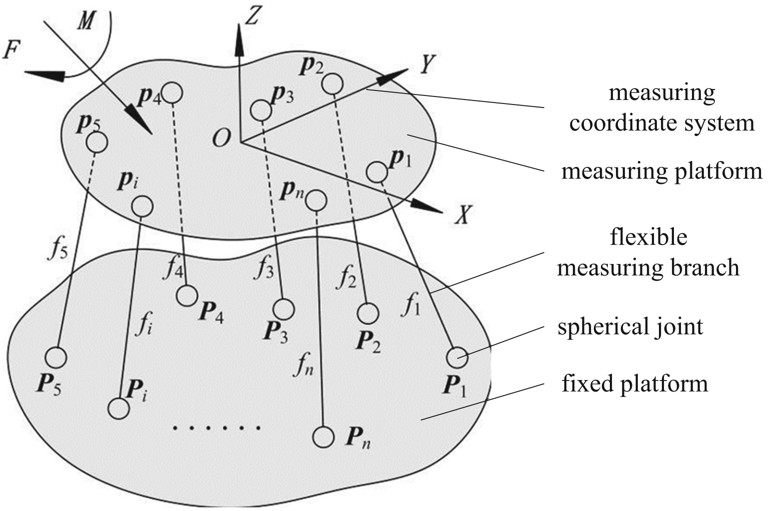


Fig. 1. Generalized six-axis PFSM

measuring branch and $\$_i$ represents the unit line vector along the axis of the i-th flexible measuring branch.

Then Eq. (1) can be expressed as

$$\boldsymbol{F}_w = \boldsymbol{Gf} \tag{2}$$

where $\boldsymbol{F}_w = [\boldsymbol{F}\ \boldsymbol{M}] = [F_x\ F_y\ F_z\ M_x\ M_y\ M_z]^T$ is the wrench acted on the measuring platform, $\boldsymbol{f} = [f_1\, f_2\, f_3\, \ldots f_n]^T$ (n $\geq$ 6) is the vector composed of the axial force of each branch, $\boldsymbol{G}$ is the force Jacobian matrix which will directly influence the performance of the force sensing mechanism. $\boldsymbol{G}$ can be expressed as:

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{S}_1 & \boldsymbol{S}_2 & \boldsymbol{S}_3 & \ldots & \boldsymbol{S}_{\mathrm{n}} \\ \boldsymbol{S}_{01} & \boldsymbol{S}_{02} & \boldsymbol{S}_{03} & \ldots & \boldsymbol{S}_{0n} \end{bmatrix} (n \geq 6) \tag{3}$$

where $(\boldsymbol{S}_i, \boldsymbol{S}_{0i})$ represents the unit line vector along the axis of the i-th branch.

2.2 Structure and Mathematic Model of 8/4-4 PFSM

The structure diagram of the 8/4-4 PFSM is shown in Fig. 2. The mechanism's model is composed of three parts, outer platform for fixing, inner platform for measuring and 8 flexible measuring branches connecting between the two platforms. The inner and outer platforms are coaxial arrangement. O is the geometric center of mechanism, A_i are the intersections of i-th branches and inner platform, B_i are the intersections of i-th branches and outer platform. A_i are distributed on the same circumference of the inner platform. B_i are alternately distributed in two circumferences of the outer platform, the distance between two circumferences is $2H$. The angle between OA_i and OB_i is θ,

