# Efficient Combinations of Rejection Strategies for Dense Point Clouds Registration

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**Abstract.** The Iterative Closest Point (ICP) algorithm has been viewed as a standard approach to registering two point clouds. In the process of point clouds registration, the eliminating incorrect point pairs has important effect on the accuracy and stability of registration. In the past two decades, numerous strategies of excluding point pairs have been developed and various combinations of them have been applied to the variants of ICP algorithm. In this paper, an efficient combination of rejection strategies is proposed. It also is compared with other heuristic combinations. As shown in our case studies, the proposed combination can realize more accurate registration without sacrificing computational efficiency.

Keywords: Point cloud registration · Iterative Closest Point · Rejection strategies

### 1 Introduction

Point clouds registration algorithm is a core technique of computer graphics. Registering two point clouds consists in finding the rotation and the translation occurred in the overlapping regions of two point clouds, and it has widespread applications, such as simultaneous localization and mapping (SLAM). Therefore, the point clouds registration algorithms have received considerable attentions. The classic solution of registering two point clouds is the Iterative Closest Point (ICP) algorithm proposed by Besl and McKey [1]. The original ICP is not suitable for the situation where the point clouds to be registered overlap partially. However, partial overlap is a very common phenomenon in engineering practices, many variants of the ICP algorithm have been developed to fulfill the practical needs.

As reported in [2], the existing methods for registering two point clouds can be categorized in two groups, namely the sparse approaches and the dense approaches. The sparse approaches focus on a few meaningful points that have obvious geometry features or visual features and match point pairs by comparing the values of these features. Although matching feature is more robust than the dense approaches, it requires additional information and computational efforts. Most dense approaches only need the Cartesian coordinate of points which can be provided by tremendous sensors, such as 3D laser scanners, stereo cameras or depth cameras, etc. The dense approaches

are, therefore, simple and easy for use. The dense approaches have to assume that a reasonable initial estimate of transform is available, while the sparse approaches don't need such information. Compared with the sparse approaches, the dense approaches are stronger in will but weaker in power with respect to the data association. The original ICP algorithm is a typical dense approach, and it establishes the point correspondences by finding the closest point in other meshes starting from the current transform estimate. It usually refers to the point-to-point alignment. The point-to-point alignment is inevitable to produce numerous spurious point pairs, resulting in the incorrect alignment of two point clouds. Several association methods [3, 4] were developed in the past years to partially remedy the limitation of the point-to-point alignment. An alternative way to cope with the limitation of the point-to-point alignment is to find suitable rejection strategies of the false point pairs.

In the last two decades, numerous strategies of excluding pairs have been developed and their various combinations have been applied to the variants of the ICP algorithm. Nevertheless, it is difficult to find a versatile combination as argued in [5]. This paper focuses on the heuristic rejection strategies used in dense approaches and compares the performances of their combinations. A simple but also efficient combination of the strategies of rejecting point pairs is proposed for accurate point cloud registration.

The remainder of this paper is organized as follow. Section 2 introduces the details of the existing strategies of rejecting pairs. Section 3 proposes a robust point clouds registration algorithm that combines strategies of excluding pairs used in the vanilla ICP and ICRP. In Sect. 4, the proposed algorithm is compared with several heuristic combinations of rejection strategies by real-world data, and it is applied to the consistent mapping. Section 5 draws a conclusion.

## 2 Previous Work

Besl and McKay [1] proposed the classic ICP algorithm which is a fundamental tool to point clouds registration. The algorithm was examined by experimental data, but it didn't consider the partial overlap between two point clouds. Various variants of the ICP algorithm have been proposed during the last two decades. Developing a suitable strategy of rejecting pairs is an important research direction of these studies. The simplest solution of rejecting incorrect point pairs is to preset a distance threshold. If the distance of the point pairs exceeds a pre-set threshold, they will be identified as noise point pairs and discarded. The original formulation of the ICP approach including a distance threshold is called the vanilla ICP approach as reported in [6]. It is reasonable that vanilla ICP algorithm decreases gradually distance threshold with the increase of the iterations. Because the distances of the correct point pairs trend to be zero with the increase of the iterations. Such strategy was used in [7], and later on, Pomerleau *et al.* [8] introduced the Relative Motion Threshold (RMT) for rejection, and a simulated annealing ratio was used to automatically change the rejection threshold. Pajdla and Van Gool [9] developed the Iterative Closest Reciprocal Point (ICRP) algorithm. Their idea is that the distances of point pairs are theoretically zero and the closest point relation is symmetrical when the exact registration has been realized. Although the motion transform breaks this relation, the distance used to represent the relation should be less than a threshold  $\varepsilon$ . Point g in source point cloud has been associated with point m in target point cloud with nearest neighbor search, and then point m is associated with its closest point g' in source point cloud. If the distance between points g and g' exceeds the threshold  $\varepsilon$ , the point pair (g, m) will be excluded.

