Fractional-Order Integral Sliding Mode Controller for Biaxial Motion Control System

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Abstract. Biaxial tables are widely applied in high performance motion control applications for high accuracy. Tracking error is one of the most significant indicators of machining precision, and tracking control is an effective means to eliminate the tracking error. In this paper, to attenuate the tracking error and reduce the chattering phenomenon in the control input simultaneously, a fractional-order integral sliding mode controller is proposed. Compared with the existing sliding mode controller, the proposed control law not only maintains the original robustness against variations but also reduces the tracking error effectively. At the same time, the overshoot can be weakened and the reaching law will converge to the sliding surface more rapidly. Experiments conducted on a biaxial table demonstrate that the proposed control scheme is easy to apply, the tracking error is smaller and the input chatter can be improved significantly compared to the integer SMC.

Keywords: Motion control · Fractional-order integral · Tracking error

1 Introduction

Although biaxial tables are widely used in high performance motion control applications for high accuracy, there are still errors in machining. Tracking error is regarded as one of the most significant indicators of machining precision. The effective approaches to improve machining accuracy contain tracking control methods, trajectory planning methods and so on.

To reduce the tracking error, there are a lot of researchers having developed several control algorithms over past few decades. Tomizuka [1] firstly proposed a zero phase error tracking controller (ZPETC) by canceling the stable dynamics of the servo drive to improve tracking accuracy. The main drawback of ZPETC is sophisticated, then a simple algorithm sliding mode control (SMC) was proposed by Altintas et al. [2], which has practical advantages in rapid tuning and implementation, but with severe chattering. Later, Sun [3] developed a new adaptive control approach to position synchronization of multiple motion axes, this new method guarantees asymptotic convergence to zero of both position and synchronization error. Barton and Alleyne [4]

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put forward a synthetic method for precision motion control by combining individual axis iterative learning control and cross-coupled iterative learning control into a single control input, which enhanced the precision motion control of the system through performance improvements in individual axis tracking. Because the effect of cross-coupled during high speed feed drives can not be ignored, a H∞ controller was designed by Yong et al. [5] to minimize the tracking error, which was augmented with integral action to achieve accurate tracking.

With the strong robustness against model uncertainties and disturbance rejection, SMC is one of the popular controllers in industrial applications all the time. However, because of the discontinuous nature of SMC, it will produce the chattering phenomenon and high-frequency oscillations in practice [6]. To counteract the chattering phenomenon in SMC, the fractional order sliding mode controller is received more and more attention. Delavari et al. [7] presented a fuzzy fractional order sliding mode controller for nonlinear systems to reduce the chattering phenomenon, and the fuzzy logical controller is used to replace the signum function at the reaching phase in the SMC. Zhang et al. [8] proposed a fractional order sliding-mode control for velocity control of permanent magnet synchronous motor, in which a fuzzy logic inference scheme is utilized to obtain the gain of switching control. Yin et al. [9] applied a fractional order sliding mode controller (FOSMC) in nonlinear systems to achieve extremum seeking. Tang et al. [10] designed a new fuzzy fractional order sliding mode controller for antilock braking system (ABS), this strategy can not only deal with the unsertainties but also track the desired slip faster than conventional SMC. In all of the above methods, it is clear to see that the robustness and chattering phenomenon can be effectively improved.

The original intention of this study is to maintain the robustness and decrease the chatter of SMC simultaneously. Thus, a fractional order integral sliding mode controller is proposed. The control law is tested on a biaxial table, and the performance of the proposed strategy is compared against the traditional SMC. Experiment results show that the tracking error is smaller and the chattering phenomenon is less than those of integer SMC.

2 Definition of Fractional Order Calculus

Fractional order calculus is a classical mathematical idea which allows to arbitrary order differentiation and integration, and it can be specified in terms of the fundamental operator ${}_aD_t^\alpha$ known as differ-integration operator [11],

$$f(t) = {}_aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & c > 0 \\ 1, & c = 0 \\ \int_a^t (d\tau)^\alpha, & c < 0 \end{cases} \tag{1}$$

where a and t are lower and upper limits, respectively, $\alpha \in R$ is the order of the fractional order operator. Among several basic definitions of arbitrary order differentiation and integration, the most popular two definitions are the Grunwald-Letnikov (G-L) and the Riemann-Liouville (R-L) definitions.

