8

How to Do "The Impossible", a Quantum Mechanics Without Observers: The de Broglie–Bohm Theory

> But in 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the 'observer', could be eliminated.
>
> John Bell [14, p. 160]

8.1 Introduction

Let us see where we are: In Chap. 5, we learned that there is no easy way to understand what the wave function means. If we try to give a meaning by analyzing measurements within quantum mechanics, we only get unphysical macroscopic superpositions. If we try to give it a statistical meaning, we run into a contradiction, because of the no hidden variables theorems. In Chap. 7 we learned that there is something nonlocal going on in the world, but we do not know what.

What is needed is a theory that gives a meaning to the wave function, beyond being a tool for predicting the results of laboratory measurements. This theory will thus have to go beyond ordinary quantum mechanics, namely it will include "hidden variables", that give a more detailed description of a physical system than the one given by the wave function, but without being refuted by the no hidden variables theorem of Sect. 5.2. Because of the EPR-Bell argument, this theory will have to be nonlocal.

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Surprisingly, not only does such a theory exist, but it was introduced at approximately the same time as the Copenhagen interpretation, even slightly before, in 1924–1927, by the French physicist Louis de Broglie. But, as we said in Chap. 1, that theory was rejected at the time of its introduction by a large majority of physicists, and ignored even by critics of the Copenhagen school, like Einstein and Schrödinger. The theory was even abandoned by its founder, only to be rediscovered and completed by the American physicist David Bohm in 1952, then further developed and advertised by John Bell.

We shall call this theory the de Broglie–Bohm theory, because although it was developed fully only by Bohm (who hadn't heard of de Broglie's work), it was introduced by de Broglie about 25 years before Bohm. Other people call that theory Bohm's theory or Bohmian mechanics [62]. We shall not argue about that.

Here is what it achieves:

- It is a "hidden variables" theory.
- Its "hidden variables" are not hidden at all (hence the expression "hidden variables" is quite a misnomer in this case).
- There is no fundamental role whatsoever for the "observer" in that theory.
- That theory is not contradicted by the no hidden variables theorems. It is a sort of statistical interpretation of quantum mechanics, but a consistent one.
- The de Broglie–Bohm theory is entirely deterministic.
- It accounts for all the observations used to justify the validity of ordinary quantum mechanics.
- It allows us to understand the "active role" of the measuring devices, meaning that a measurement in general does not record some pre-existing value of the system being "measured", as the "no hidden variables" theorems imply. But it does so without making it a philosophical a priori.
- It explains to some extent where the nonlocality of the world comes from.

It would seem that, given all the claims to the effect that such a theory is impossible, and given what it accomplishes, its mere existence should be a subject of considerable interest, but this is not the case. Although interest in the de Broglie–Bohm theory is probably increasing, it is still widely ignored or misrepresented, even by experts on foundations of quantum mechanics (we postpone the discussion of why this is so to Chap. 10).

From this comment, the reader can guess that the claims of this chapter are *not at all* universally accepted by physicists. In fact it represents a very minority view. We explain it because we think that the de Broglie–Bohm theory does solve the "mysteries" of quantum mechanics, but we emphasize that is not a standard opinion. We also think that this is the only existing way to solve those mysteries, but that is even less a generally accepted view, since there are a number of other solutions on the market. We shall discuss one of those alternative solutions in the next chapter and briefly mention the other ones.

As in the rest of this book, we shall avoid formulas and only rely on drawings to explain how the de Broglie–Bohm theory works.[1] We shall proceed slowly and step by step:

- Explain what the de Broglie–Bohm theory says about the world and how it deals with the double-slit experiment.
- What are "measurements" in that theory.
- The previous points concern the deterministic behavior of a given system. But one has also to explain where the statistical predictions of quantum mechanics come from in the de Broglie–Bohm theory.
- One has also to see what is the status, in the de Broglie–Bohm theory, of the reduction or collapse of the wave function.
- Finally, how the de Broglie–Bohm theory allows us to understand nonlocality.

But once this is done, we will have explained, within the de Broglie–Bohm theory, all the phenomena discussed in Chaps. 2 and 7 and we will have given a meaning to the formalism of Chap. 4.

Then, all the quantum mysteries will be clarified; in particular, the loose talk about the moon not being there when nobody looks at it or the cat being both alive and dead, will simply disappear. There will no longer be this incomprehensible duality of rules for the evolution of the wave function (depending on whether one measures something or not). Nonlocality of course will remain baffling but at least its origin will be clearer.

8.2 The de Broglie–Bohm Theory in a Nutshell

In the de Broglie–Bohm theory, particles have positions at all times, and therefore trajectories, and thus also velocities, independently of whether one measures them or not. The positions are, by convention, called the "hidden variables" of the theory, because they are not included in the purely quantum

[1] See, e.g., [1, 190] for elementary introductions the de Broglie–Bohm theory and [7, 18, 27, 62, 93, 105, 189] for more advanced ones. There are also pedagogical videos made by students in Munich, available at: https://cast.itunes.uni-muenchen.de/vod/playlists/URqb5J7RBr.html.

description, given by the wave function Ψ. But here, the word “hidden” is silly since the positions are not hidden at all: they are the only things that are directly “seen”. For example, in the double-slit experiment, one detects particle positions on the second screen in Fig. 2.6. Actually, as we shall explain in the next section and in Appendix 8.A, the particle positions are also the only things that are directly “seen” in *any* experiment.

In the de Broglie–Bohm theory, the *complete physical state* of a particle or a system of particles is given both by its wave function, which is the same as in ordinary quantum mechanics, *and* the positions of the particles.

They both change in time, in the following way:

1. The wave function evolves according to the usual laws, but *nothing special happens to it during measurements.*
2. The motion of the particles is guided by their wave function. This means that the velocity of a particle is a function of its wave function and its position, if we consider a single particle. If we consider a system composed of several particles, the velocity of each one of them is a function of the wave function of that system and of the positions of all the other particles. We will illustrate this motion below.

The de Broglie–Bohm theory is sometimes called the “pilot wave” theory, because the wave function tells the particle how to move.

Now, we *finally* have given a physical meaning to the wave function! It is not only something that allows to “predict results of measurements”, but it has a clear physical role *outside the laboratories*: there are particles that move and the wave function guides the motion of the particles.

That’s all! The de Broglie–Bohm theory is simply a theory of matter in motion, just like Newton’s theory. Of course, the way particles move is different (the phenomena to be explained are radically different!) than in Newton’s theory, but there is nothing philosophically new.

All we have to do now is to explain what this new kind of motion is and how it accounts for the strange quantum phenomena.

One should first stress an important fact about the motion of the particles, that will be used repeatedly in this chapter: if a wave function is composed of two non-overlapping parts, as in Figs. 4.5–4.7, then only the part of the wave function where the particle actually is matters as far as the guidance of the particle is concerned.[2] If the particle is in the left part of the wave function

[2]When we say that a particle is “in” a part of the wave function, we mean that it is where that part of the wave function is non-zero. But we shall use the expression “in the wave function” as a shorthand.

of Fig. 4.5, then it will be guided only by that part of the wave function, and similarly if the particle is in the right part of that wave function. If the two parts are recombined and overlap again, as for example in Fig. 4.10, then the particle will be guided by this recombined wave function.

We shall now illustrate, via simple examples, how such guiding works, starting with the double-slit experiment, described in Sect. 2.1.

In Figs. 8.1, 8.2, 8.3 and 8.4, we show solutions of that experiment within the de Broglie–Bohm theory. Each (wavy) line represents the trajectory of a single particle (since particles are sent one by one, there is no interaction between the particles). Different lines correspond to different initial positions behind the two slits. Since we have many trajectories, those initial positions are not easy to distinguish visually, but one can think that each point behind the slits corresponds to the initial position of a trajectory.

In the de Broglie–Bohm theory, each particle goes through only one slit, but the wave function goes through both slits when they are both open (see Fig. 4.10), and this in turn affects the motion of the particle, since the wave function guides it. This is rather easy to understand intuitively: the wave function propagates like a wave. Obviously, a wave beyond the slits will be different

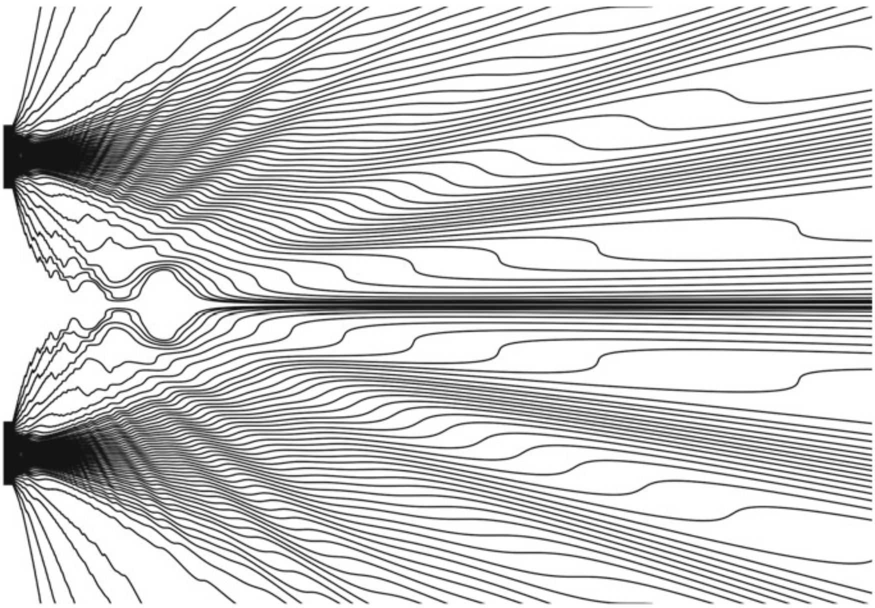

Fig. 8.1 De Broglie–Bohm trajectories computed for the double-slit experiment. Each (*wavy*) *line* represents the trajectory of a single particle. See Fig. 8.2 for another example of de Broglie–Bohm trajectories, with a different distribution of particles behind the slits (A. Gondran cc by-sa 4.0)

