Trajectory Shaping Guidance Law Based on Downrange-to-Go Polynomial

Namhoon Cho, Youdan Kim, Hyo-Sang Shin and Antonios Tsourdos

1 Introduction

Ground or naval targets are usually equipped with the defensive measures and anti-missile systems such as armoured shield for self-protection, electronic jammer, close-in weapons system, etc. However, those measures cannot provide isotropic and uniform defensive power over all directions due to limited coverage. In other words, a certain direction around the target is more vulnerable to incoming attack than others. From the view of offensive missile, it will be advantageous to have a capability of hitting the target from a desired impact direction.

On the other hand, it is desirable in most cases to regulate the lateral acceleration to zero as the missile approaches to the target. This is to maximize the hit probability and the destructive power of warhead, to reduce the angle-of-attack at the moment of impact, and to allow small correction near the end of engagement. Most importantly, zero terminal acceleration constraint is necessary to avoid command saturation at the end of homing phase, because the control authority and the margin for manoeuvre in response to external disturbances can be maintained by ensuring this constraint.

To meet the above requirements, several guidance laws have been developed to cope with terminal impact angle constraint while reducing terminal manoeuvre

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demand at the same time [1–10]. Time-to-go Polynomial Guidance law (TPG) presented in [2, 4] is of concern in this study. TPG is a class of guidance laws with terminal impact angle and acceleration constraints, which is a general framework of trajectory shaping guidance law design. To design the TPG, the form of acceleration command is given by a polynomial of time-to-go, and the coefficients of the polynomial are determined by terminal boundary conditions. TPG is known as a guidance scheme that provides several promising results in theoretical analysis, flexibility of tuning, and good performance. Moreover, TPG is a general form of trajectory shaping guidance laws for terminal impact angle control, and weighted linear quadratic optimal guidance laws with terminal impact angle constraint can be regarded as a specific type of TPG.

In this study, a modified approach is proposed to improve several shortcomings of the TPG. In [2, 4], TPG was designed by designating acceleration command as a polynomial of *time-to-go*. Note that TPG has been developed on the basis of linearized engagement kinematics with constant speed assumption. Also, the guidance command is given by an explicit function of time-to-go for which an approximate estimate is only available rather than the exact one. Unlike the TPG, in this study, the desired crossrange trajectory is constructed using a polynomial of *downrange-to-go*, which is the first step of a new trajectory shaping guidance law design. By doing this, the proposed guidance law does not depend on (1) linearization of engagement kinematics, (2) assumption of constant speed missile, and (3) inaccurate time-to-go estimate. In summary, the proposed approach overcomes the shortcomings of TPG while sharing similarities in its design philosophy.

The key idea of the proposed approach is to assign a desired *crossrange* pattern as a function of *downrange-to-go*, instead of other options for the independent variable such as time-to-go, range-to-go, or path-length-to-go. If time-to-go is chosen as the independent variable for describing the desired trajectory, then it is hard to deal with time-varying speed case and the final command will depend on inaccurate time-to-go estimate. If path-length-to-go is chosen, then the constant speed assumption can be relaxed. However, the design in this case is identical to the previous one using the time-to-go except the change-of-variable in engagement kinematics, and it is difficult to represent the path-length-to-go as an exact form. Or, if range-to-go is used to describe the desired trajectory, then it will be difficult to consider the terminal impact angle constraint. To treat the shortcomings of the previous methods, downrange-to-go is used to describe the desired trajectory in this study. Also, the proposed approach does not require linearization of engagement kinematics. Note that this approach differs from performing feedback linearization of engagement kinematics first and then designing the virtual control as a polynomial of downrange-to-go.

The rest of this paper is organized as follows: The problem of stationary target interception with terminal constraints is formulated in Sect. 2. The trajectory shaping guidance law is proposed and its properties are discussed in Sect. 3. Numerical simulation is performed to demonstrate the effectiveness of the proposed guidance law and the results are shown in Sect. 4. Concluding remarks are summarized in Sect. 5.

2 Problem Formulation

In this section, assumptions to design a guidance law, equations of planar engagement kinematics, and problem statement are described.

2.1 Assumptions

The following assumptions are considered to design and analyse the guidance law proposed in this study.

Assumption 1 The target is stationary, and the velocity and acceleration vectors of the missile lies on a plane for all time, i.e., the engagement is two-dimensional.

Assumption 2 The missile is a lag-free point-mass such that the actual lateral acceleration equals to the commanded lateral acceleration without time-delay and distortion.

Assumption 3 The information on the position and velocity of the missile and the position of the target is available from sensors without time-delay and noise.

Assumption 4 The (initial) velocity of the missile is within $\pm 90\,\mathrm{deg}$ from the desired terminal impact direction.

Note that Assumption 2 does not influence on the tangential acceleration which is related to the change of speed. The lateral (normal) acceleration is only related to the change of flying direction and therefore to the geometric shape of the curve flown by missile.

2.2 Planar Engagement Kinematics

Figure 1 shows the planar homing engagement geometry considered in this study. In Fig. 1, (X_I, Y_I) and $\left(X_f, Y_f\right)$ denote the inertial coordinate system and the impact coordinated system, respectively. The impact coordinate system has its origin on the stationary target T, X-axis is aligned to the desired terminal impact direction, and γ_{f_d} is the flight path angle for the desired terminal impact direction. For the missile M, (x, y) is the position with respect to the impact coordinate system, V_M is the speed, γ_M is the flight path angle, and a_M^n is the lateral acceleration. The downrange-to-go denoted by x_{go} is the remaining distance to the target along X_f-axis, and $x_{go} = -x$ by construction. The flight path angle error is defined as follows