The α th-order Riemann-Liouville (R-L) fractional-order integration of continuous function *f*(*t*) is given by,

$$ {}_aI_t^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}}d\tau \tag{2} $$

where $\alpha \in (0, 1)$, $\Gamma(\bullet)$ is the Gamma function and,

$$ \Gamma(\alpha) = \int_0^{\infty} e^{-u}u^{\alpha-1}du \tag{3} $$

The αth-order Riemann-Liouville (R-L) fractional-order derivative of continuous function *f*(*t*) is defined as,

$$ {}_aD_t^{\alpha}f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_a^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}}d\tau, & m-1<\alpha<m \\ \frac{d^m f(t)}{dt^m}, & \alpha = m \end{cases} \tag{4} $$

where $m-1<\alpha\leq m,\ m\in N$, m is the minimum integer number in the value which are larger than α.

3 Controller Design

3.1 Dynamics of Biaxial Table Feed Drive

The dynamics of a single feed drive system of biaxial table shown as Fig. 1, which can be described as the following differential equation,

$$ \frac{J}{K_aK_tR_g}\ddot{x}(t) + \frac{B}{K_aK_tR_g}\dot{x}(t) = u(t) - \frac{1}{K_aK_t}T_d(t) \tag{5} $$

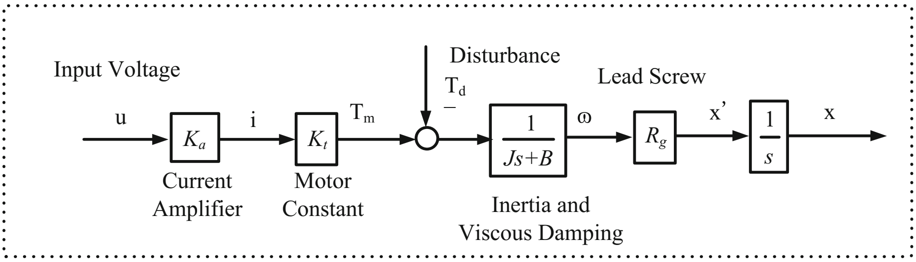

Fig. 1. A single feed drive system of the biaxial table

where J is the inertia and B is the viscous damping coefficient, both of them can be identified in advance. K_a is the current amplifier and K_t is the motor constant, R_g is the lead screw. $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ are the position, velocity and acceleration of the actuator, respectively. T_d is the external disturbance torque, u is the control input.

For simplicity, the coefficients of the dynamics can be rewritten as,

$$M = \frac{J}{K_a K_t R_g}, C = \frac{B}{K_a K_t R_g}, u_d = \frac{1}{K_a K_t} T_d(t) \tag{6}$$

So the dynamics differential equation can be expressed as,

$$M\ddot{x}(t) + C\dot{x}(t) = u(t) - u_d \tag{7}$$

In practice, our control objective is to drive the tracking error to zero asymptotically under any initial conditions. Because the exact knowledge of M, C and u_d are not known, there is only nominal or identified model can be available to design the controller, meanwhile these uncertainties would have a bad influence on robustness of system. Conventional SMC belongs to a class of nonlinear control strategies, which is robust to such uncertainties and time variations in the drive system [6]. So we can believe that fractional-order integral sliding mode controller is also robust to these uncertainties for it is a part of SMC essentially.

3.2 Design of Fractional Order Integral Sliding Mode Controller

The block diagram of fractional order sliding mode controller is shown in Fig. 2, the actual position is measured from an encoder, and the actual velocity and acceleration is estimated by utilizing the method of digital differentiation.

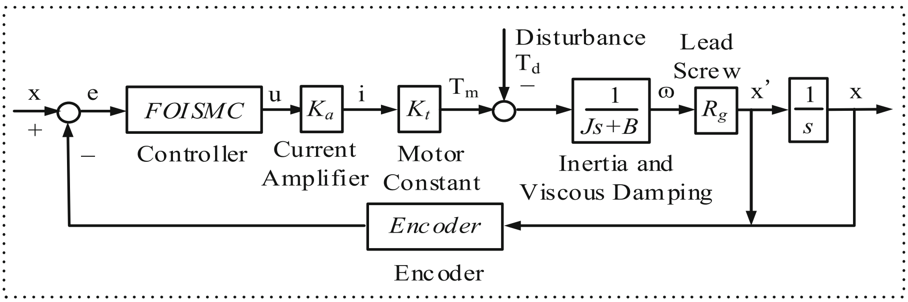

Fig. 2. Fractional-order sliding mode control scheme

Like designing a traditional SMC, there are also two fundamental steps in fractional-order sliding mode controller, one is the selection of a sliding surface and the other is the control law.