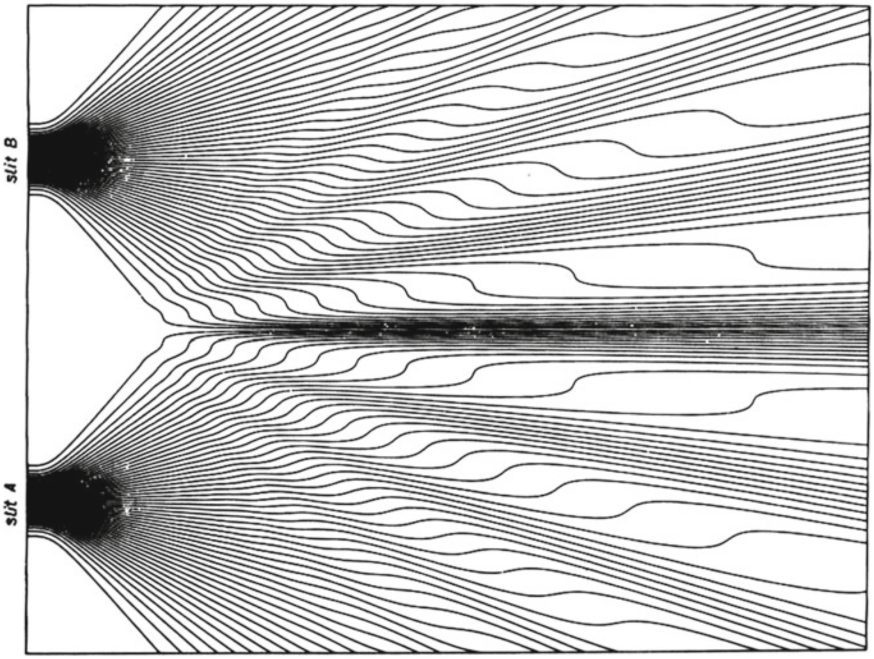

Fig. 8.2 An example of de Broglie–Bohm trajectories computed for the double-slit experiment, with a different distribution of particles behind the slits than in Fig. 8.1. Each (*wavy*) *line* represents the trajectory of a single particle. Reproduced with the kind permission of *Società Italiana di Fisica* and the authors, from C. Philippidis, C. Dewdney, B.J. Hiley: Quantum interference and the quantum potential, *Il Nuovo Cimento B* **52**, 15–28 (1979)

if it has two sources (one for each slit) or one. Thus, the behavior of the particle (depicted in Figs. 8.1, 8.2, 8.3 and 8.4 when both slits are open) is affected by the fact that the slit *through which it does not go* is open or not.

As an analogy, imagine a water wave coming through the slits, and a small, light object being carried by that water wave; evidently, the form of the wave behind the slits will depend on whether one slit is open or both, and that will affect the motion of the object, even though the latter goes through only one slit.

Looking at Figs. 8.1, 8.2, 8.3 and 8.4 should dispel the "mystery" of the double-slit experiment, if we accept the idea that a particle can be guided by a wave. There is no sense in which the particle "goes through both slits" if they are both open, as one says too easily. The particle, *being a particle, always goes through only one slit*. The wave that guides its motion, *being a wave, goes through*

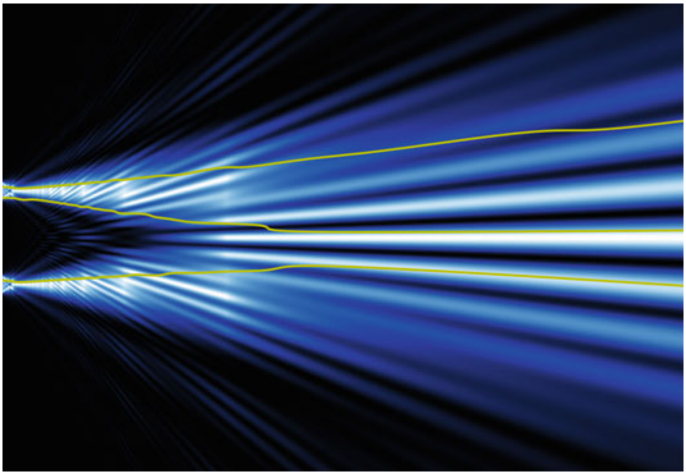

Fig. 8.3 De Broglie–Bohm trajectories computed for the double-slit experiment. The *white* and *blue* areas indicate places where the wave function is non zero and their intensity is proportional to the square of the wave function (*white* more intense, *blue* less intense). The *yellow lines* are three particular trajectories (A. Gondran cc by-sa 4.0)

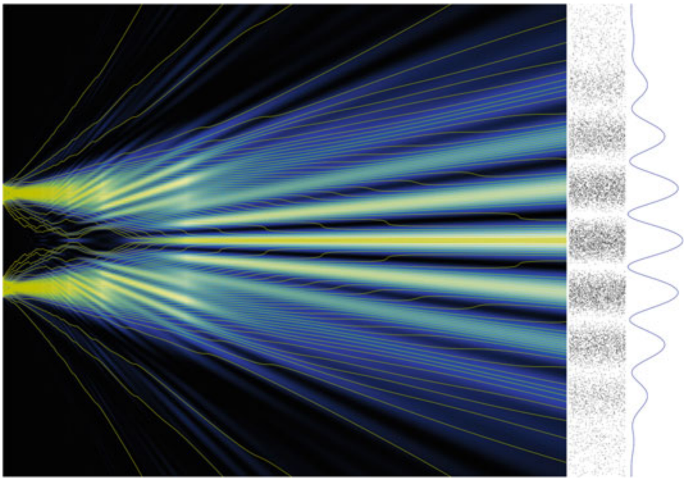

Fig. 8.4 De Broglie–Bohm trajectories computed for the double-slit experiment. The *white* and *blue* areas indicate places where the wave function is non zero and their intensity is proportional to the square of the wave function (*white* more intense, *blue* less intense). There are one hundred *yellow lines* indicating particular trajectories. The *blue curve* on the right of the figure indicates the density of particles detected on the second screen (A. Gondran cc by-sa 4.0)

both slits if they are both open, and that affects the motion of the particle behind the slits. What is surprising here?[3]

Note also that the particles do not cross the line in the middle of Figs. 8.1 and 8.2 (this is a property of the de Broglie–Bohm equations in this situation, but we shall not prove it). As a consequence, one can determine a posteriori through which slit the particle went, by detecting the particle on the screen, since the particle ends up in the upper part of the picture if and only if it goes through the upper slit. Note that this knowledge is obtained without "looking" directly through which slit the particle goes.

Now, if one puts a detector behind one of the slits, say the lower one as in Fig. 2.7, that allows us to record through which slit the particle went, this changes the part of the wave function going through that slit in such a way that this part no longer guides the motion of the particle that goes through the upper slit.[4] Then, if one considers only the events *where the detector does not detect the particle* and where one therefore knows that the particle goes through the upper slit, the pattern on the second screen, where the landing of the particle is detected, will be as if the lower slit was closed, i.e., as in part (a) of Fig. 2.6.

But there is nothing related to our knowledge in itself that influences the behavior of the particle; the detector interacts with the part of the wave function that guides the particles and that changes the future motion of the particle. Of course, that interaction also allows us *to know* which slit the particle went through, but the behavior of the particle is entirely guided by physical laws and does not depend at all on an "observer" looking at it.

Finally, if one considers the delayed-choice experiment of Sect. 2.2, there is again nothing surprising from the point of view of the de Broglie–Bohm theory and there is no action whatsoever of the future on the past! The particle always goes through one slit and is simply guided by the part of the wave function in which it finds itself. Because of the lenses in Figs. 2.9 and 2.10, the wave function behaves in a somewhat different way than in the usual double-slit experiment, but that's all: the region where the plate P may or may not be inserted is the region where the two wave functions, coming from the upper and the lower slits overlap and that gives rise to the interference pattern (Fig. 2.10). If one does not insert the plate P, those two wave functions continue their

[3]The interested reader may look at [111], where an indirect measurement of trajectories of particles leads to a picture qualitatively similar to Figs. 8.1, 8.2, 8.3 and 8.4.

[4]In the usual approach, one says that the effect of the detector is to collapse the wave function, see Sect. 4.2. In the de Broglie–Bohm theory, there is never a real collapse but there is something playing a similar role, which is explained in Sect. 8.4.3.

propagation towards the counters C_1 and C_2, but there, they do not overlap any more and thus no interference pattern is observed (Fig. 2.9).

Actually, there is one small surprise in the delayed-choice experiment: if one computes the trajectories of the particles in the de Broglie–Bohm theory, one finds that the particles detected by the counter C_1 in Fig. 2.9 went through the lower slit while those detected by counter C_2 went through the upper one, contrary to what one might naively expect and what is usually said in presentations of the delayed-choice experiment (and that we followed in Sect. 2.2). The reason for that somewhat strange behavior is a property of the de Broglie–Bohm dynamics (not proven here): the particles cannot cross the horizontal line in the middle of Fig. 2.9, just as they could not do it in Figs. 8.1 and 8.2[5]; so they bounce back when they hit that middle line and go to the counter that is on the same side of that middle line as the slit that they went through.

Remember that in ordinary quantum mechanics particles do not follow any paths whatsoever, so, if one adopts the orthodox viewpoint, one should not be surprised by this counterintuitive behavior.