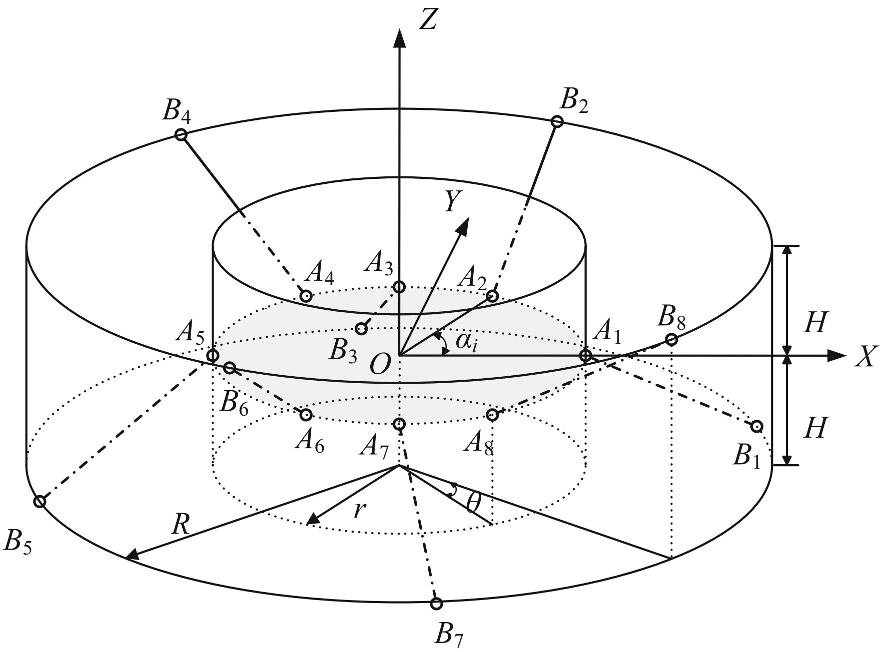


Fig. 2. Structure diagram of the 8/4-4 g of the 8/4-4 PFSM

8 branches are radial and tilted to one side, as shown in Fig. 3. According to the right-hand rule, the references coordinate system in Fig. 2. is established with O as the origin, OA_1 as the positive direction of axis X, and the coordinates of A_i and B_i can be expressed as

$$\begin{aligned} \boldsymbol{A}_i &= \begin{bmatrix} r\cos\alpha_i & r\sin\alpha_i & 0 \end{bmatrix} \\ \boldsymbol{B}_i &= \begin{bmatrix} R\cos(\alpha_i+\theta) & R\sin(\alpha_i+\theta) & (-1)^i H \end{bmatrix} \\ \alpha_i &= \frac{\pi}{4}(i-1) \end{aligned} \tag{4}$$

where r and R represent the inner and outer platform radius respectively, α_i is the angle between A_i and X axis, $\alpha + \theta$ is the angle between B_i and X axis.

Based on the screw theory, the force Jacobian matrix of the mechanism is obtained:

$$\boldsymbol{G} = \begin{bmatrix} \frac{\boldsymbol{A}_1-\boldsymbol{B}_1}{|\boldsymbol{A}_1-\boldsymbol{B}_1|} & \frac{\boldsymbol{A}_2-\boldsymbol{B}_2}{|\boldsymbol{A}_2-\boldsymbol{B}_2|} & \frac{\boldsymbol{A}_3-\boldsymbol{B}_3}{|\boldsymbol{A}_3-\boldsymbol{B}_3|} & \cdots & \frac{\boldsymbol{A}_8-\boldsymbol{B}_8}{|\boldsymbol{A}_8-\boldsymbol{B}_8|} \\ \frac{\boldsymbol{B}_1\times\boldsymbol{A}_1}{|\boldsymbol{A}_1-\boldsymbol{B}_1|} & \frac{\boldsymbol{B}_2\times\boldsymbol{A}_2}{|\boldsymbol{A}_2-\boldsymbol{B}_2|} & \frac{\boldsymbol{B}_3\times\boldsymbol{A}_3}{|\boldsymbol{A}_3-\boldsymbol{B}_3|} & \cdots & \frac{\boldsymbol{B}_8\times\boldsymbol{A}_8}{|\boldsymbol{A}_8-\boldsymbol{B}_8|} \end{bmatrix} \tag{5}$$