Since the oscillation induced by sliding surface may excite high frequency unmodeled dynamics of the system and damage the performance of the system, it is necessary to reduce the chattering phenomenon of the control input. To achieve such a goal of tracking accuracy, the fractional-order integral sliding surface is selected as,

$$s = \dot{e} + k_0 D_t^{-\alpha} \dot{e}(t) \tag{8}$$

where k is a positive gain and is the obtainable and desired tracking bandwidth of the drive. ${}_0D_t^{-\alpha}$ is the function of fractional-order integral operator $(0 < \alpha < 1)$, e and $\dot{e}$ are the position and velocity tracking error of the drive system, respectively,

$$e = x_r - x_a \tag{9}$$

$$\dot{e} = \dot{x}_r - \dot{x}_a \tag{10}$$

Taking the derivative of s with respect to time yields,

$$\dot{s} = \ddot{e} + k\left({}_0D_t^{-\alpha} \dot{e}(t)\right)' \tag{11}$$

where $\ddot{e}$ is the tracking error of acceleration,

$$\ddot{e} = \ddot{x}_r - \ddot{x}_a = \ddot{x}_r - \frac{1}{M}(u(t) - u_d - C\dot{x}_a(t)) \tag{12}$$

Here, the reaching law is selected as,

$$\dot{s} = -k_1 s - k_2 \text{sgn}(s) \tag{13}$$

where $k_1, k_2 \in R^+$ are sliding mode coefficients, sgn (s) is the sign function of s, which can be expressed as,

$$\text{sgn}(s) = \begin{cases} 1, & s > 0 \\ 0, & s = 0 \\ -1, & s < 0 \end{cases} \tag{14}$$

Combined Eqs. (11) and (12) with (13), the control law can be selected as,

$$u(t) = M\left(\ddot{x}_r(t) + k\left({}_0D_t^{-\alpha} \dot{e}(t)\right)' + k_1 s + k_2 \text{sgn}(s)\right) + u_d + C\dot{x}_a(t) \tag{15}$$

After designing the control input, the second step is stability analysis. Stability analysis has to satisfy the reaching condition of proposed fractional-order switching surface, which means wherever or whatever initial conditions state, the control output could drive initial states to switching surface. Here, the fundamental Lyapunov function is chosen as,

$$V = \frac{1}{2}s^2 \tag{16}$$

According to Eq. (16), the derivative of V can be expressed as:

$$\dot{V} = s\dot{s} \tag{17}$$

For stability of nonlinear systems, the derivative of the Lyapunov function must be negative so as to fit the principle of conversation of energy.

By substituting s from Eq. (13) and sign function of s from (14), we can represent Eq. (17) as,

$$\dot{V} = -k_1 s^2 - k_2 s\ \mathrm{sgn}(s) \begin{cases} = 0, & s = 0 \\ < 0, & s \neq 0 \end{cases} \tag{18}$$

It is obvious that the Eq. (18) guarantees the asymptotic stability. So, next we will prove that the control law drives system to converge to switching surface in finite time.

When the initial state satisfies $s(t_0) > 0$, we can represented Eq. (13) as,

$$\dot{s} = -k_1 s - k_2 \tag{19}$$

Solving the Eq. (19), we can gain the solution of s,

$$s = \frac{-k_2 + [k_2 + k_1 s(t_0)]e^{-k_1(t-t_0)}}{k_1} \tag{20}$$

When the Eq. (20) equals to zero, the system can converge to switching manifold, and the time can be expressed as,

$$t = -\frac{1}{k_1}\ln\frac{k_1}{k_2 + k_1 s(t_0)} + t_0 \tag{21}$$

Similarly, when the initial state is $s(t_0) < 0$, the system can also converge to switching manifold when the time satisfies the following expression,

$$t = -\frac{1}{k_1}\ln\frac{k_1}{k_2 - k_1 s(t_0)} + t_0 \tag{22}$$

So, once the time is longer than the following expression,

$$t > -\frac{1}{k_1}\ln\frac{k_1}{k_2 + k_1|s(t_0)|} + t_0 \tag{23}$$

the system will converge to switching surface at any initial state. From the Eq. (23), we can draw a conlusion that the converging time is associated with k_1 and k_2. What's more, we can also know that larger k_1 and smaller k_2 can make time shorter. In experiment, the parameters k_1 and k_2 can be determined by gradually increasing its value from zero to satisfactory tracking accuracy, and then we get the parameter of k_1 in fractional-order sliding mode controller is larger than that in traditional SMC and k_2 is smaller. For the parameter α, the tracking error will increase when it tends to be large, but when it becomes small, the chattering phenomenon will become serious, thus the selection of α should be a compromise between tracking accuracy and chattering elimination so as to get better performance.