Only in a more complete theory, like the de Broglie–Bohm one, where one assigns trajectories to particles, can one meaningfully ask and answer questions such as: through which slit did the particle go? That the answer to that question is counterintuitive is no argument against the de Broglie–Bohm theory: why should physics in the microscopic scale satisfy our intuitions?

Here is how John Bell summarized the de Broglie–Bohm theory in the case of the double-slit experiment:

> Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave–particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.
>
> John Bell [14, p. 191]

It is interesting to compare this statement with what is claimed in a standard quantum mechanical textbook, which we already quoted in Chap. 2:

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[5]We did not explicitly draw that line in Fig. 2.9, but it is clear that there is an horizontal line in the middle of that figure, with respect to which the parts of the figure above that line and below it are symmetric images of each other.

> It is clear that [the results of the double-slit experiment] can in no way be reconciled with the idea that electrons move in paths. [...] In quantum mechanics there is no such concept as the path of a particle.
>
> Lev Landau and Evgeny Lifshitz [114, p. 2]

And, after describing the double-slit phenomenon, Feynman wrote:

> Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it.
>
> Richard Feynman, [79, p. 145]

What is surprising is the dogmatic assurance of those statements: how does one know that an experiment *can in no way* be reconciled with a concept or that no "deeper explanation" of a phenomenon than its description can be given?

We will now turn to natural questions that the reader may ask about the de Broglie–Bohm Theory. We will separate those questions into those whose answers are relatively simple, discussed below, and those whose answers are relatively more complicated, discussed in the Appendices. Sometimes we will invoke facts, about quantum mechanics or the de Broglie–Bohm theory, that we cannot prove without getting into the mathematical formalism.

8.3 How Do "Measurements" Work in the de Broglie–Bohm Theory?

We have seen in Sect. 5.2 that measurements in general cannot possibly reveal pre-existing properties of quantum systems. In particular, one cannot assign values to both the positions and the velocities of individual particles in such a way that their statistical distribution agrees with the quantum mechanical predictions.

But in the de Broglie–Bohm theory, particles do have a position and a velocity at each instant! Isn't that a plain contradiction?

No, because the no hidden variables theorem refer to *results of measurements* and the positions and the velocities in the de Broglie–Bohm theory refer to *properties of particles independently of measurements.*

To understand what is going on, we need to analyze how measurements work in the de Broglie–Bohm theory.

We will discuss here what "measurements" of velocities mean in the de Broglie–Bohm theory and we will discuss "measurements" of spin in

Appendix 8.A. We will come back to the relation between the de Broglie–Bohm theory and the no hidden variables theorem in Sect. 8.4.1.

8.3.1 "Measurements" of Velocities in the de Broglie–Bohm Theory

The simplest example is the measurement of velocities: how does one do that? One measures the difference of the positions at two different times and one divides by the length of the time interval.[6]

So, measurements of velocities are in the end dependent on measurements of positions. Now consider a particle in a box, like the one we introduced in Sect. 7.2. It turns out that, in the de Broglie–Bohm theory, for many wave functions associated to a particle in a box, the particle is actually at rest: it has a well-defined velocity, but which is equal to zero! Yet, quantum mechanics predicts that results of measurements of velocities will have a probability distribution which is quite different from zero (another quantum fact that one has to accept without proof).

But how does one measure the velocity of the particle in the box? One cannot just look at it with God's eye so to speak and *see* that it is at rest. One way to measure this velocity is to open the box, let the particle move and detect its position after some time. Then, one obtains its velocity, as we said, by computing the difference between the positions at the initial time and the final one and dividing by the length of the time interval.[7]

But, in the de Broglie–Bohm theory, opening the box changes the wave function of the particles in the box; that in turn causes the particles in the box to start to move (remember: that's because the wave function guides the particles), and they move in such a way that, if we measure the positions of the particles after some time, and compute their velocity with the above method, we obtain results whose statistical distribution agrees with the quantum mechanical predictions.[8]

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[6]In formulas, if t_1 and t_2 are the two times, we get $v = \frac{x(t_2)-x(t_1)}{t_2-t_1}$. To be precise, this formula defines an average velocity. Below, we will let $t_2 \to \infty$.

[7]The reader might worry that we do not know the initial position with infinite precision and that measuring it might disturb the wave function of the particle. That is true, but we can assume that the box is relatively small and that if one measures the later position after a long enough time, the uncertainty about the initial position within the box will not affect very much the final result for the velocity: if t_1 is the initial time, t_2 the final one and $x(t_1)$ the initial position, $\lim_{t_2\to\infty} \frac{x(t_1)}{t_2-t_1} = 0$, so that $v = \lim_{t_2\to\infty} \frac{x(t_2)-x(t_1)}{t_2-t_1} = \lim_{t_2\to\infty} \frac{x(t_2)}{t_2-t_1}$ does not depend on $x(t_1)$.

[8]However, describing how this "measurement of velocities" works in detail would take us beyond the scope of this book. See [27, Sect. 6.3], [36, Sect. 5.1.4] for more details.

But this means that our "measurement of velocity" did *not* measure the initial velocity of the particle (which was zero!).

Note also that, unlike what Heisenberg's inequality is often taken to mean, not only do particles have both a position and a velocity at all times, but one can *know* both with arbitrary precision, at least in the example of the particle in the box: indeed, we know from the de Broglie–Bohm theory, that the velocity is zero and we can, in principle, measure independently its position with arbitrary precision.

But if we "measure its velocity" by the above procedure (opening the box and measuring the position later), then we obtain something entirely different, and if we take the size of the box as related to the "spread" of the initial positions (see Sect. 4.4) and consider the "spread" of the statistical distribution of results of what are called the "measurements of the velocities", those two "spreads" will satisfy Heisenberg's inequality, simply because the "measured" statistical distributions agree with the ones predicted by the quantum mechanical formalism and the Heisenberg inequality is just a mathematical consequence of that formalism.

But in the de Broglie–Bohm theory, particles do have positions and velocities at all times, and the values of those variables, being well-defined, would *not* satisfy Heisenberg's uncertainty relations. The latter are satisfied by the results of measurements of positions and velocities, but at least for the velocities, these measurements results from interactions with the particle being measured (letting it move) and do not reveal the true velocity of the particle (here equal to zero).

8.4 Things Not Discussed in Detail

There are several natural questions that the reader may raise about the de Broglie–Bohm theory: what is the relationship of that theory with the non-locality discussed in Chap. 7? How does a deterministic theory, like the de Broglie–Bohm one, account for the statistical predictions of quantum mechanics? If the wave function does never collapses in the de Broglie–Bohm theory, what happens with that rule?

The first question will be answered in Appendix 8.B, and the two others below; but we will start with the relationship between the de Broglie–Bohm theory and the no hidden variables theorem.

8.4.1 Why Isn't the de Broglie–Bohm Theory Refuted by the No Hidden Variables Theorem?

We just answered that question in the previous Sect. 8.3.1: the no hidden variables theorem of Sect. 5.2 says that one cannot introduce "hidden variables" simultaneously for both positions and velocities *in such a way that their statistical distribution coincides with the quantum mechanical predictions*.

In the de Broglie–Bohm theory, we do introduce both positions and velocities, but, and that is the important point, *their statistical distribution does not coincide with the quantum mechanical predictions*. Indeed, we just saw that in the example of the particles in a box, we can have particles that are at rest (hence, their velocity is zero), while the quantum mechanical prediction for the *"measurements" of velocities* is not zero! The crucial word here is "measurements" (with scare quotes). Those "measurements" do not measure any pre-existing property of the particles but are the results of interactions with those particles, as we explained in Sect. 8.3.1. And the de Broglie–Bohm theory does predict correctly the statistical results of those measurements.

So, there is no contradiction with the no hidden variables theorems. As we explain in Appendix 8.A, the same thing is true for the "measurements" of the spin (they are the results of interactions and do not reveal a pre-existing property of the particle).

Here is how Bell summarized the situation:

> […] the word [measurement] comes loaded with meaning from everyday life, meaning which is entirely inappropriate in the quantum context. When it is said that something is 'measured' it is difficult not to think of the result as referring to some pre-existing property of the object in question. This is to disregard Bohr's insistence that in quantum phenomena the apparatus as well as the system is essentially involved.
>
> John Bell [12, p. 34]

Bell is here referring to statements of Bohr such as:

> […] the *impossibility of any sharp distinction between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear*.
>
> Niels Bohr [31, p. 210], quoted in [14, p. 2] (italics in the original)

However, in the de Broglie–Bohm theory, the fact that measurements do not in general measure an intrinsic property of the particle follows from the equations of the theory and not from some more or less a priori notion or some "intuition" suggested by the strange behavior of quantum particles.

What precedes cannot be emphasized strongly enough: *the de Broglie–Bohm theory is a "hidden variables" theory where the hidden variables (the positions) are not hidden (they are the only thing that we ever detect) and that is not refuted by the no hidden variables theorems.*

8.4.2 Where Does "Randomness" Come from in the de Broglie–Bohm Theory?*

So far, we have discussed how various individual trajectories behave in the de Broglie–Bohm theory, but what about the statistics of the results? How can a deterministic theory reproduce apparently random results?

To start, think of coin tossing: each toss of a coin is a perfectly deterministic process (if you are worried about the free will of the tosser, let us replace him by a machine) and is entirely determined by the initial properties of the coin, namely its position, velocity, the way it rotates etc. When we want to explain why the results of tossing several coins look random (random was defined in Sect. 3.1.1), we have to say that these initial properties are also random; one of the reasons that they are random is that a slight change in those initial properties (a little more velocity, a faster rate of rotation) will make the coin fall heads instead of tails and vice-versa; therefore, one cannot control those initial properties with the precision that would be needed in order to obtain a definite result.

Something similar happens in the de Broglie–Bohm theory. Let us consider, for simplicity, systems composed of a single particle, like in the double-slit experiment; the extension to systems with many particles is rather easy, but will not be discussed in detail here. Let us also first discuss the quantum mechanical predictions for measurements of positions.