Rusinkiewicz and Levoy [10] compared several strategies of excluding pairs, and they concluded that rejecting pairs containing points on mesh boundary is especially useful for avoiding erroneous pairing when the overlapping between two point clouds is not complete.

Chetverikov *et al.* [5] developed the Trimmed Iterative Closest Point (TrICP) algorithm. It used the degree of overlapping to determine whether the point pairs should be retained. Assume that the degree of overlapping is known and represented by a variable  $n_o$ . By the TrICP, point correspondences are firstly found with point-to-point alignment method. The distances of point pairs are arranged in ascending order. The  $n_o$  pairs at the front of the queue are treated as the exact point pairs. Comparing it to the ICRP algorithm, the authors claimed that the TrICP outperformed the ICRP when the degree of overlapping is less than 50%.

Zinßer *et al.* [11] proposed the Picky ICP algorithm. The algorithm attempted to reject the point pairs except the point pair with smallest distance when a point appeared in different point pairs. May *et al.* [6] proposed the frustum ICP. The concept of *frustum* came from the field of computer vision, and applied to the ICP algorithm. It created a vision cone to describe degree of overlapping, and it showed that the algorithm was more robust than the vanilla ICP with respect to rejecting pairs. However, the cone is not suitable for some sensors, such as laser scanner. In other words, its versatility is weak.

Serafin and Grisetti [2] described a full approach, namely the Normal Iterative Closest Point (NICP), that can directly operate on raw 3D sensor data, either depth images or 3D laser scans. The NICP rejected false pairs using the combination of three rejection strategies: distance threshold, compatibility of normal and compatibility of curvatures.

All the aforementioned algorithms have important roles in point clouds registration. Many attempts have been made to compare performances of these algorithms to find out the best algorithm. There is no one attested that it is able to reject completely noise pairs. The existing algorithms actually are complementary or superfluous. May *et al.* [6] noted that the frustum ICP can be used together with other algorithms. However, they did neither give any strategy for combination nor analyze the difference of the algorithm with others except the vanilla ICP. Pomerleau *et al.* [8] combined several rejection strategies with the picky ICP to test their proposed algorithm. Various combinations of rejection strategies were used. Nevertheless, there is no reported work that was devoted to analyzing the performances of different combinations between the existing rejection strategies. This paper aims at comparing the performances of different

combinations of rejection strategies, and then, proposes a robust point clouds registration algorithm which combines strategies of rejecting pairs used in the vanilla ICP and the ICRP. In the ensuring sections, our proposed algorithm will be firstly presented and compared to other heuristic combinations. Because the new algorithm combines the strategies of excluding pairs used in the vanilla ICP and the ICRP, it is called the vanilla Iterative Closest Reciprocal Point (vanilla-R ICP) algorithm in this paper. As Rusinkiewicz and Levoy [10] classified the variants of the ICP algorithm into six stages, i.e., selection of points, matching points, weighting the corresponding pairs appropriately, rejecting certain pairs, assigning an error metric and minimizing it. We, therefore, follow this framework to introduce the new algorithm.

## 3 The Vanilla-R ICP Algorithm

### 3.1 Selection of Points

The computational overhead of the ICP algorithm depends mainly on the number of points. The exhaustive pairing often is computationally unaffordable. The existing methods for data reduction include the random sampling, uniform sampling, normal-space sampling, covariance sampling, and other sampling methods using extra information such as color and intensity of points. Although the normal space sampling proposed by Rusinkiewicz and Levoy [10] may work better on accuracy, we choose the random sampling is that computing surface normals of points in certain meshes is not necessary, and it will reduce the computational effort. Another reason is that there are often enough points attached to different surfaces such that the distribution of normals among selected points is not too small. Sampling from the normal space of points might be more suitable in the case where the existing poor normals occurs. In our experiment, we randomly sample points is up to a pre-set threshold  $n_e$ , and this sampling methods in other algorithms.