4 Experiments Validation

The proposed fractional-order integral sliding mode controller is applied in a real time platform of biaxial table as shown in Fig. 3. A computer with matlabR2013a is used to achieve feedrate planing, interpolation and controller design, then the computer transmits the real-time information to dSPACE DS1103 controller platform by hardware fiber bus and software ControlDesk. Experiments are carried out in the current-control loop of Yaskawa AC servo motor system. For translational axes, motors will be coupled with a 10 mm/pitch lead screw, respectively. The dynamic parameters shown in Table 1 are from [12], whose identification method was proposed by Erkorkmaz [13].

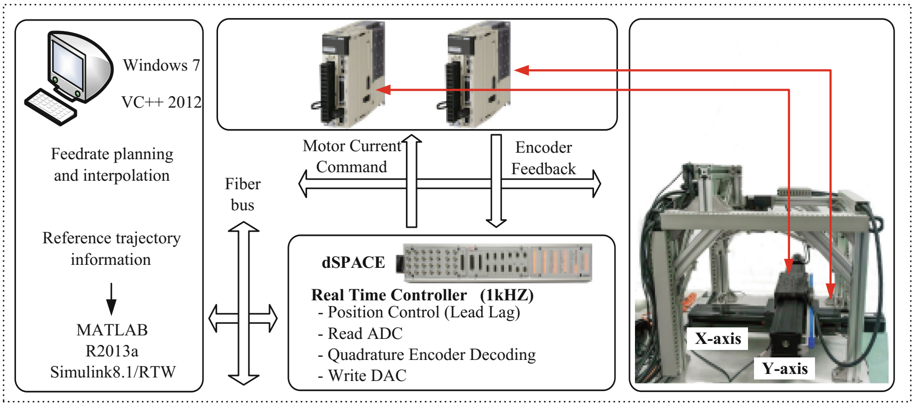

Fig. 3. Experimental platform of the biaxial table

Table 1. Dynamic parameters for biaxial table

Parameters	X-axis	Y-axis
m (Vs^2/m or Vs^2/rad)	0.1101	0.0271
c (Vs/m or Vs/rad)	0.4001	0.3871

The actual position is obtained from the encoder, and the actual velocity and acceleration are estimated by taking the derivative of the measured position from the linear encoder, so there may be noisy in velocity and acceleration. In this paper, we classify these uncertainties as external disturbance and give equivalent torque compensation, the relevant plants are given in Table 2.

Table 2. Disturbance parameters for biaxial table

Parameters	X-axis	Y-axis
u_d (if vel > 0)	0.11847	0.11881
u_d (if vel < 0)	−0.11466	−0.11151

In experiment, the parameters of X-axis and Y-axis are tuned separately and properly as shown in Table 3.

Table 3. Controller paremeters for biaxial table

Parameters	X-FOISMC	X-SMC	Y-FOISMC	Y-SMC
α	0.85	1	0.85	1
k	55	55	75	65
k_1	65	33	70	50
k_2	0.01	0.05	0.01	0.05

In order to facilitate the comparison of integer and fractional sliding mode controllers, the SMC strategy proposed in [2] and the method proposed in this paper are experimented simultaneously on biaxial table. The results of X-axis are shown in Fig. 4.

For a comparative analysis of the performance of integer and fractional-order controllers, we plot the control signals and tracking errors in the same picture in Fig. 5, respectively. From Fig. 5(a), it can be seen that the initial overshoots and chattering phenomenon of the proposed control signal are smaller than those of the integer SMC. In addition, it can be also seen from Fig. 5(b) that the tracking error of the presented method is smaller and smoother than that of the conventional SMC.