Consider a large number of independent particles, as we did in the double-slit experiment. And assume that each particle has the same wave function as the others. The word "independent" means here that they are sent one by one so that they cannot interact with each other.

If one has a large number of particles, distributed in some random fashion, one can define the statistical distribution of that set of particles, as we did in Sect. 3.4.1. Now, consider Fig. 8.5 in which only a few points are indicated on the left (for better visibility) but where the continuous curve is supposed to represent the statistical distribution of the particles that would be obtained if one had a large number of particles.

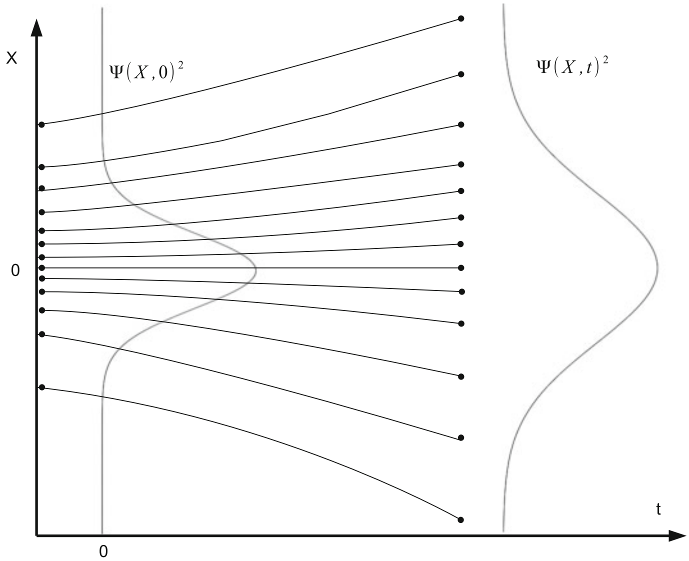

Fig. 8.5 Illustration of properties of the $\Psi(x,t)^2$ distribution, in one dimension, for a Gaussian Ψ. Each *dot* represents the position of a particle, both at time 0 and at time t, connected by trajectories. The statistical distribution of particles is given by $\Psi(x,0)^2$ on the left of the picture and by $\Psi(x,t)^2$ on the right

Of course, this notion becomes precise only in the limit where the number of particles tends to infinity, but we can use that concept "approximately" when the number of particles is large enough. After all, remember that the number of particles in a small quantity of matter is on the order of Avogadro's number, namely on the order of 10^{23} (1 followed by 23 zeros), so that we shall always feel free to assume that the number of particles is so large that the approximation we make by "taking it to infinity" does not matter.

Now suppose that we have a large number of particles, whose initial statistical distribution is given. Each initial position $X(0)$ of a particle gives rise to a unique trajectory, hence to a unique position $X(t)$ at any given later time t. This is illustrated in Fig. 8.5, where the lines going from left to right correspond to trajectories and the curve on the right gives the distribution of the positions $X(t)$ at the later time t.

Hence, if we start with an initial statistical distribution of the particles,[9] we shall have a well-defined statistical distribution of the particles at all later times:

[9]See Fig. 3.2 for an illustration of that concept.

each value of $X(0)$ gives rise to a unique value of $X(t)$; so, if we have a large number of values of $X(0)$, with a certain statistical distribution, we shall have a corresponding set of values of $X(t)$, which will also have certain statistical distribution. This statistical distribution is uniquely defined, once we know the initial statistical distribution of the particles and the way particles move.

Now, an important property of the de Broglie–Bohm theory (which we shall not prove) is that, if we have a large set of particles, having an initial statistical distribution equal to the square of the wave function at time 0, $\Psi(x, 0)^2$, then, at any later time t, the statistical distribution of the particles that we just defined will be also equal to the square of the wave function, but at time t, $\Psi(x, t)^2$. This is illustrated in Fig. 8.5.

This property of the de Broglie–Bohm theory can also be illustrated by Figs. 8.1 and 8.2. There, one assumes a statistical distribution of particles (i.e., of initial positions just behind the slits, which are not easily visible in Figs. 8.1 and 8.2, because of the large number of lines starting behind the slits) given by $\Psi(x, 0)^2$, 0 denoting the time of passage through the slits. Then, the right side of the figure indicates the place where the particle lands on the screen and these dots have a statistical distribution given by $\Psi(x, t)^2$, t being the time of arrival on the screen. This is similar to what one saw in Chaps. 2 and 4, for example in Fig. 4.10.

So if we assume that the statistical distribution of the particles in some specific situation is determined at some initial time by $\Psi(x, 0)^2$, this will be true at all later times and therefore coincide with the usual quantum predictions.

Now, consider measurements of other quantities than positions, for example velocities. As we already said, when we "measure" the velocity, we do it, indirectly, by measuring positions. So, if one predicts correctly (meaning, in agreement with the usual quantum predictions) the statistics of the positions of particles, we automatically predict the correct statistics for the results of velocity measurements and in fact also for any other measurement.[10]

Now comes a deeper question: if assuming that, at some initial time, the distribution of the particles being determined by $\Psi(x, 0)^2$ implies the correct quantum prediction at later times, what justifies that assumption about the statistical distribution of the particles at that "initial" time?

Well, we can simply assume that this assumption held also at some earlier time t, say $t = -1$: at that time, the distribution of particles would be given by

[10]The same thing is true for the "measurement" of the spin (see Appendix 8.A): what we see directly is only the particle going out of a box through the upward hole or the downward one. But that means that the only thing that we directly observe are *positions*. So, if we correctly predict the results of detection of positions (and the de Broglie–Bohm theory does predict correctly the positions), we also correctly predict the results of "measurements" of spin.

$\Psi(x,-1)^2$. Then, the previous reasoning would then imply that this assumption at time -1 implies that the statistical distribution of the particles at time 0 is given by $\Psi(x,0)^2$.

But it is obvious that we then get into a "chicken and egg" problem, since the assumption at time $t=-1$ will be justified by a similar assumption at time $t=-2$, etc., and, ultimately, we have to make assumptions that go back to the beginning of the Universe. And making assumptions about the beginning of the Universe is something that always makes some people (including this author) uneasy.

But since the de Broglie–Bohm theory is deterministic, "randomness" can only come from assumptions about initial conditions and the latter always ultimately refer to those of the Universe.

This argument may seem to some readers to be an instance of what is called "GIGO", or "garbage in, garbage out", namely that we just assume what is to be proven. But that is not quite true; our argument relies on a non-evident property of the de Broglie–Bohm theory, namely that assuming that the statistical distribution of the particles at some "initial" time is given by $\Psi(x,0)^2$ guarantees that this statistical distribution will be given by $\Psi(x,t)^2$ at all later times.

Let us come back to our initial example of coin tossing. When we explained why the results of tossing several coins look random we said that the initial conditions of each coin that determine its motion are also random.

But our Universe could be different: certain machines tossing coins could be so finely tuned that they would produce far more heads than tails for example. So, if we want to explain why the machines tossing the coins produce random results, we have to go back in time and examine how they were built. And then, how whatever was used to build them was built etc. But this is also a "chicken and egg" problem going back further and further in time.

Thus, even to explain the simplest random results, such as those of coin tossing, we are logically brought back to the initial properties of the Universe!

It is no different in the de Broglie–Bohm theory, except that the "random" initial distribution of particle positions is assumed to be given by $\Psi(x,0)^2$.

Unfortunately, going further into this discussion will be too technical for this book. Let us simply conclude by saying that the properties of the de Broglie–Bohm theory give a perfectly coherent understanding of the random nature of the quantum predictions simply by making assumptions on the initial distribution of the positions of the particles whenever one repeats many times the same experiment, with the same initial wave function.

In practice, when one explains the random nature of the results of coin tossing, one does not go back to the origin of the Universe, but one is happy to refer to the uncontrollable initial conditions when coins are tossed, and we can take a similar attitude with respect to the de Broglie–Bohm theory.

8.4.3 What About the Collapse of the Wave Function?*

The short answer, which we shall elaborate in this section, is that there is never any collapse of the wave function in the de Broglie–Bohm theory, but there is a collapse "in practice", which coincides with the one in ordinary quantum mechanics, at least when the latter is unambiguous. This collapse is often called an "*effective collapse*", which just means in practice but not in principle.

As we saw, if a wave function is in a superposed state, namely a sum of two (or more) terms corresponding to different physical situations, like going through one slit or the other in the double-slit experiment, then one has to keep both terms in order to predict the future behavior of the system correctly.

Indeed, even when the two parts of the wave function are initially far apart from each other, like when one part goes through one slit and the other part through the other slit in the double-slit experiment (see Fig. 4.10, just after the slits), they may recombine later, in the sense that the two parts of the wave function will overlap (see Fig. 4.10, further to the right), and then both terms will affect the behavior of the particle.[11]

This is what happens in the double-slit experiment and gives rise to the interference pattern on the second screen (see Fig. 4.10 and Figs. 8.1, 8.2, 8.3 and 8.4 for the behavior of the particles in the de Broglie–Bohm theory).

But what happens when we "observe" a quantum system? We already described what happens in the quantum formalism in Sect. 5.1: in order to observe something, we need that the particle interacts with a macroscopic system, because that is the only sort of thing that we can directly perceive. Such a system could be any detector in a laboratory, a pointer pointing up or down, or a cat that can be alive or dead, but must be composed of a large number of particles.

Suppose that we have two terms, each of which corresponds to macroscopically distinct situations, for example the wave function (5.5) of Sect. 5.1 which is:

$$\text{a superposition of the state } \varphi^{\uparrow}\Psi_1 \text{ and of the state } \varphi^{\downarrow}\Psi_2\ , \tag{8.1}$$

where Ψ_1 and Ψ_2 correspond to the wave functions localized near the upper and the lower slit respectively and $\varphi^{\uparrow}$ and $\varphi^{\downarrow}$ are the wave functions associated

[11] See the Glossary for a formal definition of the notion of overlap of two wave functions.

to the last two pictures in Fig. 5.3, namely to macroscopic bodies (the pointers) detecting through which slit the particle goes.