$$\bar{\gamma}_M \triangleq \gamma_M - \gamma_{f_d} \tag{1}$$

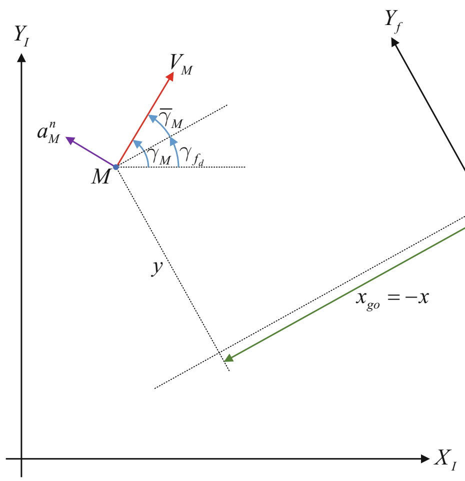

Fig. 1 Planar engagement geometry

The motion of the missile can be represented in the impact coordinate system as

$$\begin{aligned} \dot{x} &= V_M \cos \bar{\gamma}_M, & x(t_0) &= x_0 < 0 \\ \dot{y} &= V_M \sin \bar{\gamma}_M, & y(t_0) &= y_0 \\ \dot{\bar{\gamma}}_M &= \frac{a_M^n}{V_M}, & |\bar{\gamma}_M(t_0)| &< \frac{\pi}{2} \end{aligned} \tag{2}$$

where $\dot{(\,)} = \frac{d}{dt}(\,)$, and $\bar{\gamma}_M(t_0) = \bar{\gamma}_{M_0} = \gamma_{M_0} - \gamma_{f_d}$. Due to Assumption 4 which implies that $\dot{x}(t) \geq 0$ for $\forall t \in [t_0, t_f]$, the downrange x can be used as an independent variable instead of the time t, then equation of motion can be rewritten as

$$y' = \tan \bar{\gamma}_M \tag{3}$$

$$\bar{\gamma}'_M = \frac{a_M^n}{{V_M}^2 \cos \bar{\gamma}_M} = \frac{a_M^n}{{V_M}^2}\sqrt{1 + \tan^2 \bar{\gamma}_M} = \frac{a_M^n}{{V_M}^2}\sqrt{1 + y'^2} \tag{4}$$

where $(\,)' = \frac{d}{dx}(\,)$. The second derivative of y with respect to x can be derived as

$$y'' = \frac{d}{dx}\tan \bar{\gamma}_M = \bar{\gamma}'_M \frac{d}{d\bar{\gamma}_M}\tan \bar{\gamma}_M = \bar{\gamma}'_M \left(1 + \tan^2 \bar{\gamma}_M\right) = \bar{\gamma}'_M \left(1 + y'^2\right) \tag{5}$$

Equation (4) can be rewritten using Eq. (5) as follows.

$$a_M^n = \frac{y''}{\left(1 + y'^2\right)^{\frac{3}{2}}}{V_M}^2 = \kappa {V_M}^2 \tag{6}$$

Note that κ in Eq. (6) is the curvature of the curve flown by missile. Additionally, the derivative of the lateral acceleration with respect to x can be derived as follows.

$$\begin{aligned}\left(a_M^n\right)' &= \frac{\dot{a}_M^n}{V_M \cos \bar{\gamma}_M} = \frac{\dot{a}_M^n}{V_M}\sqrt{1+y'^2} \\ &= \frac{y'''\left(1+y'^2\right) - 3y'y''^2}{\left(1+y'^2\right)^{\frac{5}{2}}} {V_M}^2 + \frac{2y''}{1+y'^2}\dot{V}_M\end{aligned} \tag{7}$$

2.3 Problem Statement

The constraints on the miss distance, impact angle, and acceleration at the terminal time can be represented as follows.

$$y\left(t_f\right) = 0 \tag{8}$$

$$\bar{\gamma}_M\left(t_f\right) = 0 \tag{9}$$

$$a_M^n\left(t_f\right) = 0 \tag{10}$$

Let t_f be the time instance when x becomes zero, i.e., $x\left(t_f\right) = 0$. Considering Eqs. (3) and (6), the terminal constraints of Eqs. (8)–(10) are equivalent to the following conditions using the changed independent variable x.

$$y\left(0\right) = 0 \tag{11}$$

$$y'\left(0\right) = 0 \tag{12}$$

$$y''\left(0\right) = 0 \tag{13}$$

Note that, if $\dot{a}_M^n\left(t_f\right) = 0$ is considered as an additional constraint, then, in view of Eq. (7), $y'''\left(0\right) = 0$ is required in addition to Eq. (13).

The problem to be solved in this study is to design a guidance law $a_{M_{\mathrm{cmd}}}^n$ with which all of the terminal constraints given by Eqs. (8)–(10) (or equivalently, Eqs. (11)–(13)) can be achieved.

3 Guidance Law Based on Downrange-to-Go Polynomial

This section is devoted to the development of a new trajectory shaping guidance law based on desired crossrange pattern given by a polynomial of downrange-to-go.

3.1 Design of Guidance Law

3.1.1 Desired Crossrange Pattern

Let the desired crossrange pattern be the polynomial of downrange-to-go which can be written as

$$y(x) = c_m {x_{go}}^m + c_n {x_{go}}^n = \left[{x_{go}}^m \ {x_{go}}^n \right] \begin{bmatrix} c_m \\ c_n \end{bmatrix} \tag{14}$$

where $x_{go} = x(t_f) - x = -x$, m and n are the design parameters satisfying $m > n \geq 2$, and c_m, c_n are the constant coefficients. The derivatives of Eq. (14) can be written as follows.

$$y'(x) = -\left[m{x_{go}}^{m-1} \ n{x_{go}}^{n-1} \right] \begin{bmatrix} c_m \\ c_n \end{bmatrix} \tag{15}$$

$$y''(x) = \left[m(m-1){x_{go}}^{m-2} \ n(n-1){x_{go}}^{n-2} \right] \begin{bmatrix} c_m \\ c_n \end{bmatrix} \tag{16}$$

It can be concluded from Eqs. (14)–(16) that the constraints of Eqs. (11)–(13), which are equivalent to the constraints of Eqs. (8)–(10), will be satisfied if m and n are chosen to be $m > n > 2$. If $m > n > 3$, then the additional constraint on the terminal jerk, namely $\dot{a}_M^n(t_f) = 0$, can be satisfied at the same time.