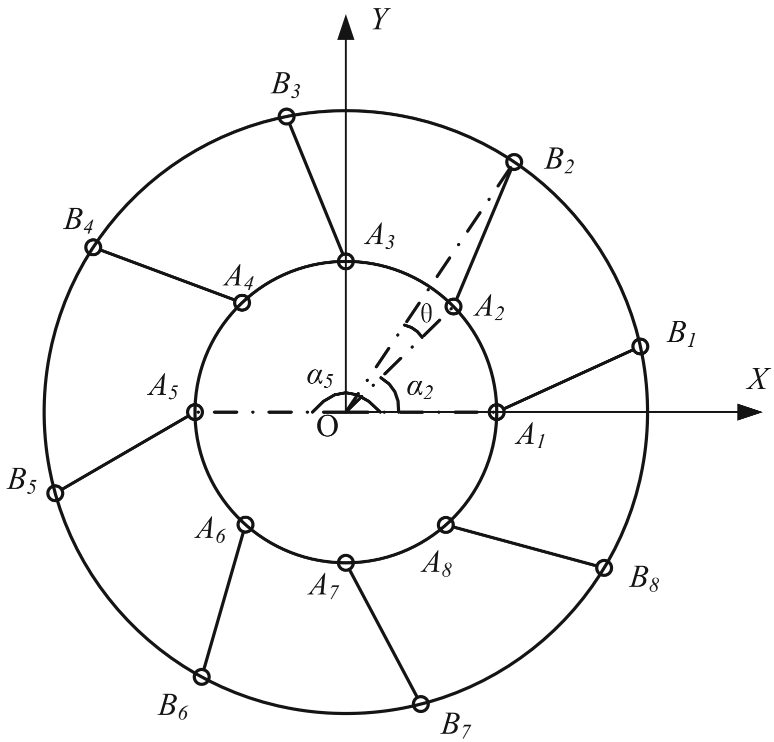


Fig. 3. Top view of the 8/4-4 PFSM

where

$$\begin{aligned} &\boldsymbol{A}_i - \boldsymbol{B}_i = [\, r\cos\alpha_i - R\cos(\alpha_i + \theta) \quad r\sin\alpha_i - R\sin(\alpha_i + \theta) \quad (-1)^{i-1}H \,] \\ &\left|\boldsymbol{A}_j - \boldsymbol{B}_j\right| = \sqrt{R^2 + r^2 - 2Rr\cos\theta + H^2} \\ &\boldsymbol{B}_{\mathrm{i}} \times \boldsymbol{A}_i = [\, (-1)^{i-1}Hr\sin\alpha_i \quad (-1)^{i}Hr\cos\alpha_i \quad -Rr\sin\theta \,] \end{aligned} \tag{6}$$

3 Decoupling Analysis of 8/4-4 PFSM

Under the action of generalized external force $\boldsymbol{F}_w$, the PFSM will have the corresponding deformation $\Delta\boldsymbol{D}$, which can be expressed as:

$$\Delta\boldsymbol{D} = \left[\Delta\boldsymbol{d}^{\mathrm{T}}, \Delta\boldsymbol{\theta}^{\mathrm{T}}\right] = [\Delta x, \Delta y, \Delta z, \Delta\alpha, \Delta\beta, \Delta\gamma]^{\mathrm{T}} \tag{7}$$

where, $\Delta\boldsymbol{d}$ is the resulting linear deformation, $\Delta\boldsymbol{\theta}$ is the resulting rotational deformation.

Meanwhile under the action of reacting force, the flexible measuring branches of the PFSM will also produce axial deformation, the relationship between the reacting force $\boldsymbol{f}$ and the axial deformation $\Delta\boldsymbol{l}$ can be expressed as:

$$\boldsymbol{f} = \boldsymbol{K}_l \Delta\boldsymbol{l} \tag{8}$$

where $\Delta\boldsymbol{l} = [\Delta l_1\ \Delta l_2\ \Delta l_3\ \Delta l_4\ \Delta l_5\ \Delta l_6\ \Delta l_7\ \Delta l_8]^{\mathrm{T}}$ is a vector composed of axial deformation of 8 flexible measuring branches. $\boldsymbol{K}_l = \mathrm{diag}(k_1\ k_2\ k_3\ k_4\ k_5\ k_6\ k_7\ k_8)$ represents the stiffness matrix of 8 flexible measuring branches, since the 8 flexible measuring branches are the same, so we suppose the 8 branches have the same stiffness k_l.

