Jean Bricmont

Quantum Sense and Nonsense

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Preface

There are many mysteries in contemporary physics, and some of them have been around for almost a century: What does quantum mechanics mean? What does it say about the world and about our place in it?

Those mysteries have been used to justify a great deal of mysticism and have been associated with all kinds of religious, pseudo-scientific, philosophical, social, or even political doctrines.

The goal of this book is to explain, in the least technical language possible, the reasons for these mysteries and a possible solution to them, as well as to dispel the mysticism that has surrounded quantum mechanics.

In order to achieve those goals, we will have also to delve into the history and philosophy of science.

Throughout the years, I have benefited from such a large number of discussions, seminars, and exchanges with so many people that thanking them all by name would scarcely be possible.

However, I must stress that I learned most of what I know about the subject through discussions with Detlev Dürr, Tim Maudlin, Nino Zanghì, and especially with Sheldon Goldstein.

Several readers of parts of this book have made useful comments and corrections, and I wish to thank them: Olivier Cailloux, Laurent Dauré, Serge Dendas, Michel Hellas, Alexis Merlaud, Richard Monvoisin, and Richard Wanliss-Orlebar.

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Louvain-la-Neuve, Belgium Jean Bricmont

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What Are the Issues Raised by Quantum Mechanics?

Although this book belongs to the "popular physics" category, its main purpose is cultural rather than scientific. We shall try to explain to the lay reader the basic principles of quantum theory, and emphasize their paradoxical nature, but our main goal is to unravel the incredible amount of confusion, pseudo-science and bad philosophy that accompanies most popular discussions of quantum mechanics.

But this will also plunge us into the deepest questions about our understanding of the world and of our place in it.

First of all, what is quantum mechanics? It is the theory of the elementary constituents of matter, such as atoms or electrons, and of radiation, that emerged in 1900 and was developed in the late 1920s. This theory has led to the most spectacularly well-confirmed predictions ever made in science. Some experimental results agree with the theoretical predictions up to one part in a billion. The theory underpins all modern electronics and telecommunications. It explains the stability of atoms and of stars, and lies at the foundation of the whole field of particle physics, as well as of solid state physics, chemistry, and thus, in principle, of biology. It is truly our most fundamental theory of nature. Yet, to quote the famous American physicist Richard Feynman, winner of the 1965 Nobel Prize in Physics, "nobody understands quantum mechanics" [79].[1]

While stunningly successful in its predictions and its practical applications, quantum mechanics has enjoyed a parallel career as alleged grounds for a wide

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[1] In this book, we shall not give too many or too long quotes. We refer those who want more precisions to my more detailed but more technical book *Making sense of Quantum Mechanics*, [36]. However, this book will be self-contained.

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range of speculations. It has been claimed that quantum mechanics proves the existence of God, free will, and the afterlife, or that it justifies belief in the direct influence of mind on matter and telepathy. There is a sort of "therapy" called quantum healing. Quantum mechanics has been linked to Jungian psychoanalysis, to vitalism, to all sorts of New Age beliefs, to Eastern mysticism and to dialectics (Hegelian or Marxist), among other systems of thought (see Chap. 11 for references).

Although most physicists dismiss these ideas as unscientific, there is no shortage of famous physicists, starting with Niels Bohr and Werner Heisenberg,[2] as well as many of their followers, who have claimed that quantum mechanics signals the end of "objective reality" or that, after the advent of quantum mechanics, physics no longer deals with reality but only with "our knowledge of it". We shall refer below to those views as those of the "Copenhagen" interpretation. This school of thought is named so because Bohr lived and worked in Copenhagen. There exists also a rather widespread impression that, thanks to quantum mechanics, a cat can be both alive and dead at the same time.

A number of physicists maintain that quantum mechanics implies the existence of multiple universes that proliferate endlessly, in which copies of ourselves live 'parallel' lives, each unaware of the others.

Another claim which is often made is that quantum mechanics shows that the deterministic world-view of classical physics is no longer tenable.[3]

To whet the reader's appetite, we shall start by quoting what some famous physicists have said about what quantum mechanics means, in particular concerning the disappearance of "objective reality". Of course, the quotes here may look strange, but we will explain later what motivates them.

Werner Heisenberg, one of the founding fathers of quantum theory wrote that:

> […] the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them […] is impossible […]
>
> Werner Heisenberg [100, p. 129]

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[2] We refer to the Glossary for a brief biographical note on the scientists mentioned in this book. Niels Bohr was Danish and received the 1922 Nobel Prize in Physics, for his work on the structure of atoms, and Heisenberg was German and received the 1932 Nobel Prize in Physics, for his work on quantum mechanics.

[3] The notion of determinism will be explained in Chap. 3.

He added: “the natural laws formulated mathematically in quantum theory no longer deal with the elementary particles themselves but with our knowledge of them.” [101, p. 15]

Concerning Niels Bohr, the founder of the “Copenhagen interpretation”, Aage Petersen, who was his assistant for many years, characterized his views as follows:

> When asked whether the algorithm of quantum mechanics could be considered as somehow mirroring an underlying quantum world, Bohr would answer: “There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.”
>
> Aage Petersen [150, p. 12]

The German physicist Pascual Jordan, who was a very important contributor in the early days of quantum mechanics insisted that, if one measures the position of an electron: “the electron is forced to a decision. We compel it *to assume a definite position*; previously, it was, in general, neither here nor there; it had not yet made its decision for a definite position […].”[4] He made a similar statement concerning the measurement of velocity.

The American physicist John Archibald Wheeler, who studied with Bohr and who made important contributions both to nuclear physics and to cosmology, is famous for saying [200, p. 192]: “No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon.” He also wrote: “The past is not really the past until it has been registered. Or put another way, the past has no meaning or existence unless it exists as a record in the present.” [47, pp. 67–68].[5]

Eugene Wigner, co-recipient of the 1963 Nobel Prize in physics for his contributions to quantum and nuclear physics stressed “the essential role played by the consciousness of the observer” [201, p. 251], because of quantum mechanics.[6]

The American physicist and Cornell university professor David Mermin, well known for his work in statistical and condensed matter physics and who also worked a lot on foundations of quantum mechanics, wrote in 1981: “We now know that the moon is demonstrably not there when nobody looks.” [124, p. 397].

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[4] Reference [109], quoted and translated by M. Jammer [108, p. 161].

[5] We will come back to the ideas of Jordan and of Wheeler in Chap. 4.

[6] To be fair to Wigner, one must add that his ideas on the role of consciousness in quantum mechanics changed over time (see Esfeld [72]).

But not everybody agreed with the Copenhagen interpretation. Its most famous critics, before World War II, were Albert Einstein and Erwin Schrödinger.

In a letter to Schrödinger, Einstein referred to Bohr as the "Talmudic philosopher" for whom "reality is a frightening creature of the naive mind" [66]. Einstein also referred to Bohr as [68] "the mystic, who forbids, as being unscientific, an enquiry about something that exists independently of whether or not it is observed [...]".

Schrödinger complained that "Bohr's [...] approach to atomic problems [...] is really remarkable. He is completely convinced that any understanding in the usual sense of the word is impossible." [171]. He also wrote: "If I were not thoroughly convinced that the man [Bohr] is honest [...] I would call it intellectually wicked." [...] [177]

Schrödinger did not even try to hide his feelings when he wrote to his friend Max Born, who was asserting "time and again that the Copenhagen interpretation is practically universally accepted": "Have you no anxiety about the verdict of history? Are you so convinced that the human race will succumb before long to your folly?" [178].

After World War II, the critique of the mainstream view was taken up for the main part by the American physicist David Bohm and the Irish physicist John Bell. The latter was once interviewed by people who recalled that: "We first asked Bell over the telephone whether he himself felt he had demonstrated that 'reality doesn't exist'. He responded by warning us that he is an impatient, irascible sort who tolerates no nonsense." [11, p. 86].

Can anybody seriously ask a physicist whether he has proven that "reality doesn't exist"?

We shall come back, in Chap. 10, to the historical disputes among physicists concerning quantum mechanics, but all this shows that there is indeed something very bizarre about quantum theory. The intention of this book is to separate the wheat from the chaff. We mean to explain, in the simplest possible terms, what is so bizarre about quantum mechanics, while also trying to show that its mysteries can nevertheless be understood in rational terms.

There are three main conceptual issues associated with quantum mechanics, to which we shall refer below as being *the three fundamental questions.*

1. *The role of the "observer".* Since Copernicus, modern science has de-centered human beings from its explanations of reality, first by realizing that the Earth is not at the center of the Universe and then by showing that humans are not the object of a special act of creation, but rather the result of a lengthy contingent evolution. Quantum mechanics seems to have put humans back

at the center of the picture: it is sometimes claimed that it abolishes the distinction between subject and object or that it gives an active role to human consciousness within the theory. But if the human observer has a role in shaping reality, one must ask how reality was shaped before humans existed. If humans got there in the first place through evolution, how did that work? Biology is based on chemistry, whose mechanisms are explained ultimately through quantum mechanics. But what role could the human subject have had during this whole process, before its appearance as *Homo sapiens*?

2. *The issue of determinism.* Determinism means that future events are determined by preceding ones. So, if a system is deterministic and if its present situation is given, all its future states are fixed (see Chap. 3 for a more detailed discussion).
   However, as we shall see, quantum mechanical predictions are essentially statistical. This means that, if the present situation of a quantum mechanical system is given, quantum mechanics only assigns various probabilities to what the future state of that system may be. Does that imply that quantum mechanics signifies the end of a deterministic world-view? Does it explain or justify "free will"?
3. *The issue of locality.* One of our most basic experiences of the world is that when we act on it, we act on it *locally*. For example, I can act on something by touching it. I can communicate with someone else by speaking; but this means that a sound wave propagates from place to place between us. Even if I use radio, TV or the Internet to communicate, all these means rely on waves propagating at a finite speed from where I am to the recipient of my message. That is what one calls locality: every action from one place to another results from a propagation of something (waves for example) at a finite speed between those places.
   There is nothing in our experience of the world that suggests that one might act *instantaneously at a distance*.
   However, in quantum mechanics the non-existence of instantaneous actions at a distance is not so obvious. So, our third issue will be whether quantum mechanics does imply the existence of instantaneous actions at a distance. If yes, does that justify beliefs such as telepathy? And does that conflict with the theory of relativity's notion that "nothing travels faster than light"?

A first goal of the book will be to explain why quantum mechanics has raised such issues and to give the traditional answers to those questions. Roughly speaking, these answers are that quantum mechanics has given a fundamental role to measurements or observations within physics and has refuted determinism. On nonlocality, the traditional answers are ambiguous and often confused.

On the other hand, the answers that we will try to defend in this book are, in a nutshell:

1. *The role of the "observer".* There is no need whatsoever to give a special role to the observer or even to observations in order to account for the quantum phenomena.
2. *The issue of determinism.* There is a way to account for the quantum phenomena in a deterministic theory, although a rather special one.
   Those answers to [1] and [2] are based on the works of Louis de Broglie, a French Nobel Prize in physics, David Bohm and John Bell.
3. *The issue of locality.* Certain facts discovered thanks to quantum mechanics do imply that there exist in Nature instantaneous actions at a distance. This discovery follows from an argument partly due to Albert Einstein, Boris Podolsky and Nathan Rosen and partly to John Bell. This does *not* justify unscientific beliefs, such as telepathy, but it does create a tension with the theory of relativity.

As we shall explain below, the main problem with the usual formulation of quantum mechanics is that it is perfectly capable of predicting, with spectacular precision, the statistical results of experiments (nobody denies that), but is not saying anything definite about what is happening in the physical world outside the laboratories. Physicists do have pictures of what is going on in the world, but those pictures are not part of the theory, which speaks only of what happens when quantum objects are being 'measured'. And, sometimes, these pictures are contradicted by logical but relatively unknown consequences of the quantum theory itself.

Since the views defended here are not considered orthodox by most physicists, there is a serious ethical problem in defending a heterodox view of science in a popular book. Why not first convince the scientific community before exposing one's own views to the general public? There are three answers to that objection: one is that I shall carefully distinguish between what is generally accepted and what is not.[7]

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[7] Chapters 2 and 4 are completely standard. Chapter 5 is based on standard but not very widely known results, and therefore the conclusions drawn there are not universally accepted. This is even more true of Chap. 7, which is based on easy-to-prove theorems and experimental facts, but where the conclusions are even less generally accepted than those of Chap. 5. Chapter 8 is the one chapter of this book that is highly controversial (but which is also central from our point of view). In Chap. 9 we criticize a popular but by no means generally accepted theory. Finally, Chaps 3, 6, 10 and 11 concern philosophy and history of science, subjects on which there is never a consensus.

The second answer is that there are many popular books explaining views different from mine and I shall refer to several of them, so that the reader can decide which view is the most plausible (see "Further Reading" at the end of the book). Finally, it is not really true that a "scientific consensus" exists on the issues discussed in this book. There used to be one, and it is still the basis of most textbooks on quantum mechanics. But right from the very beginning of the theory, there were famous dissenters, notably Einstein, but also de Broglie and Schrödinger. Later, David Bohm and John Bell were also critical of the orthodoxy, even though their voice was barely audible. But now, any conference on "foundations" of quantum mechanics will see a variety of positions and interpretations confronting each other, and none of them can claim to be either the orthodox view as presented in textbooks or a new orthodoxy.

It should also be emphasized that, contrary to popular books praising, say, alternative medicines, there is nothing "anti-scientific" in this book: we are not denying any application or experimental prediction of quantum mechanics. We are only concerned with what quantum mechanics means, not with its empirical correctness.

There is a cast of characters that will appear repeatedly in this book: Einstein, Bohr, Heisenberg, Schrödinger, de Broglie, Bohm, Bell, Feynman, Wheeler, Wigner, and many less important figures, arguing among themselves about the issues raised here.

By showing that science does not necessary produce a consensus over every major subject, in particular the one treated here, we hope to give a more positive image of science as an open endeavor rather than as a producer of dogmas. The uncertainties are challenges rather than weaknesses of the endless work in search of scientific knowledge. There is nothing anti-scientific in this view of science.

This book is not written especially for physicists, but if aspiring physics students read it, they are likely to be told during their studies that the issues raised here are irrelevant or "purely philosophical" or even "metaphysical". These claims are also found in the writings of physicists defending the mainstream views. Two arguments are often given to justify these claims:

1. The quantum theory works perfectly well, in all known circumstances; it is not contradicted by any experiment and leads to many technical applications.
2. The goal of physics is solely to predict results of experiments performed in laboratories and to produce technical applications.

The first point is correct, but it is precisely *because* it works so well that trying to understand why it works makes sense. Obviously if quantum mechanics worked half of the time, so to speak, there would be no reason to try to understand it in depth. Many models in physics are known to be applicable within certain limits and, once we know that, there are no further questions to be raised about those models. But quantum mechanics works on all known scales and is not contradicted by any experiment whatsoever.[8] Isn't it therefore worthwhile to ask *why* it works so well?

For the second point, there are several answers. The goal of science has always been, at least in part, to understand the world. Otherwise, why would anybody worry about the origin of the Universe or about distant galaxies? Certainly such studies, by themselves, have no technological applications. And of course, celestial mechanics, which gave rise to modern science, had no applications at all in its early development. Moreover, the theory of evolution had no application when it was introduced, even though it greatly changed our understanding of the world.

In fact, for most people, what is interesting in science is what it tells us about our vision of the world and of ourselves in it.

The idea that the only goal of physics is to predict results of experiments performed in laboratories inverts the means and the goal. Experiments are needed to test our theories in order to avoid falling into idle speculation or "metaphysics", but our theories are about the world, not about the experiments themselves.

Of course, it may be that it is simply impossible to understand the quantum world and that we have to content ourselves with predicting results of experiments. One could say that our experiments amount to "asking questions" about Nature; we do get answers and they can be predicted, at least statistically, but nothing more can be said; in particular, one cannot understand what is going on inside the experimental apparatuses.

Why not? After all, who are we but somewhat evolved creatures? Why should we expect to be able to understand the world as it is? Isn't the fact that quantum mechanics looks ununderstandable simply a consequence of the limitations of our minds? That may be the case, but one needs some argument to reach this pessimistic conclusion, rather than simply settling for the assertion.

Besides, there is a serious issue of coherence raised by the notion that the only goal of physics is to predict results of experiments performed in laboratories. If indeed that was all there is to physics, why do experiments in the first place?

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[8] Putting aside the problem of having a quantum theory of gravitation, which is a deep unsolved problem, but that cannot be regarded as a refutation of quantum mechanics.

The need to finance costly experiments is "sold" to politicians and the public by saying that we are discovering the fundamental laws of Nature. But if, in quantum mechanics, we give up the idea of understanding the world and restrict physics to be "exclusively about piddling laboratory operations", as John Bell puts it [12, p. 34], then how can we claim that we are trying to find the fundamental laws of Nature? What would the funders say if they read the statements by physicists who claim that their goal is merely to predict results of experiments performed in laboratories, and nothing else? Wouldn't they at least be puzzled and ask for some clarification? Isn't it therefore simply a matter of intellectual honesty to ask ourselves how we would clarify those statements? Physicists may have answers to those questions, but it may be worthwhile to see what they are and to discuss them.

Although nothing in this book will be very technical and we refer to [36] (and references therein) for more details, we shall put some extra material in the footnotes and appendices, either for the sake of precision or to provide the reader with more references.

We should also warn the reader that there will be nothing "fancy" in this book: no Big Bang, no Black Holes, no String Theory, no Quantum Gravity.... It is our contention that many of these fancy topics, about which several popular books have already been written, are difficult to grasp if the basic conceptual problems of quantum mechanics (the subject of this book) are not clarified first. Furthermore, we claim that, in order to achieve that clarification, it is sufficient to study the simplest physical situations.

We shall not quite follow the preacher's maxim: "First, tell them what you are going to tell them, then tell them, then tell them what you have told them", but we shall not refrain from repeating ourselves. This may be bad style, and we apologize to the reader who may be annoyed by repetitions, but we believe that it is easy enough to overlook a crucial point if it is only made once.

To keep a difficult subject as clear and simple as possible, we shall mainly rely on elementary drawings, illustrating both experiments and theory. The only mathematical concept that we use is the one of function, but only in very simple cases. The drawback of this approach is that it obliges us to ask the reader to take for granted some mathematical results, that will be stated verbally, without formulas.

Moreover, the book can be read in different ways. We put an asterisk on the title of sections that can be skipped at first reading and give a detailed summary at the end of each chapter for those who find that chapter either too difficult or too easy in order to allow them to continue with the rest of the book. Some readers might find it useful to start by reading the summary before reading the details of the chapter. Finally, all the main theses of the book will be summarized in Chap. 12 (the reader may want to jump to it to see where we are headed).

The intention of this book is not to give final answers to the conceptual problems of quantum mechanics, but rather to open the reader's mind to the possibility that answers can be given beyond what is taught in standard quantum mechanics courses or in most popular books on the subject. The student I once was, who could not understand what he was told about what quantum mechanics *meant*, would have been delighted to read such a book.

Before getting started, and although our goal is *not* to explain quantum mechanics as a physical theory but only to discuss its conceptual aspects, it may be useful to explain briefly where quantum mechanics came from (but reading this section is not necessary to understand the rest of the book).

1.1 Historical Background*

1.1.1 Pre-quantum Physics

Painting in broad strokes, we shall distinguish four periods in the history of physics, before quantum mechanics. First, the Newtonian revolution in the seventeenth and eighteenth centuries gave us laws that govern the motion of planets, projectiles, satellites etc. It all relied on the law of universal gravitation saying that bodies attract each other through a force proportional to the product of their masses and decreasing with the square of their distance. This, plus the idea that the acceleration of a body is proportional to the force exerted on it by other bodies allowed Newton and his followers to derive the trajectories of the planets in the solar system, that had been previously stated by Copernicus, Kepler and others.[9]

This was one of the major conceptual revolutions in the history of mankind: while previously various disciplines could record empirical regularities, it was only Newton (and other scientists at that time) who could use mathematical formulas (that Newton largely developed himself)[10] to compute and predict how objets will behave in hitherto unobserved circumstances (like launching a new projectile).

In the nineteenth century, there were two new developments. First, the discovery of new forces that were not of a gravitational nature: electricity and

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[9] See Appendix 7. A for more details.

[10] This was the theory of derivatives and integrals, also developed independently by Leibnitz.

magnetism. After several stages, the laws governing those forces were unified by the Scottish physicist James Clerk Maxwell into a theory called electromagnetism. The latter postulates the existence of waves, also called fields, that are created by charged particles but that also guide their motion. Light is an example of an electromagnetic wave, but there are many others, like X-rays, or radio and TV waves. Roughly speaking, the emitter of a radio or TV station transforms words and pictures into electromagnetic waves and the latter produce in your radio or TV a motion of charged particles, electrons, that then generates sound and light.

One could think of those waves as water waves on which a little boat (the charged particle) floats, except that (and that is a big difference!) here there is no water: those waves are supposed to propagate in a vacuum. This, as well as the nature of the gravitational force which also acts without any medium, is quite mysterious and we shall come back to that in Chap. 6.

The third period was the development of statistical methods. The industrial revolution was advanced by the steam engine and thermodynamics was describing how those engines work.[11] During the second half of the nineteenth century, thanks to the combined work of the Austrian physicist Ludwig Boltzmann, the American physicist William Gibbs and of Maxwell and Einstein, one managed to explain the laws of thermodynamics by applying statistical reasonings to the motions of myriads of molecules or atoms, whose very existence was disputed at that time.[12]

The last stage, which was quite revolutionary, was marked by the two theories of relativity, the so-called special one, developed in 1905, due mainly to the works of the Dutch physicist Hendrik Lorentz, the French mathematician Henri Poincaré, and Albert Einstein, and the general one developed around 1915, due mainly to Einstein and the German mathematician David Hilbert.

The special theory of relativity basically modified Newton's laws of motion in order to make them compatible with the newly discovered laws of electromagnetism. Indeed, when electromagnetism was developed, it was realized that this theory was not compatible with some aspects of the classical laws of Newton and that was a major problem. On the other hand, the general theory of relativity replaced Newton's theory of gravitation. We shall briefly discuss the meaning of the special theory of relativity in Sect. 7.7.

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[11] The most famous law of thermodynamics is the one saying that entropy increases, but we shall not discuss it.

[12] The existence of atoms was definitively established by Einstein in 1905 when he derived the laws of motion of objects that are very small but visible with a microscope, bombarded by those invisible atoms (this motion is known as Brownian motion).

1.1.2 Quantum Physics

Quantum physics emerged from troubles within the classical world view sketched in the previous subsection. Those came from different sources. One of them was the so-called specific heats of solids, which is the way the temperature of a body changes when it absorbs a certain amount of heat. There was a well defined classical prediction for those quantities that turned out to be completely wrong at low temperatures. Similarly, there was a type of electromagnetic waves[13] whose behavior was radically at variance with what was classically expected.

However, these worries did not look serious enough to cause a major revolution.

That second problem was "solved" in 1900 by the German physicist Max Planck, who decided, in a completely ad hoc way, to treat those waves, whose energies were previously thought to take a continuum of values, as if they were made of integer multiples of a fixed amount of energy, called "quanta" of energy, and related to the frequency of the wave; in that way, he was able to deduce the observed behavior of those waves. That was a major progress, but with no understanding of why it worked. Planck received the Nobel Prize in Physics in 1918 for that discovery.

A next step was taken by Einstein in 1905, with his explanation of the photoelectric effect, namely the fact that light can kick electrons out of atoms only if it has a sufficiently high frequency (classically, one would expect that this kicking out phenomena would depend on the intensity of the light and not on its frequency). This was explained by Einstein by postulating that light is made of some sort of particles, quanta of light, called photons, whose energy is proportional to their frequency, so that a high frequency would mean a high energy of the photons, hence an ability to kick out electrons. Einstein was awarded the Nobel Prize in Physics in 1921 for "his discovery of the law of the photoelectric effect".[14]

In 1907, Einstein applied Planck's method of quanta of energies to account (more or less) for the specific heats of solids. His method was refined in 1912

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[13] Called the black body radiation, namely the radiation inside a closed cavity whose walls are opaque to radiation.

[14] One might wonder why Einstein did not get the Nobel Prize for his two theories of relativity. This is because they were still considered somewhat philosophical, even though, by 1921, they had several experimental confirmations. In particular the criticism of relativity by the philosopher Henri Bergson played a role in the attitude of the Nobel committee, even though Bergson's objections were based on a complete misunderstanding of Einstein's theory, see http://nautil.us/issue/35/boundaries/this-philosopher-helped-ensure-there-was-no-nobel-for-relativity and [38].

by the Dutch physicist Peter Debye, with results quite in agreement with observations.

In 1913 came Bohr's model of the atom, which is still taught in elementary physics and chemistry classes. This model accounted for the fact that the radiation emitted by atoms was again taking a discrete set of values rather than a continuous one (a discrete set is, for example, any finite set or the set of integers $1, 2, 3\dots$ or their inverses $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$). His model described the atom as a miniature solar system, with the nucleus in place of the sun and the electrons circling around it like the planets. But they circled on a well-defined set of orbits, having different energies. The discrete values of the emitted energies came from electrons jumping from one orbit to another and emitting the energy difference between those orbits.

This was another major success in accounting for the observed phenomena, but still without any theoretical understanding of why it worked.

Things were so puzzling that in 1911 Einstein, seeing an insane asylum in Prague, said to the physicist and philosopher Philip Frank: "These are the madmen that do not occupy themselves with the quantum theory."[15]

What is described here is called the old quantum theory, or the pre-quantum theory. The breakthrough came during the years 1924–1927. First Louis de Broglie suggested that, in the same way that waves such as light might be associated with particles, the photons (as was suggested by Einstein), matter particles such as electrons might be associated with waves, but he did not have a full-fledged theory about those waves (we shall discuss this idea in depth in Chap. 8).

Then, independently of de Broglie and of each other, first Werner Heisenberg, and slightly later Erwin Schrödinger, developed the modern quantum theory. Heisenberg found a way to compute in a concrete situation the discrete values taken by the energy of a system. This was then generalized by him together with other German physicists, Max Born and Pascual Jordan. Important work in that direction was also due to the Swiss physicist Wolfgang Pauli and the British physicist Paul Dirac.[16]

Schrödinger on the other hand associated to physical systems a mathematical concept, the wave function (discussed in Chap. 4) and wrote down equations telling how this object changes over time. He could also show that his method and the one of Born, Heisenberg and Jordan led to the same results.[17]

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[15] See [83, p. 98].

[16] Their method was basically algebraic and relied on the use of matrices, with which most physicists were not very familiar at that time.

[17] Schrödinger's method was more analytic and was based on differential equations. It is basically his method that is taught and used nowadays.

For a while, there was a great puzzlement about the meaning of the newly introduced concepts. But things were wrapped up, so to speak, at the Solvay Conference, held in Brussels in October 1927, where all the great physicists of the time met and the so-called "Copenhagen interpretation" was generally accepted.[18]

The transistor, on which modern electronics is based, was discovered thanks to quantum mechanics, which allows sometimes electrons to jump over a barrier that they could not go through classically. Lately quantum mechanics has found applications in cryptography, as well as in teleportation of information and quantum computing; we shall briefly discuss these applications in Sect. 7.6.

Quantum mechanics also allowed chemists to understand how atoms are bound together in molecules and it lays at the basis of atomic and nuclear theory.

After World War II, quantum mechanics was further extended to a quantum theory of electromagnetic waves and is at the basis of all the physics of elementary particles. It has also found numerous applications in astrophysics and cosmology.

1.2 Outline of the Book

Let us first say that this outline can be skipped over, as it is mainly intended to be a useful reference for the reader.

The first mystery raised by quantum mechanics is that, as physicists often say, quantum objects can be in two places at once or be in a "superposition" of states with different and mutually incompatible properties. There are good experimental and theoretical reasons why they employ this language and we shall explain them in Chap. 2.

This idea of superposition is at the basis of the notion that 'observations' play a central role in the physical theory, since, when an observation is made on a system which is in a superposed state, the system is supposed to 'jump' or to 'collapse' into one of those states (we never see objects having mutually incompatible properties). In other words, since we never observe directly a superposed state, observation is supposed to destroy these superpositions.

This will do justice to the way physicists often speak about quantum phenomena. The phenomena are strange and all the talk about observations

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[18] We shall discuss it throughout the book, but especially in Chaps. 4, 5 and 10.

affecting reality is not purely arbitrary, nor based on pure prejudice, although, as we shall see later, it is not inevitable either.

Of course, an obvious question is: who is "observing"? A purely physical device in the laboratory that records the result of an experiment or a human subject? This question is often quite central in popular discussions of quantum mechanics.

In Chap. 3, we will make a small "philosophical" detour in order to define what determinism means, what it implies concerning "free will" and how probabilities are used in physics.

Next we shall explain, in Chap. 4, the language used by physicists to predict what happens with these "superposed states".

We shall then try to understand, in Chap. 5, what that language means. We shall see that some natural ways to understand it unfortunately run into very serious difficulties.

Indeed, it is in relation to the phenomenon of superposition that Schrödinger introduced his famous *Gedankenexperiment*[19] of a cat who, if one follows the prescriptions of quantum mechanics to their logical conclusions, would be in a "superposed" state of being "both alive and dead". Of course, Schrödinger regarded this as a *reductio ad absurdum* of the quantum mechanical formalism.

In Chap. 6, we will make another "philosophical" detour in order to clarify certain notions such as "realism" and "observations", that are often encountered in discussions of quantum mechanics. We will argue that there is no way to solve the problems of quantum mechanics by modifying our philosophy, for example by giving up "realism".

In Chap. 7, we shall turn to the second mystery: the fact that quantum mechanics does imply that there exists a certain subtle form of action at a distance in the world. We shall also discuss to what extent this is compatible with Einstein's theory of relativity.

In Chap. 8 we shall explain, without mathematics, how to formulate quantum mechanics without referring to "observers". This is a theory due to the French physicist Louis de Broglie and the American physicist David Bohm. In two words, that theory says that 'matter moves' or, more precisely, that quantum particles do have positions at all times and therefore also have trajectories and velocities.

Contrary to what is often alleged, this is not contradicted by any known quantum facts or arguments.

The theory was introduced at approximately the same time as ordinary quantum mechanics and its "Copenhagen" interpretation, between 1924 and

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[19] This means a thought experiment, viz. an experiment that illustrates the theory and is not really performed in laboratory, but using a German word in English always makes things look more serious.

1927, by Louis de Broglie, but it was rejected at the time by a large majority of physicists, and ignored even by critics of the Copenhagen school, like Einstein and Schrödinger. The theory was then abandoned by its founder, only to be rediscovered and completed by David Bohm in 1952, then further developed and advertised by John Bell.

In this theory, there is no special role given to the observer or to the "measuring device". All the usual quantum predictions are recovered in the de Broglie–Bohm theory, and the nonlocality of the world is also to some extent explained by that theory.