3.1.2 Determination of Coefficients

Equations (14) and (15) can be augmented into matrix form as follows.

$$\begin{bmatrix} y(x) \\ y'(x) \end{bmatrix} = \begin{bmatrix} {x_{go}}^m & {x_{go}}^n \\ -m{x_{go}}^{m-1} & -n{x_{go}}^{n-1} \end{bmatrix} \begin{bmatrix} c_m \\ c_n \end{bmatrix} \tag{17}$$

The coefficients c_m and c_n can be determined by the initial conditions. Since downrange is used as a new independent variable in this study, the initial conditions given in Eq. (2) can be rewritten as follows.

$$\begin{aligned} y(x_0) &= y_0 \\ \bar{\gamma}_M(x_0) &= \bar{\gamma}_{M_0} \end{aligned} \tag{18}$$

Using Eqs. (3) and (18), the initial slope of the crossrange pattern can be calculated as

$$y'(x_0) = \tan \bar{\gamma}_{M_0} \tag{19}$$

Considering Eqs. (18) and (19) in Eq. (17), the coefficients can be obtained as follows

$$\begin{aligned}\begin{bmatrix} c_m \\ c_n \end{bmatrix} &= \begin{bmatrix} {x_{go_0}}^m & {x_{go_0}}^n \\ -m{x_{go_0}}^{m-1} & -n{x_{go_0}}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} y_0 \\ \tan\bar{\gamma}_{M_0} \end{bmatrix} \\ &= \frac{1}{m-n} \begin{bmatrix} -n{x_{go_0}}^{-m} & -{x_{go_0}}^{-m+1} \\ m{x_{go_0}}^{-n} & {x_{go_0}}^{-n+1} \end{bmatrix} \begin{bmatrix} y_0 \\ \tan\bar{\gamma}_{M_0} \end{bmatrix} \\ &= \frac{1}{m-n} \begin{bmatrix} -\left(ny_0 + x_{go_0}\tan\bar{\gamma}_{M_0}\right){x_{go_0}}^{-m} \\ \left(my_0 + x_{go_0}\tan\bar{\gamma}_{M_0}\right){x_{go_0}}^{-n} \end{bmatrix}\end{aligned} \tag{20}$$

where $x_{go_0} = -x_0 > 0$.

3.1.3 Closed-Form Solution

Substituting Eq. (20) into Eq. (17) and considering Eq. (3) yields the closed-form solution for the desired crossrange trajectory in terms of downrange-to-go.

$$\begin{bmatrix} y(x) \\ \tan\bar{\gamma}_M(x) \end{bmatrix} = \begin{bmatrix} {x_{go}}^m & {x_{go}}^n \\ -m{x_{go}}^{m-1} & -n{x_{go}}^{n-1} \end{bmatrix} \begin{bmatrix} {x_{go_0}}^m & {x_{go_0}}^n \\ -m{x_{go_0}}^{m-1} & -n{x_{go_0}}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} y_0 \\ \tan\bar{\gamma}_{M_0} \end{bmatrix} \tag{21}$$

Because the coefficients obtained in Eq. (20) are constants, Eq. (21) can be reinterpreted as the existence of an invariant quantity **c** throughout the engagement.

$$\begin{aligned}\begin{bmatrix} {x_{go}}^m & {x_{go}}^n \\ -m{x_{go}}^{m-1} & -n{x_{go}}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} y \\ \tan\bar{\gamma}_M \end{bmatrix} &= \begin{bmatrix} {x_{go_0}}^m & {x_{go_0}}^n \\ -m{x_{go_0}}^{m-1} & -n{x_{go_0}}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} y_0 \\ \tan\bar{\gamma}_{M_0} \end{bmatrix} \\ &= \mathbf{c}\end{aligned} \tag{22}$$

3.1.4 Guidance Command

Equation (6) can be rewritten using Eqs. (3) and (16) as

$$\begin{aligned} a_M^n(x) &= y''(x)\cos^3\bar{\gamma}_M(x)\,{V_M}^2 \\ &= \left[m(m-1){x_{go}}^{m-2} \;\; n(n-1){x_{go}}^{n-2}\right] \begin{bmatrix} c_m \\ c_n \end{bmatrix} \cos^3\bar{\gamma}_M(x)\,{V_M}^2 \end{aligned} \tag{23}$$

The open-loop form guidance command can be obtained by substituting Eq. (20) into Eq. (23) as follows.

$$
\begin{aligned}
a^n_{M_{\text{cmd}}} &= \left[m\,(m-1)\,{x_{go}}^{m-2}\;\; n\,(n-1)\,{x_{go}}^{n-2}\right]\begin{bmatrix} c_m \\ c_n \end{bmatrix}\cos^3\bar{\gamma}_M {V_M}^2 \\
&= \left[m\,(m-1)\,{x_{go}}^{m-2}\;\; n\,(n-1)\,{x_{go}}^{n-2}\right]\begin{bmatrix} -n{x_{go_0}}^{-m} & -{x_{go_0}}^{-m+1} \\ m{x_{go_0}}^{-n} & {x_{go_0}}^{-n+1} \end{bmatrix}\begin{bmatrix} y_0 \\ \tan\bar{\gamma}_{M_0} \end{bmatrix} \\
&\quad\cdot \frac{1}{m-n}\cos^3\bar{\gamma}_M {V_M}^2 \\
&= -\left[\frac{mn}{x_{go}^2}\left\{(m-1)\left(\frac{x_{go}}{x_{go_0}}\right)^m - (n-1)\left(\frac{x_{go}}{x_{go_0}}\right)^n\right\}y_0\right. \\
&\qquad \left. + \frac{1}{x_{go}}\left\{m\,(m-1)\left(\frac{x_{go}}{x_{go_0}}\right)^{m-1} - n\,(n-1)\left(\frac{x_{go}}{x_{go_0}}\right)^{n-1}\right\}\tan\bar{\gamma}_{M_0}\right] \\
&\cdot \frac{1}{m-n}\cos^3\bar{\gamma}_M {V_M}^2
\end{aligned}
\tag{24}
$$