By literature [21], the relation between $\Delta\boldsymbol{l}$ and $\Delta\boldsymbol{D}$ can be expressed as:

$$\Delta\boldsymbol{l} = \boldsymbol{G}^{\mathrm{T}} \Delta\boldsymbol{D} \tag{9}$$

Combining Eqs. (2), (8) and (9), the relation between $\boldsymbol{F}_w$ and $\Delta\boldsymbol{D}$ can be obtained:

$$\boldsymbol{F}_w = \boldsymbol{K}\Delta\boldsymbol{D} = \boldsymbol{G}\boldsymbol{K}\boldsymbol{G}^{\mathrm{T}}\Delta\boldsymbol{D} \tag{10}$$

Substituting Eqs. (5), (6) into Eq. (10), the stiffness matrix $\boldsymbol{K}_s$ and the relation between $\boldsymbol{F}_w$ and $\Delta\boldsymbol{D}$ of the 8/4-4 PFSM can be obtained:

$$\boldsymbol{F}_w = \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \boldsymbol{K}_s \Delta\boldsymbol{D} = k_l \begin{bmatrix} \Lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Lambda_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Lambda_4 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\alpha \\ \Delta\beta \\ \Delta\gamma \end{bmatrix} \tag{11}$$

where

$$\begin{aligned} \Lambda_1 &= 4(R^2 + r^2 - 2Rr\cos\theta) \\ \Lambda_2 &= 8H^2 \\ \Lambda_3 &= 4H^2 r^2 \\ \Lambda_4 &= 8R^2 r^2 \sin^2\theta \end{aligned} \tag{12}$$

Equation (11) is the analytical solution of 8/4-4 PFSM. It can be seen that the stiffness matrix is a diagonal matrix, which proves the decoupled characteristic of the 8/4-4 PFSM. When a certain direction force/moment is applied on the origin of reference coordinate system, the deformation will also be on this certain direction but will not interfere with other directions. No matter what the value of the parameters are, as long as the geometric constraints in the Sect. 2.2 are satisfied, the decoupled characteristics of 8/4-4 PFSM will not be affected.

4 Isotropy Analysis of 8/4-4 PFSM

Based on the analysis of decoupled performance of 8/4-4 PFSM, it is found that when the decoupled performance is satisfied, the deformation of each direction is independent and easy to compare. Therefore, the decoupled characteristics and its constraints are considered as the premise and basis of isotropy. In order to obtain better isotropy properties, the decoupled performance must be considered at the beginning. However, stiffness decoupling does not mean that the isotropy is satisfied, a parametric model with isotropy constraints is still needed for the isotropy configuration.

When the force sensing mechanism is fully isotropy, the sensitivity of the force/moment in the three-dimensional direction is consistent. From this, we can conclude that the Eq. (13) must be satisfied when 8/4-4 PFSM is fully isotropy.

$$\begin{cases} \Lambda_1 = \Lambda_2 \\ \Lambda_3 = \Lambda_4 \end{cases} \tag{13}$$

Substituting Eq. (12) into Eq. (13), the parametric equation can be obtained:

$$\begin{cases} 4(R^2 + r^2 - 2Rr\cos\theta) = 8H^2 \\ 4H^2r^2 = 8R^2r^2\sin^2\theta \end{cases} \tag{14}$$

Simplifying Eq. (14), we acquire:

$$R^2 + r^2 - 2Rr\,\cos\theta = 4R^2\sin^2\theta \tag{15}$$

Dividing the Eq. (15) with the R^2:

$$1 + (\frac{r}{R})^2 - 2(\frac{r}{R})\cos\theta = 4\sin^2\theta \tag{16}$$

Suppose $t = r/R$, then Eq. (16) can be simplified to the Eq. (17) which takes θ as the independent variable and t as the dependent variable:

$$t^2 - 2t\cos\theta + 4\cos^2\theta - 3 = 0 \tag{17}$$

Solving Eq. (17) for t, we get:

$$t = \cos\theta \pm \sqrt{3}\sin\theta = 2\sin(\frac{\pi}{6} \pm \theta) \tag{18}$$