The de Broglie–Bohm theory is far from being the only theory proposed as an alternative to ordinary quantum mechanics. A rather popular alternative is the idea of "many-worlds", which says that the Universe constantly splits into zillions of copies of itself, and, in each of these worlds, a copy of each of us lives an independent life, while being unaware of what happens in the other worlds.

While the "many-worlds" idea may have a certain science fiction kind of attraction in the minds of some people (hence, its popularity), we shall show in Chap. 9 that it is not really consistent.

It is natural to ask why the views advocated here, those of de Broglie, Bohm and Bell are often ignored. To explain that, one has to re-examine the history of quantum mechanics, which is what we shall do in Chap. 10. We claim that the ideas of Einstein, Schrödinger, de Broglie, Bohm and Bell were not really understood and, for that reason, were never really refuted. In their famous debate, Bohr did not really reply adequately to Einstein; Schrödinger's cat paradox was ignored; and the theories of de Broglie and Bohm were rejected without being examined. Finally, Bell's result on nonlocality was almost universally misunderstood.

Because of the philosophical problems to which it is linked, quantum mechanics has had a much greater cultural impact than most scientific theories. It has certainly inspired postmodernist philosophy as well as cultural studies, various pseudo-sciences and even contemporary art. In Chap. 11, we will make a brief tour of the ways in which quantum mechanics has been mixed with pseudo-science, mysticism, religions, philosophy, politics, ideology and social sciences and discuss to what extent this impact is due to misunderstandings and misrepresentations of quantum mechanics.

Finally, we will summarize our main theses of this book (Chap. 12) and provide a glossary of the main concepts encountered in the book, as well as some biographical details about the scientists encountered here. We will also give to the reader a bibliography of "further readings", including books written from a perspective different from ours.

2

The First Mystery: Interference

In *Alice Through the Looking-Glass*, the Queen says to Alice that she used to believe six impossible things before breakfast. In this book, we'll ask you to believe only two "impossible things" (before or after breakfast, as you wish): the *superposition of states* or the idea that particles can be in two different mutually incompatible states at once and *nonlocality*, which means that, in some circumstances, one can act at a distance, arbitrarily far, and instantaneously.

But, unlike what the Queen was speaking about, these "impossible things" are well-established facts. The first "impossible thing" will be explained in this chapter and in Chap. 4. We shall see in Chap. 8 that this "impossibility" is not only possible, but is not really that surprising. The second "impossible thing" will be explained in Chap. 7 and will be partly clarified in Chap. 8, but will nevertheless remain baffling.

We invite the reader to put herself or himself mentally in the shoes of a Sherlock Holmes or a Hercule Poirot and pay attention to what is really proven in the experiments below as opposed to what is often asserted in loose talk about them, and that we shall also explain.

As in every good detective novel, the reader has to be patient before learning about the denouement of the plot, which will come only in Chap. 8. Until then, we shall not try to demystify quantum mechanics too much, but rather explain the language physicists use and why they use it.

Indeed, before being demystified, the reader has to understand what is strange in quantum mechanics and, in some sense, "mystifying".

J. Bricmont, *Quantum Sense and Nonsense*, DOI 10.1007/978-3-319-65271-9_2

2.1 The Double-Slit Experiment

To discuss the double-slit experiment, we shall follow the presentation by Feynman [78], and consider the behavior of three types of objects, bullets, waves and electrons, in a situation where they move towards a wall with two slits or holes in it, and where there is a second wall, behind the first one, on which the arrival of those objects is recorded.

Let us start with bullets. This is illustrated in Fig. 2.1: the little box on the left of each part (a, b, c) of the figure emits the bullets. They are sent one by one towards a wall with two slits in it, each of which can be open or not. If they miss the slits, they are absorbed by the first wall. If they pass one of the slits, they arrive somewhere on a second wall, behind the first one and that arrival is recorded. There is no aiming of the bullets towards one of the slits.

Since the bullets cannot always be sent with exactly the same initial position or the same initial velocity, there is some "random" distribution of the bullets on the second wall. In Fig. 2.1 one sees what happens when a set of bullets are sent when only the upper slit is open (part a), when only the lower slit is open (part b), and when both slits are open (part c). Each blue dot on the second wall represents the detection of one bullet, and the blue curves indicate the density of impacts of the bullets.

In part (c) of Fig. 2.1, when both slits are open, the density of impacts of bullets is simply the sum of the densities when each slit is open (see part (a) and (b) of Fig. 2.1).

There is no particular mystery here.

Consider now a wave, say a water wave, sent through the slits. Then, we get interference effects, shown in Figs. 2.2, 2.3 and 2.4. To understand that, think of throwing two pebbles in a pond, at some distance from each other. One

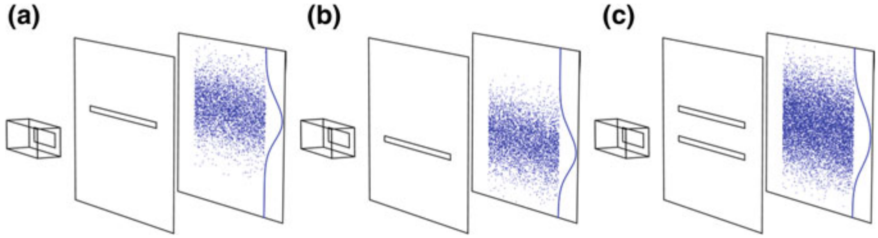

Fig. 2.1 The double-slit experiment with bullets. The little box on the left of each part **a**–**c** of the figure emits the bullets. Part **a** shows what happens when only the upper slit is open, part **b** when only the lower slit is open, and part **c** when both slits are open. Each *blue dot* on the second wall represents the detection of one bullet, and the *blue curves* indicate the density of impacts of the bullets (A. Gondran cc by-sa 4.0)

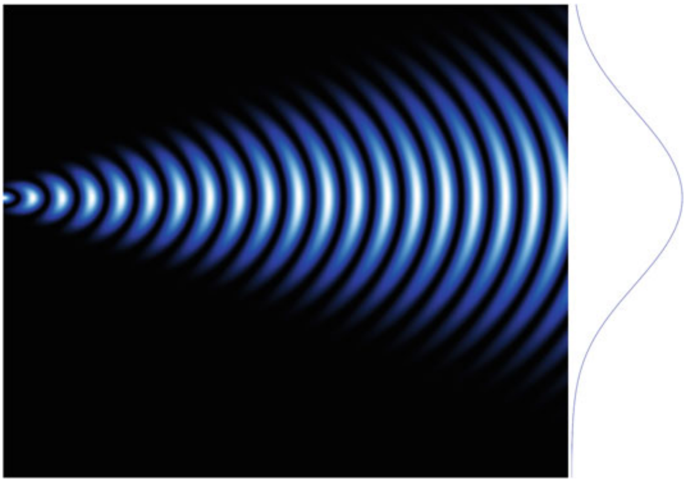

Fig. 2.2 The double-slit experiment with waves when only the upper slit is open. The intensity of the wave is shown in *white* (more intense) and *blue* (less intense). The *blue curve* on the right of the figure indicates the intensity of the wave on the second wall (A. Gondran cc by-sa 4.0)

Fig. 2.3 The double-slit experiment with waves when only the lower slit is open. The intensity of the wave is shown in *white* (more intense) and *blue* (less intense). The *blue curve* on the right of the figure indicates the intensity of the wave on the second wall (A. Gondran cc by-sa 4.0)

can see that the waves generated by the pebbles interfere: at some places the interference is constructive and the waves add to each other, at other places it is destructive and the waves subtract each other.

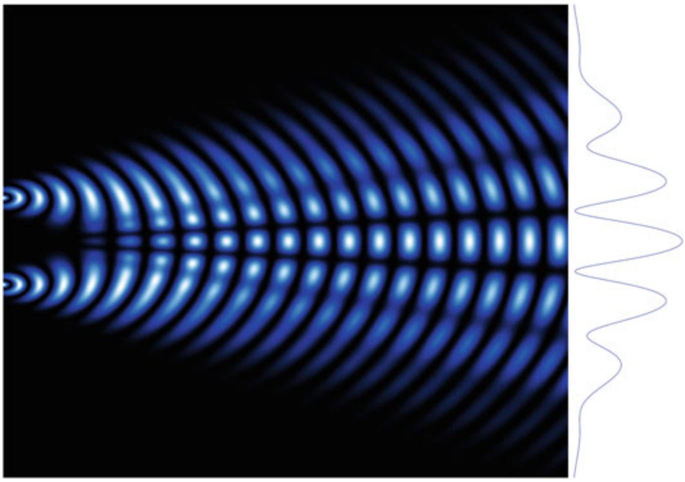

Fig. 2.4 The double-slit experiment with waves when both slits are open. The intensity of the wave is shown in *white* (more intense) and *blue* (less intense). The *blue curve* on the right of the figure indicates the intensity of the wave on the second wall. This exhibits an interference effect (A. Gondran cc by-sa 4.0)

In the two slits experiment with waves, if only one slit is open, we get the results of Figs. 2.2 and 2.3, which are not very different from what one gets with bullets, see parts (a) and (b) of Fig. 2.1.

But, when both slits are open, the wave goes through both slits and each slit acts as a source of a wave propagating towards the second wall (like the two waves produced by the two pebbles). The intensity of the wave indicated by the blue curve on the right of Fig. 2.4 is *not* the sum of the intensities of the waves indicated by the blue curves on the right of Figs. 2.2 and 2.3. Note that at some places the intensity of the wave on the second wall is less than what it is when only one slit is open, but that is exactly what one would expect for waves, because of interference.

In order to make the comparison with what happens with bullets easier, we have included in Fig. 2.5 a three dimensional picture with all three situations arising with waves (only the upper slit open, only the lower slit is open, and both slits open).

So far, so good; there is nothing surprising here and these two behaviors are called "classical": one for particles (bullets), one for waves.

The surprises come when one does the experiment with electrons. Electrons are little particles with a negative electric charge, and they surround the nucleus in atoms. When moving freely, they carry electricity.

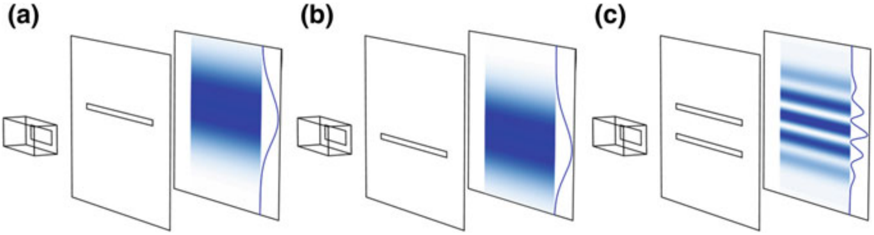

Fig. 2.5 The double-slit experiment with waves in three dimensions. Part **a** shows what happens when only the upper slit is open, part **b** when only the lower slit is open, and part **c** when both slits are open. The *blue areas* on the second wall in each part correspond to the detection of the intensity of the wave. The *blue curves* indicate the intensity of the wave on the second wall. This exhibits an interference effect (A. Gondran cc by-sa 4.0)

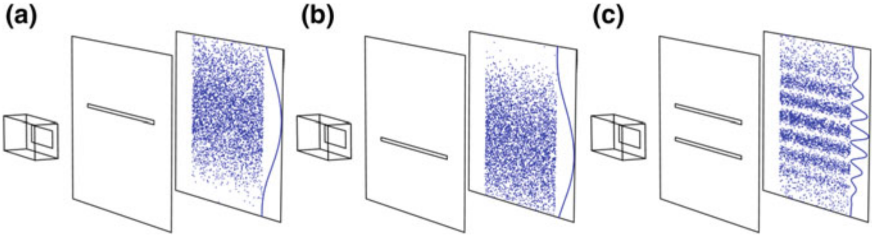

Fig. 2.6 The double-slit experiment with electrons, shown in three dimensions. Part **a** shows the detections of electrons on the second wall when only the upper slit is open. Each *dot* on the second wall corresponds to the detection of one electron. Part **b** shows the detections of electrons on the second wall when only the lower slit is open, and part **c** shows the detections of electrons on the second wall when both slits are open. In all three parts, the *blue curves* indicate the density of impacts of the electrons on the second wall (A. Gondran cc by-sa 4.0)

For the double-slit experiment with electrons, we show the situation in three dimensions, see Fig. 2.6, when only the upper slit is open, when only the lower one is open and when both slits are open.

When only one slit is open, one obtains the results of parts (a) and (b) of Fig. 2.6, where the blue curves describe the densities of the impact of the electrons on the second wall. The results are similar to what one obtains with bullets, see parts (a) and (b) of Fig. 2.1.

Now, *when both slits are open*, one obtains the results of part (c) of Fig. 2.6, which is similar to what one obtains with waves, see Fig. 2.4 and part (c) of Fig. 2.5. Note that there are places where the density of particle impacts is *less* when both slits are open than when only one of them is.

This phenomenon and many other related phenomena are called *interference phenomena*, because whether one slit is open or not seems to interfere with the behavior of the particles going through the other slit. This is the essence of the first quantum mystery!

Note that electrons are sent (in principle) one by one, so that no explanation of their behavior can be based on a possible interaction between the electrons. Note also that one detects always the electrons in one piece and at a precise location, no matter where the second wall is placed.

The mystery thickens if one puts a detector behind one of the slits, say the lower one, that would allow us to determine whether the particle goes through that slit.

Then, the interference pattern disappears (see Fig. 2.7)! And that is true even if one considers only the events where the detector *does not* detect a particle; which means that, in order for the interference pattern to disappear, it is enough that *we are able to know* through which slit the particle went (here, through the upper one), simply by checking that it does not go through the other slit.

Another way to remove the interference pattern is to put the second wall sufficiently close to the first one. Then, the interference pattern observed in part (c) of Fig. 2.6 essentially disappears, see Fig. 2.8.

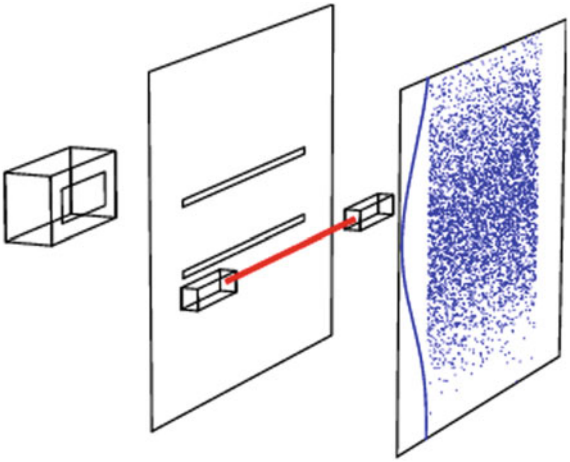

Fig. 2.7 The double-slit experiment with electrons when both slits are open, but where one puts a detector that detects particles going through the lower slit. The detector is indicated by the *red line* behind the first wall. Each *blue dot* on the second wall corresponds to the detection of one electron, which means that it was not detected after the first wall, and thus that it went through the upper slit. The *blue curve* indicates the density of impacts of the electrons on the second wall. We see that the interference pattern observed in part **c** of Fig. 2.6 disappears and that one gets results similar to what happens when only the upper slit is open (part **a**) of Fig. 2.6) (A. Gondran cc by-sa 4.0)

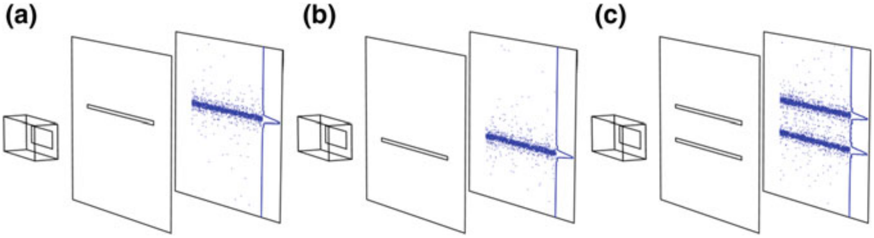

Fig. 2.8 The double-slit experiment with electrons, similar to Fig. 2.6, but where one puts the second wall sufficiently close to the first one (the distance between the walls is less by a factor of ten than in Fig. 2.6, although we have rescaled the figure so that this distance looks the same as in Fig. 2.6). We see that the interference pattern observed in part **c** of Fig. 2.6 essentially disappears (A. Gondran cc by-sa 4.0)

This is sometimes expressed by saying that, if we *look* or if we *know* through which slit the particle went, then it behaves like a particle, but if we do not know through which slit it went, it behaves like a wave.

And this leads to the famous expression: *the wave-particle duality*. Electrons are supposed to have a dual nature: sometimes they are particles, sometimes waves. Moreover, which "nature" they have seems to depend on whether we "look" at them or not!

In the age of Twitter, Sean Carroll, a theoretical physicist at the California Institute of Technology, who is also a cosmologist and author of several popular books, considers that the best answer to "how to summarize quantum mechanics in five words?" comes from the physicist and science writer Aatish Bhatia whom Carroll quotes:

> Quantum mechanics in 5 words. Don't look: wave. Look: particle.
>
> Sean Carroll [40, p. 35]

This is the first time that we refer to *our knowledge* of something as having apparently an impact on a physical situation. What this really means is of course one of the main subjects of this book!

To summarize, the double-slit experiment with electrons leads to an apparent *dead end* : indeed, what does each particle do?

1. Does it go through one slit? But then, why does the opening of the other slit affect the density of particles detected at a given point on the second wall? How come that this density is lower at some places when both slits are open than when only one slit is open?

2. Does it go through both slits? No, because one always detects the particle in one piece at a given place (there is no half-electron), and, by placing the second wall right next to the first one, one can determine through which slit the particle went, see Fig. 2.8.

These phenomena are sometimes described by saying that the particle goes through both slits when they are both open and through one slit otherwise. But what does it mean for a particle to go through two slits whose separation is far greater than the size of that particle? And how does the electron, while moving towards the wall with the slits, "know" whether one or both slits will be open, so as to know whether it should behave as a wave or as a particle?

This double-slit experiment is an example of what Niels Bohr called "complementarity": we can either check through which slit the particle went, when both slits are open, and then the particle behaves as a particle, or we can ignore which slit the particle went through, and then the particle behaves as a wave. But we cannot combine both pictures into a single coherent whole.

Note that "complementary" is used here in a non-habitual fashion: the word usually means that two pictures, say of a person viewed from the front and from the back, may "complement" each other in the sense that they yield a more precise image of that person. But one must stress that, for Bohr, the wave description and the particle one are "complementary" in the sense that they *exclude* each other.

In any case, these "ways of speaking" do not cast much light on what is really going on.

That the double-slit experiment is mysterious is acknowledged by most physicists. For example, in a standard textbook of quantum mechanics, written by two famous Soviet physicists, Lev Landau and Evgeny Lifshitz, one reads:

> It is clear that [the results of the double-slit experiment] can in no way be reconciled with the idea that electrons move in paths. [...] In quantum mechanics there is no such concept as the path of a particle.
>
> Lev Landau and Evgeny Lifshitz [114, p. 2]

And, after describing the double-slit phenomenon, Richard Feynman wrote:

> Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it.
>
> Richard Feynman, [79, p. 145]

Coming back to the three fundamental questions raised in Chap. 1, what does the double-slit experiment suggest?

1. It suggests some sort of "reality created by the observer", since knowing through which slit the particle goes (by putting a detector behind one of the slits) seems to affect its behavior. But nothing can be concluded so far, as to what sort of "observer" we are talking about. Does it have to be a human subject or merely some purely physical interaction with the detector behind the slit in the first wall?
2. As for determinism, it seems that one cannot predict or control where the particle will be detected on the second wall. But that in itself may not be terribly surprising, since it is also true for the bullets, unless one is able to control very precisely their initial position and initial velocity. But one could expect that, the smaller the particle, the more difficult it is to control those values and that our incapability to predict where the electrons will be detected is thus not that strange and, by itself, does not prove indeterminism.
3. There seems also to be something nonlocal going on, since opening one slit affects the behavior of the particle going through the other slit (when only that slit is open), even if both slits are quite distant from each other. But there is no proof that the effect is instantaneous. Moreover, one can check that the size of the effect (the interference pattern) depends on how far apart the slits are and where the second wall is placed. For example, if the slits are far apart, or if the second wall is close to the first one, the interference effects will be small (see Fig. 2.8). So, we cannot conclude from this that any genuine nonlocality, namely any instantaneous action at a distance, exists.

2.2 Delayed Choices

John Wheeler invented a clever experiment, called the "delayed-choice" experiment, that makes the mystery of interferences even more troubling.

One can modify the double-slit experiment as follows (see Figs. 2.9 and 2.10): insert lenses behind the slits that will focus the incoming particles toward two counters C_1 and C_2 that may detect them. If one detects the particle on one of those counters, one will be tempted to conclude that the particle went through the upper slit if counter C_1 detects it, and that it went through the lower slit if counter C_2 detects it.[1]

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[1] As we will see in Chap. 4, quantum mechanics does not assign paths to particles so that saying that the particle went through one slit or the other is not really allowed by the usual formalism. Besides, we will see in Sect. 8.2 that, in a theory that does assigns paths to particles, one can determine through which slit the particle went, but the result is not the one stated here.

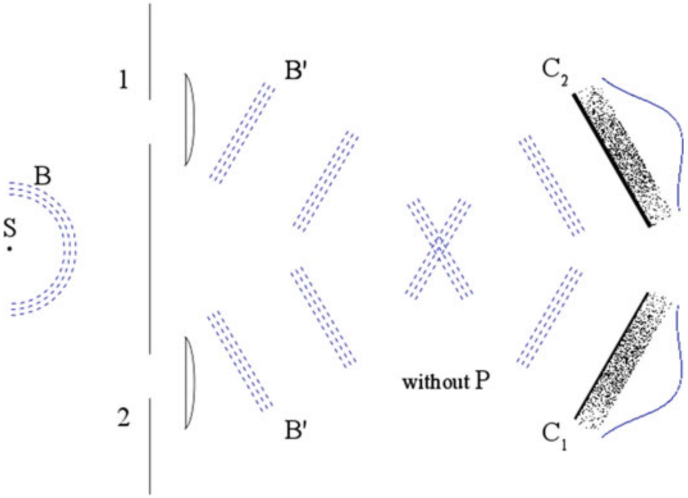

Fig. 2.9 The delayed double-slit experiment with electrons when both slits are open. The source indicated S sends a beam of particles denoted B towards both slits. One inserts lenses, behind the slits, that will focus the incoming beams of particles, denoted B', toward two counters C_1 and C_2 that may detect them. The resulting density of detections of particles on each counter is similar to what one obtains when only one slit is open, namely there are no interference effects. If one detects the particle on one of those counters, one will be tempted to conclude that the particle went through the upper slit if counter C_1 detects it, and that it went through the lower slit if counter C_2 detects it (A. Gondran cc by-sa 4.0)

But one can also insert a detection plate in the region where what appears to be the particles trajectories cross each other, as in Fig. 2.10 (the plate is denoted by P in that figure). Then, one will see an interference pattern as in part (c) of Fig. 2.6, and according to the standard way of speaking, one will say that the particle went through both slits.

But one can choose to insert the detection plate *after* the passage of the particles through the slits. So, it looks like we can decide whether the particle went through both slits or through only one of them by inserting or not the detection plate after the particle had supposedly decided to go through one slit or both!

This is the basis of the claim by Wheeler, that "the past is not really the past until it has been registered" [47, p. 68].

Moreover, Wheeler invented an ingenious scheme where such "experiments" would not take place in the laboratory, but on a cosmic scale: light sent by distant quasars can pass on either side of a galaxy [198]. The experiment here concerns photons instead of electrons, since light is composed of the former, but the phenomena are similar in both cases. The two sides of the galaxy are

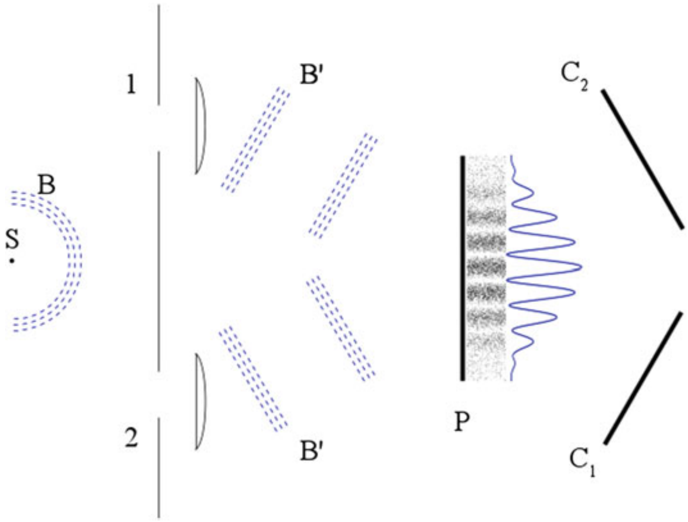

Fig. 2.10 The delayed double-slit experiment with electrons when both slits are open. As in Fig. 2.9, the source indicated S sends a beam of particles denoted B towards both slits. One inserts lenses, behind the slits, that will focus the incoming beams of particles, denoted B', toward two counters C_1 and C_2 that may detect them. But, contrary to what happens in Fig. 2.9, here one also inserts a detection plate (denoted by P in the figure) in the region where what appears to be the particles trajectories cross each other. Then, one sees an interference pattern as in part c of Fig. 2.6, and, according to the standard way of speaking, one will say that the particle went through both slits. Since one can choose to insert the detection plate P *after* the passage of the particle through the slits, it looks like we can decide whether the particle went through both slits or through only one of them (as in Fig. 2.9) by inserting or not the detection plate after the particle had supposedly decided to go through one slit or both (A. Gondran cc by-sa 4.0)

like the two slits here. Then, when the photon reaches the Earth, one can choose to either put some equivalent of the detection plate or not to put it: if we do not put it, we can detect on which side of the galaxy the light went, and, if we do put it, we can "observe" that it went on both sides at once.

If we accept Wheeler's reasoning, this implies that we could decide *now*, by choosing which kind of experiment to perform on the light coming from distant quasars, what happened billions of years ago! In other words, the choices we are making now do not only "create reality", but they also "create" the past. If this were true, it would give us, humans, a more fantastic role in Nature than what most of science fiction can imagine.

2.3 Summary

The phenomenon of interference can be intuitively grasped by thinking of two waves generated by two pebbles thrown in a pond at some distance of each other. At any given place away from the pebbles, the waves will add or subtract each other, which is also expressed by saying that they interfere constructively or destructively.

The first mystery of quantum mechanics is that particles seem to interfere with themselves, although, when one detects them, one always find them localized at some precise place and not spread out as waves are.

We illustrated that with the double-slit experiment: when quantum particles such as electrons are sent towards a wall with two slits in it, and their arrival is recorded on a second wall, beyond the first one, one gets the statistical results illustrated in parts (a) and (b) of Fig. 2.6, when only one slit is open (the blue curves represent the densities of the impacts of the detected particles). This is not mysterious: one should expect a certain “randomness” in the distribution of the initial positions and velocities of the electrons going towards the first wall and thus a certain random distribution of the impacts of the particles on the second wall.

What is mysterious is that, when both slits are open, the density of the impacts of the detected particles is described in part (c) of Fig. 2.6. This looks like Figs. 2.2, 2.3 and 2.4, which shows what happens with waves, and not at all like Fig. 2.1, which shows what happens with classical particles such as bullets.

Since the particles are sent in this experiment one by one and are detected in one piece and at precise locations on the second wall, one would naturally assume that they pass through one slit or the other. But, if they do, one would expect the density of the impacts of the particles on the second wall to be the sum of the corresponding densities when each slit is open, as in part (c) of Fig. 2.1. But that is not at all what part (c) of Fig. 2.6 shows.

This is the first example of interference: opening one slit seems to affect the behavior of particles that pass through the other slit. Besides, if we add a detector behind one of the slits to see whether the particle goes through it or not, then the interference pattern disappears. This happens when one considers the events where the detector does not detect the particle, namely when one considers only the particles that are known to go through the other slit (see Fig. 2.7). This is sometimes expressed by saying that, if we know through which slit the particle goes, then it behaves like a particle, but if we do not know through which slit it goes, then it behaves like a wave. But unless the word “know” here is better explained and clarified, this sort of summary only adds to the mystery of the situation.

This easily leads to the idea of “delayed-choices”: one can insert a detection plate in the modified double-slit experiment of Figs. 2.9 and 2.10 and detect an interference pattern, or not insert it and then “see” through which slit the particle went by placing some detectors further away. In that way, we decide whether the particle “goes through both slits at once” or “goes through only one slit” *after* the particle has apparently “decided” to do one or the other.

This looks truly fantastic: we, humans, “create reality”, not only in the present but also in the past!

3

"Philosophical" Intermezzo I: What Is Determinism?

3.1 Definitions

Since we used the word "determinism" in the previous chapters, it might be good to pause for a moment and to define what this word means and what it implies.

3.1.1 Determinism and Randomness

To define determinism in a physical theory, one can simply say that, if the state of a physical system is given at some time, then there are laws specifying what this state will be at all later times.

When we say that "there are laws", we do not assume that we know those laws but simply that certain systems follow certain laws. After all, we should have no doubt that the planets were moving according to the laws of gravitation before we discovered those laws or even before humans existed.

Let us first define the *state* of a physical system. If one considers the sun and one planet (forgetting about everything else in the Universe), and if the precise positions and velocities of both bodies are given, then their positions and velocities are determined for all future times by Newton's laws of motion, and they can even be expressed in a rather explicit formula.[1] If one considers the sun and all the planets (again, forgetting about everything else) and one specifies the precise positions and velocities of all these bodies at a given time,

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[1] The reason why one need to specify both positions *and* velocities and not positions alone is due to a property of Newton's laws, that we will not go into.

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then their positions and velocities are again determined for all future times by those laws of motion, but there does not exist an explicit formula expressing what those positions and velocities will be.

This extends in principle to the whole Universe, as long as one considers only gravitational forces and one remains within the framework of classical physics.

This also extends to the tossing of a coin, the throwing of a dice or a roulette ball on the wheel: if one specifies exactly the way those objects are thrown (their position, velocity, the way they rotate etc.), then again their future is determined.

In these examples, the state of the systems under consideration is defined by the exact positions and velocities of all the bodies of the system at a given time. Once such a state is specified, the laws of motion determine the state of the system at all later times.

The important word here is "exact": everyone knows that, if the coin, the dice or the roulette ball are thrown a little bit differently, the result may be heads instead of tails, a 6 instead of a 2, or the ball landing on the red instead of on the black.

The essential point that defines a deterministic dynamics is that it associates to each exact state of the physical system at a given "initial" time a *unique* state for all future times. We will call *initial conditions* the exact state of the system at that initial time.

In physics, one also considers non-deterministic dynamics. This is defined as a dynamics where each initial state does *not determine* a unique later state, but several possible states.