Because of the relation shown in Eq. (22), finally, the guidance command can be rewritten in closed-loop feedback form as follows.

$$
a^n_{M_{\text{cmd}}} = -\left(\frac{mn}{{x_{go}}^2}y + \frac{m+n-1}{x_{go}}\tan\bar{\gamma}_M\right)\cos^3\bar{\gamma}_M {V_M}^2 \tag{25}
$$

3.2 Properties of Guidance Law

Using the guidance law of Eq. (25), the missile will follow the desired trajectory given by Eq. (14). The critical points and the inflection points of $y\,(x)$, at which y' and y'' equals to zero, respectively, occurs at

$$
\begin{aligned}
y' = 0: &\qquad x_{go} = 0, \\
&\qquad x_{go_{crt}} = \left[-\frac{c_n n}{c_m m}\right]^{\frac{1}{m-n}} = \left[\frac{n\left(my_0 + x_{go_0}\tan\bar{\gamma}_{M_0}\right)}{m\left(ny_0 + x_{go_0}\tan\bar{\gamma}_{M_0}\right)}\right]^{\frac{1}{m-n}} x_{go_0} \\
y'' = 0: &\qquad x_{go} = 0, \\
&\qquad x_{go_{inf}} = \left[-\frac{c_n n\,(n-1)}{c_m m\,(m-1)}\right]^{\frac{1}{m-n}} = \left[\frac{n\,(n-1)\left(my_0 + x_{go_0}\tan\bar{\gamma}_{M_0}\right)}{m\,(m-1)\left(ny_0 + x_{go_0}\tan\bar{\gamma}_{M_0}\right)}\right]^{\frac{1}{m-n}} x_{go_0}
\end{aligned}
\tag{26}
$$

Note that the missile with trajectory $y\,(x)$ will change its turning direction at the critical points, and the lateral acceleration will be zero at the inflection points. It can be observed in Eq. (26) that the critical and inflection points of the desired trajectory occurs at the points with certain ratios to the initial downrange-to-go.

According to the extreme value theorem, the maximum $\left|y''\right|$ may occur at $x_{go} = x_{go_0}$, $x_{go} = 0$, or $x_{go} = x_{go_{inf}}$. If $m > n \geq 2$, it is trivial that $y''\,(0) = 0$ by the proposed design, and therefore,

$$\max_{x} \left| y'' (x) \right| = \max \left(\left| y'' (x_0) \right| , \left| y'' \left(-x_{go_{inf}} \right) \right| \right) \tag{27}$$

For the curvature κ given in Eq. (6), the following is always true.

$$|\kappa (x)| = \frac{\left| y'' (x) \right|}{\left(1 + y' (x)^2 \right)^{\frac{3}{2}}} \leq \left| y'' (x) \right| \tag{28}$$

Therefore, from Eqs. (27) and (28), the boundedness of curvature can be guaranteed.

$$|\kappa (x)| \leq \max \left(\left| y'' (x_0) \right| , \left| y'' \left(-x_{go_{inf}} \right) \right| \right) \tag{29}$$

Note from Eq. (6) that the upper bound on curvature implies that of lateral acceleration, and consequently, Eq. (29) can be used to adjust the desired trajectory considering the manoeuvrability limit of missile.

The desired terminal constraints can be achieved with the proposed trajectory shaping guidance law based on the desired crossrange pattern given by a polynomial of downrange-to-go. The characteristics of engagement can be controlled with the choice of design parameters m and n. As mentioned in introduction, the proposed guidance law of Eq. (25) does not depend on (1) linearized engagement kinematics, (2) constant speed assumption, and (3) inaccurate time-to-go estimate. Exact nonlinear engagement kinematics equations are considered in the design process of the proposed guidance law without constant speed assumption. Also, the downrange-to-go entering into the guidance command can be obtained without any approximation.

4 Numerical Simulation

Numerical simulation is performed to demonstrate the effectiveness of the proposed guidance law. The simulation results are described in this section.

4.1 Simulation Environment

The closed-loop form guidance command is utilized to demonstrate the proposed method. The horizontal plane is assumed to be the engagement plane for the simulation. To include the effect of speed change in the simulation, the following simple model is utilized

$$\dot{V}_M = -\frac{C_{D_0} \rho S}{2\bar{m}} {V_M}^2 \tag{30}$$

Table 1 Simulation parameters for the missile and the target

Parameter	Value	Unit
$\left(X_{M_I}(t_0), Y_{M_I}(t_0)\right)$	(0, 1000)	m
$\left(X_{T_I}, Y_{T_I}\right)$	(4000, 0)	m
$V_M(t_0)$	200	m/s
C_{D_0}	0.02	–
$\bar{m}$	90.035	kg
ρ	1.2041	kg/m^3
S	0.2	m^2

Table 2 Guidance parameters, initial flight path angle, and terminal impact angle

	(m, n)	γ_{M_0} [deg]	γ_{f_d} [deg]
Case 1	(5, 4)	0	$-75:15:0$
Case 2	(5, 4)	$-90:15:30$	-45
Case 3	(6, 5), (5, 5), (5, 4), (5, 3), (4, 3), (3, 2)	0	-45

where C_{D_0} is the zero-lift drag coefficient, ρ is the atmospheric density, $\bar{m}$ and S are the mass and reference area of the missile, respectively.