Substituting $t = r/R$ into Eq. (18), the parameters' solution for fully isotropy configuration is obtained:

$$\begin{cases} r = 2R\,\sin(\frac{\pi}{6} \pm \theta) \\ H = \pm\sqrt{2}R\,\sin\theta \end{cases} \tag{19}$$

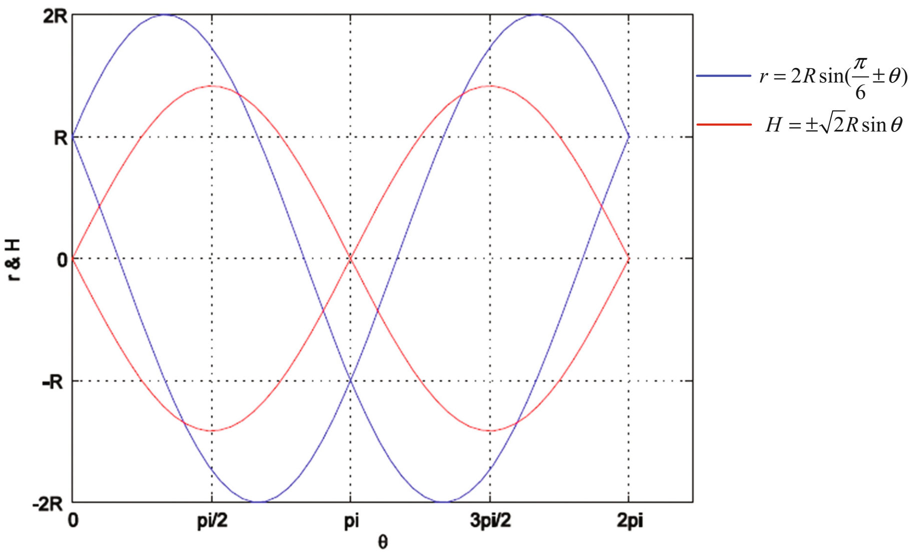


Fig. 4. The fully isotropy solution

According to Eq. (19), the isotropy of 8/4-4 PFSM is determined by r, R, H and θ, and the fully isotropy solution is a set of analytical solutions. In order to express the parameters' constraint relation more intuitively, the fully isotropy solution is drawn in Fig. 4. It can be seen that when R is determined, each θ (except at the intersection point) corresponds to the two groups' solution of R and H, that is, each θ (except the intersection point) corresponds to the four fully isotropy configurations.

Although all the solutions satisfying the isotropy can be expressed in Fig. 4, there also exists some repeated solutions due to the strong symmetry and repeatability of the Fig. 4, and the central symmetry of the 8/4-4 PFSM. It is necessary to simplify the configuration solutions obtained in Fig. 4. First, parameter H is simplified, because of the central symmetry of the 8/4-4 PFSM, $H = \sqrt{2}R\,\sin\theta$ or $H = -\sqrt{2}R\,\sin\theta$ does not have any real impact on the 8/4-4 PFSM, so $H = \pm\sqrt{2}R\,\sin\theta$ can be simplified as $H = \sqrt{2}R\,\sin\theta$. Then, parameter r is simplified, according to Fig. 4, we found that for $r = 2R\sin\,(\pi/6 + \theta)$ and $r = 2R\sin\,(\pi/6 - \theta)$ two cases, the change of r is a completely opposite process. The change of $r = 2R\sin\,(\pi/6 + \theta)$ from 0 to 2π is actually the same process of $r = 2R\sin\,(\pi/6 - \theta)$ from 2π to 0, so $r = 2R\sin\,(\pi/6 \pm \theta)$ can be simplified as $r = 2R\sin\,(\pi/6 + \theta)$. The parameters' solution for fully isotropy configuration is simplified as:

$$\begin{cases} r = 2R\,\sin(\frac{\pi}{6} + \theta) \\ H = \sqrt{2}R\,\sin\theta \end{cases} \tag{20}$$