It is a dynamics defined by assigning to each state a series of other states with certain probabilities.

The simplest examples are again given by coin tossing and dice throwing. Suppose that we do not know (as is the case in practice) the initial conditions in a particular tossing or throwing. Then, we assign certain probabilities to the outcomes.

In the case of a coin, assuming that there are no tricks or bias, the probability of heads or tails is $\frac{1}{2}$, irrespective of what happened previously. For a dice, each face has probability $\frac{1}{6}$, again irrespective of what happened previously.

But one could imagine a biased coin, where heads would fall with probability 1/4 and tails with probability 3/4. Or a biased dice where one face would appear with probability 1/2 and each of the five other faces with probability 1/10. Obviously, the sum of the probabilities of the different possible outcomes must always be equal to 1.

Almost all of the applied sciences use probabilistic dynamics, of course more fancy ones than the examples given here, but based on the same idea.

The outcomes of such non-deterministic processes are often called "random". To define (intuitively) a random sequence of results, consider the repeated tossing of a non biased coin, with results noted H (heads) and T (tails). This sort of experiment is typical of what one calls random and thus can be used to explain that notion.

What does one expect when one tosses many times such a coin? First of all, one expects both H and T to appear about half of the time.[2] But one expects also each successive pair of the form HH, HT, TH, TT to appear about a quarter of the time. Each of the eight series composed of three successive results HHH, HHT, HTH, etc. should appear one-eight of the time. More generally, in a random sequence, every finite series of symbols, made of H's and T's, should appear with a frequency that depends only on its length (the longer the length, the smaller the frequency)[3] so that two finite series of the same length appear with the same frequencies.[4]

This notion of random sequence may become more intuitive if one considers examples of sequences that are not random. For example, the sequence HTHTHTHT…is just the repetition of the pair HT, which therefore occurs more than a quarter of the time, and is not random. Another example is given by the sequence HHTTHTHHHTTHTHHHTTHTHHHTTHTHHHTTHTH…, which is obtained by the repetition of the series HHTTHTH, and is also not random (the series HHTTHTH is of length 7 and thus should occur, in a random sequence, only a fraction $\frac{1}{2^7} = \frac{1}{128}$ of the time).

We will call *apparently random* (for reasons that will be clear below) a sequence satisfying the above definition.

Suppose we are given a sequence that is apparently random according to the above definition. We can raise a fundamental question. Is that sequence *truly random*, or, to use a synonym, *intrinsically random*? What these expressions mean is that no conceivable deterministic explanation of the appearance of this random sequence could be given. This notion has to be contrasted with the previous definition (apparently random) that referred to a statistical property of the sequence (two finite series of the same length appear with the same

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[2] This expectation will be justified trough the law of large numbers in Sect. 3.4.1.

[3] To define the frequency with which some series of symbols occurs in a sequence of results, one counts the number of those series of symbols in that sequence and one then divides that number by the total number of results.

[4] Thus, every finite series of results of length n will occur a fraction of the time equal to $\frac{1}{2^n}$. This of course makes sense only for an infinite random sequence. Since, in practice, every sequence has a finite length, the definition given here has to be considered as an idealization.

frequencies), while the notion of truly or intrinsically random refers to the impossibility for the sequence to be produced by a deterministic mechanism.

We already saw that the results of coin tossing are not *truly* random. This gives a simple example of a sequence of results that appears random, but that is in reality deterministic: if we give a more detailed description of the system, namely the exact initial conditions of the state of the coin for each tossing, then the results are determined by those initial conditions.

Conversely, any deterministic system can look indeterministic if we do not describe it in sufficient detail. The coin tossing and the dice throwing described above provide examples of that situation. If we give the initial conditions in detail, then those processes are deterministic, but if we do not, then they may *appear to be* random.

One might ask a further question: can one find a criterion that would allow us to determine, whenever the behavior of a system is *apparently random*, whether it is *truly random*? The answer is "no": although we shall not prove it, the examples of the coin tossing and the dice throwing illustrate why this is so: one cannot think of anything more apparently random than the throwing of a coin or a dice. In fact these examples epitomize the notion of randomness. But if even those system are in fact deterministic, once one gives a more complete description of their states, how can one hope to find a criterion that would prove that a system is truly random and does not simply look random because of our incomplete description of their states?

This does not mean that there cannot be any truly random systems in Nature, but that, in order to prove that fact, one has to give other arguments than simply observe that they appear random.

3.1.2 Determinism and Predictability

This brings us to another important distinction, the one between *determinism* and *predictability*. In the examples of the coin tossing or the dice throwing, we said that, if we knew the initial conditions with enough precision, we could in principle calculate on which face they would fall. But here and in many other phenomena, we cannot, in practice, know the initial conditions with enough precision to be able to predict the face on which they will fall.

It may also happen that, although the initial conditions can be known with great precision, the calculation of the evolution of the system between its initial state and its final one may be too complicated to be carried out.

Yet another possibility is that *there are deterministic laws* governing a given physical situation, but that we do not know them.

To summarize the difference between determinism and predictability, the first notion refers to the nature of the (possibly unknown) laws governing a given phenomenon and the second one to our ability to know and use those laws to predict the future. So, predictability does not depend only on the nature of the physical laws (whether they are deterministic or not), but also on human abilities, both our capacity to make precise measurements (of initial conditions) and to make possibly complicated calculations.

The French mathematician Henri Poincaré explained nicely the difference between predictability and determinism in the following passage:

> Why have the meteorologists such difficulty in predicting the weather with any certainty? Why do the rains, the tempests themselves seem to us to come by chance, so that many persons find it quite natural to pray for rain or shine, when they would think it ridiculous to pray for an eclipse? We see that great perturbations generally happen in regions where the atmosphere is in unstable equilibrium. The meteorologists are aware that this equilibrium is unstable, that a cyclone is arising somewhere; but where they can not tell; one-tenth of a degree more or less at any point, and the cyclone bursts here and not there, and spreads its ravages over countries it would have spared. This we could have foreseen if we had known that tenth of a degree, but the observations were neither sufficiently close nor sufficiently precise, and for this reason all seems due to the agency of chance. Here again we find the same contrast between a very slight cause, unappreciable to the observer, and important effects, which are sometimes tremendous disasters.
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> Henri Poincaré [153, p. 398] (original [152])

Events that look random or chancy to us, like the weather, may very well be as determined as the eclipses and this appearance of randomness then reflects only our cognitive limitations.

Because of what is said above, some readers may draw the conclusion that physical laws *are necessarily deterministic* if one describes the physical situation in sufficient detail or, in other words, that the notion of having fundamental physical laws that are indeterministic is unthinkable. But we made no such claim.

Indeed, although it is difficult to give a criterion showing that some systems behave in a "truly random" fashion we should not assume that a fundamentally "random" Universe is impossible. It is perfectly conceivable that the fundamental laws of Nature are not deterministic. Who are we to tell Nature how to behave?

People who do not like determinism sometimes accuse it of being a "metaphysical assumption". Since we don't assume determinism or demand a priori that a physical theory be deterministic, this accusation cannot be used against the point of view defended here.

3.2 Determinism and Physics

Of course, the coin tosses and dice throwings discussed above are too simple to be of any interest in physics.

When Newton and his successors discovered what is known as classical mechanics (basically the laws of motions of planets, projectiles, satellites) they introduced the archetypical example of *deterministic physical laws*.

This led the French physicist and mathematician Pierre Simon Laplace to write, at the end of the 18th century:

> Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it — an intelligence sufficiently vast to submit these data to analysis — it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.[5]
>
> Pierre Simon Laplace, [116, p. 4]

This is the clearest formulation of the universal determinism of the laws of classical physics. But, to come back to the distinction that we made between determinism and predictability, note that Laplace added immediately after that sentence that *we* shall "always remain infinitely removed" from this imaginary "intelligence" and its ideal knowledge of the "respective situation of the beings who compose" the natural world; that is, in modern language, ideal knowledge of the exact initial conditions of all the particles in the Universe.

Laplace distinguished clearly between what Nature does and our knowledge of it. Moreover, he stated this principle at the beginning of an essay on *probability*. But, as we shall discuss below, probability for Laplace is nothing but a method that allows us to reason in situations of partial ignorance.

The meaning of Laplace's quote is completely misrepresented if one imagines that *he* hoped that one could arrive someday at a perfect knowledge and a universal predictability, for the aim of his essay was precisely to explain how to proceed in the absence of such a perfect knowledge.

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[5] This intelligence is often referred to as the "Laplacian demon". (Note by J.B.).

However, celestial mechanics dealt only with the motion of bodies attracting each other through gravitational forces.

In the 19th century, physicists discovered laws governing electric and magnetic phenomena. As we said in Sect. 1.1.1, James Clerk Maxwell, expressed those laws through a mathematical theory, known as electromagnetism, postulating the existence of electromagnetic waves, that are both created by the particles and that guide their motion.

The system consisting of particles and waves interacting with each other is still deterministic in the same sense that classical mechanics is. The only novelty is that one has to specify not only the initial positions and velocities of all the particles in the system, but also the initial state of the electromagnetic waves.

Physics was further modified in the beginning of the 20th century by the two theories of relativity, the special and the general ones, due mainly to the works of Lorentz, Poincaré, Einstein and Hilbert.

These theories basically changed Newton's laws of motion (in order to make them compatible with the newly discovered laws of electromagnetism) and changed also Newton's theory of gravitation.

But as far as determinism is concerned, nothing substantial changed.[6]

The real break with determinism in physics came with quantum mechanics. In the previous chapter, we saw that we cannot predict the exact place where the particle is going to end up on the second screen in the double-slit experiment.

In the next chapter, we shall see that the formalism of quantum mechanics incorporates this unpredictability in its very formulation: one associates to physical systems certain states and, given such a state, one can compute the probabilities of jumping into another state after a "measurement". But, at least in ordinary quantum mechanics, there is no information that would, *even in principle*, allow us to predict with certainty which state one will jump into. Thus, it seems that, with quantum mechanics, we have a candidate for a physics governed by intrinsically random laws.

Of course, and now we come back to what we said about the coin and the dice: could it be that, by providing a more complete description of the physical systems than the one of ordinary quantum mechanics, one could recover a deterministic theory?

So, the issue of determinism and of whether a more detailed description than the usual quantum one is possible, are intimately linked.

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[6] There are several caveats that should be made here, in order to be rigorous, but it would go far beyond the scope of this book.

And these two questions were at the heart of the discussions between Einstein, Schrödinger, Bohr, Heisenberg, Pauli and others.

But we shall return to these issues later and discuss now the reason why determinism seems to many people to have great negative "philosophical" consequences.

3.3 Determinism and Free Will

The notion of universal determinism of physical laws has provoked much hostility because it seems to contradict our notion of free will. To discuss that issue, we must first try to define free will: free will is not just the feeling that I act without external constraints, without anybody forcing me to do or not do something, but that "I" choose to do one thing rather than another.

The choice can be trivial, like between two kinds of deserts, or serious, like which profession to embrace or whether or not to commit a crime. But in our everyday life, we constantly feel that we have to make choices and, when we are not under external constraint, that we do so freely.

This notion of free choices has very many moral and legal implications. Courts routinely distinguish between people who are responsible for a crime that they committed and those who are not. The first ones are sent to jail, the others to mental hospitals.

But suppose that someone tells you that those who commit crimes apparently "of their own free will" are actually determined to commit those acts and are not "really" free not to do them.

How could that be possible? Let us first discuss a form of "determinism" which is very common in social and political discussions, but which is not relevant here: social and psychological determinism. For example, one might say that some person had a bad childhood or grew up in a crime-prone environment and thus became a criminal. The problem with this form of "determinism" is that it is not strict: one can always find counterexamples, namely people who have had the same psychological or sociological background and that did not become criminals. Even if one adds what one knows about genetic determinism to the picture, the result is still not strict determinism and, presumably, even a more detailed knowledge of genetics than the one we have now would not lead to a strict determinism.

In fact, these forms of "determinism", or rather of lack of strict determinism, are again examples where this lack is due to an incomplete description of the systems.

To give a more detailed description, for which determinism might hold, one should describe the state of every brain cell and every connection between cells in every detail, maybe down to the state of the last atom of our brain, and also describe all the interactions between the brain, the rest of the body and the outside world. Such detailed states would almost certainly differ between people of the same socio-psychological background, and that would lead one of them to commit a murder and the other not.

But then, of course, one may say that the person who commits the murder is not more free not to do it than the person who does not commit it: in both cases, we have zillions of neurons interacting with each other and producing different results, but that is just like two computers executing different programs. There is no genuine free will in either situation.

Of course, such a detailed description would in practice be impossible to obtain, but remember that we discuss determinism here, not predictability.

This leads us to the view sometimes known as the "clockwork universe": at the time of the Big Bang, the world started in a certain state and then it evolved deterministically according to physical laws.

This view has all kinds of strange implications: while I try to find the next words to write in this book, they were in fact all determined at the time of the Big Bang.

Moreover, our strong feeling that some criminals are really guilty, because they acted out of their own free will, while others may simply be mentally disturbed, would become an illusion if what is said above were true. It would only be that our knowledge of brain states is insufficient to explain what goes wrong in the brains of the people considered "really guilty". Note that, in the same way that there is no test that can show that some events are "truly random", nobody has ever given a way to test whether some individual's actions are really due to his or her "free will".

But if one accepts what precedes, and although we may deplore the acts of criminals, it would be natural to do it in the same way that we deplore the effects of hurricanes. If the Universe is deterministic, there is no reason to hold the worst criminals responsible for their actions; after all, we do not hold hurricanes responsible for their effects. If there is no genuine free will, what is the difference? While we are at it, why hold Hitler responsible for his actions?

In the same spirit, one might also wonder why saints and heroes deserve to be admired ? After all, the difference between them and the worst criminals is simply that they have bunches of neurons in different states. One could admire them in the same way that one admires a computer playing chess very well, but that is usually not the same admiration as the one extended to humans.

One could reply that this does not make any difference in practice: nobody could even dream of ever predicting what an individual is going to do (in general circumstances) by analyzing its brain and "computing" its future behavior.

Besides, our legal judgments, our sentiments of love or hatred, of admiration or contempt, are just as determined by Nature as anything else; so there is nothing that we can change about them.[7]

But this does not remove the uneasiness created by the conflict between determinism and free will.

And this is where quantum mechanics comes in: if quantum mechanics shows that Nature is truly random and if, when we come down to the microscopic structure of anything, including our brains, we encounter atoms or electrons, and if the latter obey the laws of a "truly random" quantum mechanics, then, says the defender of free will, the argument going from universal determinism to "free will is an illusion" collapses.[8]

This is correct, but a universe whose ultimate laws are truly random will not give us a picture of the world in which free will exists either. Our feeling of free will does not mean that there are some random events in our brains, but that *conscious choices* are made.

Those have nothing to do with random events. The problem of free will is part of what philosophers call "the mind-body problem": we see ourselves as having feelings and sensations and making free choices, while our view of the physical universe has no such properties: the world outside ourselves looks like a machine and whether it has some random element in its functioning does not change anything so far as that is concerned.[9]

One "solution" to that problem is dualism: the body and the mind belong to two different substances (in the Western religions, the mind is identified with the soul, which is supposed to be immortal, but one can be a dualist without believing in the immortality of the soul/mind). But that solution creates problems of its own: does the mind interact with the body? If it interacts, does it violate physical laws in doing it? If it does not interact, what is the point of introducing the mind as a substance separate from the body?

Besides, if this is supposed to explain free will, how and when did the latter appear during evolution? One can have longer or shorter arms or necks, but it does not seem to make sense to have half of genuine free will. Either you have

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[7] Nevertheless, one can argue that the development of a scientific world view has led to an increased skepticism with respect to the doctrine of free will and that has led to changing attitudes with respect to education and to criminal law: "rewards and punishments" have been increasingly viewed as a practical matter rather than a question of principle based of distinguishing true merit and true guilt.

[8] For example, the distinguished physicist Nicolas Gisin links the supposed lack of determinism in quantum mechanics to free will, see [90, 119].

[9] We will come back to the mind-body problem in Sect. 11.5.1.

it or you don't. Were some animals devoid of free will, while their children had it? Or is every living being endowed with free will? What about plants?

The point of this digression is only to show the depth of the problem, which we have no intention of solving or even of discussing any further.[10]

Indeed, our only goal here is to "free" physics, so to speak, from concerns about free will. We do not want to be distracted by the frequent feeling that an indeterministic physical theory is preferable to a deterministic one, because it would allow us to "save free will". There is nothing in the character of physical theories that will make free will more or less understandable or plausible. So let us pursue our investigation of physical theories and let the chips fall where they may, without philosophical prejudices or worries.

Besides, as the British logician and philosopher Bertrand Russell observed, scientists should look for deterministic laws like mushroom seekers should look for mushrooms. Deterministic laws are preferable to non-deterministic ones because they give a way to control things more efficiently (at least in principle) and because they give more satisfactory explanations of why things are the way they are. Looking for deterministic laws behind the apparent disorder of things is at the heart of the scientific enterprise.

Declaring some events truly "random" is basically saying that we have no explanation for them and that is not what science hopes to do (although, of course, one may be forced to concede that we fail to find an explanation for certain events).

Whether we succeed or not in that search for deterministic laws depends in a complicated way both on the structure of the world and on the structure of our minds. But the opposition to determinism tends to make people feel that the project itself is doomed to fail or even hope that it will fail. And that state of mind does run counter to the scientific spirit.

3.4 Probabilities and Determinism

Now, let us go back to the idea of deterministic laws and ask ourselves, why and how do physicists use probabilities, as they do in practice, if laws are deterministic?

The answer was already given when we mentioned the limitations of our abilities: probabilities are used because of our ignorance: either because we do not know the initial conditions of the system or because, even if we knew them, the calculation of the future would be too complicated.

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[10] The philosopher Colin McGinn has developed the interesting idea that the problem of "free will" may lie beyond the limits of human understanding [123].

But how does one assign probabilities to an event? If we consider simple examples, like coin tossing or dice throwing, we can use the symmetry of the situation: there are two faces for the coin and six for the dice and we do not see any reason why one would be more likely than the other, so we assign probability one-half for each face of the coin and one-sixth for each face of the dice.[11]

As we said, in any single throw of the coin or the dice, the result is determined by the initial conditions and would be predictable if one could know them with sufficient precision. The fact that we assign probabilities to such events does not mean that we deal with something "truly random", but simply that the phenomena in question are not controllable or computable in detail by us.

In more complicated situations, where there is no symmetry between the different outcomes, assignment of probabilities is more difficult and we shall not explain in detail how one does it. What one tries to do is to apply Laplace's "principle of indifference", namely one tries to reduce the situation to a set of cases about which we are "equally ignorant", i.e., the information that we do have does not allow us to favor one case over the other. In other words, we try to get as close as possible to the situation of the tossing of a coin or the throwing of a dice, where there is a symmetry between the faces.

What one wants to avoid, when assigning probabilities, is to introduce bias in our judgments, or "information" that we do not really have or is illusory, like people who believe in lucky numbers.

In physics, if one deals with a deterministic dynamics, but where the initial conditions are in practice unknown, one assigns a probability distribution to those initial conditions and that determines a probability distribution on future events: the probability of those events is simply the probability of the initial conditions that led to them through the deterministic laws.[12]

But simply quantifying our ignorance, which is what assignments of probabilities do, may not sound very useful. In order to obtain useful probabilistic statements, we need the law of large numbers.

3.4.1 The Law of Large Numbers*

That law says, roughly speaking, that, even if a sequence of events are random, there is some regularity that emerges when the same random events occur many times.

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[11] In case our observations systematically deviate from what one would expect on the basis of this assignment of probabilities, namely that all faces fall with equal frequencies, we will suspect the coin or the dice to be biased and then revise our probabilities.

[12] This notion will be important in Sect. 8.4.2 when we discuss the de Broglie–Bohm theory.

To explain that law through a simple example, consider coin tossing: if one tosses a coin many times, say n times, one gets a sequence of results, for example: H, T, T, T, H, H, ..., (with H = heads, T = tails). Each such sequence has the same probability, namely $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \cdots = (\frac{1}{2})^n$, since each individual result has probability $\frac{1}{2}$ and there are n tosses of the coin.

For example, if $n = 4$, we have $(\frac{1}{2})^4 = \frac{1}{16}$; if $n = 10$, we have $(\frac{1}{2})^{10} = \frac{1}{1024}$.

Now, let us compute the probability that the coin always falls heads. That has also probability $(\frac{1}{2})^n$, since there are n tosses of the coin and, in each case, falling heads has probability $\frac{1}{2}$. The same holds for the coin always falling tails.

On the other hand, let us compute the probability of the coin falling heads half of the time (and thus tails also half of the time), assuming n to be an even number? Unlike the situation where the coin always falls heads or tails, here there are many different sequences of outcomes where the number of heads is $\frac{n}{2}$. For example, for $n = 4$, one can have: H, T, T, H, or H, T, H, T and four other possibilities.

There is a mathematical formula that counts the number of sequences of outcomes where the coin falls heads half of the time.[13] There is also a formula that counts the number of sequences of outcomes where the coin falls heads k times and tails $n - k$ times.[14]

Now, using those formulas one can show that the number of sequences of outcomes where the coin falls heads (and tails) more or less half of the time is, for n large, almost equal to the total number of possible sequences.[15]

That may sound strange at first sight, but it is just the result of a computation: for n large, the overwhelming majority of sequences of results have a number of heads more or less equal to the number of tails and both are more or less equal to $\frac{n}{2}$.

But that means that the probability that a given sequence of results has a number of heads more or less equal to the number of tails is almost equal to one, or, in other words, is almost certain.

This result is the simplest example of the law of large numbers (which we will not state in general, but only through examples).

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[13] We shall not use that formula, but we give it for the interested reader: writing $n = 2m$, since n is even, that number of sequences is: $\frac{n \times (n-1) \times (n-2) \cdots \times (m+1)}{1 \times 2 \times 3 \cdots \times m} = \frac{n!}{m!^2}$, where, for a number m, m! is a shorthand notation for $1 \times 2 \times 3 \cdots \times m$.

[14] Again, for the interested reader, that number is: $\frac{n!}{k!(n-k)!}$.

[15] Which is equal to $2 \times 2 \times 2 \cdots = 2^n$. We shall not be precise about what "large", "more or less" and "almost equal" mean here. For a mathematician, n "large" means that one considers a limit where n tends to infinity, "more or less" means that the frequencies of outcomes where the coin falls heads is very close to $\frac{1}{2}$, and "almost equal" means that the equality becomes exact in the limit where n tends to infinity.

Another example of the law of large numbers is that, in the overwhelming majority of sequences of results, each of the series HH, HT, TH, TT occurs more or less one quarter of the time. And each of the eight series HHH, HHT, HTH, etc. occurs, in the overwhelming majority of sequences of results, one-eight of the time.

Let us define the *statistical distribution* of a sequence of results obtained by repeating a large number of times the same experiment: one collects all the frequencies with which different events occur in a given sequence of results.

For coin tossing, events would be a H or a T, a series made of a pair HH, HT, TH,TT, or a series made of a triple HHH, HHT, HTH, etc. So, one counts the number of H and T, the number of pairs HH, HT, TH,TT, of triples HHH, HHT, HTH, etc. and one divides that number by the total number of results. We call that set of data (all the frequencies of occurrence of finite series of H and T) the statistical distribution of the given sequence of results.[16]

The general law of large numbers says that, when one repeats a large number of times the same experiment, associated to a given probability distribution (here, for coin tossing, each face has probability one-half), in the overwhelming majority of sequences of results, each event (like a H or a T, or a series HT, HHT etc.) will occur with a frequency equal to its probability: the probability of a single symbol H or T is one-half, the probability of a series of two symbols is $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$, because each tossing is independent of the other and the probability of a series of three symbols is $\frac{1}{8} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, which corresponds to the frequencies mentioned above.

More generally, suppose that there is an experiment whose results are associated to a given probability distribution $P(x)$, as in Fig. 3.1. If one repeats many times that experiment, one will obtain a sequence of results. To define the statistical distribution of those results, divide the x axis into small intervals and count the number of particles in each interval, then divide that number by the total number of particles. This allows us to define a curve, denoted $D(x)$ in Fig. 3.2, such that the area between that curve and an interval A on the x axis gives the fraction of particles in that interval.

There are of course many possible sequences of results, but, in the overwhelming majority of them, the statistical distribution of the results, denoted $D(x)$ in Fig. 3.2, will coincide with the probability distribution, provided of course that this probability distribution has been adequately chosen to reflect the physics of the problem.[17] Of course, since, in practice, the sequences of

---

[16] Again, this notion becomes precise in the (idealized) limit where the number of results tends to infinity.

[17] In fact, one way to verify that the chosen probability distribution was correctly chosen is through this coincidence between the frequencies of events in most of the sequences of results and that probability.

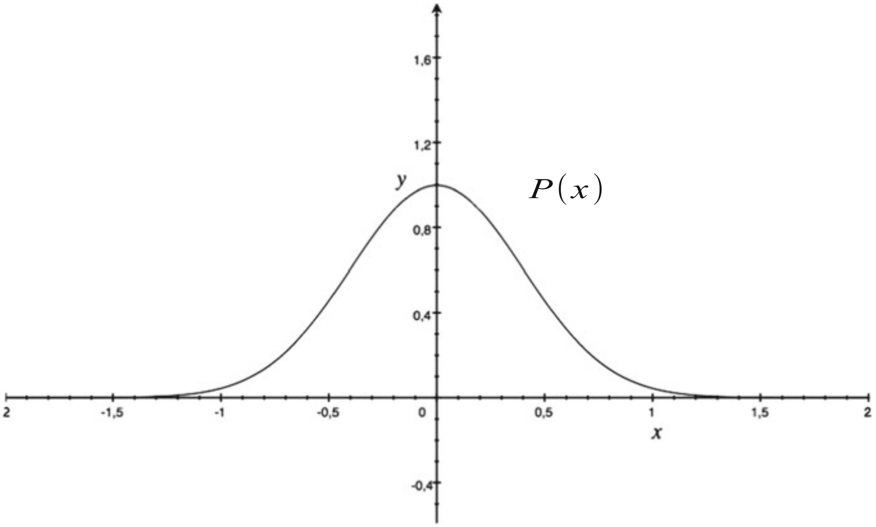

Fig. 3.1 An example of a (Gaussian) probability distribution. See Fig. 3.2 for its relation to statistical distributions

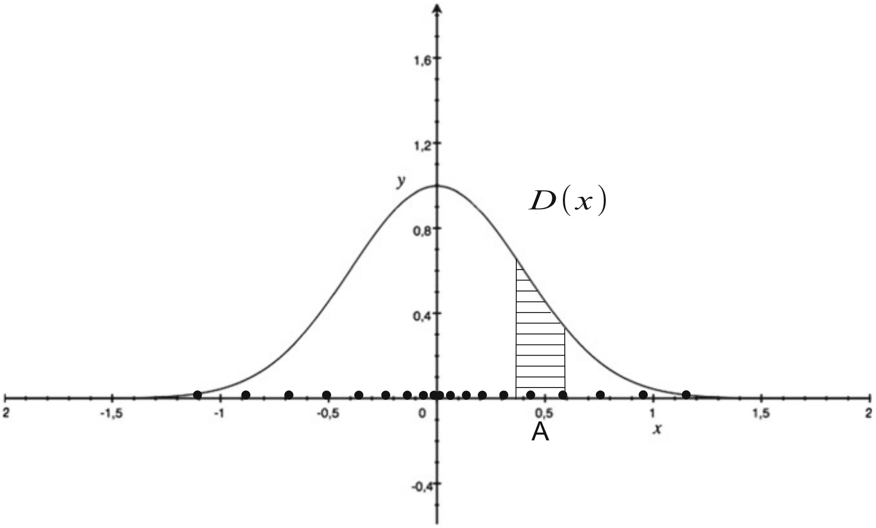

Fig. 3.2 An example of a statistical distribution of a sequence of results of experiments associated to the probability distribution $P(x)$ of Fig. 3.1, each *dot* representing one result. The *curve* $D(x)$ is obtained by dividing the x axis into small intervals and counting the number of particles in each interval, then dividing that number by the total number of particles. The area between the *curve* $D(x)$ and a given interval A on the x axis gives the fraction of particles in that interval

results are always finite, the coincidence between the frequencies and the probability is only approximate, the approximation improving with the number of results.

Coming back to the curves discussed in Chap. 2 and describing the densities of impacts of particles, for example the blue curves in Fig. 2.1 or in Figs. 2.6–2.8, we can also view them as applications of the law of large numbers: each individual particle will land at a certain place, which will be different from one experiment to the next. But, if one sends many particles, one always gets the same curves for the densities of the detected particles, irrespective of whether the experiment is done today in Paris or next week in New York.

This law of large numbers applies to many more complicated situations and is at the basis of most usages of probabilities in physics and in the natural sciences: for example, the whole field of statistical physics, which derives properties of gases, liquids and solids from those of their microscopic constituents (atoms or molecules) is entirely based on the law of large numbers. The properties of gases, liquids and solids look deterministic and are certainly reproducible, but they nevertheless result from the application of the law of large numbers to myriads of atoms or molecules whose motions are not known in detail and are therefore treated as being "random", even though the laws governing their motion may be deterministic and our use of probabilities is only due to our ignorance. The law of large numbers is also used in many applied sciences (insurances for example).

So, even though probabilities are only a way to reason in situations of ignorance, we recover some degree of (almost) certainty when one considers a large number of similar events and one is interested in some "average" behavior. And it is mostly through this law of large numbers that probabilities are useful in physics. We will see how this law can be applied to understand the quantum statistics in Chap. 8.

3.5 Summary

First, we explained that there can be two different sorts of physical laws, deterministic and indeterministic ones. All pre-quantum mechanical laws, those of Newtonian mechanics, of electromagnetism, and of special and general relativity are of the first sort. Only quantum mechanics is a potential candidate for a sort of physical laws where randomness enters at a fundamental level.

(Footnote 17 continued)
For example, if a coin is biased, one will observe a deviation between the observed frequencies and the assigned probabilities, and this should lead us to revise those probabilities.

But we also saw that a process like coin tossing may appear random only because we do not know or do not control the initial conditions with enough precision.

Therefore, the supposedly fundamental randomness of quantum mechanics depends on whether quantum mechanics is "complete" – namely on whether or not a more detailed description of physical systems can be given than the one of ordinary quantum mechanics.

There is a strong opposition to determinism in physics because of our feeling of "free will" and our desire to believe that this free will is genuine and not an illusion. But we argued that even an indeterministic physics would not make free will more genuine. The issue of free will is part of the mind-body problem and lies outside the scope of physics.

Finally, even if the physical laws are deterministic, it does not mean that probabilities do not have to be used. In fact, we have to use them because of our ignorance or because of our computational limitations. When we assign probabilities to events, we try to quantify our ignorance, on the model of a coin tossing or a dice throwing, where there is a symmetry between the faces and we thus assign the same probability to each face.