It is important to notice that the symbols $\varphi^{\uparrow}$ and $\varphi^{\downarrow}$ refer to systems having a large number of particles on the order of Avogadro's number $\sim 10^{23}$, and that each particle of those bodies has its own wave function, so that each of the symbols $\varphi^{\uparrow}$ and $\varphi^{\downarrow}$ actually corresponds to a large number of wave functions for individual particles and is an aggregate of all those wave functions.[12]

The particle will be "in" only one of these terms (meaning that only one of the two terms in (8.1) will be non-zero where the particle is): either the particle goes through the upper slit and the term $\varphi^{\uparrow}\Psi_1$ is the one which is non-zero where the particle is, or it goes through the lower slit and the term $\varphi^{\downarrow}\Psi_2$ is the one which is non-zero where the particle is.

As we said, in principle, we must keep both terms because they may overlap later and produce interference effects as in Fig. 4.10. Thus, keeping only one term (the one in which the particle is) could lead to different predictions.

The interference pattern would be destroyed if the two wave functions did not overlap as they do in that figure. However, when one considers *macroscopic* systems, namely systems composed of a large number of particles, like a pointer or a cat, in a superposed state like (8.1), we need the overlap to occur for *each wave function of each particle* in the system represented by $\varphi^{\uparrow}$ or $\varphi^{\downarrow}$. But, and that is another fact that we will not prove, making the two parts of the wave function of each particle overlap is *in practice* impossible if the number of particles is very large. As an analogy, suppose that you try to control the tossing of a coin so that it falls heads. If you are clever enough you might be able to do it once, twice, maybe ten times, but to do it on the order of 10^{23} times would be in practice impossible.

Thus, if we can be sure that no overlap will occur in the future between the two terms in a wave function like (8.1), because they refer to macroscopic objects, we can simply keep the term in which the particle happens to be (and we know which one it is because of the coupling between the particle and the macroscopic device, by simply looking at the latter: the pointer is up or down), as far as the predictions for the future behavior of the system are concerned.

Reduction or collapse of the wave function in the de Broglie–Bohm theory is as simple as that. It is just a practical impossibility, not an "in principle" one. This reduction is a matter of degree: as the number of particles increases it becomes more and more difficult to make the two parts of the wave function overlap in the future, but there is no fixed number for which there would be a sharp jump from a non-reduced wave function to a reduced one.

[12]This is a simplification, but it will be adequate for our argument.

So, in some sense, we do “collapse” the wave function when we look at the result of an experiment. But this is only a practical matter. We can still consider that the true wave function is and remains forever given by the time evolution of the full wave function (8.1). It is simply that one of the terms of the wave function no longer guides the motion of the particle, either now or at any time in the future and it would just be cumbersome to keep it in our calculations, but the results would be the same if we did.

The measuring process here is an entirely physical process, with no role whatsoever left to the observer. And the latter only uses the reduction of the wave function as a practical tool for further calculations on the system.

Finally, let us stress that there is a rather common misconception about this “in practice” reduction of the wave function,[13] namely that this phenomenon not only allows us to make sense of the effective collapse within the de Broglie–Bohm theory, but that it is also, on its own, sufficient to account for the collapse rule, within ordinary quantum mechanics. The idea is roughly that, if the different terms in the sum involving a macroscopic object like (8.1) do not overlap, then we just pick up the one we see at the end of the experiment in order to predict the future behavior of the system.

The crucial difference between that view and the de Broglie–Bohm theory is that, in the latter, there is a fact of the matter as to where the particle *is* and as to whether the pointer *points* up or down. Then, we learn where the particle is by looking at the macroscopic measuring device, and use that information to predict the future behavior of the particle in a simpler way than if we kept the whole wave function. But we learn something that exists in the world, independently of whether we look at it or not.

But if we do not reason within the de Broglie–Bohm theory and remain within ordinary quantum mechanics, there is no fact whatsoever that distinguishes one term from the other in a sum like (8.1), except our *observations*.

So, we then go back to square one: putting our observations in the very formulation of our physical theories, which is exactly what we have been trying to avoid all along and that the de Broglie–Bohm theory manages to do.

There are more sophisticated ways to try to make the practical impossibility of interference between macroscopic wave functions the cornerstone of a solution to the measurement problem, for example, the many-worlds interpretation of quantum mechanics, but this will be discussed in Chap. 9.

[13]This is related to what is called *decoherence* in the literature.

8.5 Is It that Simple?*

By simply assuming that particles have positions (hardly a revolutionary idea, although not a generally accepted one) and that their motion is guided by the wave function (an idea which is also not really revolutionary, but also not generally accepted), we have accounted for the interference phenomena in the double-slit experiment. By doing so, we have completely eliminated the role of the observer and we have done that within a deterministic theory.

Could it be that the solution to all the conceptual problems of quantum mechanics is that simple? The answer is again yes and no. If one is interested in what is called non relativistic quantum mechanics (namely the part of quantum mechanics that leaves aside the theory of relativity), which basically covers most of physics, like atomic, molecular and solid state physics, the foundations of chemistry and all applications to modern electronics, then the answer is yes.

But there is a part of physics dealing with waves rather than particles, like the electromagnetic waves. And there is a quantum theory for those electromagnetic waves, related to high energy physics, the sort of physics tested in accelerators such as the ones at CERN. This is a spectacularly successful part of physics, with a correspondence between experimental observations and theoretical predictions superior to anything else in science.

Moreover, it is crucial for that part of physics to take into account the theory of relativity. Therefore, a natural question for the de Broglie–Bohm theory (and a frequent objection raised against it) is whether there is an extension of de Broglie–Bohm theory to the quantum theory of the electromagnetic waves, and whether this extension incorporates the theory of relativity.

A detailed answer to that question is unfortunately too complicated to be given here. The brief answer is that, yes there is a way to extend the de Broglie–Bohm theory to quantum electromagnetic waves, but there is no unique way to do that and it is not clear which extension is the best.

As for relativity, the problem is the same as the one discussed in Sect. 7.7, namely the nonlocal effects whose reality is proven by the EPR-Bell argument, irrespective of one's views on quantum mechanics.

If we consider our discussion of nonlocality in Appendix 8.B, we see that, in the situation illustrated in Figs. 8.8 and 8.9, one measures the spin of the *A* particle before one measures the one of the *B* particle. But, because of the relativity of simultaneity, "before and after" are relative to the state of motion of the system in which the system is described, see Sect. 7.7. And that is a serous problem if we want to have a causal view of the world, where causes precede their effects in an absolute sense.

But the situation is not any better in ordinary quantum mechanics. There, the nonlocal effects are due to the collapse of the wave function, which in the EPR-Bell situation, is nonlocal, since a measurement on the A particle collapses the wave function of both the *A* and the *B* particles, no matter how far apart those particles are. Of course, since the status of the wave function is unclear, that difficulty can be swept under the rug, which is often done, as we explained in Sect. 7.5.

If one looks at books on quantum field theory or relativistic quantum mechanics, the collapse rule is almost never mentioned, although it is supposed to be a basic tenet of any quantum theory. The reason is that the collapse rule cannot be treated in a relativistic fashion, precisely because, in the EPR-Bell situation, it is a nonlocal operation. Indeed, as explained in Sect. 7.7, relative to one state of motion, the measurement of the *A* particle will occur before the one of the *B* particle and it is that measurement that will induce the collapse of the wave function of both the *A* and the *B* particles. Relative to another state of motion, the measurement of the *B* particle will occur before the one of the *A* particle and it is that measurement that will induce the collapse of the wave function of both the *A* and the *B* particles. So, if collapses of wave functions are real physical operations, it is not at all clear how to reconcile causality with the fact that the chronological order of those operations depends on the state of motion relative to which they are described.

So, the problem of combining causality, nonlocality and relativity is not just a defect of the de Broglie–Bohm theory, since nonlocality is an unavoidable feature of Nature. How to fully reconcile quantum nonlocality and the theory of relativity is an open problem, but for everyone, not just for defenders of the de Broglie–Bohm theory, although most physicists refuse to admit that this problem is a real one.

8.6 A Last Look at Traditional Questions

At the risk of repeating ourselves (the impatient reader may skip this section) we want to discuss again two of the main questions that we have raised in this book: does quantum mechanics imply the "death of determinism" in physics and is quantum mechanics a complete theory?

8.6.1 So, Does God Play Dice After All?

If there is one sentence of Einstein that anybody who has an interest in quantum mechanics must have heard, it is: "God does not play dice" [35, p. 91]. We will discuss that sentence in its historical context in Sect. 10.1, but here we want to reformulate that question: does quantum mechanics prove that the Universe is indeterministic? In Chap. 3, we claimed that it is not easy to prove such a statement, because apparent indeterminism can always be due to an incomplete description of physical systems. One way to "prove" indeterminism is to claim that quantum mechanics is both intrinsically indeterministic and complete, but its completeness is precisely what has to be demonstrated.