Three simulation cases are considered in this study. In Case 1, simulation is performed using the proposed guidance law with $(m, n) = (5, 4)$ for various terminal impact angles with fixed initial flight path angle, and vice versa in Case 2. In Case 3, simulation is performed for various design parameters with fixed terminal impact angle and initial flight path angle. Initial position of the missile and the target for Cases 1–3, and the physical parameters of the missile are summarized in Table 1. Guidance parameters (m, n), initial flight path angle, and the terminal impact angle for Cases 1–3 are summarized in Table 2.

4.2 Simulation Results

4.2.1 Case 1: Various γ_{f_d}, Fixed γ_{M_0} and (m, n)

Figures 2, 3, 4, 5 and 6 show the trajectories in the inertial coordinate system, the lateral acceleration commands, the speed histories, the crossranges with respect to the impact angle coordinate system, and the flight path angle errors, respectively.

Simulation results of Case 1 shows that various desired terminal impact directions can be handled with the proposed guidance law.

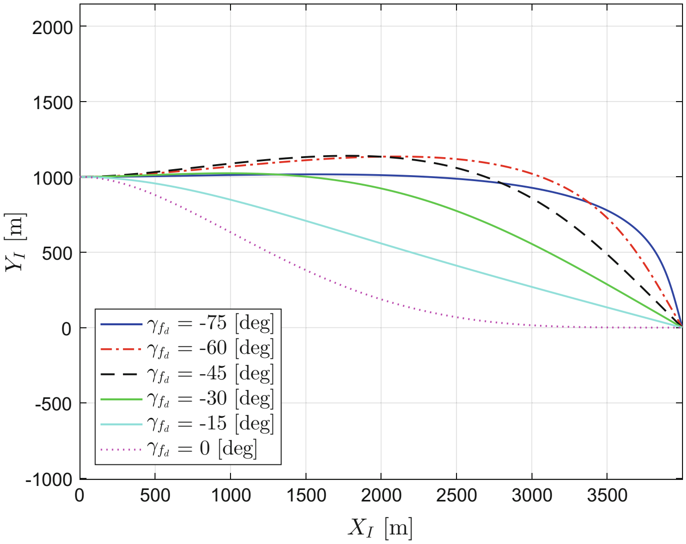

Fig. 2 Case 1: X_I-Y_I trajectory

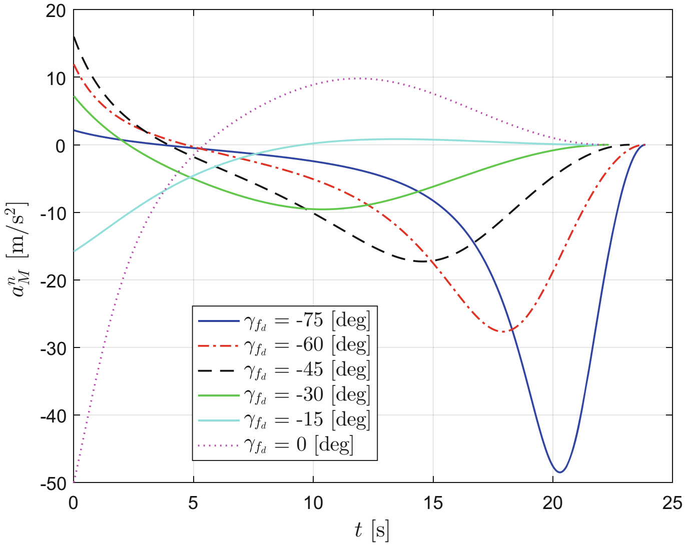

Fig. 3 Case 1: lateral acceleration a_M^n

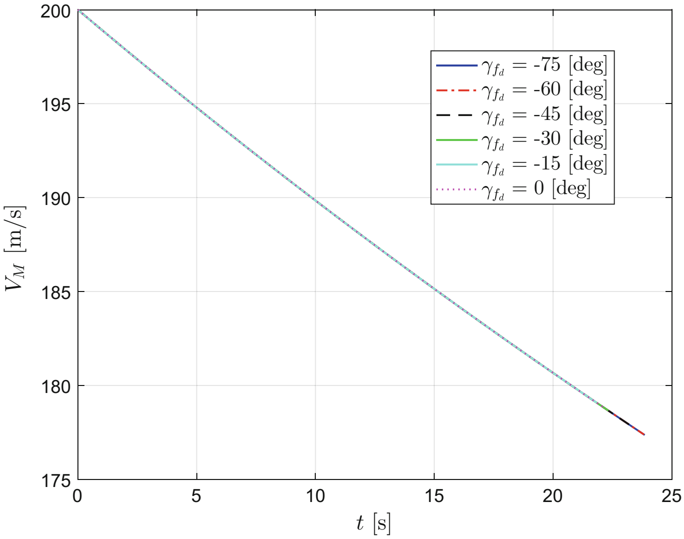

Fig. 4 Case 1: speed V_M

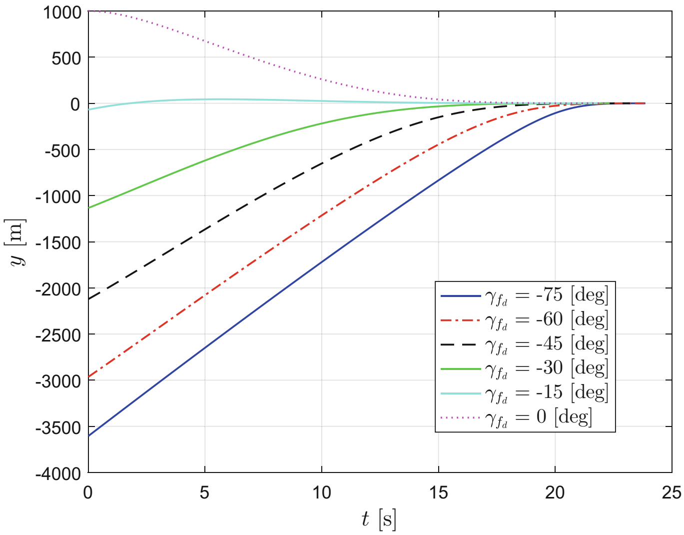

Fig. 5 Case 1: crossrange y

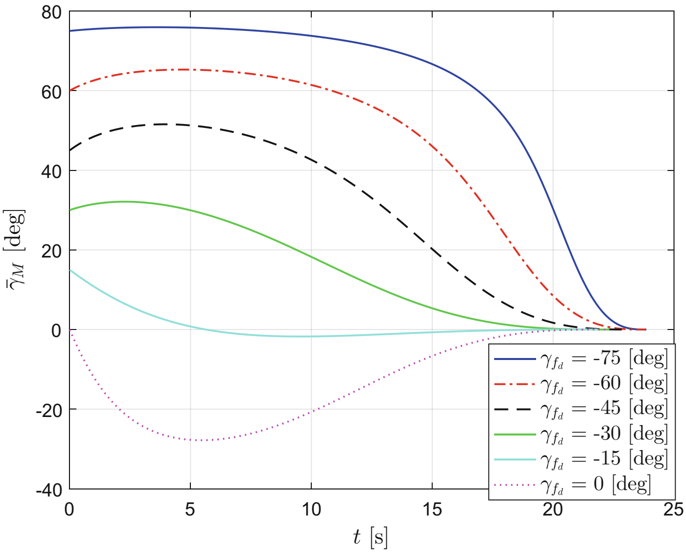

Fig. 6 Case 1: flight path angle error $\bar{\gamma}_M$

4.2.2 Case 2: Various γ_{M_0}, Fixed γ_{f_d} and (m, n)