### 3.2 Matching Points

Rusinkiewicz and Levoy [10] summarized most correspondence finding methods. The commonly used methods include finding the closest point in the other mesh [1] and searching the point projected from source point cloud to destination mesh in the destination range image. The later might use a metric based on point to segment distance, point to plane distance, point to facet distance, the related mathematical tools were presented in [12]. Finding the correspondences through projection has an obvious shortcoming of efficiency due to the large amount of data to be manipulated. Serafin and Grisetti [2] introduced the concept of index image to reduce the memory movements. In this paper, we choose the point-to-point approaches due to two reasons: (1) There are enough numbers of points so that the influence of the discrete is small for data association, and (2) finding the closest point in the other meshes is much faster

than finding the correspondences through projection [7]. In our algorithm, point correspondences are found by:

$$\min_{y_{j\in T}} f(y_j) = \left\| y_j - (\mathbf{R}x_i + \mathbf{t}) \right\|_2 \tag{1}$$

where point  $x_i$  is a selected point from source points, whereas point  $y_j$  belongs to target point cloud. If the criterion described in Eq. (1) is reached, the pair  $(x_i, y_j)$  will be treated as the candidate pair. In our algorithm, the *k*-d tree was used to accelerate this computation. There are available variants of *k*-d tree to improve computational efficiency, such as the cached *k*-d tree [13].

#### 3.3 Rejecting Pairs

The rejection principles of pairs used in the vanilla ICP and the ICRP are combined to realize accurate and robust registration. This combination is called the vanilla-R criterion for simplification in this paper. As to be demonstrated in Sect. 4, it works better than other algorithms introduced in Sect. 2. The method excludes the point pairs which have lager distance than the pre-set distance threshold that would degrade with iteration. It is noted that the distance of point pair refers to the distance between the point transformed by current transform estimate and its mate. Transform between two point clouds consists of a rotation **R** and a translation **t**. **R**<sub>k</sub> and **t**<sub>k</sub> are obtained in the *k*th iteration of the algorithm. If the distance of point pair is lower than the distance threshold, the algorithm will decide whether the pair should be retained using the rejection strategy of the ICRP [9]. A candidate pair  $(x_i, y_j)$  is discarded if one of the following criterions holds.

The distance of pair is larger than the threshold  $d_k$  which gradually decrease with iteration according to value of *step*:

$$\left\|\mathbf{y}_{j} - (\mathbf{R}_{k}x_{i} + \mathbf{t}_{k})\right\|_{2} > d_{k}$$

$$\tag{2}$$

$$d_k = d_{k-1} - step \tag{3}$$

The distance between  $x_i$  and reciprocal point  $x'_i$  in the source point cloud *S* exceeds the pre-set threshold  $\varepsilon$ :

$$\min_{x_{i}^{'}\in\mathcal{S}}f\left(x_{i}^{'}\right) = \left\|y_{j}-\left(\mathbf{R}_{k}x_{i}^{'}+\mathbf{t}_{k}\right)\right\|_{2}$$

$$\tag{4}$$

$$\left\|x_{i}-x_{i}^{'}\right\|_{2}>\varepsilon\tag{5}$$

The parameter settings are presented in Table 1.

Algorithms	Error	Run time	Related parameters
Vanilla ICP	37627.6	1.9035	$d_0 = 1010, step = 25, n_e = 5000$
ICRP	29698.1	2.76656	$\varepsilon = 10, \ n_e = 1000$
TrICP	24094	8.02731	$n_o = 20000$
Picky ICP	39966.9	2.88134	$n_e = 500$
Vanilla-R ICP	13126.1	1.86003	$d_0 = 1010, \ step = 25$
			$\varepsilon = 10, n_e = 500$
Vanilla-Picky ICP	25038.1	1.67218	$n_e = 100$
Trimmed-R ICP	25599	9.14937	$n_o = 8000, \ \varepsilon = 10$
Trimmed-Picky ICP	31787	4.8373	$n_o = 8000$

**Table 1.** Parameters were set as optimal values derived by sampling the parameter space. The Vanilla-R ICP obviously outperforms others on accuracy and its computational efficiency is slightly lower than that of the vanilla-picky ICP.

### 3.4 Error Metric and Minimization

This step uses all the pairs filtered by rejection strategies to calculate the misalignment error and find the transform between two point clouds. Our algorithm uses the point-to-point error metric, and finds the transform using a method based on singular value decomposition [14].