But now, we can say more: we have a theory that does complete quantum mechanics and that is deterministic, so that the claim that quantum mechanics proves indeterminism is surely false. However, determinism in the de Broglie–Bohm theory is a special sort and has two properties that make it somewhat different from what one might expect from a deterministic theory in the setting of classical physics:

(1) First of all, the de Broglie–Bohm theory is nonlocal. This means that, even if one wants to determine the future of what happens only in a given region of space, denoted A, one has in principle to specify the physical state of the entire Universe, since events in regions that are arbitrarily far from region A might influence instantaneously what happens in the latter.
This does not contradict the deterministic nature of the theory, but one would naively expect that, in a deterministic theory, it would be sufficient to know the initial conditions in a neighborhood of region A in order to predict the future in that region, at least for short times. But that is not true in the de Broglie–Bohm theory.
Of course the same thing happens in Newton's theory, since gravitational forces also act arbitrarily far and in principle instantaneously; but at least their effects decrease with distance, which is not true for the EPR-Bell nonlocal effects.
What remains true is that the correlations between distant particles that give rise to nonlocality are difficult to maintain in practice over large distances, so that, again in practice, the determinism of the theory would hold even if one forgot about events very distant from region A in our specification of the initial conditions of the Universe. But that is an "in practice" statement not an "in principle" one.
(2) Secondly, the de Broglie–Bohm theory contains in its very formulation an element of radical uncertainty that one might not expect in a deterministic theory. Indeed, the best analogy is to think of the initial conditions of quantum systems as being like the ones of a large number of coins that are being tossed.

> Although, in principle, the end result of each coin tossing can be determined if one knew the initial conditions with sufficient precision, in practice it is impossible to do. For quantum systems, this impossibility is even more an "in principle" one, but the simplest way to explain the situation is through this analogy.

So, coming back to Einstein's famous quote, no, God does not play dice or at least there is no argument based on quantum mechanics that indicates that he does. The idea of determinism can be maintained, thanks to the de Broglie–Bohm theory, but it is of rather special type.

8.6.2 Is Quantum Mechanics Complete?

We have repeatedly asked that question in this book, but now we can give it a clear answer: no, ordinary quantum mechanics does not give a complete description of physical systems, and one can give a more complete description of them than the one given by the wave function, in which the "observer" looses entirely its special status. Moreover, because that more complete theory, the de Broglie–Bohm one, introduces only the particle positions as "hidden variables" and accounts for the measurements of everything else in terms of interactions between the particle and some apparatus, it avoids being refuted by the no hidden variables theorems of Sect. 5.2.

But there is a weaker sense in which ordinary quantum mechanics is complete, namely as far as empirical predictions are concerned; one might call it "predictively complete". That is simply because, in the de Broglie–Bohm theory, one cannot control the initial conditions of the particles well enough to be able to make more precise statistical predictions than the usual ones. We have sketched an explanation of why this is so in Sect. 8.4.2. The simplest way to understand this situation is by analogy with a set of tossed coins whose initial conditions could not be controlled sufficiently well, so as to produce different statistics than the usual ones (half heads, half tails).

But the value of the de Broglie–Bohm theory lies in its explanatory power, not in its predictions.

8.7 Conclusion: The Merits of the de Broglie–Bohm Theory

First of all, let's ask: what is the relationship between the de Broglie–Bohm theory and ordinary quantum mechanics? The quick answer to this question is that it is *not a different theory*! More precisely, the de Broglie–Bohm theory *is* a theory, while ordinary quantum mechanics is not. Indeed, quantum mechanics doesn't even pretend to be a theory, but rather claims to be an algorithm allowing us to compute "results of measurements".

Another way to say this is that ordinary quantum mechanics is the algorithm used to compute results of measurements that can be *derived* from the de Broglie–Bohm theory: in that theory, measurements do not really measure anything (except for detections of positions) but are interactions between a macroscopic system and a microscopic one. Once one understands that, the mystery of the ever present "observer" of standard quantum mechanics disappears.

One might also say that ordinary quantum mechanics is simply a truncated version of the de Broglie–Bohm theory or that the de Broglie–Bohm theory is a completion of ordinary quantum mechanics: in the latter, one ignores the particle trajectories, but since the empirical predictions of the de Broglie–Bohm theory are statistical, and are the same as those of ordinary quantum mechanics, there are no practical consequences of that omission.

Thus, ordinary quantum mechanics is sufficient "for all practical purposes" to use Bell's expression [12], for which he even invented an acronym: FAPP. But it is the de Broglie–Bohm theory that *explains* why ordinary quantum mechanics is sufficient FAPP, something that is true but mysterious without de Broglie–Bohm.

These remarks also provide a reply to a frequent objection raised against the de Broglie–Bohm theory: what are the new predictions made by that theory compared to ordinary quantum mechanics? Once we understand that the de Broglie–Bohm theory is just a way to make sense of ordinary quantum mechanics, which, on its own, does *not* make sense as a theory about the world outside the laboratories, that objection collapses.

In fact, it is excellent news that the de Broglie–Bohm theory does *not* make new predictions with respect to those of ordinary quantum mechanics or, at least, that it does not make predictions at variance with those of the latter. Otherwise, it would simply be refuted by experiments, given the incredible empirical success of ordinary quantum mechanics. What the de Broglie–Bohm theory does is to explain what goes on in the world that makes ordinary quantum mechanics successful, but not contradict or complement the latter's predictions.

To physicists making the "no new predictions" objection to the de Broglie–Bohm theory, one should retort: what does ordinary quantum mechanics say about the world outside the laboratories? The answer is likely to run into difficulties for the reasons discussed in Chap. 5: either because of the existence of macroscopic superpositions or because of the no hidden variables theorems. And if the answer is that ordinary quantum mechanics does not say anything about the world outside the laboratories, the next question should be: "Are you satisfied with that state of affairs? And, if yes, why do you build laboratories then, if they do not lead to any knowledge of the world outside the laboratories?".

If people reply that the many extremely successful technological applications of quantum mechanics show that no question should be asked about the latter, the answer that we have already given is that, the more "it works", the more it is natural to ask ourselves "why does it work so well?".

As we saw, the de Broglie–Bohm theory eliminates the dual nature of the time evolution in quantum mechanics: one between observations and one during observations. It also explains in a natural way why, as the no hidden variables theorems show, one cannot introduce hidden variables for both positions and velocities (or for the spin values, see Appendix 8.A). Finally, even the strangest aspect of all of quantum mechanics, nonlocality, is made more understandable thanks to the de Broglie–Bohm theory (see Appendix 8.B).

Moreover, since it is a truism that a single counterexample is enough to refute a general claim, the de Broglie–Bohm theory is a counterexample to three claims that have been almost universally accepted by physicists, commented by philosophers, taught in classes, and sold to the general public:

1. That quantum mechanics signals the end of determinism in physics.
2. That quantum mechanics assigns a special role, in its very formulation, to the "observer". There has been quite some debate as to whether this "observer" is a set of laboratory instruments or a human consciousness, but the debate would never have got under way if the central role of observations in quantum mechanics had not been accepted to start with.
3. That quantum mechanics is something that "nobody understands", to quote Richard Feynman [79]; that quantum mechanics is mysterious and requires a far more drastic revision in our ways of thinking than any previous scientific revolution.

By simply existing, being deterministic, and describing the "measurements" as purely physical processes, the de Broglie–Bohm theory constitutes a refutation of claims 1 and 2.

A further quality of de Broglie–Bohm theory is its perfect clarity, which refutes claim 3. In that theory, we just deal with matter in motion, just as in classical physics, but of course with very different laws of motion than the classical ones, which is to be expected, since the phenomena to be explained (like interference) are radically different from the classical ones.

Given that there are endless bookshelves of confused talk about the role of the "observer" in physics or about the death of determinism, or about the radical incomprehensibility of the quantum world, this is no small feat, especially given the number of times that this accomplishment has been declared impossible.

Finally, coming back to the three fundamental questions raised in the beginning of this book (indeterminism, the role of the observer and nonlocality), we have already answered how the de Broglie–Bohm theory answers the first two. As for nonlocality, the merit of the de Broglie–Bohm theory is to make it explicit: when the wave function of a pair of particles is like the one described in Sect. 7.4, the motion of these particles is coordinated in such a way that, acting on the wave function of the pair near where one particle is, may affect the behavior of the other particle, even if that particle is arbitrarily far away from where the action takes place.

This is also often considered an objection to the de Broglie–Bohm theory, but since Bell has shown that nonlocality is here to stay, even if quantum mechanics was superseded some day by another theory, far from being a defect, the natural account of nonlocality within the de Broglie–Bohm theory is one of its greatest merits.

Let us leave the last word to John Bell:

> Bohm's 1952 papers on quantum mechanics were for me a revelation. The elimination of indeterminism was very striking. But more important, it seemed to me, was the elimination of any need for a vague division of the world into "system" on the one hand, and "apparatus" or "observer" on the other. I have always felt since that people who have not grasped the ideas of those papers ... and unfortunately they remain the majority ... are handicapped in any discussion of the meaning of quantum mechanics.
>
> [...]
>
> Why is the pilot wave picture ignored in textbooks? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism are not forced on us by experimental facts, but by deliberate theoretical choice?
>
> John Bell [14, pp. 173, 160]

One possible answer to that last question could be that "the pilot wave picture" is just one "interpretation" of quantum mechanics among many, so why pay attention to that one alone? The next chapter will deal with that objection.

8.8 Summary

The de Broglie–Bohm theory is a theory of matter in motion, just like the whole of "classical" physics is (meaning the whole pre-quantum physics, including the theories of relativity). In the latter, particle move under the influence of forces or of electromagnetic waves or because of the structure of space-time. In the de Broglie–Bohm theory, there is an object, the wave function, that guides the motion of the particles. That motion is illustrated in Figs. 8.1, 8.2, 8.3 and 8.4 and is very non classical but it accounts for the observations.

One of the most important aspect of the de Broglie–Bohm theory is that it explains what happens in what are called "measurements": first of all, the latter are always in the end measurements of positions. This is both true for measurements of velocities, that depend on measuring the distance between two positions at different times or the measurement of spin, which depends through which hole a particle exits from a box.

In the de Broglie–Bohm theory, measurements of velocities do not reveal a value that is "already there": for velocities, in some situations, the true pre-measurement velocity is zero, but the "measured" one is not.[14] Measurements are interactions between particles and some macroscopic objects and that interaction is described by the de Broglie–Bohm theory.