Figures 7, 8, 9, 10 and 11 show the trajectories in the inertial coordinate system, the lateral acceleration commands, the speed histories, the crossranges with respect to the impact angle coordinate system, and the flight path angle errors, respectively.

Simulation results of Case 2 shows that the proposed guidance law can cope with various initial flight path angles, as long as $\left|\bar{\gamma}_{M_0}\right| < \frac{\pi}{2}$. A limitation of the proposed guidance law is that the missile flies in a pattern that x_{go} is always decreasing with respect to time, but this is not a severe restriction.

4.2.3 Case 3: Various (m, n), Fixed γ_{M_0} and γ_{f_d}

Figures 12, 13, 14, 15 and 16 show the trajectories in the inertial coordinate system, the lateral acceleration commands, the speed histories, the crossranges with respect to the impact angle coordinate system, and the flight path angle errors, respectively.

For a given initial flight path angle and a terminal impact angle, Case 3 shows that the engagement trajectory can be adjusted by the choice of design parameters m and n. The case of $(m, n) = (3, 2)$ is included in Case 3 to show that the terminal acceleration constraint can be met only if $m > n > 2$. Also, the cases of $(m, n) = (5, 3)$, $(4, 3)$ are included in Case 3 to show that the terminal jerk constraint can be met only if $m > n > 3$. In addition, Fig. 13 shows that the magnitude of the initial lateral acceleration increases with greater $m + n$. Furthermore, interception is achieved while the missile is decelerating in all cases. Therefore, it can be concluded