This may not seem very useful, but the law of large numbers allows us to make statements that are almost certain when one considers a large number of random events, as one does in statistical physics. In that sense, we recover some certainty in the midst of uncertainty.

4

How Do Physicists Deal with Interference?

So, we have seen, in Chap. 2, one of the two "impossible things that we have to believe before breakfast" namely things being apparently in two different states before being measured or before one looks at them.

There is a mathematical language that allows to predict the "impossible" phenomenon of the double-slit experiment. We shall describe this language without mathematics.[1]

This language should be regarded, for the moment, only as a "recipe" for predicting those phenomena, without worrying about what it "means" physically. The main problem for the reader is probably *not to ask, for the moment*: what does this language mean (beyond being an efficient recipe for predictions of observations)? We shall come to the question of the physical meaning of this language in the next chapter, but in this chapter, we shall only explain how the recipe works.

4.1 The Wave Function

In order to predict the behavior of electrons in the double-slit experiment, we need a notion that is central in the whole of quantum mechanics: the *wave function*.[2] As the name indicates, it is a function, which is usually denoted by

---

[1] See the Appendix for somewhat more precise mathematical treatment; however *it is not necessary to read the Appendix in order to follow the rest of the arguments*.

[2] Note for the advanced reader: throughout this book, we shall not distinguish between the wave function of a physical system and its quantum state, which is a more general notion.

J. Bricmont, *Quantum Sense and Nonsense*, DOI 10.1007/978-3-319-65271-9_4

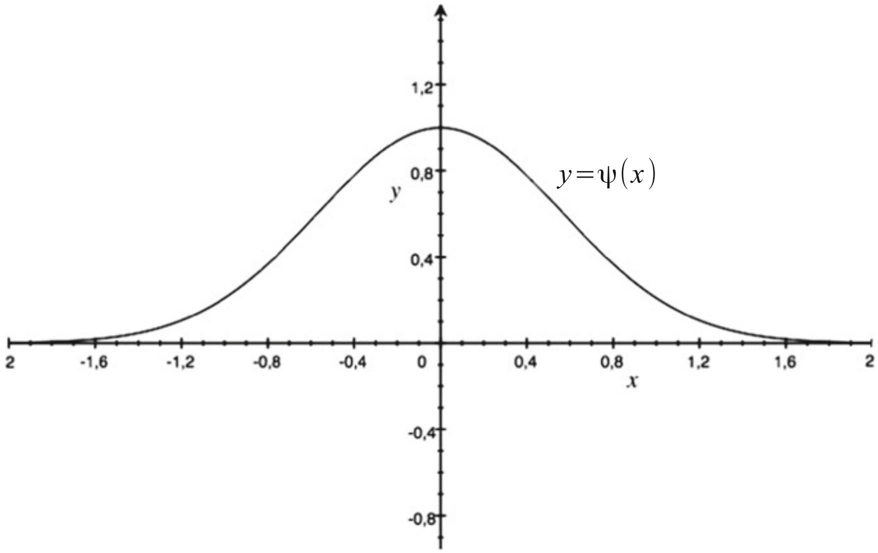

Fig. 4.1 An example of a wave function $\Psi(x)$. Each point on the *curve*, with coordinate (x, y) has a y coordinate equal to $\Psi(x)$

the Greek letter Ψ and such a function is associated in principle to any physical system.

To simplify matters, we shall consider Ψ as being a function of one variable denoted x: $\Psi = \Psi(x)$, which corresponds to a single particle moving on a line.[3] For a graphical representation of wave functions, see Figs. 4.1 and 4.2. We will give other examples of wave functions below.

The physical meaning of $\Psi(x)$, in orthodox quantum mechanics, is simply that the square of the wave function $\Psi(x)^2$, determines the probability of finding the particle somewhere if one "measures" its position: the probability of finding the particle in a set A is given by the shaded area in Fig. 4.3 under the curve $\Psi(x)^2$. We assume that the total area under the curve in Fig. 4.3 is equal to one, so that the probability of the particle being found *somewhere* is equal to one, as it should. Another example of a function $\Psi(x)^2$ is given in Fig. 4.4.[4]

---

[3] In mathematics, a function is usually denoted $f(x)$, but for the wave function the notation $\Psi(x)$ is almost universal.

[4] A caveat is necessary here: the number $\Psi(x)$ is in reality a complex number and one should write $|\Psi(x)|^2$ instead of $\Psi(x)^2$, see the Appendix.

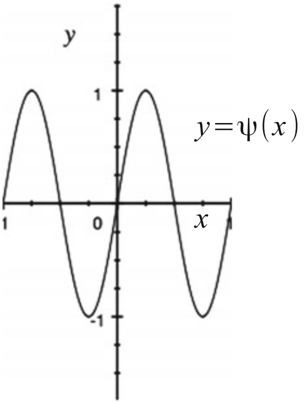

Fig. 4.2 An example of a wave function $\Psi(x)$ that takes both positive and negative values. Each point on the *curve*, with coordinate (x, y) has a y coordinate equal to $\Psi(x)$

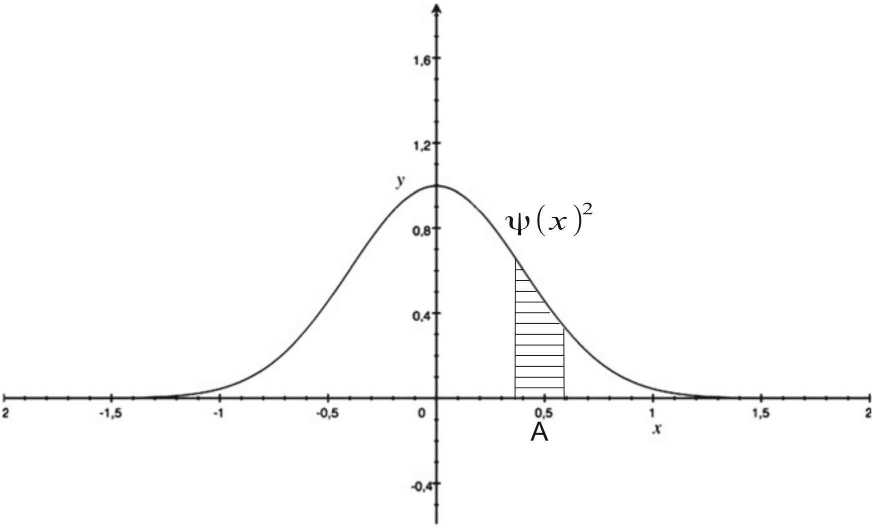

Fig. 4.3 An example of a wave function $\Psi(x)^2$. Each point on the *curve*, with coordinate (x, y) has a y coordinate equal to $\Psi(x)^2$. The function $\Psi(x)^2$ is the square of the function $\Psi(x)$ drawn in Fig. 4.1. The probability of finding the particle in the region A is equal to the *shaded area* (we used the same curve as in Fig. 3.2)

So, if one considers many particles, all having the same wave function $\Psi(x)$, and one detects their individual positions, the statistical distribution of those positions will be given by the curve $\Psi(x)^2$.[5]

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[5] This fact follows from the law of large numbers, discussed in Sect. 3.4.1.

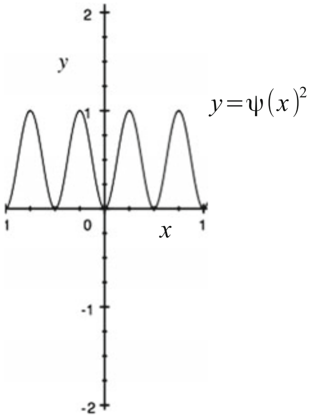

Fig. 4.4 Another example of a wave function $\Psi(x)^2$. Each point on the *curve*, with coordinate (x, y) has a y coordinate equal to $\Psi(x)^2$. The function $\Psi(x)^2$ is the square of the function $\Psi(x)$ drawn in Fig. 4.2

An immediate question may occur: if the physical meaning of $\Psi(x)$ is given by $\Psi(x)^2$, why talk about $\Psi(x)$ instead of talking directly and only about $\Psi(x)^2$? That will become clear soon, when we explain below that, in some circumstances, one must add different wave functions and not their squares.

Another natural question is, why use the square of $\Psi(x)$, and not $\Psi(x)$ itself or its cube $\Psi(x)^3$, or its fourth power $\Psi(x)^4$, or any other power? Why not $\Psi(x)$ is easy to answer: $\Psi(x)$ in general is not a positive number and probabilities have to be positive numbers (see Fig. 4.2)! The same answer holds for $\Psi(x)^3$ (the cube of a negative number is also negative), but for the other powers, there is no easy answer: the fact is that only $\Psi(x)^2$ leads to experimentally correct results. We will simply accept that as a fact.

There are *two mistakes* to be avoided when one thinks about the wave function in the orthodox fashion:

1. The first mistake is to think that $\Psi(x)^2$ describes some density of "stuff", for example some density of matter or of electric charge. In classical physics, particles were supposed to be "point particles", namely localized at an exact point in space, which works at least as an idealization. It is certainly more intuitive to think of them as being somewhat spread out and if $\Psi(x)^2$ corresponded to a density of matter or of electric charge, that would be appealing (actually this is how Schrödinger first thought of the meaning of $\Psi(x)^2$).

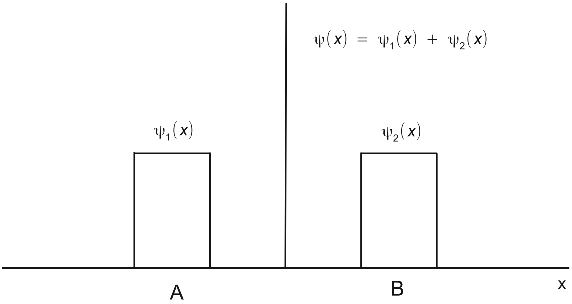

Fig. 4.5 A wave function being the sum of two wave functions that are non-zero over non-overlapping regions

The problem is that, as we see in the example drawn in Fig. 4.5, the wave function $\Psi(x)$ can be a sum of two other functions:

$$\Psi(x) = \Psi_1(x) + \Psi_2(x), \tag{4.1}$$

where the sum of the functions $\Psi_1(x)$ and $\Psi_2(x)$ is easy to understand: we add, for each value of the variable x the numbers $\Psi_1(x)$ and $\Psi_2(x)$. Here, $\Psi_1(x) = 0$ where $\Psi_2(x) \neq 0$ and vice-versa, so that the sum $\Psi(x)$ is simply equal to $\Psi_1(x)$ on the interval A where $\Psi_1(x) \neq 0$, to $\Psi_2(x)$ on the interval B where $\Psi_2(x) \neq 0$, and is equal to 0 everywhere else.

The two regions A where $\Psi_1(x) \neq 0$ and B where $\Psi_2(x) \neq 0$ can be as far apart from each other as one wants (we do not indicate the distance between A and B in Fig. 4.5, but we can imagine it to be large) and then the density of stuff picture does not work, because the particle will always be detected either in the region A where $\Psi_1(x) \neq 0$ or in region B where $\Psi_2(x) \neq 0$, but not in both.

2. The other mistake is to think that $\Psi(x)^2$ determines the probability of the particle *being* in a region like A in Fig. 4.3. But that is *not* what orthodox quantum mechanics says. The latter always defines $\Psi(x)^2$ as determining the probability of the particle *being found in a region like A in Fig. 4.3 if one measures its position.*

   Of course the two notions may look identical: after all, measuring something means that we measure some property (here the particle's position) that is there before one measures it. If one measures the length of a table, one assumes that the table has a length and that, when we measure it, we simply learn what it is.

   But in ordinary quantum mechanics it is not so simple, because, a priori, a measurement might affect the system being "measured" (which of course implies that using the word "measurement" in this context could be misleading).

That is why ordinary quantum mechanics only speaks of *results of measurements*, but says nothing about what happens outside of them or before a measurement. Results of measurement are, by definition, sufficiently macroscopic so that they can be directly perceived by us. Quantum mechanics does not say that particles have a position before being "observed".[6]

As already mentioned in Chap. 1, someone who expressed that view very clearly was Pascual Jordan, one of the founder of quantum mechanics and an adherent of the Copenhagen interpretation of quantum mechanics, who wrote:

> In a measurement of position, "the electron is forced to a decision. We compel it *to assume a definite position*; previously, it was, in general, neither here nor there; it had not yet made its decision for a definite position [...] If, in another experiment, the *velocity* of the electron is measured, this means: the electron is compelled to decide itself for some exactly defined value of the velocity; and we observe *which* value it has chosen."
>
> Pascual Jordan [109], quoted and translated by M. Jammer [108, p. 161] (italics in the original)

Putting aside those mistakes (which are made quite naturally), let us come back to the meaning of $\Psi(x)$. Equation (4.1) and Fig. 4.5 is the first example encountered in this book of what is called a *superposition* or a *superposed wave function* which is an expression used whenever the wave function is a sum of wave functions corresponding to different physical situations.

A superposition is usually described by saying that the particle is *both in A and B*, which means only that the particle will be detected either in A or in B if ones measures its position and that the theory refuses to claim that it is either in A or B before being measured. This, of course, gives a special role to measurements and is the source of this special role in the quantum formalism.

The latter statement is very similar to the claim that the particle goes through both slits in the double-slit experiment; we will discuss that experiment using the language of wave functions in the next section.

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[6] In Chap. 8 we shall see that $\Psi(x)^2$ can actually be understood as being related to the probability of the particle *being* at point x, but that will be possible only within a more complete theory than ordinary quantum mechanics.

At this point, we beg the reader *not to try to understand* what the wave function means, beyond what is said here: it gives the probability distribution of results of measurements and that's it![7]

The wave function $\Psi(x)$ also changes with time, more or less like a wave, hence the name wave function. The equation that governs the way $\Psi(x)$ changes in time, *when no measurements are made* is famously known as *Schrödinger's equation*. To indicate the fact that Ψ evolves in time, we write Ψ as a function of both the position x and the time t: $\Psi(x, t)$.

We shall not write down Schrödinger's equation, but we will list here two fundamental properties of the way $\Psi(x, t)$ evolves in time:

1. This evolution is *deterministic*, i.e. if, at some initial time, noted 0, we give ourselves a function $\Psi(x, 0)$, then this determines a unique function $\Psi(x, t)$ for all later times t. We explained the notion of determinism in Chap. 3. The function $\Psi(x, 0)$ corresponds to what we called the initial condition in that chapter. Once it is given, the functions $\Psi(x, t)$ for later times are determined in a unique way.
2. This evolution is *linear*: if the initial wave function at time 0 is a sum of two other wave functions $\Psi_1(x, 0)$ and $\Psi_2(x, 0)$, as in (4.1):

$$\Psi(x, 0) = \Psi_1(x, 0) + \Psi_2(x, 0),$$

then for all later times,

$$\Psi(x, t) = \Psi_1(x, t) + \Psi_2(x, t), \tag{4.2}$$

where $\Psi_1(x, t)$ is the result of the deterministic evolution with initial wave function $\Psi_1(x, 0)$, and $\Psi_2(x, t)$ is the result of the deterministic evolution with initial wave function $\Psi_2(x, 0)$.

This describes all we need to know about the time evolution *when no measurements are made*.

But what happens to $\Psi(x, t)$ if we measure the position of the particle? To give a simple example, suppose that we have, as in Figs. 4.5 and 4.6, a wave function $\Psi(x, t) = \Psi(x) = \Psi_1(x) + \Psi_2(x)$, with the region A, where $\Psi_1(x) \neq 0$, and the region B, where $\Psi_2(x) \neq 0$, being different and with t being the time when the measurement is made.

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[7] We will discuss in the next chapter some apparently natural ways to understand what the wave function means (and see that they run into problems). We will later give a physical meaning to the wave function, in Chap. 8.

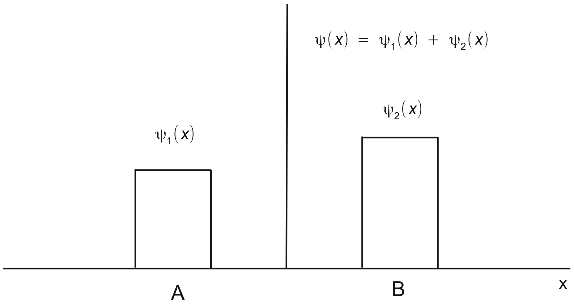

Fig. 4.6 A wave function being the sum of two wave functions that are non-zero over non-overlapping regions, but with unequal probabilities for the particle to be found in region A and in region B

Then, for $\Psi(x)$ as in Fig. 4.5, we shall find the particle in the region A with probability $\frac{1}{2}$, and in region B with probability $\frac{1}{2}$ (because of the symmetry in Fig. 4.5). After the measurement, the wave function *"collapses" or "is reduced"* to either $\Psi_1(x)$ or $\Psi_2(x)$, depending on the result. For an example where there is no symmetry and the probabilities to find the particle in region A and in region B are not equal, see Fig. 4.6.

The time evolution "during measurements" has two properties that are the exact opposite of what happens "outside of measurements":

1. This evolution is *non-deterministic*, i.e. it gives only the probabilities for the particle to be found in regions A or B and, depending on the result, the wave function jumps to either $\Psi_1(x)$ or $\Psi_2(x)$. In Fig. 4.5, we chose a symmetric situation, where both probabilities are equal to $\frac{1}{2}$, but in principle any other probabilities could be obtained, see Fig. 4.6.
2. This evolution is *non-linear*: in our example, we start with a wave function $\Psi(x, 0) = \Psi_1(x) + \Psi_2(x)$, see Fig. 4.5. After the measurement, we get $\Psi(x, t) = \Psi_1(x)$ or $\Psi(x, t) = \Psi_2(x)$, where t is a time right after the measurement. This is a *not* a sum as in the linear evolution (4.2).

Now, the fact that we have incompatible rules for the time evolution "outside of measurements" and "during measurements" raises an obvious question: what kind of physical processes qualify as measurements? And why do the physical laws change when measurements occur? As we have said, this question will be with us throughout this book, but we see here why it enters into ordinary quantum mechanics. And, since the word "measurement" implies an "observer" who does the measurement, we see why ordinary quantum mechanics puts the observer on center stage.

The problem posed by this duality of rules was expressed ironically by John Bell:

> What exactly qualifies some physical systems to play the role of "measurer"? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system …with a PhD?
>
> John S. Bell [12, p. 34]

Finally, we should stress that it is *only through this second rule, the one valid "during measurements" that probabilities enter into quantum mechanics.* The Schrödinger evolution, valid when no measurements are made, is perfectly deterministic.

Thus, both the central role of the observer and the apparent indeterminism of quantum mechanics have their roots in this collapse rule. We will see in Chap. 7 that the problem of nonlocality is also related to this rule.

4.2 The Double-Slit Experiment

Let us now discuss the double-slit experiment, using the language of wave functions.

To do that, we shall rely on Fig. 4.10. Let 0 denote the time when the wave passes through one or two slits in the first wall and let $\Psi_1(x, 0)$ be the wave function right beyond the upper slit. Let $\Psi_2(x, 0)$ be the wave function right beyond the lower slit.

To illustrate this via a simple example, consider Fig. 4.7, where we have drawn the wave functions of Fig. 4.5, but with the variable x on the vertical axis: one can think of $\Psi_1(x)$ and $\Psi_2(x)$ as qualitatively similar to the wave functions $\Psi_1(x, 0)$ and $\Psi_2(x, 0)$ after the slits in the two slits experiment of Fig. 4.10.

The wave arriving on the second wall when only the upper slit is open is given by $\Psi_1(x, t)$, where t is the time at which the particles are detected, and where $\Psi_1(x, t)$ is the solution of the usual time evolution when one starts with $\Psi_1(x, 0)$, see Fig. 4.8. The wave coming on the second screen when only the lower slit is open is given by $\Psi_2(x, t)$, where t is the time at which the particles are detected, and where $\Psi_2(x, t)$ is the solution of the usual time evolution when one starts with $\Psi_2(x, 0)$, see Fig. 4.9.

If only the upper slit is open, the density of particles detected at a point x on the screen will be given by $\Psi_1(x, t)^2$, since that density is given by the *square* of the wave function, see the curve on the right of Fig. 4.8. Similarly, if

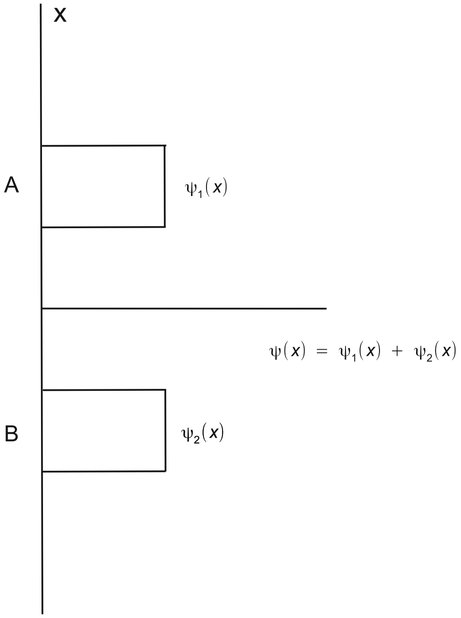

Fig. 4.7 The same wave function as in Fig. 4.5, but drawn vertically

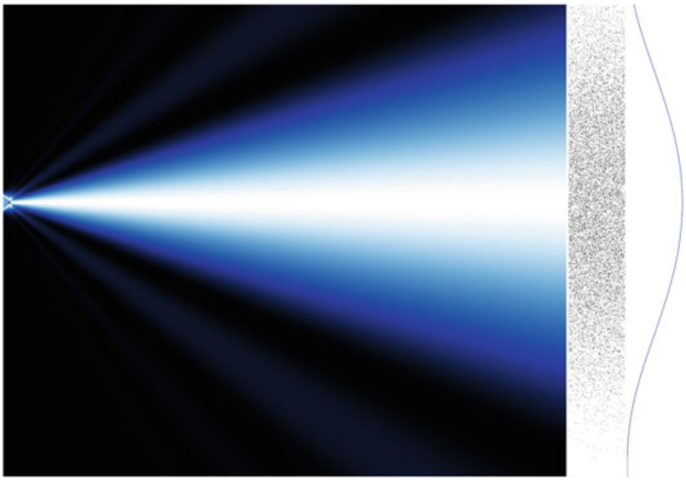

Fig. 4.8 The time evolution of the wave function in the situation of the double-slit experiment when only the upper slit is open (the time evolution goes from *left* to *right*). The *white* and *blue* areas indicate places where the wave function is non zero and their intensity is proportional to the square of the wave function (*white* more intense, *blue* less intense). *Dots* on the right indicate the impact of particles. The *blue curve* on the right indicates the density of such impacts. This figure corresponds to part (a) of Fig. 2.6, but in two dimensions and interpreted in the language of the wave function (A. Gondran cc by-sa 4.0)

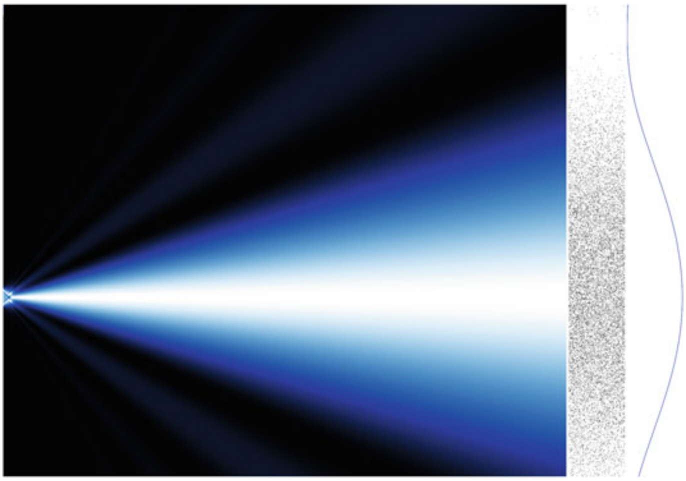

Fig. 4.9 The time evolution of the wave function in the situation of the double-slit experiment when only the lower slit is open (the time evolution goes from *left* to *right*). The *white* and *blue* areas indicate places where the wave function is non zero and their intensity is proportional to the square of the wave function (*white* more intense, *blue* less intense). *Dots* on the right indicate the impact of particles. The *blue curve* on the right indicates the density of such impacts. This figure corresponds to part (b) of Fig. 2.6, but in two dimensions and interpreted in the language of the wave function (A. Gondran cc by-sa 4.0)

only the lower slit is open, the density of particles detected at a point x on the screen will be given by $\Psi_2(x,t)^2$, see the curve on the right of Fig. 4.9.

But if both slits are open, then the initial wave function, just beyond the slits, will be the sum of the wave function associated with the upper slit and the one associated with the lower slit, $\Psi(x,0) = \Psi_1(x,0) + \Psi_2(x,0)$.

And, because of the fundamental property of *linearity* of the evolution of the wave functions, the wave function arriving on the second screen when both slits are open will be the sum of the two wave functions, $\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t)$.

So, the density of particles detected at a point x on the screen will be given by the square of $\Psi(x,t)$, namely by $(\Psi_1(x,t) + \Psi_2(x,t))^2$, see the curve on the right of Fig. 4.10. But that square *is not equal* to the sum of the squares $\Psi_1(x,t)^2 + \Psi_2(x,t)^2$.[8]

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[8] To see this, consider the following example:

$$(3+4)^2 = 7^2 = 49 \neq 3^2 + 4^2 = 9 + 16 = 25. \tag{4.3}$$

In general, for real numbers a and b, we have:

$$(a+b)^2 \neq a^2 + b^2, \tag{4.4}$$

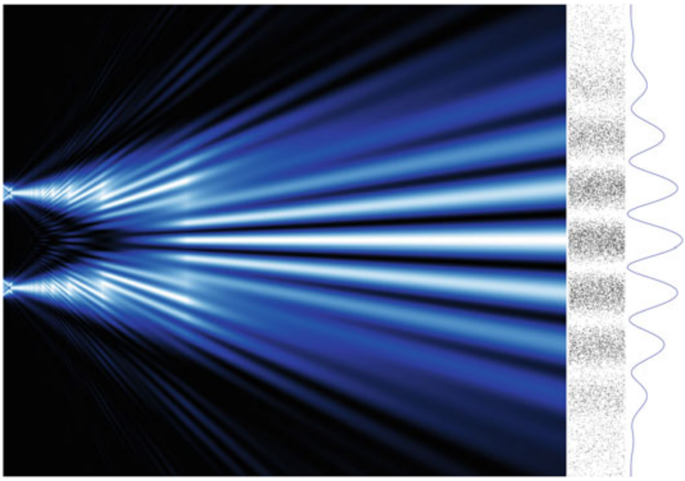

Fig. 4.10 The time evolution of the wave function in the situation of the double-slit experiment when both slits are open (the time evolution goes from *left* to *right*). The *white* and *blue* areas indicate places where the wave function is non zero and their intensity is proportional to the square of the wave function (*white* more intense, *blue* less intense). We see that at some places, the waves coming from both slits interfere constructively, and at other places, destructively. *Dots* on the right indicate the impact of particles. The *blue curve* on the right indicates the density of such impacts. This figure corresponds to part (c) of Fig. 2.6, but in two dimensions and interpreted in the language of the wave function (A. Gondran cc by-sa 4.0)

Since the quantum mechanical rules predict that the distribution of electrons detected on the second screen will be given by the square of the wave function corresponding to one slit being open, or the other one, or both, depending on what is open or not, we can understand, at least qualitatively, why the density given by the curve on the right of Fig. 4.10 can be less than the sum of the densities given by the curves on the right in Figs. 4.8 and 4.9 at some points (and larger at others).

We will illustrate this phenomenon via a simple mathematical example in the Appendix.

On the other hand, if we put a detector behind the first screen, as in Fig. 2.7, we can tell through which slits the particle went, so that the wave function

---

(Footnote 8 continued)
or, with $a = \Psi_1(x, t)$ and $b = \Psi_2(x, t)$,

$$(\Psi_1(x,t) + \Psi_2(x,t))^2 \neq (\Psi_1(x,t))^2 + (\Psi_2(x,t))^2, \tag{4.5}$$

namely the square of a sum is not equal to the sum of the squares!

collapses to either $\Psi_1(x, 0)$ or $\Psi_2(x, 0)$, because the detector performs a measurement and thus the collapse rule applies, and the interference pattern disappears: the resulting density of particles detected on the second wall will be given either by $\Psi_1(x, t)^2$ or $\Psi_2(x, t)^2$.

In the situation of Fig. 2.7, if we *do not* detect the particle going through the lower slit, it means that we "know" that it went through the upper one and thus the wave function collapses to $\Psi_1(x, 0)$ and the density of particles detected on the second wall will be given by $\Psi_1(x, t)^2$.

Finally, in the delayed-choice situation of Sect. 2.2, when no detection plate is inserted, the wave functions pass each other and, if t is now the time of arrival at the counter C_1 and C_2, the resulting detections will be given by $\Psi_1(x, t)^2$ or $\Psi_2(x, t)^2$, where $\Psi_1(x, t)^2$ will be concentrated at C_1 and $\Psi_2(x, t)^2$ will be concentrated at C_2 (see Fig. 2.9). On the other hand, with the detection plate P inserted, one gets the usual interference pattern of $(\Psi_1(x, t') + \Psi_2(x, t'))^2$, with t' now being the time when the waves reach the detection plate (see Fig. 2.10).

This finishes our explanation of how the quantum mechanical rules concerning the wave function predict the observed behavior in the double-slit experiment. But we do not claim to have *explained* what happens in the experiment. In fact, ordinary quantum mechanics usually emphasizes the fact that the theory does not explains what happens but only predicts it.

4.3 Einstein's Early Worries

At the Solvay Congress of 1927, which was a historical landmark in the discussions about the meaning of quantum mechanics, Einstein considered a particle going through a hole, as shown in Fig. 4.11. In the situation described in the picture, the wave function spreads itself over the half circle,[9] would but one always detects the particle in one piece at a given point, somewhere on the detection surface denoted by P in Fig. 4.11.[10]

Einstein raised the following objection:

> But the interpretation, according to which $(\Psi)^2$ expresses the probability that *this* particle is found at a given point,[11] assumes an entirely peculiar mechanism

---

[9] We speak of a half circle because the picture is two dimensional, but of course in three dimensions the spreading would be over a hemisphere.

[10] The situation is not very different than what happens in the two slits experiment when only one slit is open (see parts (a) and (b) of Fig. 2.6).

[11] Here Einstein writes $(\Psi)^2$ of what we write $\Psi(x)^2$, x being the point to which he refers (Note by J.B.).