That measurements are interactions and not just passive observations is what one would expect on the basis of the no hidden variables theorems of Sect. 5.2, and is also one way to understand the Copenhagen view, but here this fact is shown to be a consequence of the theory, not an a priori claim.

One might wonder how does one recover the statistical quantum predictions in a deterministic system such as the de Broglie–Bohm theory. This can only be done via suitable assumptions on the initial conditions of the system. It turns out that those assumptions are rather natural: one has only to assume that the initial positions of the particles of any system are distributed according to the quantum mechanical statistics. This may seem like assuming what has to be proven, but it is not, since the validity of this statement depends on a non

[14] And for the spin, there is simply no pre-existing value of the spin being "measured", see Appendix 8.A.

obvious property of the de Broglie–Bohm theory: if the initial positions of the particles are distributed according to the square of the wave function $\Psi(x, 0)^2$ at some "initial" time, they will be distributed at a later time according to the square of the wave function at that time $\Psi(x, t)^2$.

In the de Broglie–Bohm theory, the wave function always evolves according to the usual Shrödinger's equation and never collapses. But then, how does one explain the practical necessity to use the collapse rule in ordinary quantum mechanics? The answer is that, when a particle interacts with a measuring device, namely a big system, the wave function of the latter becomes coupled to the one of the particle: this leads to macroscopic superpositions, where one part of the wave function of the pointer is up and the other down.

But then, because macroscopic objects contain many particles, in order for the two parts of the wave function (up and down) to interfere with each other, the wave function of each particle of the up part of the pointer would have to overlap with the corresponding wave function of the same particle in the down part of the pointer. However, this requires too many overlaps so that it becomes impossible in practice (even if not in theory) to make the up and down parts of the wave function of the pointer interfere with each other.

Then, one can just use for later purposes the wave function of the part of the pointer which we see – either up or down. But, unlike in ordinary quantum mechanics, here there is a fact of the matter as to whether the pointer is up or down at the end of the experiment and then, looking at the result has no physical significance whatsoever.

In Appendix 8.B, we will explain why the de Broglie–Bohm theory is nonlocal: in the situation discussed in Chap. 7, a single wave function may guide simultaneously two particles together. So that, acting on one part of the wave function, may influence the way both particles are guided, no matter how far apart those particles are.

Of course, because of the EPR-Bell result, this nonlocality is a quality rather than a defect.

Finally, we stressed that, even if the de Broglie–Bohm theory is not the final word on quantum mechanics, particularly when it comes to a quantum theory of waves and fields and the theory of relativity, it has the merit of completely eliminating the observer from quantum mechanics, clarifying the paradoxes surrounding the notion of measurement, restoring determinism and making the unavoidable nonlocality of the Universe somewhat more understandable.

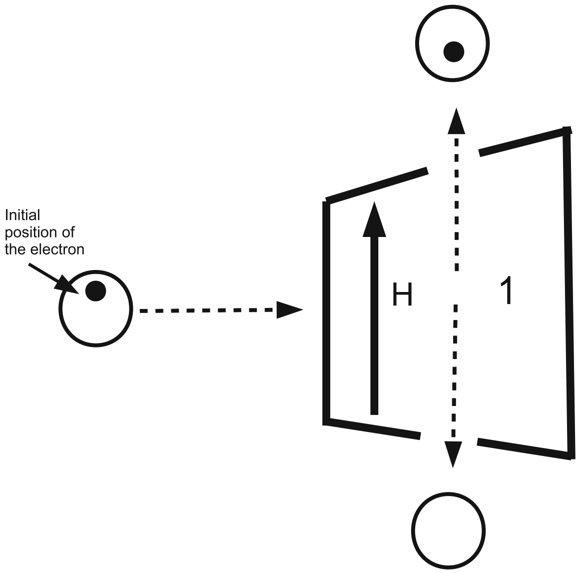

Fig. 8.6 An idealized spin measurement: A particle is sent towards a *box*, which is perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1. The wave function associated to the particle is represented by a disk. In the *box*, the wave function splits into two parts, one going upward in the direction of the field, the other going downward, in the direction opposite to the one of the field. The particle position is indicated by a *dark dot*. In the de Broglie–Bohm theory, if the particle starts initially above the horizontal line in the middle of the figure (at the level of the rightward pointing *arrow*), it will always go in the upward direction, namely here in the direction of the field. This figure corresponds to the situation described in Fig. 7.3, but within the de Broglie–Bohm theory

Appendices

8.A "Measurements" of the Spin in the de Broglie–Bohm Theory

We will describe here how a spin measurement works in the de Broglie–Bohm theory.[15] In Fig. 8.6, we show the wave function, represented by a disk moving towards the box with a magnetic field H in it. As far as the wave function is concerned, it splits itself in two parts, one going in the direction of the field,

[15]It should be stressed that all the "experiments" are only meant to illustrate the theory, not to explain how real experiments are performed. Some ideas of this appendix come from Chap. 7 of David Albert's book *Quantum Mechanics and Experience* [1].

the other in the direction opposite to the one of the field. Those two parts are represented by two disks.[16]

But all we directly observe is the final position of the particle. As shown in Figs. 7.3 and 7.4, it will either go in the direction of the field, or in the direction opposite to the one of the field. One can show that, in the de Broglie–Bohm theory, if the particle starts initially above the horizontal line in the middle of the Fig. 8.6 (at the level of the rightward pointing arrow), it will always go in the upward direction, namely in the direction of the field in this figure (again, a property of the de Broglie–Bohm theory that we shall not prove).

Now, here is something a priori surprising, but which is fundamental if one wants to understand the de Broglie–Bohm theory. Suppose that we reverse the direction of the magnetic field, relative to its direction in Fig. 8.6, as is done in Fig. 8.7.

And let us start with *exactly* the same wave function and *exactly* the same particle position, as in Fig. 8.6.

One can show that the particle will again go in the upward direction, see Fig. 8.7. But now, this is the direction *opposite to the one of the field*.

In the situation of Fig. 8.6, one would say that the spin is "up", in Fig. 8.7 that it is "down" (up just means "in the direction of the field", down means "in the direction opposite to the field"). But the only difference between the two figures comes from the orientation of the field. As far as the particle is concerned, its complete physical state, namely its wave function and its position are exactly the same in both situations.

In other words, the value up or down of the spin that actually results from the "measurement" does not depend only on the wave function and the initial position of the particle (which, remember, in the de Broglie–Bohm theory, is the *complete* description of the physical state of any system), but on the concrete arrangement of the "measuring" device. Here the scare quotes that we used all along when speaking of measurements are finally understandable: there is no intrinsic property of the particle that is being "measured", in general, in a "measurement", except for measurements of positions.

Of course, since the system is deterministic, once we fix the full initial state (the wave function and the position) of the particle *and* the experimental device, the result of the experiment is pre-determined. But that does not mean that the spin value that we "observe" is pre-determined, because, as we saw, we can measure the spin by orienting the magnetic field in one direction or

[16]At least for some wave functions, which we will assume are those associated with the particles here. With some simplification, one may assume that each part of the wave function takes a constant value on those disk and vanishes elsewhere.

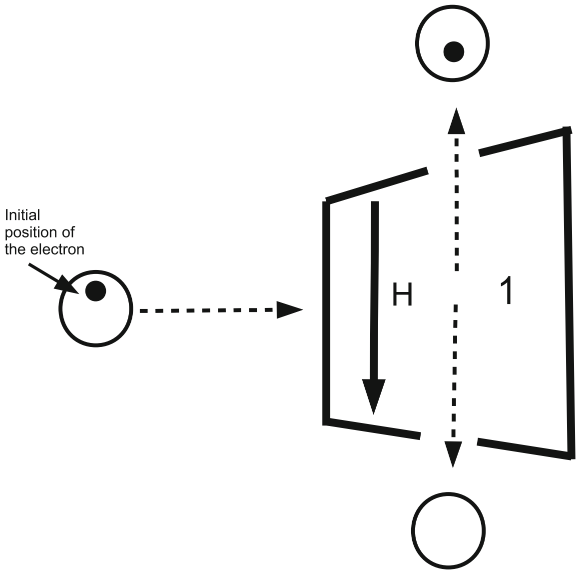

Fig. 8.7 An idealized spin measurement with the field reversed relative to Fig. 8.6: A particle is sent towards a *box*, which is perpendicular to the plane of the figure, and in which there is a magnetic field H oriented downwards along the vertical axis, denoted 1. The wave function associated to the particle is represented by a disk. In the *box*, that wave function splits into two parts, one going downward in the direction of the field, the other going upward, in the direction opposite to the one of the field. The particle position is indicated by a *dark dot*. In the de Broglie–Bohm theory, if the particle starts initially above the horizontal line in the middle of the figure (at the level of the rightward pointing *arrow*), it will always go in the upward direction, namely in the direction opposite to the one of the field in this figure, as opposed to what happens in Fig. 8.6

the opposite one. So the value of the "spin" of the particle that results from a measurement depends on our conventions, which means that it does not exist as an intrinsic property of the particle.

8.B How Does the de Broglie–Bohm Theory Account for Nonlocality?

As we saw in Chap. 7 there exist nonlocal effects in Nature.[17] But we do not know what these effects are, because in ordinary quantum mechanics nonlocality manifests itself through the "collapse rule", and the meaning of

[17] Some ideas of this appendix come from Chap. 7 of David Albert's book *Quantum Mechanics and Experience* [1].

that rule depends on the meaning of the wave function, which itself is unclear. But in the de Broglie–Bohm theory, the wave function has a clear meaning: it guides the motion of the particles.