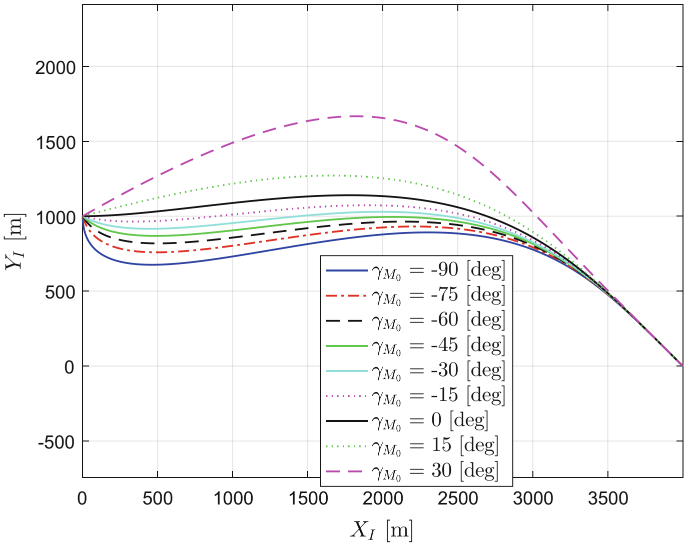

Fig. 7 Case 2: X_I-Y_I trajectory

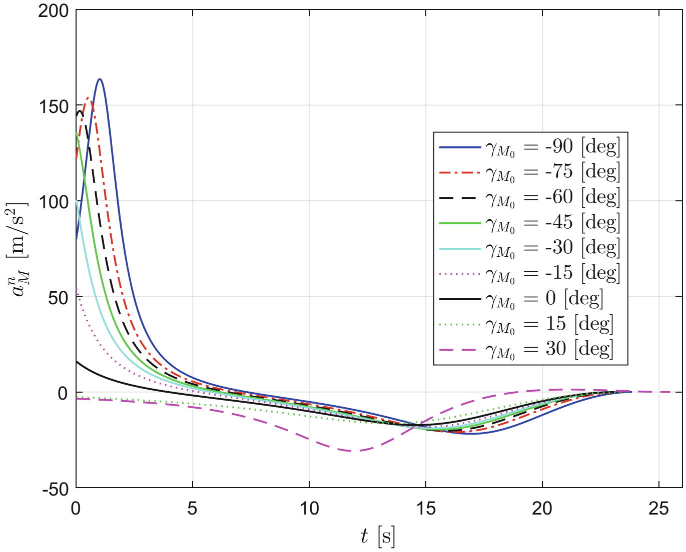

Fig. 8 Case 2: lateral acceleration a_M^n

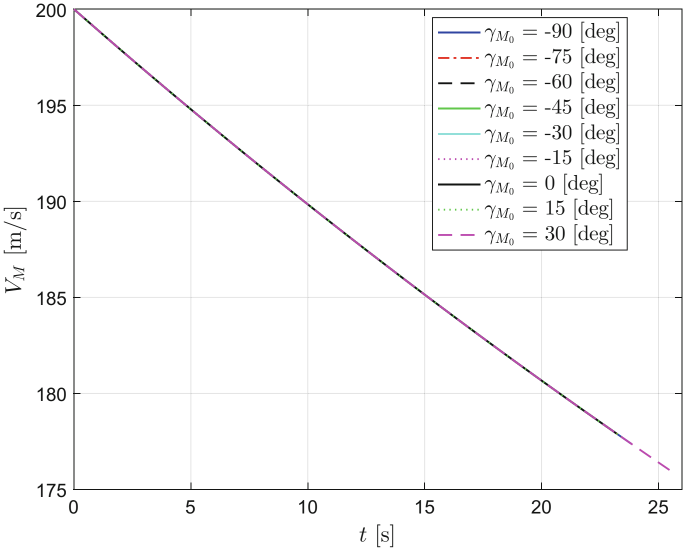

Fig. 9 Case 2: speed V_M

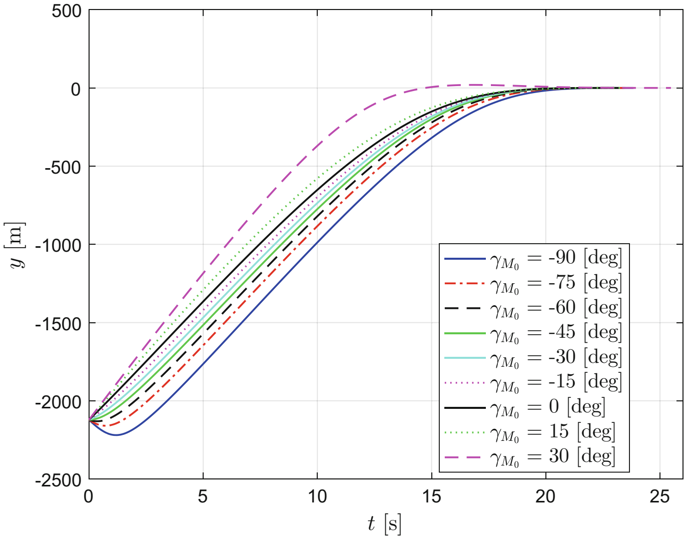

Fig. 10 Case 2: crossrange y

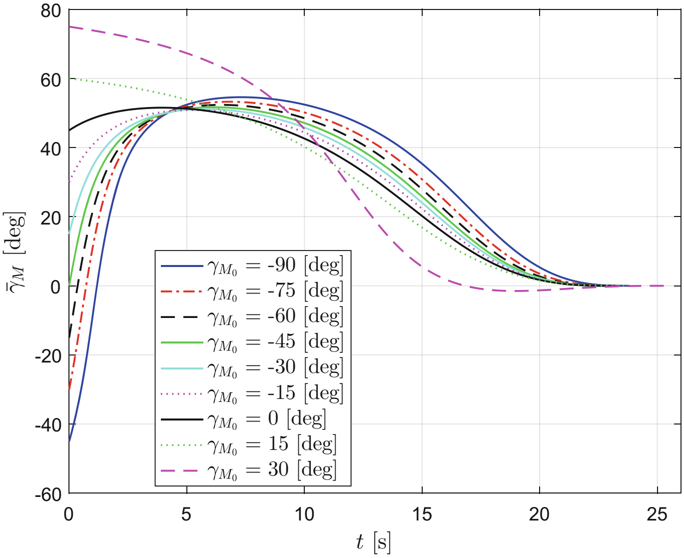

Fig. 11 Case 2: flight path angle error $\bar{\gamma}_M$

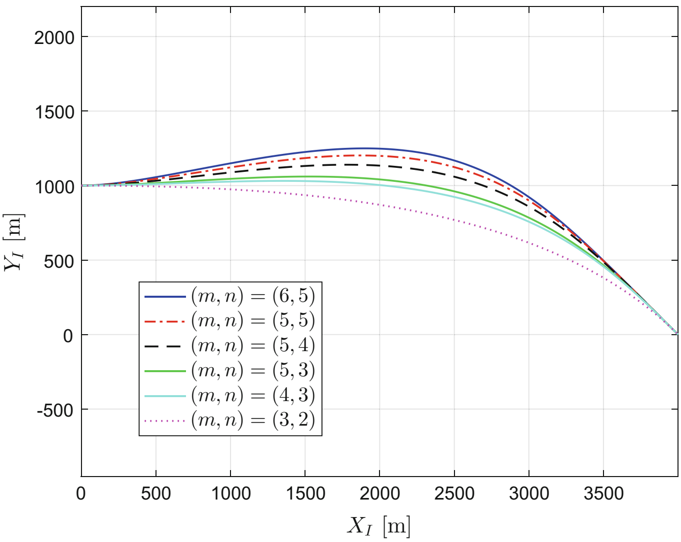

Fig. 12 Case 3: X_I-Y_I trajectory

from the results of Cases 1–3 that the proposed guidance law can achieve stationary target interception with given terminal impact angle and acceleration constraints, together with the flexibility of shaping.