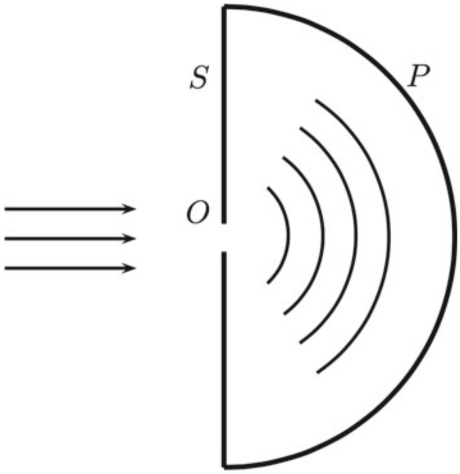

Fig. 4.11 A version of Einstein's objection at the 1927 Solvay Conference. Drawing by Travis Norsen. See [7, p. 440], or [182, p. 254] for the "original" (published in French translation at the time of the Solvay Conference)

> of action at a distance, which prevents the wave continuously distributed in space from producing an action in *two* places on the screen.
>
> Albert Einstein [7, p. 441]

Indeed, if the wave function is spread out in space before the particle is detected, then the fact that the latter is always detected at a given point implies that the wave function collapses on that point (or on a small neighborhood of that point), i.e. that it vanishes everywhere else instantaneously. Thus some sort of action at a distance (detecting the particle at one place makes the wave function vanish everywhere else) must be taking place. Einstein adds:

> In my opinion, one can remove this objection [action at a distance] only in the following way, that one does not describe the process solely by the Schrödinger wave, but that at the same time one localises the particle during the propagation.
>
> Albert Einstein [7, p. 441]

Here Einstein stresses the essential ambiguity concerning the meaning of $\Psi(x)^2$: does it describe the probability of the particle *being* somewhere or of the particle *being detected* somewhere if we detect its position, but not being anywhere before that? And Einstein also stresses that the second meaning, which is the orthodox view of quantum mechanics, implies this "peculiar mechanism of action at a distance", namely the collapse of the wave function at a point.

Einstein could not accept this "action at a distance" (nor did anybody else at that time), for reasons that will be discussed in Chaps. 7 and 10. In Chap. 8 we will see that the situation described by Einstein does *not* imply any action at a distance, but that this will be true only within a more complete theory than ordinary quantum mechanics, a theory in which "one localises the particle during the propagation".

The essence of Einstein's objection to the orthodox view of quantum mechanics, which is often misunderstood, was always based on this nonlocal aspect of the orthodox view.

4.4 Heisenberg's Inequality or "Uncertainty Principle"

Heisenberg's inequality is one of the mathematical consequences of the quantum formalism that is often presented as being one of the main "mysteries" of quantum mechanics (although we do not see it that way).

To explain this inequality, we have to mention that, besides measurements of positions, one can also measure velocities and that, given a wave function, one can compute the probability distribution of results of "measurements of the velocity".[12] Of course, since we stressed that, in ordinary quantum mechanics, particles do not have trajectories, the reader may wonder how they can possibly have a velocity, since the velocity simply quantifies the way the position of a particle changes along its trajectory. But this has again to do with the fact that we speak of "results of measurements", not of what exists independently of measurements and measurements may not simply measure some intrinsic property of the particle being measured, but may perturb the particle or interact with it in an uncontrollable way.

In 1927, right at the time when the quantum theory was being developed, Heisenberg discovered an inequality relating the probability distribution of the position measurements to the probability distribution of the velocity measurements.[13]

To be more precise, Heisenberg wrote an inequality relating how "spread out" the probability distribution of the position measurements is compared to how "spread out" the probability distribution of the velocity measurements

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[12] We will come back to what measurements of velocity really are in Sect. 8.3.

[13] Heisenberg, and physicists generally, speak of momentum measurements rather than velocity measurements, but momentum is simply defined as the product of the mass times the velocity.

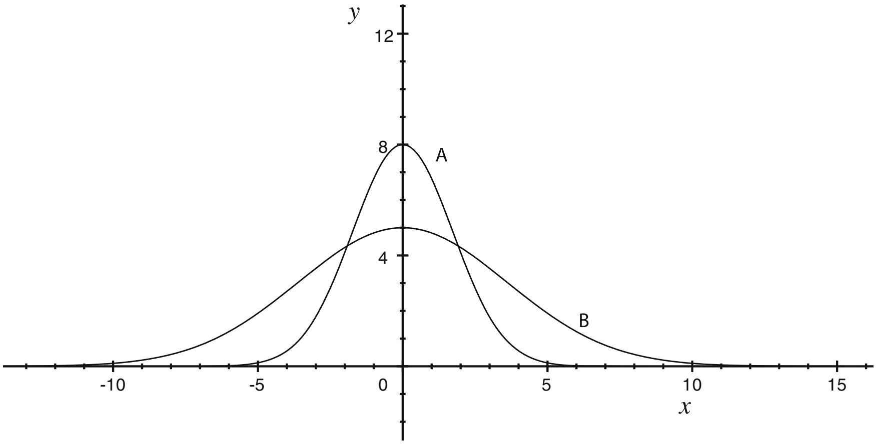

Fig. 4.12 The *curve B* is more "spread out" or more "flat" than the *curve A*

is. The notion of a probability distribution being "spread out" or "flat" is illustrated in Fig. 4.12.

In words, Heisenberg's inequality says that the less "spread out" the probability distribution of the position measurements is, the more "spread out" the probability distribution of the velocity measurements must be, and vice versa.[14]

But what does this inequality mean? Sometimes it is interpreted as showing that particles *do not have* well-defined positions and velocities before the latter are measured (which, of course, again puts back measurements on center stage).

But that is not what the inequality means, strictly speaking; it only relates how spread out the probability distributions of certain *measurements* (of positions and velocities) are and says nothing whatsoever about what happens outside of measurements or before them. Since a priori "measurements" might affect or disturb the system being measured and not simply reveal some property of it, we cannot conclude anything, at this stage, about what Heisenberg's inequality implies concerning what goes on outside of measurements.

---

[14] For the more mathematical reader: there is a standard way to measure how spread out a probability distribution is and it is given by the *variance* of that probability distribution. Let $\mathrm{Var}(x)$ denote the variance of the distribution of the measurements of the position x and $\mathrm{Var}(p)$ the variance of the distribution of the product of the mass m times the measurements of the velocity v: $p = mv$. What Heisenberg showed is that their product cannot be made arbitrarily small and satisfies a lower bound:

$$\mathrm{Var}(x)\mathrm{Var}(p) \geq \frac{1}{4}, \tag{4.6}$$

where the value $\frac{1}{4}$ depends on a choice of physical units that we shall not discuss. But, independently of this value, what this lower bound implies is that, if $\mathrm{Var}(x)$is very small, then $\mathrm{Var}(p)$ must be very large, and vice-versa.

Let us also mention that calling Heisenberg's inequality a "principle" (like in "uncertainty principle" or sometimes "indeterminacy principle") is misleading because it suggests that this is a principle independent of other principles of the quantum theory, while it is in fact a mathematical consequence of ordinary quantum mechanics (the purpose of this remark is only to clarify the status of this inequality, and obviously not to minimize the value of Heisenberg's discovery, which was quite extraordinary).

4.5 Conclusions

Coming back to our three fundamental questions, what does the duality of rules described here imply?

1. It suggests some sort of "reality created by the observer", since the rules of physics are different when one performs a measurement and when one does not.
2. Moreover, the rules are fundamentally statistical: one predicts the probabilities of events, but nothing is said about what might determine the behavior of each individual particle. Hence, quantum mechanics looks "intrinsically random".
3. There seems to be something nonlocal going on, which is what bothered Einstein already in 1927.

As we saw, the mysterious behavior described in Sects. 2.1 and 2.2 can easily be predicted by the quantum formalism. But we then face a basic puzzle: how can one understand all this talk about wave functions, and this duality in their rules of evolution, that singles out the role of measurements?

In the next chapter, we shall examine some natural sounding answers to those questions.

4.6 Summary

In this chapter, we introduced, without real formulas, the mathematical formalism used by physicists to predict the results described in Chap. 2. We emphasized that this formalism has to be viewed, at this point, only as an efficient recipe, but nothing more. All questions concerning its meaning will be discussed later.

The central concept is the one of the wave function Ψ, which, in quantum mechanics, is associated to any physical system.

The wave function $\Psi = \Psi(x)$ is a function of one variable for a single particle in one dimension of space, and its sole meaning is that $\Psi(x)^2$ determines the probability of finding the particle somewhere, when one detects its position. This is illustrated in Fig. 4.3.

It is important to understand that $\Psi(x)^2$ does *not* represent a density of "stuff" (mass or electric charge) nor does it determine the probability of the particle *being* somewhere, when its position is not measured. Indeed a wave function can in principle be as in Figs. 4.5 and 4.6. Since the regions A and B can be far apart, the density of stuff picture does not make sense. In that situations, one says that the wave function is a superposition (or a sum) of two wave functions $\Psi_1(x)$ and $\Psi_2(x)$.

One could of course say that the particle is either in region A or in region B *before* being observed. But that is not what the theory says. It is agnostic as to what happens before observations.

The reason for that attitude, which sounds paradoxical, is that, in order to account for the double-slit experiment, one has to assume that the wave function, when both slits are open, is a superposition (meaning a sum as in (4.1)) of two wave functions, one behind each slit, at the time when the "particle" (or the "wave") reaches the first wall, with the two slits in it. And, in that situation, quantum mechanics does not commit itself to saying that the particle goes through one slit or the other.

The evolution of the wave functions puts observations on center stage: when no observations are made, they propagate smoothly, and deterministically, like waves and their evolution is linear (see (4.2)). But when observations are made, they suddenly jump or get reduced: for example, if we measure whether the particle is in region A or B in Fig. 4.5, the wave function becomes $\Psi_1(x)$ or $\Psi_2(x)$, depending on the result. This is a sudden jump, which moreover is "random" in the sense that one can only attribute probabilities to its outcome (here one-half for each possibility, $\Psi_1(x)$ or $\Psi_2(x)$).

In the double-slit experiment, one has two wave functions, one behind each slit, when both slits are open. The interference pattern that we see in part (c) of Fig. 2.6 is obtained by letting these two wave functions evolve as waves between the two walls. Then, at some places, they interfere constructively, at other places, destructively.

But, if one observes through which slit the particle goes, for example by putting a detector behind one of the slits (see Fig. 2.7), then the wave function collapses and is reduced to the wave function behind the slit through which the particle goes: either the upper one if the particle is detected there or the lower one if it is not. Then, that remaining wave function evolves between the walls, but, since the other wave function is now absent, it no longer contributes to the production of an interference pattern.

We then mentioned an early objection of Einstein, that he repeated throughout his life: consider Fig. 4.11; here the wave function spreads itself on the half circle P, yet the particle is always detected at a single point. For Einstein, either the wave function is not the complete story and the particle has, besides its wave function, also a well defined position or the wave function is the complete story but then the fact that the particle is detected at a single point means that the wave function suddenly collapses at that point and thus vanishes everywhere else. But that is a nonlocal effect or "action at a distance" that Einstein could not accept.

We added a remark on Heisenberg's inequality, which is a mathematical relation saying that the less spread out the results of measurements of the position of the particles with a given wave function are, the more spread out will be the results of measurements of their velocities, and vice versa. Although this inequality is often interpreted as showing that particles do not have definite positions or velocities before being measured, we emphasized that precisely because the status of the wave function and the role of the measurements is unclear in ordinary quantum mechanics, no such definite conclusion can be drawn.

This shows how physicists deal with the phenomenon of interference, but it does not really explain anything, because the physical meaning of the wave function, and particularly of superpositions is unclear: it is an extremely accurate tool for predicting results of measurements, but, so far, nothing else.

Appendix

4.A The Wave Function

The first precision to be made about the wave function is that $\Psi(x)$ is in general a complex number and, to be correct, one should have written everywhere $|\Psi(x)|^2$ instead of $\Psi(x)^2$ in Sects. 4.1 and 4.2, where, for a complex number $z = a + ib$, $|z|^2 = a^2 + b^2$.

The fact that the total area under the curve in Fig. 4.3 is equal to one means that $\int_{\mathbb{R}} |\Psi(x)|^2 dx = 1$. This ensures that the probability of finding the particle somewhere is equal to one, as it should!

The probability of finding the particle in a region A is therefore $\int_A |\Psi(x)|^2 dx$, see Fig. 4.3.

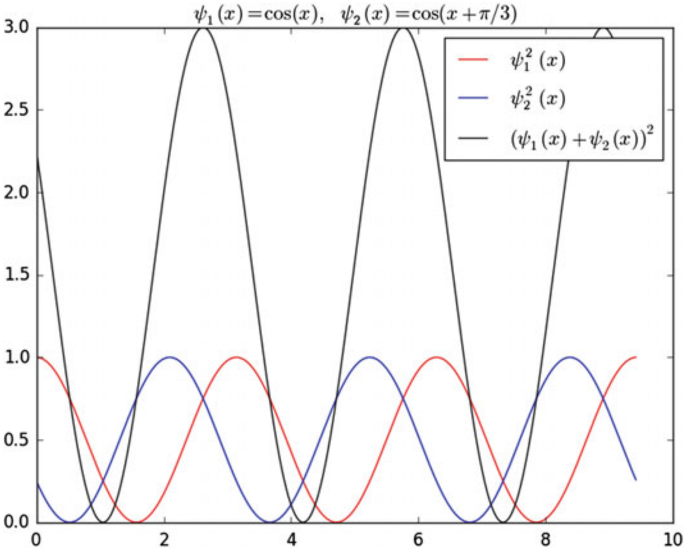

Fig. 4.13 The graphs of $\Psi_1(x)^2$, $\Psi_2(x)^2$ and $(\Psi_1(x) + \Psi_2(x))^2$, for two different wave functions $\Psi_1(x)$ and $\Psi_2(x)$ (for those who are familiar with trigonometric functions, we chose $\Psi_1(x) = \cos x$ and $\Psi_2(x) = \cos(x + \frac{\pi}{3})$, but this is not important) (A. Gondran cc by-sa 4.0)

Finally, in order to keep that constraint, each of the collapsed wave functions, after a measurement, must also satisfy $\int_{\mathbb{R}} |\Psi(x)|^2 dx = 1$. In Fig. 4.5, the situation is symmetric and, since the regions where $\Psi_1(x)$ and $\Psi_1(x)$ are non-zero do not overlap, one has $\int_{\mathbb{R}} |\Psi(x)|^2 dx = \int_{\mathbb{R}} |\Psi_1(x)|^2 dx + \int_{\mathbb{R}} |\Psi_2(x)|^2 dx = 1$ and thus $\int_{\mathbb{R}} |\Psi_1(x)|^2 dx = \int_{\mathbb{R}} |\Psi_2(x)|^2 dx = \frac{1}{2}$. So, the collapsed wave function is not $\Psi_1(x)$ or $\Psi_2(x)$, as we said in Sect. 4.2, but rather $\sqrt{2}\Psi_1(x)$ or $\sqrt{2}\Psi_2(x)$, that satisfy $\int_{\mathbb{R}} |\sqrt{2}\Psi_1(x)|^2 dx = \int_{\mathbb{R}} |\sqrt{2}\Psi_2(x)|^2 dx = 1$.

To illustrate the phenomenon of constructive and destructive interferences, consider Fig. 4.13, where the three curves represent the functions $\Psi_1(x)^2$, $\Psi_2(x)^2$ and $(\Psi_1(x) + \Psi_2(x))^2$ (we suppress the variable t here). We chose those functions (with the x axis drawn horizontally), so that they resemble (qualitatively) the wiggly blue curve on the right of Fig. 4.10. We note that the function $(\Psi_1(x) + \Psi_2(x))^2$ may vanish at points x where neither $\Psi_1(x)^2$ nor $\Psi_2(x)^2$ vanish. It can also be larger than the sum $\Psi_1(x)^2 + \Psi_2(x)^2$ for other x's. In the latter case, one says that the waves interfere constructively and in the former one that they interfere destructively.

5

Schrödinger's Cat and Hidden Variables

We have seen, in a very simple example, how the quantum formalism works. Let us emphasize that it works in far more complicated situations and works spectacularly well, but always using the same basic principles.

Now we have two problems to deal with:

(1) What does it mean exactly to associate a wave function to a physical system?
(2) Why are there two different rules of evolution of the wave function of physical systems, one when there is no measurement and another one when a measurement is made?

We shall see that none of these problems are easy to solve, but we shall start by considering a possible solution to the second one.

5.1 The Problem of Schrödinger's Cat

Could one suggest that a measurement, viewed as a physical process, perturbs the quantum particle in such a way as to "create" the result and collapse the state when the latter has the form of a superposition such as (4.1)? There is nothing a priori irrational or even strange about this idea: the measuring device is necessarily macroscopic (otherwise we would not be able to see the result) and the object being measured is microscopic. The huge difference in size between the measuring device and the system being "measured" leaves ample room for the macroscopic object to affect the microscopic one. To appreciate the difference of size, note that there are on the order of 10^{23} (1 followed by

J. Bricmont, *Quantum Sense and Nonsense*, DOI 10.1007/978-3-319-65271-9_5

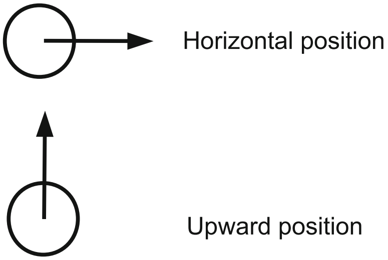

Fig. 5.1 Evolution of the pointer during a measurement when the state of the particle is Ψ_1

23 zeros) atoms or molecules in a gram of ordinary matter[1] and that the size of an electron is much smaller than the one of the atom.[2]

To further examine this idea, we need to be able to treat the process of measurement within the quantum mechanical formalism, but without introducing a priori the reduction of the wave function as an independent postulate (since we want that reduction to emerge from the rest of the quantum formalism). This quantum analysis of measurements was carried out in 1932, by the mathematician John von Neumann, who published in German a groundbreaking work on the *Mathematical Foundations of Quantum Mechanics* that was translated into English in 1955 [193]. In that book, von Neumann gave a fully quantum mechanical treatment of a system composed of a particle and its measuring device.

Consider the wave function of (4.1):

$$\Psi(x) = \Psi_1(x) + \Psi_2(x), \tag{5.1}$$

where $\Psi_1(x)$ is a wave function localized near the upper slit in the double-slit experiment and $\Psi_2(x)$ a wave function localized near the lower slit: in Fig. 4.10, one shows, in white and blue, the intensity of the square of those wave functions behind each slit.

Consider also a measuring device, represented only by a pointer, see Figs. 5.1, 5.2 and 5.3 which detects through which slits the particle went. It could be the detector in Fig. 2.7, but we will forget about the detector and simply assume that it can be coupled to the pointer. The pointer is initially in a horizontal position, see the top of Figs. 5.1 and 5.2, and ends up being in a vertical

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[1] What is called *Avogadro's number*, more or less given by the number of atoms in one gram of hydrogen, equals approximately 6×10^{23} namely 6 followed by 23 zeros.

[2] To be precise, the notion of "size of an electron" is somewhat ill-defined, but the huge difference of scale between electrons and objects that are directly observable by us is not in question.

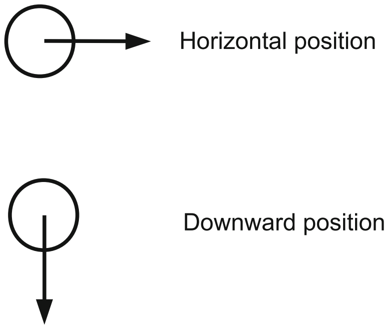

Fig. 5.2 Evolution of the pointer during a measurement when the state of the particle is Ψ_2

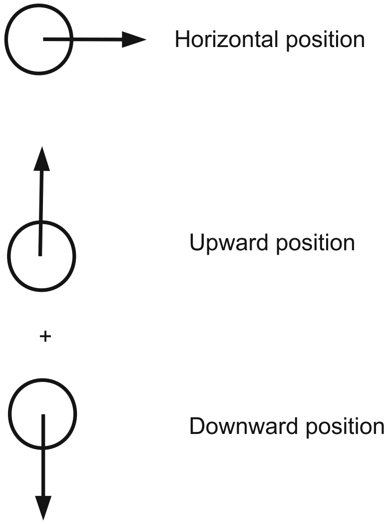

Fig. 5.3 Evolution of the pointer during a measurement when the state of the particle is $\Psi_1 + \Psi_2$

position, up or down, depending on whether the particle went through the upper or the lower slit, see the bottom of Figs. 5.1 and 5.2.

We give in the Appendix the description of measurements in the quantum mechanical formalism, but the idea is very simple and can be explained with pictures. Suppose that we first do the experiment with the lower slit closed. Then the particle goes through the upper slit and its wave function is simply Ψ_1. Then the pointer will necessarily indicate that the particle went through the upper slit, as at the bottom of Fig. 5.1.

If, on the other hand, we close the upper slit, the particle goes through the lower slit and its wave function is Ψ_2. Then the pointer will necessarily indicate that the particle went through the lower slit, as at the bottom of Fig. 5.2.

But now if both slits are open, since everything in the quantum formalism is linear (if we do not include the reduction of the wave function as an independent postulate), the result must be the "sum" of the two previous results or a "superposition" of the pointer being up (indicating that the particle went through the upper slit) and down (indicating that the particle went through the lower slit); this is illustrated in Fig. 5.3 and explained more formally in the Appendix.

The problem is that we never see the pointer in such a superposed state: we see it *either* up *or* down, but not both, and it is hard to understand what such a superposed state could even mean.

Thus, the natural conclusion to draw from this is that the quantum prediction in this case is simply *wrong*: it does not correctly predict the state of the measuring device at the end of the experiment, since it unambiguously predicts a superposed state, and this is not what is observed.

Of course, since the situation is now macroscopic, one may just *look* at the result. If the pointer points upward, we take the state of the particle to be Ψ_1. If the pointer points downward, the state becomes Ψ_2. One thus reduces the wave function, which is now indicated by the state of a visible object, just by looking at it.

But this puts again the "observer" quite explicitly at the center of the physical theory.

Schrödinger's rendered this situation more dramatic by replacing the pointer by a cat,[3] in his famous thought experiment [174]: suppose that a cat is in a sealed box and that there is a purely classical mechanism linking the pointer to a hammer that will break a bottle containing some deadly poison if the pointer is up, but not if it is down. If the poison is released, it kills the cat (see Fig. 5.4). Then, following the same reasoning as above, including now the state of the cat, we get that, after the measurement, the cat is in a superposed state "the cat alive and the cat dead".

Of course, we never see a cat in such a state. We do not even know what it could mean. But the cat example just dramatizes a problem that occurs already with the pointer: ordinary quantum mechanics predicts that the state of some macroscopic objects will be superpositions of states corresponding to two different physical situations, and such superpositions are simply not observed and are even hard to conceive.

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[3] See Sect. 10.2 for a discussion of Schrödinger's idea in a historical perspective and for the quote of the text where the cat example appears.

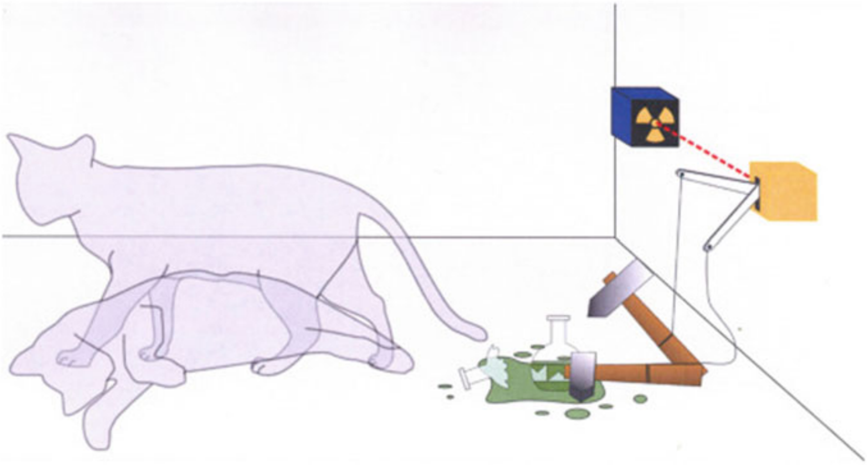

Fig. 5.4 The cat which is both alive and dead. By Dhatfield (own work) [CC BY-SA 3.0 (http://creativecommons.org/licenses/by-sa/3.0)], via Wikimedia Commons

Some people think that quantum mechanics has proven that the poor cat is both alive and dead before anybody looks at it, and that looking at the cat "collapses" its wave function. But what does "looking" mean? There are many different ways to look at an object. With the help of binoculars, or with a telescope, one could look from far away. One could peek through a small hole made in the box where the cat is, etc. None of this changes anything regarding the result of course: the pointer is always up or down, the cat is alive or dead. Since all the different physical ways of looking do not make any difference, isn't it reasonable to think that looking does not have any physical effect on the system itself and that by "looking" we simply *learn* something about the state of the system, without changing it? In other words, this situation would be analogous to tossing a coin and first hiding the result, by covering the coin with one's hand; when we later look at the coin, we see whether it is heads or tails, but of course the coin was heads or tails before we looked. This analogy is the common-sensical solution to the cat problem, and it is the one that Schrödinger had in mind.

Moreover, as long as "looking" is described in physical terms, with eyes, brains, etc., then, if one follows the linear evolution of ordinary quantum mechanics, one only gets more macroscopic superpositions: in the end, the whole universe has a wave function corresponding to a superposition of the pointer being up and the observer seeing it up "plus" the pointer being down and the observer seeing it down (at least as long as the observer is considered to be a physical object), and everything linked to the observer being in such a superposed state. One then arrives at a sort of infinite regress (everything physical being in a superposed state) and one has to appeal to a nonphysical consciousness to collapse the wave function. But even if we accept the existence of a conscious mind entirely independent of the laws of physics, which consciousness should it be? Mine when I observe? Someone else's consciousness when I tell them the results? A universal mind?

Eugene Wigner underlined the problem with the idea known as "Wigner's friend": suppose you are in a room where you can see whether the cat is alive or dead, but you have a friend outside that room who does not see the state of the cat. When does the state of the cat collapse for your friend? When you become conscious of the state of the cat or when you tell your friend what that state is? You could imagine that, before your friend learns about the state of the cat, his state describing the situation is a superposition of: "the cat is dead and the person in the room sees the cat dead" and "the cat is alive and the person in the room sees the cat alive".

The wave function now becomes a subjective affair and all this leads us to "theories" that are very poorly defined, assuming that they make any sense at all, but this explains why some people consider that quantum mechanics has proven that human consciousness intervenes directly into the world. While most physicists would reject such an idea as "nonsense", it is not clear how they refute it if they stick to the orthodox view of quantum mechanics.

And Eugene Wigner himself thought that it was "not possible to formulate the laws of quantum mechanics in a fully consistent way without reference to the consciousness." [205, p. 169].

Another line of thought is to say that the wave function never collapses and that the two situations, the cat alive and the cat dead, simply correspond to two different universes that co-exist: in one universe the cat is alive and my brain is in a state that sees her alive, and in another universe, she is dead and I see her dead. Of course, it means that the universe splits into two universes whenever a quantum experiment having several possible results takes place. This is called the "many-worlds interpretation" and will be discussed in Chap. 9.

When Schrödinger introduced his cat example, he called the situation "quite ridiculous", and saw it as a *reductio ad absurdum* of the quantum orthodoxy [174]. Indeed, the orthodox view is that the wave function is a *complete* description of any physical system. But the word "complete" is ambiguous: does it mean that this is all we can say about the system or does it mean that the system does not have any properties other than the ones described by the wave function? In other words, does the word "complete" refer to the limitations of our knowledge or to what exists?

Schrödinger wanted to show that the quantum formalism does *not* provide a complete description of at least *some* systems, because, in the case of the cat, we know that the latter is either alive or dead, and not both.

This was also the view of Einstein: ordinary quantum mechanics is an *incomplete* description of quantum systems. Some other variables must be introduced in order to get a complete description.

For the cat a more complete description would be that she is either alive or dead, even before one looks at it, just like a coin which is heads or tails. Of course, the same would be true for the pointer, which would be up or down even when nobody looks at it.

This means that we introduce, for the cat or the pointer, what are called "*hidden variables*". By definition (at least this is how we shall define them here, but this terminology is rather standard), this expression refers to any variables that complete the quantum description, i.e. that are not included in the wave function, for example the fact that the cat is alive or dead independently of whether one looks at it or not. If one follows that idea, the wave function of the cat would still be a superposition, but it would not be a complete description of the state of the cat.

The expression "hidden variables" is usually considered a bad word, but only because it goes against the orthodox view of quantum mechanics, that considers the quantum description as being complete and therefore such "hidden variables" as unnecessary, but Schrödinger's cat shows that the quantum description is not complete.

If we think in those terms, we give what is called *a statistical interpretation* of the wave function of the pointer, meaning that we consider that the pointer is up with probability $\frac{1}{2}$ and down with probability $\frac{1}{2}$, irrespective of whether one looks at it or not. And of course, the same would be true for the cat.

However, if we give a statistical interpretation of the wave function of the pointer, why not do the same for the wave function of microscopic objects described by wave functions like those in Chap. 4? In other words, why not introduce "hidden variables" all the way down so to speak, even for microscopic systems?

Since we have seen in this section that, if we stick to the quantum formalism, we cannot get rid of the duality of rules of evolution of the wave function (one when there are observations, and another one when there aren't any), nor of the central role of the observer, let us see where this idea of hidden variables leads us.

5.2 Hidden Variables

So, let us now assume that systems of all sizes have properties (like positions, velocities, energies etc.), before they are measured, and that a measurement *simply reveals a pre-existing value* associated to that property, which is what the word "measurement" suggests.

That assumption does not mean that we necessarily have a theory about those properties or variables or that we can predict or control them. It simply means that we can think of them as existing prior to our measurements and as being revealed by them.

The meaning of the wave function, if one thinks in terms of hidden variables, is to determine the statistical distribution of those variables. For example, if we prepare particles in a state such as (5.1), the assumption of hidden variables means that a fraction $\frac{1}{2}$ of such particles goes through the upper slit and a fraction $\frac{1}{2}$ of such particles goes through the lower one.

If one adopts that view, probabilities in quantum mechanics are not really different from classical probabilities: they just reflect our ignorance, and so does the wave function.

This is essentially what Einstein thought, when he summarized his position about quantum mechanics in 1949:

> I am, in fact, firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this (theory) operates with an incomplete description of physical systems [...].
>
> Albert Einstein [70, p. 666]

If we adopt a statistical interpretation of the wave function, the collapse rule is not so strange: we modify the wave function (and thus our probabilities) according to what the measurement reveals about the system.

To come back to the analogy with a coin, if we learn that it is heads or tails (a property that the coin had before looking at it), we change our probabilities for heads and tails from $\frac{1}{2}$ for heads and $\frac{1}{2}$ for tails (before looking) to 1 for heads and 0 for tails or vice-versa, depending on the result. In a statistical interpretation of quantum mechanics, the collapse of the wave function would be similar to that adjustment of probabilities. Of course, for the coin it is obvious that giving the probabilities $\frac{1}{2}$ for heads and $\frac{1}{2}$ for tails is *not* a complete description of the state of the coin.