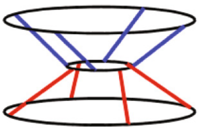
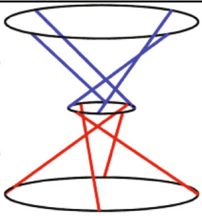
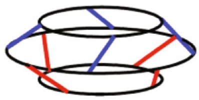
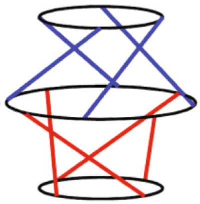
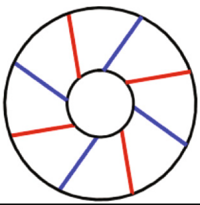
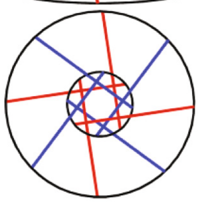
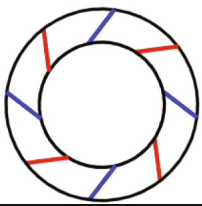
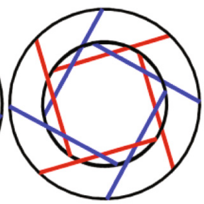
Finally, the definition domain of θ is simplified, by observing the curves of Eq. (20) in Fig. 4, it is found that if $\theta = \theta_1$, $H = H_1$, $r = r_1$ is the solutions of Eq. (20), $\theta = \theta_1 + \pi$, $H = -H_1$, $r = -r_1$ is also the solutions of Eq. (20). For the central symmetry structure, $\theta = \theta_1$ and $\theta = \theta_1 + \pi$ have the same configuration. So the definition domain of θ can be simplified as $(0, \pi)$.Combined with Eq. (20), the parameters' solution for fully isotropy configuration after simplification can be expressed as:

$$\begin{cases} r = 2R\,\sin(\frac{\pi}{6} + \theta) \\ H = \sqrt{2}R\,\sin\theta \end{cases} \quad \theta \in (0, \pi) \tag{21}$$

Then, according to Eq. (21), the fully isotropy 8/4-4 mechanism cluster is obtained.

The 8/4-4 mechanism cluster is classified according to the parameters and configuration. First, according to the relationship between R and r, the mechanism cluster is divided into $r < R$ and $r > R$ two categories. Then subdivide again according to whether the branches intersect with the smaller platform on the top view. Finally, the configuration of four groups which can represent the 8/4-4 cluster structure is obtained, as shown in Table 1. The 8/4-4 PFSM proposed in Sect. 2 is included in the first class. Compared with the previous 8/4-4 PFSM, the fixed platform of the 8/4-4 mechanism cluster is the upper and lower platforms, the measuring platform is the middle platform, the flexible measuring branches interlaced distribute among three platforms. It is worth mentioning that the 8/4-4 mechanism cluster is a derivative of the previous 8/4-4 PFSM. So the cluster also has the stiffness decoupling characteristics.

Table 1. Schematic diagram of 8/4-4 mechanism cluster.

Classification	$r < R$		$r > R$	
Structure diagram				
Top view				

The proposal of 8/4-4 PFSM and 8/4-4 mechanism cluster, fundamentally eliminates the interference of coupling elements to isotropy, simplifies constraints and provides a simple isotropic mechanism cluster for the isotropy design of sensor's elastic body.

5 Conclusion

This paper presents an 8/4-4 stiffness decoupling PFSM. Its structure model and mathematic model are analyzed. The physical meaning and constraint conditions of decoupling are given. The 8/4-4 PFSM's decoupled characteristics is verified based on its mathematic model. The fully isotropic constraint conditions of 8/4-4 PFSM are deduced and analyzed. The parameters' solution for fully isotropy configuration is obtained. According to the solution, the fully isotropy 8/4-4 mechanism cluster is proposed, which enriches and simplifies the isotropy design of sensor's elastic body.

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