The de Broglie–Bohm theory for a single particle is essentially local: the particle is always guided by the part of the wave function in which it finds itself. There is some weak form of nonlocality, if one wants to use that term here, because the motion of a particle going through one slit in the double-slit experiment may be affected by the part of its wave function going through the other slit, as we saw in Figs. 8.1, 8.2, 8.3 and 8.4.

But, when one discusses one particle, everything is still *local*, in the sense that those effects are felt only when one part of the wave function comes back and becomes superposed with the other part, the one in which the particle is. This leads to interference phenomena, but there is no instantaneous action at a distance here, since the effect will take the time needed for the two wave functions to be recombined.

The same holds for Einstein's boxes in the de Broglie–Bohm theory: the particle is always in one of the half-boxes and we simply *learn* in which box it is by opening one of them. In that situation, the wave function is partly in each of the boxes, and that might have an effect if, instead of opening the boxes far away from each other, one were to bring them together again and then recombine those two parts of the wave function.

So, in the thought experiment of Einstein's boxes, there is no action at a distance whatsoever, from the point of view of the de Broglie–Bohm theory. But, of course, coming back to the dilemma of Sect. 7.2 (either there are actions at a distance or quantum mechanics is incomplete), the de Broglie–Bohm theory is based on the idea that quantum mechanics *is incomplete*!

But we learned in Chap. 7 that there are nonlocal effects when we deal with at least two particles.

It would go far beyond the scope of this book to really explain how nonlocality appears in the de Broglie–Bohm theory, but we will sketch what happens in the EPR-Bell situation discussed in Chap. 7.

We will first describe what happens when one measures the spin of two particles far away from each other, when the wave function is as in Sect. 7.4.2, and then explain what is nonlocal in those experiments.

Consider the left part of Fig. 8.8. If we measure first[18] the spin of particle A (the box in which it is measured is closer to where the particles came from than the one measuring the spin of particle B), we shall get the up result, since, again, there is an horizontal line in the middle of Fig. 8.8 (at the level

[18]We discussed in Sect. 8.5 the problems that this notion of "first" implies if we take into account the theory of relativity.

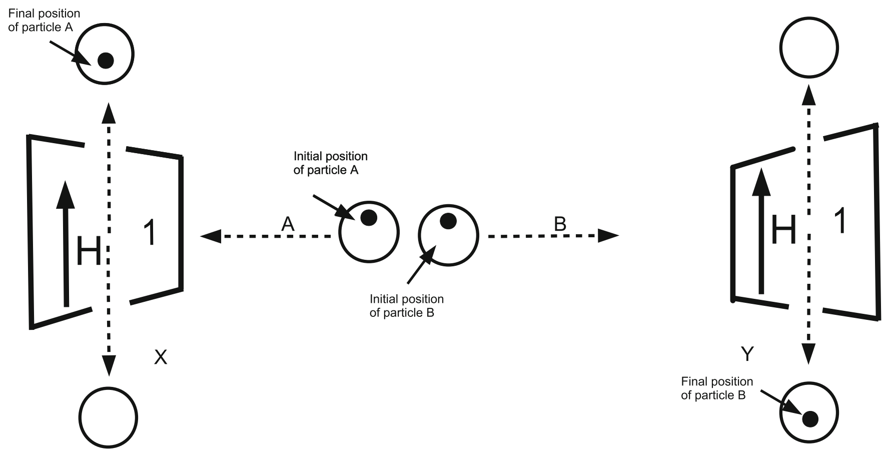

Fig. 8.8 Two particles, A and B are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1. The wave functions associated to the particles are represented by disks. In the *boxes*, the wave functions split into two parts, one going upward in the direction of the field, the other going downward, in the direction opposite to the one of the field. The particle positions are indicated by *dark dots*. Suppose we measure the spin of the A particle first (the *box* in which it is measured being closer to where the particles came from than the one measuring the spin of particle B). In the de Broglie–Bohm theory, if the A particle starts initially above the horizontal line in the middle of the figure (at the level of the two *arrows*), it will always go in the upward direction, namely in the direction of the field. But then, since the wave functions of the two particles are such that they are (anti)-correlated, the B particle will have to go in the direction opposite to the one of the field namely downwards. This figure corresponds to the situation described in Figs. 7.5 and 7.6, but within the de Broglie–Bohm theory

of the two arrows) that particles cannot cross, as was the case in Figs. 8.6 and 8.7. Since the A particle starts above that line, it will have to go up. So the A particle will go in the direction of the magnetic field. By definition, its spin will be "up".

But then, since particle B always goes in the direction of the field opposite to the one taken by the A particle, it will have to go down, that is in the direction opposite to the magnetic field, since the field is oriented in the same way in the boxes at X and at Y, see the right part of Fig. 8.8.

Note that this behavior of the B particle is independent of where it starts: above (as in Fig. 8.8) or below the horizontal line in the middle of Fig. 8.8 (at the level of the two arrows). That is because, once the spin of particle A has been measured, there is no symmetry any more between the top and bottom halves of the figure and that implies that the line in the middle can be crossed

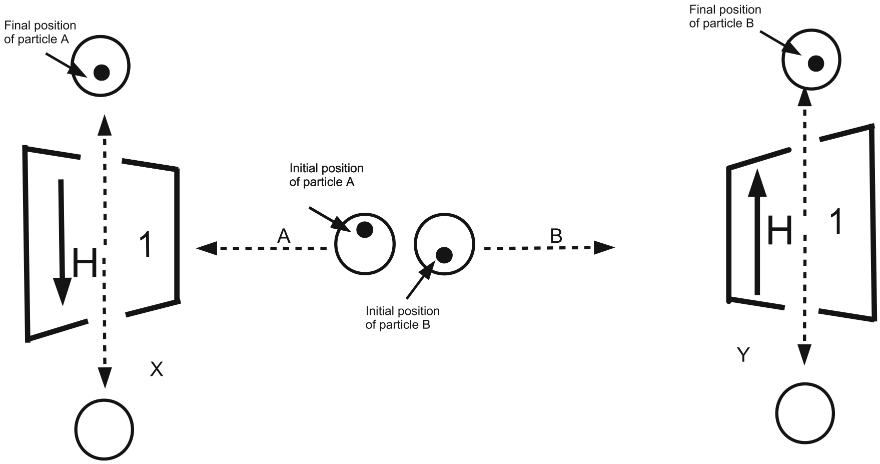

Fig. 8.9 Measurement of the spin on the left first, with the field reversed on the left hand side relative to the one of Fig. 8.8: two particles, A and B are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1 on the right and downwards on the left. The wave functions associated to the particles are represented by disks. In the *boxes*, the wave functions split into two parts, one going in the direction of the field, the other going in the direction opposite to the one of the field. The particle positions are indicated by *dark dots*. Suppose we measure the spin of the A particle first (the *box* in which it is measured being closer to where the particles came from than the one measuring the spin of particle B). In the de Broglie–Bohm theory, if the A particle starts initially above the horizontal line in the middle of the figure (at the level of the two *arrows*), it will always go in the upward direction, namely in the direction opposite to the one of the field. But then, since the wave functions of the two particles are such that they are (anti)-correlated, the B particle will have to go in the direction of the field, namely upwards

by the B particle (again, a fact about the de Broglie–Bohm theory that we cannot explain in detail).

Now, suppose that one reverses the direction of the field on the left side of Fig. 8.8, but that one does not reverse it on its right side, which measures the spin of particle B, see Fig. 8.9. Then, let us measure first the spin of particle A, as in Fig. 8.8. Since there is an horizontal line in the middle of Fig. 8.9 (at the level of the two arrows) that particles cannot cross, if the A particle starts above that line, as in Fig. 8.9, it will have to go up.

Thus the A particle goes now *in the direction opposite to the magnetic field*, and, by definition, its spin will be "down". But then, in that situation, particle B must go in the direction of the field, since the two particles are (anti)-correlated.

But that means that the B particle must now go up, see Fig. 8.9, instead of down, as it did in Fig. 8.8, and its spin will be "up".[19]

So by changing the orientation of the field on the left of Fig. 8.9, relative to Fig. 8.8, while doing nothing whatsoever on the right of Fig. 8.9, we affect the trajectory of particle B (in one situation, it goes up, in the other one it goes down) which may be arbitrarily far away from the A particle. This is the way the action at a distance manifests itself in the de Broglie–Bohm theory.

This action does not allow the transmission of messages, because, in the situation of Figs. 8.8 and 8.9, if one repeats the experiment many times, the A particle will start half of the time above the horizontal line in the middle of Fig. 8.8 (at the level of the two arrows) and half of the time below it. When it is above the middle line, it will go up in Fig. 8.8 and particle B will go down. If the A particle starts below the middle line, it will go down and the B particle will go up. If one reverses the field on the left, as in Fig. 8.9, and if the A particle starts above the middle line, both particles will go up. If the A particle starts below the middle line, both particles will go down.

So, there is a genuine action at a distance here, since acting on the A particle (by choosing how to measure its spin) instantly affects the behavior of particle B.

However, since there is no way to control whether the A particle will start above or below the middle line in Figs. 8.8 and 8.9, there is no way to control whether changing the orientation of the magnetic field at X will make particle B go up or down at Y. So there is no way, by playing with the orientation of the magnetic field at X, to send a message at Y (which one could of course do if one could decide, by choosing the orientation of the field at X, to make particle B go up or down).

The fact that the de Broglie–Bohm theory is nonlocal is a quality rather than a defect, since Bell showed that any theory accounting for the quantum phenomena must be nonlocal. Moreover, the nonlocality is of the right type, i.e., just what is needed because Bell's results, but not more, where "more" might be a nonlocal theory allowing the instantaneous transmission of messages.

[19]As in Fig. 8.8, this holds irrespectively of the initial position of the B particle, since, once the spin of the A particle has been measured, there is no longer a line in the middle of the figure that the B particle cannot cross.