In Sect. 4.4, we discussed the connection between the probability distribution of measurement of positions and the probability distribution of measurements of velocities, expressed by Heisenberg's uncertainty relation. We stressed that this relation did not prevent us from thinking that particles do have both a position and a velocity, which, by definition, would be "hidden variables", since their values would not be specified by the wave function (the latter giving only probabilities of "results of measurements" of positions and velocities).

However, there is a serious problem with the introduction of such "hidden variables", namely that it is *inconsistent*. Indeed, one can prove the following:

No Hidden Variables Theorem

Assuming that measurements of positions and velocities of particles such as electrons reveal values of those variables pre-existing to their measurement and, moreover, that the statistical distributions of those values agree with the predictions of quantum mechanics, leads to a contradiction.

Unfortunately, the reader has to take us on faith here. This theorem, although not terribly difficult to prove for mathematicians or physicists, is too technical to be explained in this book, even in an Appendix.

Moreover, this is not a real theorem, because it would go beyond the scope of this book to state its hypotheses precisely, but that can be done.[4] Of course, if a proposition (here the pre-existence of the values of the positions and velocities of particles) implies a contradiction, then that proposition must be false. So, one cannot assume that, in general, particles have pre-existing values of positions and velocities, whose statistical distributions agree with the predictions of quantum mechanics, and that those values are revealed by their "measurements".

It must be emphasized that this result is "derived from the facts alone", insofar as anything is derived from facts. In particular, to prove the above theorem, we do not need to assume that the whole quantum theory is exactly true, but only that certain empirical predictions of quantum mechanics are correct. And those predictions have been amply verified experimentally. Of course, we would not know those predictions if we did not have quantum mechanics to start with, but once the predictions are made and verified, one can reason using those facts alone, without having to assume that the full quantum theory is correct. So that, even if quantum mechanics is superseded some day by another theory, the no hidden variables theorem will still hold.

Let us stress that a misleading reaction to this theorem would be to say that there is nothing new here, since, as we have seen, the wave function does not assign a given value to the positions and the velocities of the particles. But that misses the point of the theorem, which is to consider the possibility that there be other variables characterizing an individual system than its wave

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[4] Remark: for the theorem to be true one needs to consider at least two particles on a line or one particle in two or more dimensions. See [36, Theorem 2.5.1] for details.

function, variables whose values would be revealed by proper measurements. The theorem shows that merely assuming the existence of those variables is impossible. Note that we are not assuming that there exists a theory about those "hidden" variables, telling us how they evolve in time for example, but merely that these variables exist and that their statistical distributions agree with the quantum mechanical predictions.

Actually, this theorem is only one version of a family of similar theorems (hence, we will speak of "no hidden variables theorems" with an s): indeed there are other quantities that can be "measured" in quantum mechanics and for which one can show that those measurements do not reveal pre-existing values of the quantity being measured.[5]

This sort of theorem leads some people to think that "nothing exists" outside of measurements or that "the moon is demonstrably not there when nobody looks." [124, p. 397]. But that is an over-interpretation!

Indeed, this result *does not mean* that a theory introducing hidden variables cannot exist, but *it does mean* that one cannot introduce such variables in a simple-minded fashion. Any theory that introduces some hidden variables has to avoid being refuted by the no hidden variables theorems and, thus, has to explain why the hidden variables that it introduces are not those "forbidden" ones (such as values for both the positions and the velocities of the particles whose statistical distributions agree with the quantum mechanical predictions). It should be stressed that the theorem *does not say* that it is impossible to introduce particle positions and trajectories, hence velocities (indeed, the de Broglie–Bohm theory to be discussed in Chap. 8 manages to do exactly that). Of course, one then has to explain what "measurements of velocities" could mean in that theory; but that is precisely what we will do in Sect. 8.3.

Another way to understand the no hidden variables results is to say that the measuring device must necessarily have an "active role" and not simply passively record some pre-existing property. The idea that a measuring device has an active role means that it affects the system when a measurement occurs.

This idea of the measuring device having an active role has been emphasized by Bohr and the "Copenhagen" school, but was stated as a sort of a priori principle, or as a *deus ex machina* which the quantum theory does not need to explain. As we shall see in Chap. 8, one of the main virtue of the de Broglie–Bohm theory is that it renders this active role quite explicit, and makes it a consequence of the equations of the theory themselves.

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[5] An example of such a quantity is the "spin" that we will discuss in Chaps. 7 and 8.

5.3 A Deeper Problem: What There Is

We can now reflect on what we have said in the previous sections and ask ourselves a more fundamental question: what can the wave function possibly mean? After all, it is a *function.* A function Ψ, such as the one introduced in Sect. 5.1, assigns a number $\Psi(x)$ to every number x. But we limited ourselves to one real variable x. In more complicated systems, one will introduce several variables, $x_1, x_2, \ldots, x_N$ one for each parameter characterizing the system. If the system is made of n particles in the usual three-dimensional space, $N = 3n$, since one needs three parameters or coordinates for each particle, and $\Psi = \Psi(x_1, x_2, \ldots, x_N)$ will be a function of all those variables, which means that it will assign a number to any given sequence $x_1, x_2, \ldots, x_N$.

For example, in one wants to be precise when one discusses the pointer or the cat, as in Sect. 5.1, one should introduce a sequence $x_1, x_2, \ldots, x_N$, where N is the number of parameters characterizing the state of the pointer or of the cat.

But what does that function Ψ mean? If one views it only as a tool to predict "results of measurements", using both rules of evolution (according to Schrödinger's equation outside of measurements and with collapses during measurements), even though they are mutually inconsistent, then nobody disagrees.

But, if one wants to go beyond what happens in a laboratory, what does Ψ mean concerning the microscopic world? If one does not give it a statistical interpretation (and, as we just saw, such an interpretation runs into difficulties) it is not clear at all. In fact, it has no clear meaning for the macroscopic world either, since cats and pointers are also described by a wave function that can be in a superposition of macroscopically different states, as we saw in Sect. 5.1.

One should not think that $x_1, x_2, \ldots, x_N$ represent the *actual values* of the parameters characterizing the state of the pointer or of the cat. Indeed, the wave function does not specify what the real values of those parameters are, and if one rejects the statistical interpretation of Ψ, then, $x_1, x_2, \ldots, x_N$ cannot be interpreted either as actual or possible values of those parameters. They only take given values when one performs a measurement in a laboratory.

This problem is sometimes called the one of "ontology" (a philosophical word that refers to what exists) or of "beables", a word invented by John Bell and derived from the verb "to be" in the same way that "observables" is derived from the word "to observe".

The issue of ontology amounts to asking which assumptions physics makes about what exists "out there", meaning outside the laboratories. Physicists of course speak of particles, photons, electromagnetic waves and all kinds of other things as if they existed “out there”. But if one sticks to the orthodox quantum mechanical description of those objects, it is not at all clear what properties, if any, those objects have (at least when they are not measured in a laboratory); and if they do not have any property whatsoever, no position, no velocity, no energy, etc., what does it mean to say that they “exist out there”?

If you ask a physicist what it means for an electron somewhere in space to have a given wave function, the only honest answer he could give is to say that, if one brings that electron in a laboratory and one does certain operations on it (called measurements), the results will be such and such with such and such probabilities, and nothing more. To say the least, this is not very satisfactory.

In Chap. 8 we will give our preferred alternative to that situation, but in order to appreciate what that alternative achieves, one must first appreciate the depth of the problem.

5.4 Conclusions

Coming back to the two questions raised at the beginning of this chapter, we have reached negative conclusions:

(1) Even if we incorporate the measuring device into the quantum formalism, we still do not get rid of the duality of rules and of the necessity of introducing “observers” into the formalism.
(2) We do not know what it means for a function to represent the state of a physical system, apart from predicting results of measurements. The statistical interpretation would solve that problem, but it leads to contradictions.

Yet, it is probable that most physicists who are not bothered by the meaning of quantum mechanics have in the back of their minds one of the following two “solutions”:

That the interaction of a microscopic system with a macroscopic one produces an unambiguous result, and this solves the problem of the duality of rules in quantum mechanics: at the end of an experiment, the pointer is up *or* down, the cat is alive *or* dead.[6]

Or, that “measurements” mean what the word suggests, namely that they reveal some property of the system, which may be uncontrollable and unknowable before the measurement, but that exists.

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[6] In [12] John Bell shows how in quantum mechanical textbooks one slips without warning from statements like “the cat is alive *and* dead” to “the cat is alive *or* dead”. The problem is that those statements are not the same: the second one is innocuous but the first one is not understandable. Unfortunately, quantum mechanics unambiguously predicts the first.

Of course, if one of those positions were tenable, the "no worry" physicists would be right not to worry, but, as we saw, neither of these "ways out" is compatible with either the quantum formalism (for the first one) or with experimental facts (for the second one).

Now we face a serious conundrum. There are three possible reactions to that state of affairs.

(1) A *first reaction* is to claim that one cannot understand the microscopic world and that one must content oneself with predicting the results of measurements, which are necessarily macroscopic, and are thus described in an intuitively understandable language.
For example, one could say that, if one prepares the particle so that it has some wave function, it will have such and such probability of ending up with some other wave function if we do this or that experiment (remember the collapse rule), where that final wave function will be described only in terms of what is indicated by the measuring devices. This is an entirely macroscopic observation (we can just look at the state of the detector, or of the pointer). Finally, one claims that this is all that one can say about microscopic systems.

(2) A *second reaction* is to introduce a sharp quantum/classical divide: the microscopic world is described by quantum mechanics, the macroscopic one by classical physics, where "classical" means simply that objects have definite properties unlike the ones in superposed states. In that approach, the problem of macroscopic superpositions does not arise, since classical objects do have definite states (pointers are up or down, cats are alive or dead) and, by postulation, pointers and cats are "classical".
The problem with that approach is that nobody knows where the quantum/classical divide occurs. If one atom or two atoms are described by quantum mechanics, what about ten atoms? A million atoms? Where is the limit between objects that do not have definite properties and those that have? This is not a sort of limit that one can get closer and closer to, but a sharp jump from not having definite properties to having them.

(3) A *third reaction* is to propose a more complete theory than quantum mechanics, which means introducing hidden variables. But the results concerning the impossibility of introducing some hidden variables suggest that such a theory is not simple to find.

The first two reactions are associated with the Copenhagen interpretation and are basically the "orthodox" positions. They are actually similar to each other.

In the next chapter we shall discuss a certain number of philosophical arguments trying to present those reactions as not simply making the best of a bad deal, but as a necessity, independently of quantum mechanics, or even as a positive development.

The de Broglie–Bohm theory corresponds to the third reaction and will be the topic of Chap. 8.

5.5 Summary

In this chapter, we discussed two different ways to try to understand the meaning of the wave function and of the collapse rule.

The first one is to apply the principles of quantum mechanics to all objects irrespective of their sizes, and in particular to measuring devices. One might a priori hope that the result of a measurement of, say, which slit a particle goes through in the double-slit experiment would indicate a well-defined result. This would then explain why a measurement reduces or collapses the wave function.

Alas, this solution does not work. As we saw this reasoning simply leads to macroscopic superpositions that are ununderstandable. If one wants to dramatize the situation, but without changing anything substantial to the problem, one can replace the measuring device by a cat, as Schrödinger imagined, and one gets the poor cat to be in a superposed state of being both alive and dead.

Another natural way to try to solve our problems would be to introduce a statistical interpretation of the wave function: the latter would describe not a single system, but a collection of "similar" systems, each of which would have properties that are revealed by their measurements. A bunch of particles whose wave function is a sum of two other wave functions, like the Ψ of (5.1) would simply be a bunch of particles half of which go through the upper slit and half of which go through the lower slit.

This would be analogous to a coin that has fallen heads or tails; before we look at it, we assign probability one-half to each outcome, and that would be like assigning a wave function that is a superposition. When we look at the coin our probabilities change to zero or one and that is like the state being reduced to either Ψ_1 or Ψ_2.

If that worked, what could be more reasonable?

It must be emphasized that this is *not* the same thing as having a wave function that is Ψ_1 *or* Ψ_2. Indeed, those positions are what are called "hidden variables", namely properties that the particles would have besides their wave function, which would still be the sum $\Psi = \Psi_1 + \Psi_2$. If one accepts that line of thought, the wave function is not a complete description of an individual system.

Likewise, a probability one-half for each face of the coin is not a complete description of the coin: it is either heads or tails after it has fallen on one side.

Alas, again, it turns out that this solution runs into logical difficulties: if one maintains that there are pre-existing values of the positions and the velocities of the particles, and that the statistical distribution of those values coincide with the quantum mechanical predictions, then one can derive a contradiction.

Finally, we summarized the problem by saying that it is not at all clear what the wave function Ψ means outside the laboratories. It is a function defined on a set of parameters characterizing a system, but if such a function is not linked to a statistical interpretation, namely if it does not give rise to a certain probability distribution on those parameters, then its status is totally unclear (apart from being a tool to predict results of measurements).

Now, we are in serious trouble, because none of the easy ways out of the problems raised so far seem to work.

Appendix

5.A Quantum Description of the Measurement Process

In order to give a quantum mechanical description of the measurement process, we must associate a wave function to the measuring device, namely here to the pointer. So, let φ_0 be the initial wave function associated to the pointer, where φ_0 means that the pointer is as in the top picture in Fig. 5.1.

One might question the assignment of a wave function to a macroscopic object such as a pointer. But this is exactly what we mean by "working within the quantum formalism" in analyzing the measurement process. In that formalism, by assumption, a wave function is associated to every object, even though we may not be able to describe that wave function in much detail. All we will need to know about this wave function is that there are differences between the wave function of the pointer in the top picture in Fig. 5.1 and the one at the bottom of that figure, and also with respect to the one at the bottom of Fig. 5.2.

Consider the following wave function for the particle (see (5.1)): $\Psi(x) = \Psi_1(x) + \Psi_2(x)$ (we suppress the argument x below), corresponding to the superposition of "the particle going through the upper slit" and "the particle going through the lower slit".

Now, let

$$\Psi_0 = \varphi_0 \times \ [\Psi_1 + \Psi_2] = [\varphi_0 \times \ \Psi_1 + \varphi_0 \times \ \Psi_2] \tag{5.2}$$

be the wave function Ψ_0 describing the initial wave function of the system composed of the particle going through the slits and the pointer. Here φ_0 is the wave function associated to the pointer in the horizontal position as in the top of Figs. 5.1 and 5.2 and × denotes the product of functions.

In order to see which wave function we shall obtain after the measurement, consider first another initial wave function, which corresponds to the lower slit being closed and thus the wave function of the pointer and the particle being Ψ_1:

$$\Psi_0^{\uparrow} = \varphi_0 \times \Psi_1 \tag{5.3}$$

Then, since the pointer will go in the up direction if the particle goes through the upper slit, the final wave function of the pointer and the particle will be:

$$\varphi^{\uparrow} \times \Psi_1,$$

where $\varphi^{\uparrow}$ corresponds to the pointer being as in the bottom picture in Fig. 5.1.

Similarly, if we start with the initial wave function, corresponding to the upper slit being closed and thus the particle having wave function Ψ_2, the initial wave function of the pointer and the particle will be:

$$\Psi_0^{\downarrow} = \varphi_0 \times \Psi_2. \tag{5.4}$$

Then, since the pointer will then go in the down direction, the final wave function of the pointer and the particle will be:

$$\varphi^{\downarrow} \times \Psi_2,$$

where $\varphi^{\downarrow}$ corresponds to the pointer being as in the second picture in Fig. 5.2.

Now, remember what we said about the most important property of the time evolution of wave functions: it is *linear*! Here, the initial wave function Ψ_0 (5.2) is a superposition of two other wave function, $\Psi_0^{\uparrow}$ (5.3) and $\Psi_0^{\downarrow}$ (5.4).

Linearity of the time evolution means that the wave function Ψ_0 will evolve into a superposition of the time evolution of $\Psi_0^{\uparrow}$ and $\Psi_0^{\downarrow}$.

This means that the time evolution of the initial wave function of the pointer and the particle Ψ_0 (5.2) has to be:

$$\varphi^{\uparrow} \times \Psi_1 + \varphi^{\downarrow} \times \Psi_2,$$

or, in words,

$$\begin{aligned}&\text{a superposition of the wave function } \varphi^{\uparrow} \times \Psi_1\\ &\text{and of the wave function } \varphi^{\downarrow} \times \Psi_2\ ,\end{aligned} \tag{5.5}$$

and this means that, as far as the pointer is concerned, it will be associated to a superposed wave function of "the pointer being up" and "the pointer being down".

Of course, that is not what is observed and is even impossible to make sense of.

6

"Philosophical" Intermezzo II: What Is Wrong with "Observations"?

The following thoughts may have crossed the reader's mind: if quantum mechanics accounts for our scientific observations, then what else can one ask for? After all, isn't all our knowledge based on observations? Indeed, we do not have access to the world directly, but only through our senses, in other words, through our observations. So, if we account for the latter, we account for everything that we have access to and everything else is "metaphysical".

When one discusses with physicists defending the orthodox view of quantum mechanics, one often encounters one or several of those ideas.

In this chapter, we will clarify the claims that we just made and explain why they do not assuage our worries about quantum mechanics.

There are two different aspects in this clarification: one is a general discussion about what is called "realism" in philosophy of science and the other is a more specific one about quantum mechanics. But, as in the discussion of free will and determinism in Chap. 3, our goal will be to show that this general philosophical discussion is not really relevant to the understanding of quantum mechanics.

However, even this modest goal is not so simple to reach: there is an enormous literature arguing that the "lesson" of quantum mechanics, or its "main innovation" in the history of science is that we must abandon realism.

Since we claim that the problems of quantum mechanics lie *within the physical theory* and are not to be solved by "changing" our philosophy, we must discuss those claims and see what arguments can be given to support them.

An important prerequisite to such a discussion is to define precisely what one means by expressions such as "realism". Indeed, much confusion in philosophy follows from the lack of precise definitions.

J. Bricmont, *Quantum Sense and Nonsense*, DOI 10.1007/978-3-319-65271-9_6

6.1 Realism Versus Idealism

The man-or-woman-in-the-street is of course a realist, at least about everyday life. He or she believes that if one sees chairs or tables, it is because there *are* chairs and tables. The same idea holds for cars, airplanes, the sun or stars.

Let us try to make this everyday life attitude a bit more precise.

We shall define realism by the following combination of ideas:

- There exists a world independent of human consciousness and this world is structured; it has its own properties. Everybody knows that, if they were born, it is because they had parents and that the process of childbirth requires the existence of highly structured organisms.
- A proposition is true or false depending on whether it reflects or not the properties of that world. This means in particular that the truth or falsehood of a proposition is independent of the person who expresses it, or of the group to which he or she "belongs".
- We can know true propositions about the world, for example, through our sensory experiences. Everyday life is full of such experiences that tell us how the world is (to some extent, of course).
- However, our senses can deceive us (think of optical illusions); therefore, we can never be absolutely certain that our knowledge is true and even the most common experiences could be illusory. I see myself typing on a computer, but I could, in principle, be dreaming.
- Our knowledge is *human*. It is the result of a specific interaction between "us" and the world; that interaction depends on our biology, but also on our history and our culture. Other species have other types of interactions, with the same world. Other cultures or people living in other historical periods will have a different knowledge from ours of the same world. The fact that our knowledge is human implies also that our knowledge can have limits, just as our means of perception or our physical abilities have limits: we see only "visible light", which is a small part of the set of electromagnetic waves and nobody is ever going to run a kilometer in less than one second.

Realism, so defined, may seem obviously true and is certainly the attitude of common sense and also the one of almost all scientists (except, sometimes, those involved in discussions about quantum mechanics). But even if realism may seem obvious, it can be challenged and, to explain why, let us start with the following (obvious) observations:

- In order to observe the world, we need our senses.
- In order to talk about it, we need our languages.
- In order to name things, we need concepts.
- In order to have theories about the world, we need conceptual schemes.

The next step is to ask whether what we call the "outside world" isn't in reality an effect produced by our senses, languages, or concepts? Thus, when we talk about the "outside world", aren't we in reality talking only about our senses, languages, or concepts?

Or to what extent does our view of the world come from the outside world and to what extent does it come from within ourselves?

Idealism can be described as a school of thought that ascribes most if not all of our discourses about the world as coming from within ourselves and not talking really about the world "out there".

The most extreme form of idealism is solipsism, namely the idea that everything is an illusion, that there is nothing outside my mind (not even my body) and that there is a sort of movie going on in my mind corresponding to my "experiences".

Solipsism is sometimes illustrated by a sentence due to the 17th–18th century Irish philosopher and bishop George Berkeley: "to be is to be perceived". Without entering into a discussion of Berkeley's philosophy, it is easy to see the problem with that view: who is the perceiver whose perceptions define what exists? Is it me, the whole of mankind, a man who is both deaf and blind, God, non-human creatures? Suppose someone said: "to be is to be perceived by ants". That notion would probably sound ridiculous; but why is it different if we replace ants by humans? Only if the perceiver was an all-knowing divinity could the sentence maybe make sense, but since we do not know what that divinity would perceive, this does not get us anywhere.

It must be noted that solipsism is not the same thing as claiming that only minds exist or that the "ultimate reality" is mental rather than material (one might call this position spiritualism). Indeed, if one shares those viewpoints, one admits that there is something outside of *my* mind, namely the minds of other people or some universal mind, being the "ultimate reality".

Solipsism sounds attractive to some people because it cannot be refuted. If someone insists that only his perceptions are real, how can one prove him wrong? Of course, by simply thinking that I exist outside of *his* mind, I do not agree with his solipsism, even if I was myself a solipsist, in which case I would consider *him* as merely something going on in *my* mind.

The issue of solipsism is actually an old one. Here is how Leonard Euler, a famous 18th century mathematician reacted to it:

> Thus when my brain excites in my soul the sensation of a tree or of a house I pronounce without hesitation that a tree or a house really exists out of me of which I know the place, the size and other properties. Accordingly we find neither man nor beast who calls this truth in question. If a peasant should take it into his head to conceive such a doubt, and should say, for example, he does not believe that his bailiff exists, though he stands in his presence, he would be taken for a madman and with good reason; but when a philosopher advances such sentiments, he expects we should admire his knowledge and sagacity, which infinitely surpass the apprehensions of the vulgar.
>
> Leonard Euler [73, pp. 428–449]

The French philosopher Denis Diderot remarked that solipsism is "an extravagant system which [...], to the disgrace of the human mind and philosophy, is the most difficult to combat though the most absurd."[58, p. 104].

In a humoristic fashion, Bertrand Russell remarks that he once received a letter from a self-described solipsist who complained not to meet more people of the same opinion.

A closely related doctrine is radical skepticism: one might admit that there is something outside of our minds, but that, nevertheless, we cannot know it, because we can only know it through our senses and since everyone agrees that they *might* deceive us, how do we know that they do not deceive us all the time? Like solipsism, radical skepticism is irrefutable.

But even if both doctrines, solipsism and radical skepticism, are irrefutable, they need not concern us, because no scientist could subscribe to them without stopping doing science, and in fact no functioning human being could subscribe to them either. Obviously, physicists, irrespective of their views on quantum mechanics, must believe in the "real existence" of an outside world, made at the very least of laboratories and scientific books and journals and they must also believe that we can have a reliable knowledge of what is in those laboratories, books and journals by simply looking at them.

Another reaction to the realism/idealism controversy is to say that there is something "out there" and that we can know it, but that, in order to study what is outside of our minds, we must *first* study our interactions with that outside world, through the study of physiology, psychology, sociology or history.

The way we acquire knowledge is of course and important issue, but it is important to stress that *any* study of our means of acquiring knowledge, whether that study refers to biology, psychology, sociology, or history, presupposes a realist attitude with respect to knowledge. We have to assume, for example, that there are biological or psychological facts that are independent of us and that explain how we perceive things. If we turn to history or sociology to analyze the emergence of scientific theories, or their replacement by new ones, we have to assume that there are real historical or sociological facts that allow us to explain these evolutions (or revolutions).

This is important for two reasons; first, because it is sometimes suggested that the problems of quantum mechanics are caused by our "language" or our concepts, and that we should analyze them first in order to solve those problems. Sometimes it is even suggested that physics is all about language, not Nature (for a popular version of this sort of ideas, close to the Copenhagen view, see [95]).

The second reason is that there has been a tendency, in recent decades, among some sociologists and historians of science, to claim that one can explain the changes of scientific theories by purely social and human factors, without referring to evidence or to what Nature really is.

But if one admits that history, psychology or sociology are empirical disciplines, just like the natural sciences, and that their results must be based on evidence, the natural and social scientists are all in the same boat. If one wants to use a sociological analysis, say, of the changes in science by appealing only to factors internal to the physics community, and without reference to an outside world existing independently of us and imposing constraints on our theories, then an easy reply is that, if physicists cannot establish objective theories about the world, why should one believe that historians or social scientists can?

Of course, it is sometimes useful to reflect on one's concepts or one's language, but this is only part of the general investigation of how the world is. In particular, it is difficult to define a priori what are the mental categories that are "necessary" for science to work, because, as science evolves, we tend to revise those supposedly necessary categories.

It is an empirical question to know how our minds work, what categories they use, and what limits those categories may impose on our ability to know the world, but studying those issues is not different from studying the natural sciences and thus cannot be the basis for an argument in favor of skepticism with respect to the latter.

However, even if one accepts common sense or daily life realism, i.e. that there really are tables, chairs, cars etc., an important question remains, which brings us closer to quantum mechanics, namely the one of scientific realism, which we have to discuss also.

6.2 Scientific Realism and "Observations"

Scientific theories postulate the existence of what are called "unobservable entities", such as the gravitational force, for example. When Newton introduced his theory, he assumed that there were forces attracting bodies towards each other, even if the latter were far apart, while ignoring the ultimate cause that produced these forces.[1]

Ever since that discovery, physicists as well as philosophers have wondered whether these forces are "real" or at least in what sense one should consider them real. After all, nobody sees these forces, one only sees their consequences through the motion of bodies. And they are not comparable to contact forces (like one body colliding with another). In fact, Newton was accused by his critics of having reintroduced medieval "occult qualities" into science.

Newton himself did not like this mysterious gravitational force. He wrote: "I have not as yet been able to discover the reason for these properties of gravity from phenomena, and I do not feign hypotheses."[132, p. 943].

The main reason why Newton introduced this gravitational force is that it worked in order to deduce the motion of celestial bodies and another is that he probably hoped that a better theory might be proposed in the future.

After Newton, physicists have introduced other, even more mysterious unobservable quantities like electromagnetic waves that are supposed to "propagate in a vacuum", meaning without any support (unlike water waves that propagate in water or sound waves that propagate in the air). In the theory of general relativity, one assumes that the geometry of space-time, which is also not directly observable, is "curved" by the presence of matter and energy.

And of course, quantum mechanics introduced an even more mysterious wave function, whose meaning is, as we saw, quite unclear.

However, the argument in favor of the existence of those unobservable entities is rather straightforward: we cannot formulate our theories without postulating those entities and the evidence for the truth of the theory counts therefore as evidence for the existence of those unobservable entities. After all, we have never directly seen living dinosaurs, black holes, or the inside of the Sun. Yet, we infer the existence and properties of those objects from more direct observations, so why not do the same with forces and other unobservable entities?

Of course, this argument works only for entities that are postulated by sufficiently successful scientific theories, those that led to many confirmed and surprising predictions and applications. That includes celestial mechanics, electromagnetism, the atomic theory of matter, the theories of relativity and also of course quantum mechanics (if we knew what the entities that it postulates, like the wave function, meant).

However, the real problem is not whether those unobservable quantities exist but rather, what do we mean when we use those words? They are *unanschaulich*

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[1] We shall explain that theory a little more in Appendix. 7.A.

(not representable) to use a German word often used in the early discussions about quantum mechanics. We can visualize dinosaurs (even though we never saw living dinosaurs), but how to visualize forces acting at a distance?

This is connected to the problem of meaning: if we say about something that it "exists", we must know what that something *means.* And since we can form pictures of mid-size objects like dinosaurs, the meaning of the words referring to them is pretty clear intuitively, whereas this is not necessarily the case for the unobservable entities, like forces or waves propagating in vacuum, introduced in physics.

It is important to note that, although many debates on quantum mechanics were centered on the non-representability of Nature according to that theory, the problem did occur long before its discovery, since it goes back to Newton.

One common temptation, in order to "solve" this problem, is to give a purely operationalist or instrumentalist meaning to the words referring to unobservable entities: let us forget completely about those unobservable entities, and let us formulate our physical theories solely in terms of "observable" quantities.

But that "solution", although it is often advocated by proponents of the quantum orthodoxy, meets serious problems.

The first problem with that idea is that the notion of "observable" is not clear at all. Surely, there are observations made with our unaided senses, but should one limit oneself to those? Can one use eyeglasses, magnifying glasses, telescopes, or (electronic) microscopes?

The second, deeper problem, is that the meaning of the words used by scientists goes far beyond what is "observable". To take a simple example, should paleontologists be allowed to speak about dinosaurs? Presumably, yes. But in what sense are they "observable"? After all, everything we know about them is inferred from fossil data, which are the only quantities ever directly "observed". Of course, all those inferences are based on some kind of evidence, but the point is that the evidence is evidence for something other than itself, e.g., bones of dinosaurs are evidence for the existence of dinosaurs, but the latter are not made only of their bones, and the meaning of the word "dinosaur" is not easily expressible in a language that would refer only to their bones.

Besides, we cannot say that, if an instrument measures an electric current, the meaning of the word "electric current" is given by the fact that the instrument readings are what they are, because those readings could be due to other causes, for example to a malfunctioning of the instrument. This shows that what we mean by "electric current", even if it is not entirely clear, is not reducible to readings on instruments designed to measure electric currents.

This point was made very clearly by Feynman in his famous *Lectures on Physics* (that are based on lectures given to undergraduate students at the

California Institute of Technology during 1961–1963). Referring to a lady who is given a ticket for speeding, he wrote:

> Many physicists think that measurement is the only definition of anything. Obviously, then, we should use the instrument that measures the speed – the speedometer – and say, 'Look, lady, your speedometer reads 60.' So she says, 'My speedometer is broken and didn't read at all.' Does that mean the car is standing still? We believe that there is something to measure before we build the speedometer. Only then can we say, for example, 'The speedometer isn't working right,' or 'the speedometer is broken.' That would be a meaningless sentence if the velocity had no meaning independent of the speedometer. So we have in our minds, obviously, an idea that is independent of the speedometer, and the speedometer is meant only to measure this idea.
>
> Richard Feynman [77, Sect. 8.2]

It is because we have a theory of electric currents or of how speedometers work that we can predict that certain observations will be made, with adequate instruments. If we did not have the theory first, there would be no meaning assigned to the observations in the first place.