Machining accuracy includes tracking accuracy and contour accuracy, in this paper we only discuss the tracking error. Considering the machining accuracy, we must take contour error into account, contour error will be decoupled to each axis in joint space, which is essentially the error control of single axis, so we conduct experiments on Y axis, too. The results are shown in Figs. 6 and 7. From Fig. 7(a), we know that the chattering phenomenon decreases on a degree but the overshoot is still more than that of proposed controller, and the tracking error is more than that of integer SMC in

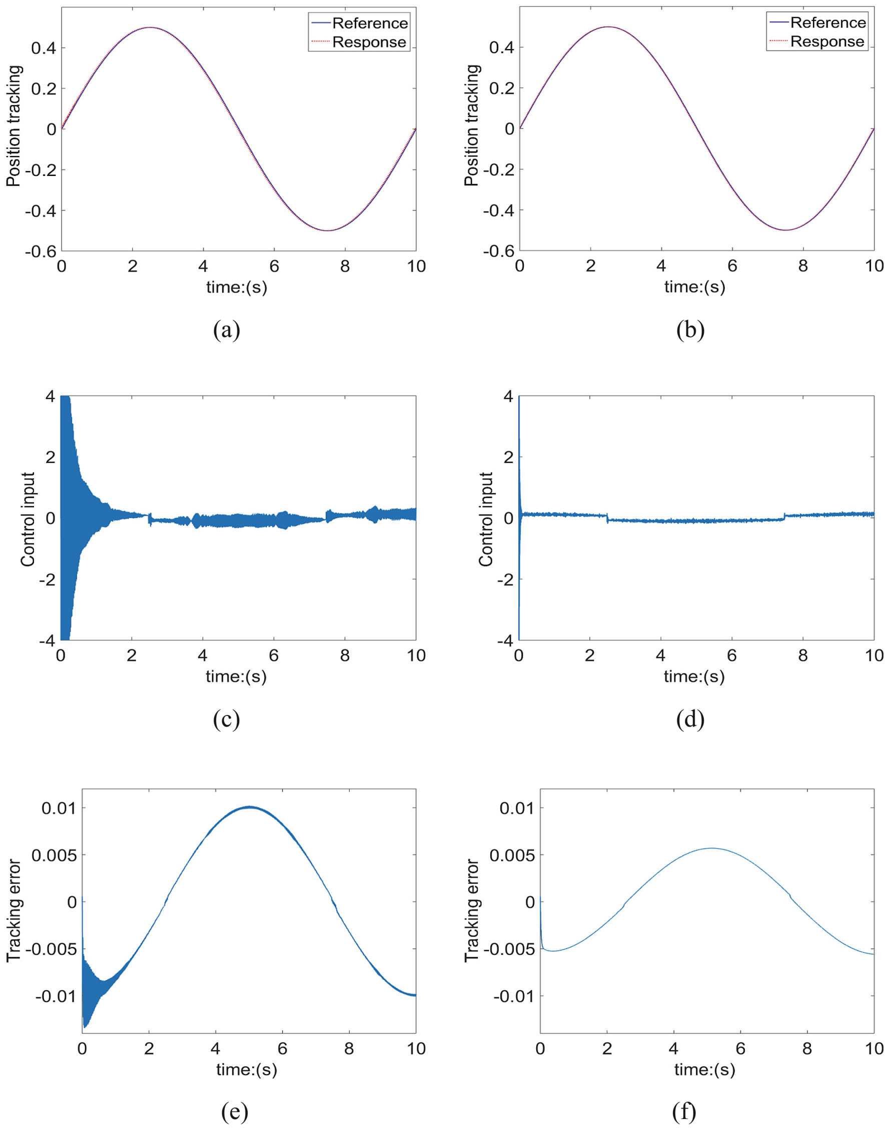

Fig. 4. Sinusoidal responses. (a), (c) and (e) Traditional SMC. (b), (d) and (f) Proposed FOISMC.

Fig. 7(b), we can make a conclusion that the fractional-order sliding mode controller obtains better performance in reducing overshoot and decreasing tracking error at the same time.

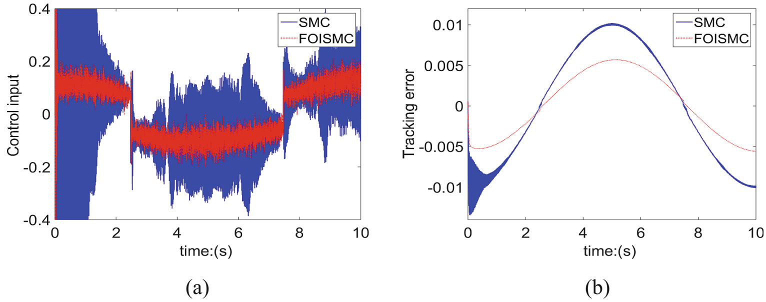

Fig. 5. Experimental results. (a) Control signal comparsion. (b) Tracking error comparsion.