### 3.5 The Design of Convergence Rules

The error usually fluctuated slightly with iterations because of the random sampling, and deceptive convergence might oftentimes occur. The convergence rule is designed to assure that our algorithm can reach an accurate result and prevent the algorithm from deceptive convergence and infinite loop. The mean registration error of eligible pairs should below an error threshold and change slightly with iteration. If the number of times the criterion has been satisfied is up to a pre-set value, the algorithm terminates. At the same time, the maximum number of iterations is designed to avoid the situation where the pre-set error threshold is too small for the algorithm to terminate, resulting in infinite loop.

## 4 Algorithm Evaluation and Application

### 4.1 Criterion of Evaluation

In this paper, we don't analyze our algorithm with experimental data. Rather, it is applied to registration of real-world data. There is no ground truth of transform between two point clouds for evaluation. The criterion of evaluation proposed here was inspired by the TrICP [5]. The sum of the registration errors from overlapping section theoretically trend to be zero when the accurate registration has been realized. Resulting registration errors of pairs are arranged in ascending order, and the result of summing registration errors of  $n_{ev}$  pairs at the front of the queue is used as the criterion of

evaluation. The value of  $n_{ev}$  was set to 5000. In general, the smaller the summing results, the better the registration. It is noted that incorrect registration theoretically might produce a small value, although the situation didn't occur in our evaluation. Visual effect of registration is an assistant means to remedy this defect of the criterion.

#### 4.2 The Algorithms for Comparison

The incorrect correspondences can be divided to two categories: (1) the points outside of the overlapping section took part in pairing, and (2) the inner points from overlapping section matched wrongly each other. The pair containing an outside point usually has a huge distance. The Vanilla ICP uses a distance threshold to reject noise pairs. The TrICP only chooses the  $n_o$  pairs with the least value of distance as correct pairs. Therefore, the vanilla ICP and the TrICP can reject this class of pairs easily. It is noted that the results of the vanilla ICP and the TrICP will be completely equivalent if the distance threshold used in the vanilla ICP is equal to the distance of  $n_o$ th pair with the least value of distance. The pairs containing outside points generally occur in boundary mesh. Thus, rejecting pairs containing points in boundary mesh also is efficient to exclude this class pairs. In fact, the strategies of excluding pairs used in the ICRP and the Picky ICP are also useful to reject the point pairs containing an outside point, but they are obviously time-consuming due to searching nearest neighbor.

Figure 1 shows a general situation where a mobile robot moved in indoor plane, and one needs to register the source point cloud into target point cloud. We assume that in Fig. 1 all the points in source space belong to inner points, the outside points have been rejected with a distance threshold or other methods above. The green section denotes the favorable pairs for registration. The yellow area describes the bad pairs which usually lead to terrible result. The brown area presents the neutral point pairs which protect the algorithm from divergence but slow down the speed of convergence. We continue to evaluate the capability of available strategies of excluding pairs in yellow and brown areas. It is obvious that the vanilla ICP and the TrICP are both incapable of rejecting the pairs in yellow and brown areas. Compared to the picky ICP, the ICRP is more competent to exclude the point pairs in the yellow area because the ICRP can find the latent pairs by searching backwards. The method based on normals can also reject the pairs in yellow area, but computing the normals of points costs at least twice amount of the time which the ICRP costs. It is therefore not considered. All the methods above are inefficient for point pairs in brown area.

According to the capability of excluding pairs, the strategies of rejecting pairs are divided into two categories: (1) more competent to reject wrong point pairs containing outside points, such as the ones used in the vanilla ICP and the TrICP, and (2) The ones which work well in terms of reducing incorrect point pairs of overlapping section, such as the ones used in the ICRP and the picky ICP. Nevertheless, the combinations that consist of the rejection strategies of one category don't improve performance of the algorithm efficiently. Hence, we only combine the rejection strategies that play different role in the process of excluding point pairs. All the algorithms to be compared are presented in Table 1. The vanilla-R ICP is the proposed algorithm that combine the rejection strategies used in the vanilla ICP and the ICRP. The Trimmed-R ICP means



**Fig. 1.** The green section depicts the favorable pairs for registration. The yellow area describes the bad pairs which usually lead to terrible result. The brown area presents the neutral point pairs which protect the algorithm from divergence but slow down the speed of convergence. (Color figure online)

that we first use the rejection strategies of the TrICP and continue to exclude the pairs with rejection strategy used in the ICRP. The Vanilla-Picky ICP and the Trimmed-Picky ICP use the same principle of combination as the Trimmed-R ICP. At the same time, we compare these combinations with the algorithms that only use an individual excluding pairs strategy.