This is something that Einstein always stressed, in particular when he met Heisenberg for the first time in 1926. At that time, Heisenberg thought that "none but observable magnitudes must go in a physical theory".

Einstein explained to Heisenberg that:

> [...] it is quite wrong to try founding a theory on observable magnitudes alone. In reality the very opposite happens. It is the theory which decides what we can observe. You must appreciate that observation is a very complicated process. The phenomenon under observation produces certain events in our measuring apparatus. As a result, further processes take place in the apparatus, which eventually and by complicated paths produce sense impressions and help us to fix the effects in our consciousness. Along this whole path [...] we must be able to tell how Nature functions [...] before we can claim to have observed anything at all. Only theory, that is, knowledge of natural laws, enables us to deduce the underlying phenomena from our sense impressions. When we claim that we can observe something new, [...] we nevertheless assume that the existing laws — covering the whole path from the phenomenon to our consciousness — function in such a way that we can rely upon them and hence speak of 'observations'.
>
> Albert Einstein speaking to Werner Heisenberg [102, pp. 63, 65]

For Einstein, observations are always "dependent on theories", which does not mean that they are arbitrarily "constructed" by us, but that we cannot use the concept "observation" as a fundamental, unproblematic concept on which to base our scientific theories or to give meaning to our scientific concepts.

Of course, observations are essential to science, in order to confirm or refute theories; nobody denies that. But the issue discussed here is the one of meaning: without theories that tell us what we observe, observations per se are meaningless.

6.3 Realism and Quantum Mechanics

We can now say what is special about quantum mechanics, compared to other physical theories: "observations" are a *deus ex machina* in that theory, because the way they work is not accounted for by the theory. In celestial mechanics, the theory tells us where a planet will be in the sky. The electromagnetic theory says how signals will propagate over the internet or via radio and TV. The special and the general theories of relativity are needed in order to make the GPS system work. In all these theories, there are, indeed, observations that are made by humans; but they occur only at the end of a long chain along which the physical theory tells us how Nature works.

In quantum mechanics, we are not at all in the same situation: there is nothing in the theory that tells us how measurements work. In particular, if we try to understand the collapse rule within the quantum formalism, then we run into Schrödinger's cat problem. If we try to think in terms of "hidden variables" that would be revealed by measurements, then we run into contradictions. So, as things stand, there is no alternative to the idea that observations play a central role in ordinary quantum mechanics and not a derived one, as in other physical theories.

That is why some physicists make statements that border on solipsism. For example, Bernard d'Espagnat, a well-known French physicist, wrote:

> The doctrine that the world is made up of objects whose existence is independent of human consciousness turns out to be in conflict with quantum mechanics and with facts established by experiment.
>
> Bernard d'Espagnat [51]

One might ask: which human consciousness? Mine (in which case one falls back into solipsism) or some "collective consciousness" of mankind? But most of mankind, even now, has no idea whatsoever about quantum mechanics.

Moreover, how could such a view be based on experiments? Are the experiments also dependent of "human consciousness"? But then, why do they establish anything objective?

Or consider the following statement by John Wheeler:

> It from bit. Otherwise put, every it – every particle, every field of force, even the space-time continuum itself – derives its function, its meaning, its very existence entirely – even if in some contexts indirectly – from the apparatus-elicited answers to yes-or-no questions, binary choices, bits.
>
> It from bit symbolizes the idea that every item of the physical world has at bottom – a very deep bottom, in most instances – an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes-no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe.
>
> John A. Wheeler [203, p. 5]

Again, who is asking these "yes-no questions"?[2] Me or someone else? In the first case, we are close to solipsism, in the second that someone is, for me, part of the "outside world"; should I reduce that person also to a series of answers to yes-no questions, posed by me of course, in which case we are back to solipsism?

In 2005, the physicist Anton Zeilinger, who has performed in Vienna some of the most remarkable quantum experiments, wrote in the prestigious scientific journal *Nature*:

> So, what is the message of the quantum? [...] I suggest that [...] the distinction between reality and our knowledge of reality, between reality and information, cannot be made.
>
> Anton Zeilinger [210, p. 743]

This quote raises the same question as the two previous quotes (and the same ambiguities about the answers): whose information are we talking about?

An even more spectacular statement is the one that we already mentioned, due to David Mermin, who wrote that "the moon is demonstrably not there when nobody looks." [124, p. 397].

It should be understood that there is nothing special here about the moon; one could have said the same thing about houses, cars etc. The origin of this expression is due to the Dutch-born physicist Abraham Pais, who wrote biographies of Bohr and Einstein and was in contact with both men. Pais recalls his conversations with Einstein who was irritated by all the talk about

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[2] The idea behind this expression is that all quantum mechanical experiments can be reduced to a series of questions whose answer is "yes" or "no", such as: does the particle go through the upper slit?.

"observations" [138, p. 907]: "We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it." Pais adds: "The rest of the walk was devoted to a discussion of what a physicist should mean by the term 'to exist'."

But if someone asserts that quantum mechanics should lead us to doubt the existence of ordinary macroscopic objects, like the moon, why believe quantum mechanics in the first place? As every scientific theory, quantum mechanics is justified by "experiments" or "observations", and the latter presuppose that objects (at least laboratory instruments) exist independently of human consciousness and of whether we look at them or not. Of course, one could argue that a scientific theory shows that some objects or properties do not exist in themselves, but are somehow produced by a specific interaction with our senses (optical illusions would be an example). But that sort of statement becomes self-refuting when it applies to the entire "outside world".

The Australian philosopher David Stove had a particularly ironical, but entirely suitable reaction to that statement from Mermin about the moon not being there when nobody looks:

> […] if it is true, then it would be irrational to believe these physicist's best theories. Fundamental physical theories never say anything about a particular macroscopic object, such as the moon; but if they did say something about the moon, then they would say the same thing about all macroscopic physical objects, hence about all land mammals, hence about the particular land mammal, Professor N. D. Mermin, who wrote the sentence I have just quoted. […] [Mammals] depend for their existence on a great many things; but somebody's looking at them is not among those things and everyone knows this.
>
> David Stove [184, pp. 99–100]

If one were demanding arguments, from d'Espagnat, Wheeler, Zeilinger, Mermin or others, supporting such strange views, they might point to some version of no hidden variables theorems discussed in Sect. 5.2, which prove that one cannot attribute a value independently of "measurements" to certain classes of "observables", such as both the position and the velocity of a particle at a given time, for example. That means, of course, that one should not be realist with respect to those values. But this has nothing to do with realism understood as a general or philosophical concept (applicable to the existence of the moon when nobody looks at it or to the existence of objects "independent of human consciousness").

The fact that certain things are not "real" does not mean that nothing is real. *But*, and that is a big "but", if one takes ordinary quantum mechanics as the final word about the Universe, with the central role played by observations in that theory as being unavoidable, and with observations meaning observations *by us*, humans (not by machines), then one can understand why physicists like d'Espagnat, Wheeler, Zeilinger and Mermin make the statements quoted above.

But the most natural reaction would be to say that there is something seriously wrong with quantum mechanics, if one is led to such views.

At the very least, a natural reaction would be to say that the theory is fantastically successful, has lots of applications, but that we really do not understand what it all means.

A less radical position than the ones of d'Espagnat, Wheeler, Zeilinger and Mermin is to claim that what happens on the microscopic scale is unknowable and that only the macroscopic manifestations of what happens on that scale can be known. But of course some argument must be given for that view and the mere fact that ordinary quantum mechanics does not give us a description of what happens on the microscopic scale does not prove that such a description is impossible.

6.4 Conclusions

There has been a line of thought, often associated with the Copenhagen interpretation of quantum mechanics, which tries to justify the centrality of observations in quantum mechanics by saying that this is what we do anyway, in all physical theories: give a meaning to our concepts in an "instrumentalist" fashion, namely only through direct observations.

But one must distinguish two different statements related to this idea:

1. The centrality of observations is a big problem, but we cannot do better.
2. The centrality of observations is not a problem at all, because science is only about observations anyway.

We have argued that this second claim is not true for all physical theories other than quantum mechanics. Classical mechanics, electromagnetism, the theories of relativity, all speak of a world existing outside the laboratories, and of our observations being derived from the theories about that world instead of being primitive concepts. But in ordinary quantum mechanics, the situation is radically different.

Concerning the first statement, one must again distinguish two very different versions of it:

1. One cannot do better for the time being.
2. One cannot do better forever.

The first statement may be true but does not imply very much, since it leaves the future open, and of course is trivially true about many open questions in science. But it is the second statement that is often made within orthodox quantum mechanics and that statement is not banal. But it is in need of an argument and does *not* follow from the first.

There is an old joke about a Frenchman visiting London, going from Waterloo station to Trafalgar Square and being puzzled by these people who give names of defeats to their public places. But, within ordinary quantum mechanics, physicists often celebrate defeats as if they were victories.

Indeed, the centrality of observations in quantum mechanics has been presented as natural, unavoidable, even as a great progress in scientific philosophy, while in reality it is a major defeat for our scientific understanding of the world, since, strictly speaking, we are no longer speaking about the world, independently of us, but only about our "interactions" with it. This situation is unique in the annals of science.

Before trying to see whether one really cannot do better, we have to discuss another quantum mystery, related to superpositions, but far more puzzling: nonlocality.

6.5 Summary

This chapter criticizes various philosophical justifications of the "no worry" attitude of many physicists with respect to quantum mechanics. We discussed in Chap. 1 one "no worry" attitude, namely "it works, let's not ask why". But another such attitude consists in saying: since we can, with quantum mechanics, predict our observations, what else can one ask for?

In order to deal with that objection, we did a detour through the eternal philosophical debate between idealism and realism, but only to conclude that physicists have to be realists at least about the readings of their own measuring devices.

Then, we explained that words like measurements or observations are in fact complicated and that, as Einstein in particular emphasized, it is "the theory which decides what we can observe". Of course, observations are crucial in science, in order to test our theories, but they cannot be taken as some kind of primitive objects on which to base our theories.

This is important for quantum mechanics, since observations or measurements function as a *deus ex machina* in that theory. What physicists should want is a theory that explains how those measurements work and ordinary quantum mechanics does not provide it.

Of course, one may fall back on the pessimistic idea that one cannot do better than what ordinary quantum mechanics does, but that is not the same thing as being happy about the situation.

7

The Second Mystery: Nonlocality

7.1 Introduction

In this chapter, we shall discuss the second "impossible thing" to believe before or after breakfast or the second quantum mystery: the existence of instantaneous actions at a distance in Nature. But what does "action" mean here? If there are such actions, do they allow an instantaneous transfer of matter? An instantaneous transfer of energy? An instantaneous transfer of messages? An instantaneous transfer of information? Does their existence contradict the idea that "nothing goes faster than light" (which is supposed to be a consequence of the theory of relativity)? Besides, if quantum mechanics shows that the mind acts directly on matter, do such actions at a distance justify telepathy? We have to discuss each of these points carefully and slowly.

We shall start by a little known, but very simple, thought experiment, known as Einstein's boxes. This example will allow us to raise and explain the issue of locality. Then we shall define precisely what we mean by nonlocality in Sect. 7.3 and give a simple proof in Sect. 7.4 of the fact that the world is nonlocal in a sense made explicit in Sect. 7.3.

We stress already that this proof combines *two* arguments, one due to Einstein, Podolsky, and Rosen in 1935 [65] (usually referred to by their initials EPR) and one due to Bell in 1964 [9]. We shall see over and over again that, if one considers only one of those arguments and forgets the other one, as many people do, then nothing spectacular follows. So, we shall always refer to the proof of nonlocality as the EPR-Bell result.

We should warn the reader that the views exposed here are *not* generally accepted. But we shall also try to convince the reader that this non-acceptance

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is due to a series of misunderstandings. When it comes to the fact that the world is nonlocal, there is really no alternative!

In Sect. 7.5 we shall discuss the significance of the EPR-Bell result and some of the misunderstandings to which it gives rise.

In Sect. 7.6 we shall sketch some technological applications of quantum mechanics, in particular of the EPR-Bell result, and in Sect. 7.7 we shall discuss the tension between nonlocality and the special theory of relativity.

7.2 Einstein's Boxes

Consider the following thought experiment.[1] There is a single particle in a box B (see Fig. 7.1), and its wave function $\Psi(x)$ is non-zero everywhere in the box B. Otherwise, the precise nature of $\Psi(x)$ does not matter; one may think of a function which is constant in B for example.

One cuts the box into two half-boxes, B_1 and B_2, and the two half-boxes are then separated and sent as far apart as one wants (we assume that we can cut the box in two without affecting the particle).

According to ordinary quantum mechanics, the state becomes

$$\text{a superposition of the state } \Psi_1 \text{ and of the state } \Psi_2,$$

where the state Ψ_1 means that the particle "is" in box B_1, and the state Ψ_2 means that the particle "is" in box B_2. Here, we put scare quotes around the verb "is" because of the ambiguity inherent in the meaning of the wave function: if it reflects our knowledge of the system, then the particle *is* in one of the boxes B_i, without quotation marks. But, as we emphasized before, this is not what ordinary quantum mechanics says: it only speaks of the probability of *observing* the particle in one box or the other. If we allow for the possibility, as one should, that observations may affect the object being "observed", this distinction is crucial to make.

The state discussed here is quite similar to the one described in Figs. 4.5–4.7.

According to ordinary quantum mechanics, if one opens one of the boxes (say B_1) and one does *not* find the particle in it, one *knows* that it is in B_2.

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[1] We base ourselves in this section on [134]; See that article or [98] for more details. We emphasize that this, like all "experiments" in this book, is a "thought experiment", meaning an experiment illustrating the theory, but not necessarily realized in practice. Some experiments described here are realized in laboratories, but when we will speak below of large distances between some subsystems, one should remember that we always assume implicitly that the subsystems under consideration are isolated from outside influences, a condition which is difficult to satisfy in practice if the separation between them is very large.

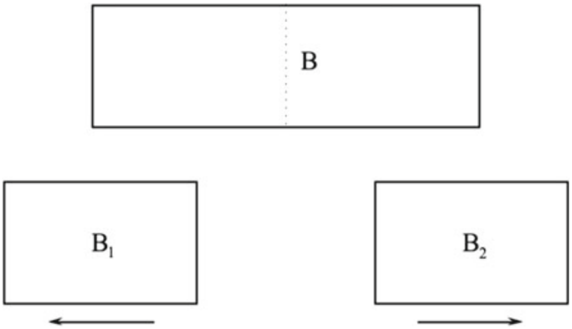

Fig. 7.1 Einstein's boxes. Reproduced with permission from T. Norsen: Einstein's boxes, *American Journal of Physics* 73, 164–176 (2005). Copyright 2005 American Association of Physics Teachers

Therefore, the state "collapses" instantaneously: it becomes Ψ_2 (and if one opens box B_2, one will find the particle in it!).

Here is the important point: since B_1 and B_2 can be as far apart as we wish, if we reject the notion of action at a distance, then it follows that acting on B_1, namely opening that box, cannot have any physical effect whatsoever on B_2. However, if opening box B_1 leads to the collapse of the wave function into one where the particle is necessarily in B_2, it must be that the particle was in B_2 all along. That is, of course, the common sense view and also the one that we would reach if the particle was replaced by any large enough object, for example a little but visible ball.

But in the situation of the particle in the box, if we reject the possibility of actions at a distance, then we must admit that quantum mechanics is *"incomplete"*, in the sense already discussed in Chap. 5: there exist other variables than the wave function that describe the system, since the wave function does not tell us which box the particle is in and we just showed, assuming no action at a distance, that the particle *is* in one of the two boxes, before one opens either of them.

In the boxes situation, the variable would simply be the label of the box in which the particle actually is. This would be an instance of what was called a "hidden variable" in Sect. 5.2.

Introducing such variables is *not* forbidden by the no hidden variables theorems of that section: the latter forbids the introduction of hidden variables for both positions and velocities of particles, but not a priori for positions alone.

This notion of action at a distance was anathema to Einstein; we saw in Sect. 4.5 that the nonlocal character of the collapse rule was already one of his objections to orthodox quantum mechanics at the Solvay Congress of 1927.

In his discussions with his colleague Max Born, Einstein wrote:

> When a system in physics extends over the parts of space A *and* B, then that which exists in B should somehow exist independently of that which exists in A. That which really exists in B should therefore not depend on what kind of measurement is carried out in part of space A; it should also be independent of whether or not any measurement at all is carried out in space A.
>
> Albert Einstein [35, p. 164]

In the example of the boxes, A and B refer here to the places where the half-boxes are (far apart) and the statement of Einstein simply means that opening one half-box cannot possibly influence the physical situation in the other half-box.

So, given his rejection of nonlocality, Einstein thought that his example of the boxes had shown that quantum mechanics is incomplete. And his reasoning was perfectly correct, if one assumes locality of course.

But if one does not reject a priori the idea of nonlocality, one should agree that Einstein had proven *at least* the following dilemma: either there exists some action at a distance in Nature (opening box B_1 changes the physical situation in B_2) *or* quantum mechanics is incomplete.

What could be nonlocal here? For example, one could think that the particle is in neither of the half-boxes before one of them is opened, and is created entirely in one of these boxes, once one of them is opened. Or, one can also think that there is one-half particle in each box and one half "jumps" instantly from one half-box to the other when one of them is opened.

This may seem extraordinarily strange (it is!), but our point here is just to indicate what seems to be an unavoidable dilemma. If you don't believe in nonlocality, then you have to accept the incompleteness of quantum mechanics.

Before discussing further this dilemma, let us consider several examples from daily life that would raise a similar dilemma and where one would side with Einstein in making assumptions, even very unnatural ones, that would preserve locality:

- Suppose that two people are located far apart, and both toss coins repeatedly but each time simultaneously. The results are completely random, heads or tails, but, at each tossing, they are always the same for both people.
- Suppose that in two casinos, far away from each other, the roulette always ends up on the red or black color, randomly, but always the same in both casinos at the same time.
- Imagine twins far apart that behave exactly in the same fashion.

In all these examples (and in many others that are easy to imagine), one would naturally assume (even if it sounded very surprising) that the two coin tossers or the casino owners were able to manipulate their apparently random results and coordinate them in advance or, for the twins, one would appeal to a strong form of genetic determinism. Who would suppose that one coin tosser immediately affects the result of the other tosser, far away, so that this other result is perfectly correlated with his own result, or that the spinning of the ball in one casino affects the motion of the ball in the other casino, again to produce a perfect correlation between both casinos, or that the action of one twin affects the behavior of the other twin? In all these cases, one would assume, even without thinking about it, a "locality" or no-action-at-a-distance hypothesis; denying it would sound even more surprising than whatever one would have to assume to explain those odd correlations.

But one thing should be a truism, namely that those correlations, if they existed, would pose a dilemma: either the results are coordinated in advance or there exists some form of action at a distance.

Note also that Einstein's assumption in the case of the boxes (that the particle is in one of the boxes before one opens either of them, which means that quantum mechanics is incomplete), is similar to the assumptions we made here about coin tossers, casinos, and twins, namely that there exists some "hidden variables" (for the coin tossers and the casinos it would be the manipulation and preparation of the results, for the twins it would be the genes) that explains the correlations. And those assumptions are very natural.

As an aside, let us mention that the example of the boxes also raises a serious question about the transition from quantum to classical physics, where "classical" just means that things have definite properties and are not in a superposed state. Indeed, if the quantum particle is replaced by a large enough object, nobody denies that the particle *is* in one of the boxes before one opens one of them. But where is the dividing line between the quantum realm and the classical one? The transition from quantum to classical physics is usually thought of as some kind of limit, like considering large masses or large energies (compared to the ones on the atomic scale); but a limit is something that one gets closer and closer to when a parameter varies, like the mass or the energy. Here, we are supposed to go from the statement "the particle is in neither of the boxes" to "the particle is in one of them, but we do not know which one". This is an "ontological" jump (meaning a radical change in what exists) and not the sort of continuous change that can be expressed by the notion of limit.

Let us now put aside the example of the boxes and ask ourselves whether there are real nonlocal effects in Nature.

The big surprise is that one can actually *prove* the existence on nonlocal effects in Nature, but not by using simply the example of Einstein's boxes. This example served only to illustrate the idea of nonlocality, which we shall now define more precisely, before proving its existence.

7.3 What Is Nonlocality?

Let us consider what kind of nonlocality or actions at a distance would be necessary, in the example of the boxes, in order to deny Einstein's conclusion about the incompleteness of quantum mechanics. So assume that the particle is in neither box, before one opens one of them. Then, opening one box, say B_1, *creates* the particle, either in B_1 or in B_2. Assume that one opens box B_1 and that one does not find the particle; therefore the particle is created in the unopened box B_2. This creation would obviously be an action at a distance; it would have the following properties:

1. *The action should be instantaneous*: opening one of the boxes creates instantly the particle in the other box.
2. *The action extends arbitrarily far*: the fact that the particle is entirely in box B_2, once we open box B_1, does not change with the distance between the boxes.
3. *The effect of that action does not decrease with the distance*: the effect is the creation of the particle in box B_2 and that effect is the same irrespective of the distance between the boxes.
4. *This effect is individuated*: suppose we have a thousand boxes, each containing one particle, and that we cut each of them into two half-boxes, then send both half-boxes far apart from each other. Then, opening one half-box will affect the state in the other half-box (coming from the cutting in two of the same box) but not in any other half-box.
5. *That action cannot be used to transmit messages*: if we open box B_1, we learn what the state becomes in box B_2, but we cannot use that to transmit a message from the place where B_1 is to the one where B_2 is. In order to transmit a message, it is enough to be able to transmit a binary signal, namely a sequence of 0's and 1's (one could use a Morse code to re-express any regular English text into such a sequence of 0's and 1's).[2] One could

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[2] The Morse code is a sequence of short and long signals that allows to code any letter or punctuation mark into a sequence of such signals. One can then also code any sentence into such a sequence. If one associates, say 0 to "short" and 1 to "long", one then converts any English sentence into a sequence of 0's and 1's.

agree that 0 correspond to the particle being found in box B_1 and 1 to it not being found in box B_1 (and, thus later found in box B_2). Now, if the person who is located where box B_1 is could decide whether the particle will be found in that box or not, she could, by repeating the experiment many times (with different half-boxes coming from the splitting in two of different boxes B, each containing a single particle), send a binary signal, i.e., a message, to the person located where box B_2 is.

However, there is no known way to choose, by acting on one box, in which of the two boxes the particle will be found. Indeed, if one repeats the experiment many times with several boxes, one obtains that the particles are sometimes in B_1, sometimes in B_2, in an apparently random and uncontrollable fashion (at least that is what quantum mechanics predicts and it corresponds to what one observes in experiments). So, there is no way to use this nonlocal mechanism (assuming that it exists) to send messages.

Of course, that also implies that one cannot transfer matter or energy, because, if such a transfer was possible, then one could use it to send messages: at each instant of time, one could decide either to send or not to send instantaneously and far away a piece of matter or of energy. One could furthermore make the following convention: sending a piece of matter or of energy is associated to sending the symbol 1 and not sending it to sending the symbol 0. And, as we just saw, sending a sequence of symbols 0 and 1 is equivalent to sending a message.

Newton's theory of gravitation had also a nonlocal aspect, but we refer to Appendix 7. A for a discussion of that aspect and a comparison with the definition given here.

The Dutch physicist Hendrik Casimir explained the fundamental problem with nonlocal actions:

> If the results of experiments on free fall here in Amsterdam depended appreciably on the temperature of Mont Blanc, on the height of the Seine below Paris, and on the position of the planets, one would not get very far.
>
> Hendrik Casimir [41], quoted in [13]

Indeed, if everything was connected with everything through nonlocal actions, then science would become impossible, because, in order to test scientific theories, one always need to assume that one can isolate some systems or some variables. For example, the results of "experiments on free fall in Amsterdam" should be independent of what happens in Paris or on the Mont Blanc.

Otherwise, one would have to take into account everything that happens in the Universe in every single experiment and that would be impossible.

Therefore, because of the problems linked with nonlocality, post-Newtonian physics has tried to eliminate property 1, the instantaneity of the physical effects. For example, in the theory of electric and magnetic field, there are waves that propagate very fast, at the speed of light (about 300,000 km/s), but at a finite speed nevertheless. The same thing is true in the general theory of relativity.

One may ask whether quantum mechanics proves that there are physical effects displaying properties 1–5 above. The example of Einstein's boxes does not allow that conclusion, because one can consistently think that the quantum description is not complete and that the particle is always in one of the boxes. Indeed, that is exactly what happens in the de Broglie–Bohm theory, as we shall see in Chap. 8. In order to prove nonlocality in the sense introduced here, i.e., a phenomenon having properties 1–5 above, we have to turn to a more sophisticated situation.

7.4 A Simple Proof of Nonlocality

That more sophisticated situation is based on a two-parts argument: one part due to Einstein, Podolsky, and Rosen, in 1935, and the other one to John Bell, in 1964. Let us first explain those arguments using an analogy. The real physical situation is explained in Sect. 7.4.2. Depending on one's taste, the reader may prefer to start with the analogy or with the real thing.

7.4.1 An Anthropomorphic Thought Experiment

The analogy is with an anthropomorphic thought experiment, but which is completely similar to what happens in real experiments and could even, in principle, be realized in the anthropomorphic form presented here. Two people, which we shall call Alice and Bob (these are the habitual names used in the field of quantum information), denoted by A and B in Fig. 7.2 are together in the middle of a room and go towards two different doors, located at X and Y. At the doors, each of them is given a number, 1, 2, 3 (let's call them "questions", although they do not have any particular meaning) and has to say "Yes" or "No" (let's call that "answers").

This experiment is repeated many times, with Alice and Bob meeting together each time in the middle of the room, and the questions and answers vary apparently at random. When Alice and Bob are together in the room,

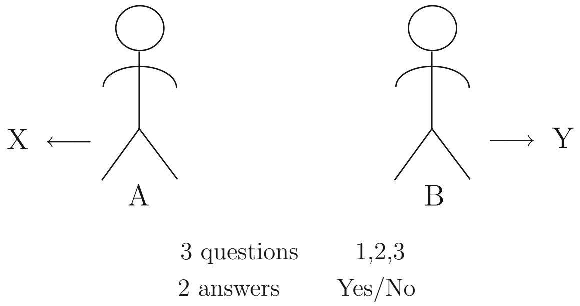

Fig. 7.2 The anthropomorphic experiment

they can decide to follow whatever "strategy" they want in order to answer the questions, namely they may coordinate their answer as they wish.

But the statistics of their answers must satisfy *two basic properties*:

1. The *first property* is that, when the same question is asked at X and Y, one always gets the same answer.
2. The *second property* is that the frequency of having the same answers on both sides when the questions are different is $\frac{1}{4}$.

How can the first property be realized? One obvious possibility is that Alice and Bob agree upon which answers they will give before moving towards the doors. They may decide, for example, that they will both say "Yes" if the question is 1, "No" if it is 2 and "Yes" if it is 3. They can choose different strategies at each repetition of the experiment and choose those strategies "at random" so that the answers will look random.

There are 3 possible questions and two possible answers to each question, so, altogether, there are 8 possible strategies:

1	2	3
Yes	*Yes*	*Yes*
Yes	*Yes*	*No*
Yes	*No*	*Yes*
Yes	*No*	*No*
No	*Yes*	*Yes*
No	*Yes*	*No*
No	*No*	*Yes*
No	*No*	*No*

Another possibility is that, when Alice reaches door X, she calls Bob and tells him which question was asked and the answer she gave. Then, of course, Bob can just give the same answer as Alice if he is asked the same question and any other answer if the question is different.

But let us assume that the answers are given simultaneously, so that the second possibility is ruled out unless there exists some instantaneous action at a distance between Alice at X and Bob at Y. Maybe Alice and Bob communicate by telepathy! Of course, this is not to be taken seriously, but that is the sort of interactions that Einstein did not consider possible. He derided them by calling them "spooky actions at a distance" [35, p. 158].

The question that the reader should ask at this point is whether there is *any other possibility*: either the answers are predetermined or a communication of some sort takes place between Alice and Bob *when* they are asked the questions. This is similar to the dilemma about the boxes: either the particle is in one of the boxes before one opens one of them, or there is some physical action between the two boxes that creates the particle in one box or the other when one opens one box. And of course it is also the same dilemma as the one for coin tossers, casinos and twins.

Note that, to pose this dilemma, one question suffices instead of three: if one question is asked in each run of the experiment, the same question at X and Y, but the answers on both sides are always the same (even though they may vary randomly between different runs of the experiment), then they must be predetermined, assuming that no communication is possible between the two sides.

Posing this dilemma is what we call the *EPR part of the argument.*

The reason that we need three possible questions is because of the *second property* of the statistics of the answers, mentioned above: when the two questions addressed to Alice and Bob are *different* (for example, question 1 is asked to Alice and question 3 is asked to Bob), then the answers must be the same *in only one quarter of the cases.*

To illustrate what we mean, we give, in Table 7.1, an example of "data": Y = Yes, N = No. These data are artificial and are meant only to give an example of what real experiments would show.

A symbol like $1N3Y$ means that the question on the left is 1 and the answer there is No, the question on the right is 3 and the answer is Yes.

When the questions on both sides are the same, the answers are always the same. They are indicated in boldface. But when the questions are different, the answers are the same only one quarter of the time. They are indicated in italics and underlined.

Table 7.1 Example of data in the "experiment" illustrated by Fig. 7.2

1 Y 1 Y	*1Y3Y*	1*Y*2*N*
1*N*3*Y*	2*N*3*Y*	**2 N 2 N**
1*N*2*Y*	3*Y*2*N*	1*Y*2*N*
1*Y*3*N*	**3 Y 3 Y**	**1 N 1 N**
2 Y 2 Y	*1N2N*	1*N*2*Y*
3*N*1*Y*	1*Y*2*N*	1*N*3*Y*
2 N 2 N	**3 N 3 N**	*1Y3Y*
1 N 1N	3*Y*2*N*	*3N2N*
1*Y*3*N*	*2Y3Y*	**1 Y 1 Y**
2*N*1*Y*	3*Y*2*N*	1*N*3*Y*
2 N 2 N	*3N1N*	**1 Y 1 Y**
2Y1Y	**1 N 1 N**	1*N*3*Y*
2*N*3*Y*	3*Y*2*N*	1*N*2*Y*
2 Y 2 Y	3*N*1*Y*	**3 Y 3 Y**
1*Y*3*N*	2*N*1*Y*	*3Y2Y*
1 N 1 N	1*N*2*Y*	3*Y*2*N*
2N1N	**2 N 2 N**	**1 Y 1 Y**
3 N 3 N	3*N*2*Y*	1*N*3*Y*

There are 54 results, with 18 (which is a third of the total) of them having the same questions on both sides[3] and 9 questions where the answers are the same with different questions on both sides, which is a quarter of the number of results with different questions on both sides ($9 = \frac{54-18}{4}$).