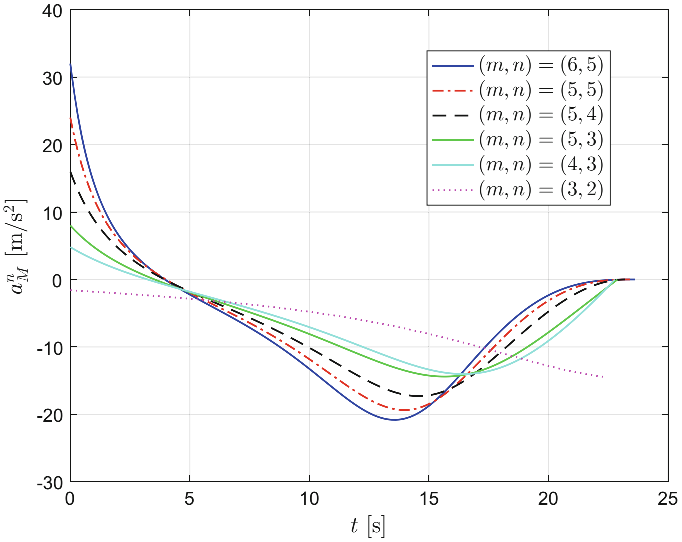

Fig. 13 Case 3: lateral acceleration a_M^n

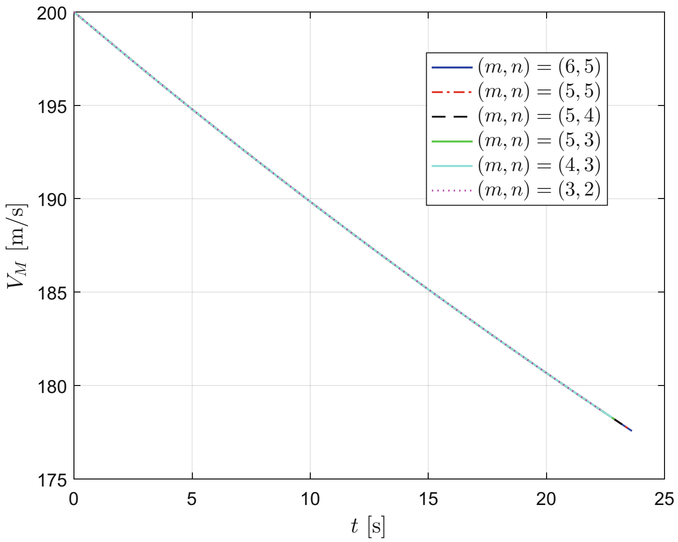

Fig. 14 Case 3: speed V_M

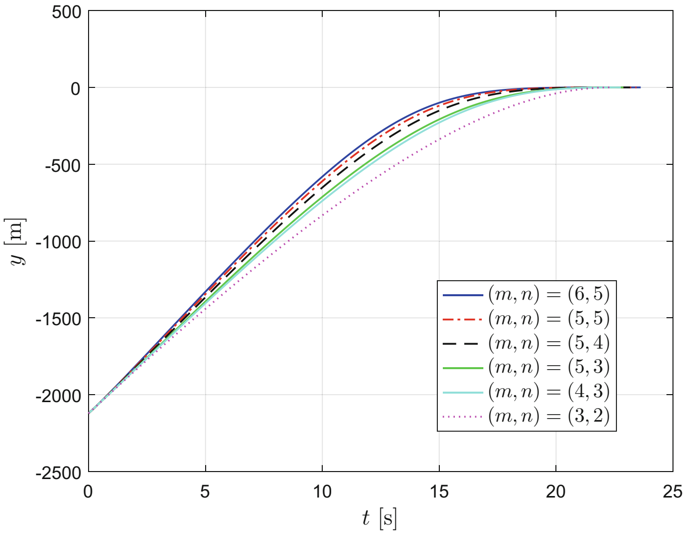

Fig. 15 Case 3: crossrange y

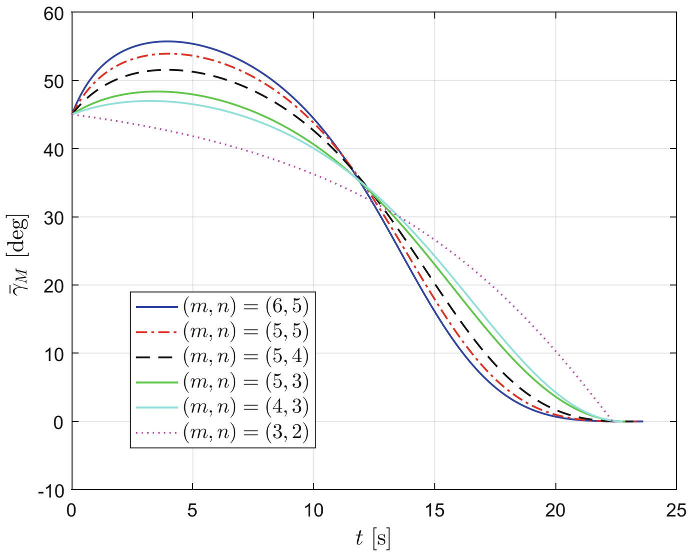

Fig. 16 Case 3: flight path angle error $\bar{\gamma}_M$

5 Conclusion

A new trajectory shaping guidance law was proposed to achieve interception of a stationary target with terminal flight path angle and acceleration constraints. The desired crossrange pattern was designed as a polynomial of downrange-to-go, which might supplement the existing trajectory shaping guidance law. The proposed guidance law was derived without linearization of engagement kinematics, and constant speed assumption. Also, the time-to-go estimate is not required to implement the proposed guidance law.

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