#### 4.3 Results

All the programs are executed on an Intel Core i5-4460 at 3.2 GHz. The dataset used here comes from the website: http://kos.informatik.uni-osnabrueck.de/3Dscans/, and the number of dataset is 5. The results presented in Table 1 are analyzed from two aspects: computational effort and accuracy.

In terms of computational effort, the vanilla-Picky ICP took the least amount of time. It benefits from the rejection strategies used in the Picky ICP, which don't need to execute time-consuming search of closest neighbor. The Vanilla-R ICP is slightly less efficient than the Vanilla-Picky ICP, which benefits from random sampling, it just needs the number of eligible pairs to be up to a pre-set threshold. The TrICP expends plenty of time in the search of closest neighbor, therefore it is very time-consuming. Comparing to the Trimmed-Picky ICP and the TrICP, we found that the combination dramatically reduces the executing time. In fact, the rejection strategy used in the Picky ICP retains a fairly small portion of the pairs filtered by the trimmed ICP. In our experiment, there are about 300 pairs left after using the rejection strategy used in the Picky ICP whereas the TrICP kept the 8000 pairs as correct pairs. According these results, we can verify that the computational effort is influenced mainly by method of selecting point. Due to the need of ensuring the integrity of the method, some low efficient sampling methods have to be remained, such as the TrICP.

In terms of accuracy, the vanilla-R ICP is superior to other algorithms. As noted in Sect. 4.2 that the rejection strategy used in the ICRP is more competent to exclude incorrect pairs of inner points than used in the Picky ICP. It is expected that the vanilla-R ICP is more accurate than the vanilla-Picky ICP. This expectation is also confirmed by the data presented in Table 1. In process of the Trimmed-Picky ICP, small number of point pairs accelerated the solution of transform relation but resulted in poor accuracy compared to the TrICP. The combinations improved the performance of individual rejection strategy except combinations with the TrICP. Figure 2 shows the visual effects of registration of the vanilla ICP and the vanilla-R ICP, the results are consistent with the indicated error data. Others have more bad or similar visual effect to the vanilla ICP, and thus, they aren't exhibited anymore.



**Fig. 2.** The left was the result of the vanilla ICP, and the right is the result of the vanilla-R ICP. The red circle highlights part where is easy to find the registration error. It can be found that the vanilla-R ICP is better than the vanilla ICP. (Color figure online)

According to the above observations, we can conclude that the vanilla-R criterion is a more efficient combination of rejection strategies than the others.

#### 4.4 Application

In this section, our algorithm was applied to realize the 3D consistent mapping. We use *incremental matching* method introduced by Chen [3]. The new scan is registered against a so-called *metascan*, which is the union of the previously acquired and registered scans. In general, using this method leads the accumulation of mapping error. To verify the consistent mapping ability of the vanilla-R ICP algorithm, we don't use any global optimization algorithm which usually improve the consistency of mapping. Figure 3 presents the visual effect of mapping the five data. No obvious error was found in the mapping, which illustrates the proposed algorithm is helpful to improve accuracy of consistent mapping. The octomap [15] was used to build the occupancy grid map which can be used to mobile robot navigation, the result is presented in Fig. 4.



**Fig. 3.** The vanilla-R ICP to 3D consistent mapping were used, and no obvious geometrical inconsistencies were found in the map. Additionally, no global optimization algorithm was used in the process of consistent mapping.



Fig. 4. Occupancy grid map. Blue represents the occupied area. (Color figure online)

## 5 Conclusion

In this paper, we summarize the available heuristic strategies of rejecting point pairs and compare the performances of various combinations of these strategies. We propose the vanilla-R ICP algorithm which combines exclusion strategies used in the vanilla ICP and the ICRP. The convergence rules are also designed to assure that the algorithm can converge to an accurate result. A new criterion for evaluating point clouds registration is designed for situation where the ground truth of transform between two point clouds to be registered is absent. Compared to other heuristic combinations of rejection strategies, the proposed algorithm can realize the more accurate registration without asking for additional computational effort.

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