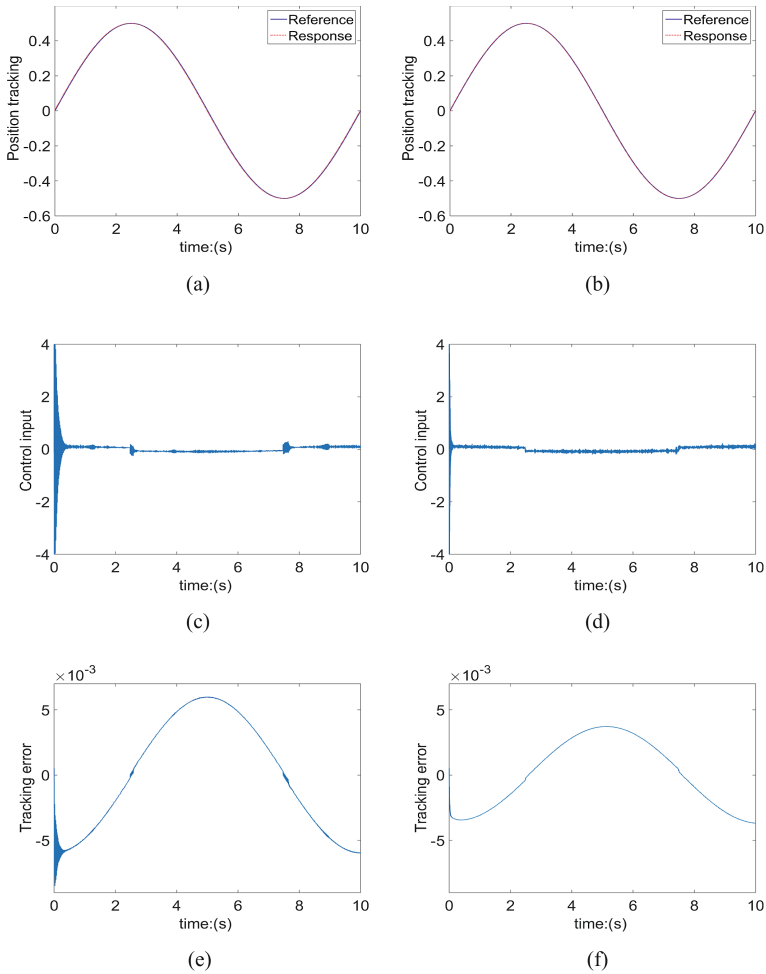

Fig. 6. Sinusoidal responses. (a), (c) and (e) Traditional SMC. (b), (d) and (f) Proposed FOISMC.

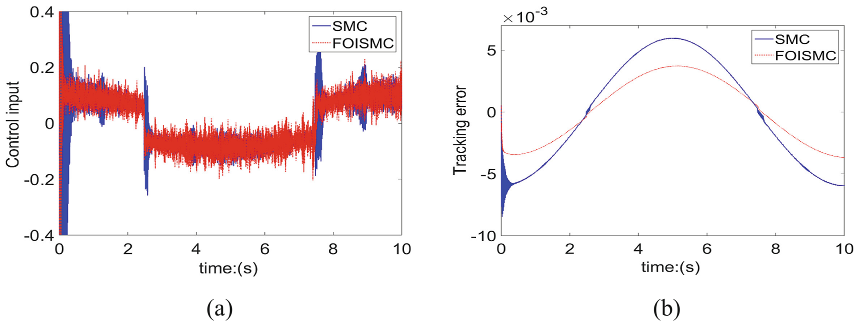

Fig. 7. Experimental results. (a) Control signal comparsion. (b) Tracking error comparsion.

5 Conclusion

A fractional-order integral sliding mode control scheme for biaxial motion system was proposed in this paper. The theoretical analysis and experiments conducted on X-axis and Y-axis shows that the tracking error of the proposed scheme is smaller and the input chatter is improved efficiently compared with the traditional SMC, furthermore, the proposed control law maintains the robustness against variations. So, we can conclude that the fractional-order integral sliding mode controller can achieve better tracking accuracy.

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