The fact that, when the questions addressed to Alice and Bob are different, the answers are the same only in one quarter of the cases, may sound innocuous.

However, this fact, combined with the idea that the properties are predetermined, leads to a contradiction:

Theorem (Bell). We cannot have these two properties together:

1. The answers are determined before the questions are asked and are the same on both sides.
2. The frequency of having the same answers on both sides when the questions are different being $\frac{1}{4}$.

Although the proof of this theorem is extremely simple, we shall defer it to Appendix 7.B.

This theorem is what we call the *Bell part of the argument.*

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[3] If the questions are asked randomly, since there are nine possible pairs of questions, three of which are the same on both side, one expects the same questions to be asked about one third of the time.

If we combine both the EPR and the Bell parts, one has to conclude that *there are nonlocal effects in Nature*. Indeed, the EPR part shows that if there are no such effects, then the answers must be pre-determined. The frequencies of having the same answers on both sides when the questions are different being $\frac{1}{4}$ is an empirical fact. Then, Bell shows that the combination of these two statements leads to a contradiction. Since nonlocality was the *only assumption* of EPR, this assumption has to be false.

Before drawing conclusions from what has been proven, let us explain why the nonlocality proven by EPR-Bell does indeed have the properties 1–5 discussed in Sect. 7.3.

1. The effect is in principle instantaneous, but one cannot check instantaneity experimentally. However, it can at least propagate at speeds far greater than the speed of light, something that can be checked experimentally (that speed is at least 50,000 times the speed of light [90]).
2. The effect extends arbitrarily far at least in principle, that is, as long as our particles are isolated, which is difficult to realize in practice for long distances.[4]
3. The effect does not decrease with the distance between X and Y. We always get perfect correlations when the questions are the same and the same statistics for different questions.
4. The effect is individuated: if one were to send a thousand pairs of people towards the doors, one would get perfect correlations between the answers in each pair but no correlation whatsoever between the pairs.
5. Finally, this effect cannot be used to send messages from X to Y. The reason for this impossibility is similar to the one applying in the case of Einstein's boxes. Each side sees a perfectly random sequence of yes and no answers and there is no way to control what the answers will be. And, for the reasons given in Sect. 7.3, this impossibility of sending messages implies that one cannot use this mechanism in order to transfer matter or energy either.

As a historical note, let us mention that, as we said, the idea that the answers must be pre-determined if they are always the same on both sides and if there is no action at a distance is due to Einstein, Podolsky and Rosen in their 1935 paper [65]. They did not express this idea in the form used here, which is rather due to Bohm [23], but the basic idea was there.

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[4] However, experiments done in 2017 by Chinese scientists show that those correlations can be maintained over more than 1,000 km [5].

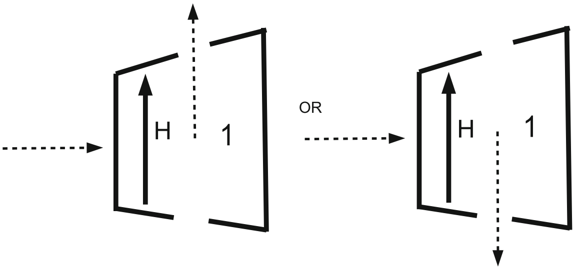

Fig. 7.3 A particle is sent towards a *box*, which is perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1. The particle will either go up, as on the left of the figure, viz. in the direction of the field, or down, as on the right of the figure, viz. in the direction opposite to the one of the field

Then, much later, in 1964, Bell [9] noticed that this property of the answers being pre-determined is incompatible with the frequency of having the same answers on both sides when the questions are different equal to $\frac{1}{4}$. Again, he did not formulate his argument in the form given here (which comes from [61]), but the basic idea was the same.

7.4.2 The Real Quantum Experiment

We will not discuss in detail how the quantum experiments work, but simply outline the basic idea. Particles such a electrons have a property called spin which, for our purposes, only means that if those particles are sent in a box with a magnetic field in it, they will either go in the direction of the field or in the direction opposite to the field. We will not need or use *any other notion about what "spin" means*. In particular, one should not try to "visualize" the electron as being some little particle spinning on itself.[5]

Moreover, the magnetic field can be oriented in any direction we choose and we always see the particle going in the direction of the field or in the opposite one. This is illustrated in Fig. 7.3 when the field, denoted H (a common notation for a magnetic field) is oriented vertically and in Fig. 7.4, when the field H is oriented horizontally.

One can prepare pairs of particles denoted A and B, coming from a common source, that are sent in opposite directions and have their spin measured by

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[5] The actual experiments are made with photons instead electrons, with the polarization of photons replacing the spin of the electrons. It will be easier for us to discuss everything in terms of electrons and spin. Moreover, the original EPR argument (see Sect. 10.1.2) did not use spin variables. This version of the argument is due to David Bohm [23].

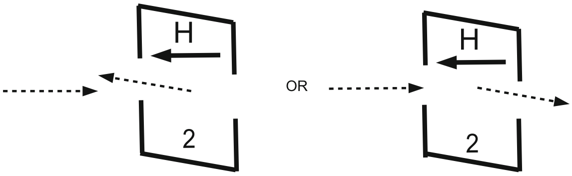

Fig. 7.4 A particle is sent towards a *box*, which is perpendicular to the plane of the figure, and in which there is a magnetic field H oriented along the horizontal axis, denoted 2. The particle will either go in the direction of the field, as on the left of the figure, or in the direction opposite to the one of the field, as on the right of the figure

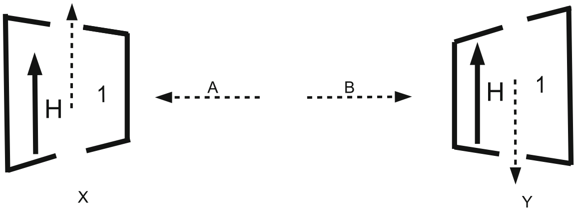

Fig. 7.5 Two particles, A and B, are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1. One possibility is that particle A goes up, viz. in the direction of the field, and particle B goes down, viz. in the direction opposite to the one of the field. The other possibility is shown in Fig. 7.6

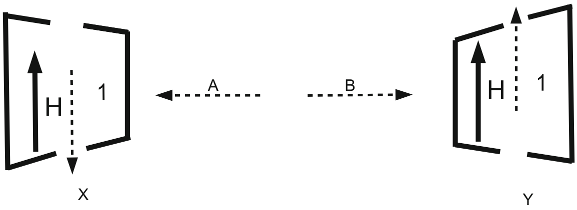

Fig. 7.6 Two particles, A and B, are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1. One possibility is that particle A goes down, viz. in the direction opposite to the one of the field, and particle B goes up, viz. in the direction of the field. The other possibility is shown in Fig. 7.5

the same detectors as in Figs. 7.3 and 7.4 but now put in the line of flight of each particle. The results will always be as Fig. 7.5 or 7.6: if particle A goes in the direction of the field, particle B goes in the direction opposite to the one of the field (Fig. 7.5) or vice-versa (Fig. 7.6).

We *never* see both particles going in the direction of the field or both going in the direction opposite to the one of the field.

The same thing happens when the field H is oriented horizontally, see Figs. 7.7 and 7.8.

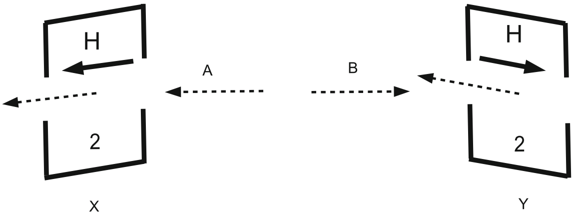

Fig. 7.7 Two particles, A and B, are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H oriented along the horizontal axis, denoted 2. One possibility is that particle A goes in the direction of the field, and particle B goes in the direction opposite to the one of the field. The other possibility is shown in Fig. 7.8

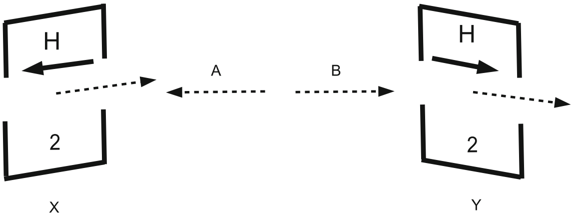

Fig. 7.8 Two particles, A and B, are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H oriented along the horizontal axis, denoted 2. One possibility is that particle A goes in the direction opposite to the one of the field, and particle B goes in the direction of the field. The other possibility is shown in Fig. 7.7

We will not write down the wave function of the pair of particles in that situation, but only describe its main properties. That wave function does not assign a given value of the spin in any direction: the particle has probability one-half to go in the direction of the field and one-half to go in the direction opposite to the one of the field. But it has also the property that the two particles are correlated: if one particle goes in the direction of the field, the other particle goes in the direction opposite to the field and vice versa, each outcome having probability one-half.

It is again an instance of a superposition; the joint wave function of the pair of particles is a superposition of two wave functions: one wave function with particle A going in the direction of the field and particle B going in the direction opposite to the one of the field, and another wave function with particle A going in the direction opposite to the one of the field and particle B going the direction of the field. And that remains true for *whichever direction* one chooses to orient the magnetic field, vertical, horizontal or any other one.

Such wave functions are called *entangled*, a word introduced by Schrödinger, which reflects the fact that, no matter how far apart the particles are, they are not independent of each other.

The nature of this joint wave function means that the spin of each particle is undetermined before the measurements, if we consider only the information contained in that wave function, but the results are perfectly correlated.

This is like the two people in Sect. 7.4.1 always giving the same answer when they are asked the same question: asking a question corresponds here to measuring the spin in a certain direction and the particle going up or down corresponds to an answer yes or no. A small caveat: for the spins, we always get opposite results: in the direction of the field on one side and in the direction opposite to the one of the field on the other, while for the two people we always get the same answer. But that is just a matter of conventions: let us decide that a result in the direction of the field at X corresponds to a Yes answer there but to a No answer at Y, while a result in the direction opposite to the one of the field at X corresponds to a No answer there but to a Yes answer at Y. In that way, we will always get the same answers at X and Y, since we get opposite results for the spin.

Below, we will speak of (anti)-correlations to refer to the perfect correlations between the results at X and Y for the spin measurements.

Now we can raise the question of EPR: if there are no actions at a distance of any sort, how come the results are perfectly (anti)-correlated, no matter how far the particles are? As we discussed already, the only possibility is that the observed values of the spin, up or down, are predetermined, for each pair of particles, and for each direction.

But then comes Bell's part of the argument: one can choose three different directions in which to measure the spin (see Figs. 7.9 and 7.10), so that the answers will always be the same when the measurements are made in the same directions on each side (meaning that, if particle A goes in the direction of the field, particle B goes in the opposite direction and vice-versa), but will be the same only $\frac{1}{4}$ of the time when the measurements are made in two different directions on each side.

That is just the result of a simple quantum mechanical computation (simple, but too advanced for this book, see for example [36, p. 127] for this calculation).

The theorem of Sect. 7.4.1 then shows that this leads to a contradiction.[6]

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[6] It is actually easy to realize the "experiment" described in Sect. 7.4.1, with Alice, Bob, and the three questions: send Alice and Bob towards X and Y and let them orient their respective magnetic field in a direction corresponding to the question that they are being asked and send towards both of them a pair of correlated particles with the quantum state described in this section. Alice and Bob can simply give an answer Yes or No depending on the result that they obtain (with our conventions), and they will then reproduce the statistics that are shown by Bell to be impossible without some form of action at a distance.

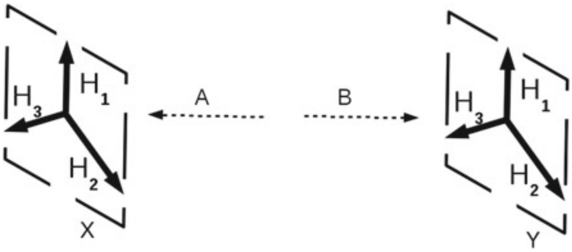

Fig. 7.9 Two particles, A and B, are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there are three possible directions for the orientation of a magnetic field, denoted H_1, H_2, H_3. One repeats several times the experiment, choosing the directions of the field on each side randomly and independently of the choice on the other side. Whenever the fields are chosen in the same directions, the two particles go in opposite directions, like in Figs. 7.5, 7.6, 7.7 and 7.8

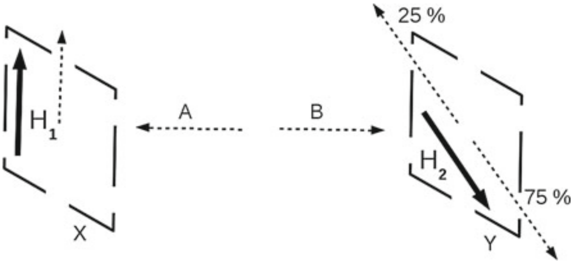

Fig. 7.10 Two particles, A and B, are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there are three possible directions for the orientation of a magnetic field, denoted H_1, H_2, H_3, see Fig. 7.9. Here one chooses direction 1 at X and direction 2 at Y. In that situation, if particle A goes in the direction of the field, as in the figure, particle B will go in the direction of the field 75% of the time and in the opposite direction 25% of the time (and vice-versa). Indeed, the directions taken by particles A and B are perfectly (anti)-correlated when the directions are the same (see Figs. 7.5, 7.6, 7.7 and 7.8) but they are (anti)-correlated only 25% of the time when the directions are different. One obtains similar results for the five other possible choices of different field orientations at X and Y

Before discussing the meaning of this theorem in the next section, it is important to understand how the EPR experiment is described in ordinary quantum mechanics. As we said, the entangled wave function of the system is a superposition of two wave functions: one with particle A going in the direction of the field and particle B going in the direction opposite to the one of the field, and another one with particle A going in the direction opposite to the one of the field and particle B going in the direction of the field.

When one measures, say, the spin of particle A and that this particle goes in the direction of the field, then the superposed wave function collapses to the wave function where "particle A goes in the direction of the field and particle B goes in the direction opposite to the one of the field". Then, particle B will necessarily go in the direction opposite to the field when the spin of particle B is measured. If particle A went in the direction opposite to the field, then the collapse would occur on the other part of the entangled wave function of the system, and particle B would necessarily go in the direction of the field.

It is this collapse operation that is nonlocal in ordinary quantum mechanics. But since the meaning of both the wave function and the collapse rule are unclear in ordinary quantum mechanics, it is not clear either what this nonlocality means, and this is the source of all the ambiguities and confusions in the discussions of nonlocality, when they are carried on within the framework of ordinary quantum mechanics.

7.5 The Meaning of the EPR-Bell Argument

As we said, the argument establishing nonlocality consists of two parts: first, the EPR part shows that, if there are no pre-existing values, then the perfect (anti)-correlations when the directions are the same imply some sort of action at a distance.

The Bell part of the argument, i.e., the theorem of Sect. 7.4.1, shows that the mere assumption that there are pre-existing values leads to a contradiction when one takes into account the statistics of the results when the questions are different on both sides (or, in the real quantum experiment, when the directions in which the spin is measured are different).

The time gap between the publications of the two parts is about 30 years (1935 for EPR and 1964 for Bell). One of the problems is that many physicists react by considering only one argument, and do not connect both together (we'll come back to this in Chap. 10 and discuss how several famous physicists have reacted to the EPR-Bell argument).

If presented with the EPR argument, they will shrug their shoulders and say, well it is not surprising that the results are correlated, since the two people Alice and Bob, or the two particles, come from the same source. But that of course means that one assumes that Alice and Bob coordinate their answers in advance, or that the source prepares the particles so that the results of the various possible spin measurements are pre-determined, which is not what ordinary quantum mechanics says, since the wave function of the pair of particles does not specify those values. In other words, they implicitly agree with Einstein that locality requires the introduction of "hidden variables", namely of pre-determined spin values in the situation invented by EPR.

If presented with Bell's part of the argument they will see it as a "no hidden variables theorem", similar to the one in Sect. 5.2, meaning simply that there are no pre-existing answers or pre-existing values of the spins.

But since the orthodoxy has told us that quantum mechanics is complete, and that "hidden variable" is a bad word, they will see nothing new in that.

Bell, however, presented his result *in combination with* the EPR argument, which shows that the mere assumption of locality, combined with the perfect (anti)-correlations when the directions of measurement (or the questions) are the same, implies the existence of those hidden variables that are "impossible". So for Bell, his result, combined with the EPR argument, was not a "no hidden variables theorem", but a nonlocality theorem, the result about the impossibility of hidden variables being only one step in a two-step argument.

Of course, it is understandable that people shrink from accepting the idea of action at a distance. How can one believe that acting in some way here will affect the physical situation arbitrarily far and instantaneously?

But the argument is logically airtight and depends only on empirical data (the perfect (anti)-correlations and the $\frac{1}{4}$ factor) that have been verified in many similar experiments with more and more possible loopholes being closed.

However, we must consider and discuss some attempts that have been made to maintain that the world is local after all:

1. Some physicists say that quantum mechanics does predict both the perfect (anti)-correlations and the $\frac{1}{4}$ factor, so where is the problem? As long as one is not willing to reason beyond the quantum formalism, there is indeed no problem. But if one starts to wonder about what explains (and not simply predicts) the perfect (anti)-correlations, then one arrives at the EPR conclusions and, with Bell's argument, to the proof of nonlocality.
2. Another strategy is to maintain that the perfect correlations between the answers when the same questions are asked is simply a coincidence that does not need to be explained.

   But the whole of science can be seen as an attempt to account for correlations or empirical regularities: the theory of gravitation, for example, accounts for the regularities in the motion of planets, moons, satellites, etc. The atomic theory of matter accounts for the proportions of elements in chemical reactions. The effects of medicines account for the cure of diseases, etc. To refuse to account for correlations, without giving any particular reason for doing so, is in general a very unscientific attitude. As Bell puts it:

> You might shrug your shoulders and say 'coincidences happen all the time', or 'that's life'. Such an attitude is indeed sometimes advocated by otherwise serious people in the context of quantum philosophy. But outside that peculiar context, such an attitude would be dismissed as unscientific. The scientific attitude is that correlations cry out for explanation.
>
> John Bell [14, p. 152]

3. A variant of the "shrugging one's shoulders" argument, is to invoke a sort of "conspiracy": for example, that both Alice and Bob have an answer to only one question but that, each time the experiment is repeated, and no matter how many times it is repeated, that happens to be the question that is being asked to them. If we make that assumption, then the theorem of Sect. 7.4 cannot be derived (for the proof of the theorem to work, we need to assume pre-existing answers for at least three questions).
   This is similar to assuming that students do well on an exam, not because they have studied the course, but because they just happen to have studied precisely the answers to the very questions that they are being asked, without knowing in advance what they would be. Sometimes that may happen (students can be lucky), but it defies imagination that it could happen for all the students, all the time, and no matter how many students there are.
   The general problem with this sort of "solution" is that, no matter what the data are, one can always save one's favorite theory (here it would be the rejection of nonlocality) if one is willing to make sufficiently ad hoc assumptions. But, again, "outside that peculiar context, such an attitude would be dismissed as unscientific". Goldstein, Norsen, Tausk and Zanghì give the following example: "if you are performing a drug versus placebo clinical trial, then you have to select some group of patients to get the drug and some group of patients to get the placebo." But for that to work, you have to assume "that the method of selection is independent of whatever characteristics those patients might have that might influence how they react to the drug" [92, Note 17]. If, by accident, the people to whom the placebo is given were exactly those that are cured spontaneously, while those to whom the drug is given are so sick that the drug has little effect on them, then of course the study would be biased. And no matter how "random" the chosen sample is, this scenario always remain a logical possibility. It will only become more and more implausible as the size of the sample increases.
   The same reasoning applies to the *ad hoc* assumption that Alice and Bob have an answer to only one question, but that this question just happens to be the one that is being asked to them, and that this occurs in each repetition of the experiment. And of course, it also applies to the particles whose spin is being measured: it would be totally unreasonable to assume that the spins

> of both particles are pre-determined and (anti)-correlated, but only in one direction and that this direction just happens to be the one in which their spin is measured.

If we reject such extreme forms of special pleading, nonlocality is there to stay. And refusing to face a problem is not the same thing as solving it. One thing is certain: nobody has yet proposed a genuinely local explanation for the perfect (anti)-correlations discussed here, and indeed nobody could do so, since Bell has proven that it is impossible.

To conclude, we have shown that some action at a distance does exist in Nature, but we have no idea what this action consists of. And we cannot answer that question without having a theory that goes beyond ordinary quantum mechanics. In ordinary quantum mechanics, what is nonlocal is the collapse of the wave function: if one does a spin measurement at X before doing one at Y, and one obtains, say, the up result, then the wave function of both particles are simultaneously reduced: it becomes the one where the spin of the A particle is up *and* the one of the B particle is down.

But that means that the wave function of the B particle instantaneously jumps when a measurement is made on the A particle. This looks like an action at a distance, but since the meaning of the wave function and its collapse is ambiguous in ordinary quantum mechanics, it is not clear that this is a real physical effect. But, as we have emphasized, if there are no physical effects whatsoever, then this means that we must have those predetermined values that lead to a contradiction.

7.6 Applications of Quantum Mechanics and of EPR-Bell

The French physicist Alain Aspect, who performed crucial experimental verifications of Bell's inequality, speaks of a "second quantum revolution", namely the one of "quantum information".[7] This includes quantum cryptography, which already exists and allows more secure encryptions than anything that can be done classically, quantum teleportation, as well as quantum computation, which is able in principle (but not yet in practice) to perform some calculations much faster than classical computers. We give below a short non technical introduction to each of these topics.

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[7] See Aspect's introduction to the 2004 edition of [14].

7.6.1 Quantum Cryptography

We saw that one cannot use the EPR-Bell effects to send messages. However, one may use them in order to safely encode messages in a way that cannot possibly be deciphered.

First of all, how does one encode messages? To start with, as we explained in Sect. 7.3, sending a message amounts to sending a sequence of 0's and 1's.

Now, suppose that Alice and Bob are far apart and that Alice wants to send a message to Bob. She wants to be sure that no spy (usually named Eve, but one might also call it the NSA) could intercept and decipher her message. Obviously, just sending the sequence of 0's and 1's that corresponds to a Morse encoding of the message (see Sect. 7.3) won't work, because a spy could decipher the message by noticing regularities (the letter a will appear in English more often than the letter z for example, but there are many other regularities) and then simply guessing (of course, there are more sophisticated ways to do that).

What Alice and Bob need is a sequence of 0's and 1's which looks random and that is known only to themselves (see Sect. 3.1.1 for the notion of "random" sequence). This sequence is called a "key", denoted by the letter k below.

Then, once they possess such a common random sequence k, Alice and Bob can code their message into a sequence of 0's and 1's that looks random also. We explain how to do that in Appendix 7.C.

Of course, Alice and Bob still have to share this sequence k. Alice could for example toss a coin many times and count 0 if the result is heads and 1 if it is tails. That would give her a random sequence, but how to share it with Bob? If she sends it by any ordinary means of communication, it can be intercepted by a spy and the whole scheme described here would become useless.

But there is a trick based onEPR-Bell that does the job: suppose that there is a machine, situated half-way between Alice and Bob that sends to each of them one of the two particles with the wave function discussed in Sect. 7.4.2, where both particles are always (anti)-correlated when their spins are measured in the same direction. And suppose that Alice and Bob can measure the spin of those incoming particles in a given direction, but the same for both of them and chosen once and for all. Suppose further that the experiment is repeated many times.

Because of the perfect (anti)-correlations of the results when the directions in which the spin is measured are the same, Alice and Bob will have the same answer Yes or No in each measurement.[8] Then they will both share the same

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[8] As we explained in Sect. 7.4.2, because of the (anti)-correlations, one makes the answer "Yes" correspond to the spin being along the direction of the field on one side and in the direction opposite to the field on the other side, and vice-versa for the answer "No".

sequence of Yes/No and they can then use that, by converting, say, each Yes into a 1 and each No into a 0, to share a common key. And, since quantum mechanics predicts that the results of the spin measurements are random, the sequence of 0's and 1's in their key will be random also.

There is still a loophole: a spy could catch the particles while they are in flight and resend them with a wave function, chosen by him, and that will produce results, when Alice and Bob do their measurements, that he can predict (and thus know). He can also arrange things so that the results look random and thus so that Alice and Bob do not notice anything strange.

But there is a way to get out of the loophole, which we will not explain in detail. Instead of measuring the spin always in the same direction, Alice and Bob can choose at each time one of the three directions considered in Sect. 7.4.2 at random. They can tell each other in which directions (1, 2, or 3) the spin measurements have been made, *without* saying what the results are. And they communicate that openly, so that a spy can listen to them, they do not care.

But then, they both know which result has been obtained on the other side when the same direction was chosen on both sides, because of the perfect (anti)-correlations, and, if the choices are perfectly random, the same direction will be chosen approximately one third of the time.[9]

So, only at the cost of more experiments, they will share a secret sequence of 0's and 1's. But, now, if the spy catches the particles in flight and re-emits them in a wave function chosen by him so that he can predict the results in one direction (say, 1), this operation will necessarily have effects on the results when the direction of measurement is not the one chosen by the spy (remember that Alice and Bob choose their directions at random, so that there is no way for the spy to know in which direction the measurements will be made).

But then, one can show that, by exchanging openly some of their results (which means that one sacrifices those results since the spy could obtain them), Alice and Bob can detect the presence of the spy.[10]

Therefore, quantum cryptography is foolproof: Alice and Bob can share a random sequence of 0's and 1's, that they may use as a key to encrypt their messages and that no spy could possibly know, without them noticing the presence of the spy.

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[9] That is because Alice and Bob have each three possible choices of directions, so there are $3.3 = 9$ choices of pairs of directions. Three of these choices will have the same direction for both Alice and Bob and $\frac{3}{9} = \frac{1}{3}$.

[10] The statements in the last two paragraphs are based on standard quantum mechanical calculations, but justifying them would go beyond the scope of this book.

7.6.2 Quantum Teleportation

The science fiction version of teleportation, à la *Star Trek*, goes like this: you enter into a machine that copies all the information contained in your body, disintegrates it and sends all your molecules and all the information in your body to some remote place where another machine reconstitutes you in a form identical to the original one. A more fancy version has your body teleported without sending any molecule through space, but by sending only information.

Of course, such machines do not exist and nobody plans to build them in the foreseeable future. Moreover, if teleportation was made at faster than light speeds, it would flatly contradict the theory of relativity (see Sect. 7.7).

However, it is not clear that anybody would want to walk into such a machine if it existed: what if there is some malfunction and only the first half of the programme, namely your disintegration, works?

Quantum teleportation, on the other hand, does exist, but does not involve any transfer of matter or of energy. Going back to the proverbial Alice and Bob, suppose Alice possesses a particle with a certain wave function. What she can do is to make sure that Bob will have a particle with the same wave function as the one she has, after some manipulations by Alice and by Bob, but without sending Alice's particle to Bob. In fact, the only thing that she has to send to Bob is one number, chosen in the set $\{1, 2, 3, 4\}$, which she can send by an ordinary open channel, meaning at a sub-luminal speed and in such a way that a spy could discover that number without being able to know which is the wave function being transferred.

To explain how this works in more detail, one would need to use the quantum formalism and that would go beyond the scope of this book. To get a rough idea of what goes on, let us say that Alice and Bob first share an "entangled" wave function such as the one discussed in Sect. 7.4.2. Then Alice carries out a certain measurement on the system composed of the wave function she wants to send to Bob and her part of the entangled wave function, which collapses that combined wave function into one of four possible wave functions.

Because of the entanglement of the wave function shared by Alice and Bob, the measurement on Alice's side also collapses Bob's wave function, and that is where the EPR-type nonlocality enters.

Now Alice sends to Bob, by an ordinary open channel, the result of her measurement. Since there are four possibilities, she just has to send a number, 1, 2, 3, 4, each number corresponding to one of the possible results. When Bob receives that number and therefore knows the result of Alice's measurement, he acts on his own wave function in a well-defined way, depending on that result, and he is guaranteed to obtain the wave function that Alice started with, so that the wave function of Alice will have been teleported to Bob.

If a spy intercepts the open channel transmission and knows which of the numbers 1, 2, 3, 4 was sent, he is not able, with that information alone, to reconstruct the state that Alice teleports to Bob.

If all this sounds a bit mysterious, it is because we refrain from using the quantum formalism. The main point is that one uses the nonlocal aspect of the collapse of the wave function in the EPR situation, which accounts for the perfect correlations discussed in Sects. 7.4.1–7.4.2. The rest are simply local quantum mechanical operations. The fact that the information sent by Alice to Bob by an ordinary channel (one of the numbers 1, 2, 3, 4) is not enough, by itself, to teleport the wave function, shows that the nonlocal aspect of the collapse of the wave function plays an essential role in that teleportation.

Wolfgang Pauli thought that Einstein's questions were ultimately always of the same kind as "the ancient question of how many angels are able to sit on the point of a needle" [35, p. 223]. But both quantum cryptography and quantum teleportation have their origins in the EPR 1935 paper, which was regarded by many people, not only by Pauli, as "metaphysical" and "irrelevant to physics". This is just another example where the history of science shows that it may take some time before one knows whether some theoretical, or even "metaphysical", idea is useful or not.

7.6.3 Quantum Computers

Suppose that, starting from New York, you want to visit both Chicago and Los Angeles, but you want to minimize the total distance of your trip. Obviously, you will first go to Chicago and then to Los Angeles. If you start from Chicago and want to visit both New York and Los Angeles, then you should first go to New York, and then to Los Angeles, since Chicago is closer to New York than to Los Angeles.

This is a very simple instance of the "traveling salesman problem": how to visit a number of cities while minimizing the total length of your trip. It is easy to see that this is an important practical problem and not only for traveling salesmen. One may want to minimize the total length of the connections between nodes of any network, for example in a computer or in the Internet.

While the solution is obvious in the examples given above, it is not at all obvious if one wants to visit, say, the capital of every state in the Unites States. Yet, there is a "simple" solution even for that problem: make a list of all those cities in some order, compute the length between each city and the next one in that list. Then add those lengths and repeat the operation for every possible ordering of all the cities, and choose the one with the smallest total length.

The problem with that solution is that it becomes incredibly time consuming when the number of cities N is large. For ten cities, there are already more than three million lists. For twenty five cities, the number of lists is a 26-digit number (in decimal notation).[11] Even the fastest computers cannot handle problems of such length, if one were to use the "simple" method given here.

That is why an important branch of mathematics was developed in order to find algorithms (which means a mechanical method that can be implemented on a computer) that solve such problems in a "reasonable" amount of time. But even the best techniques require an amount of computing that is prohibitively large when the number of cities is large. The hope, at least for the future, is that quantum computers can reduce the amount of time needed to solve problems such as that of the traveling salesman.

A problem, simpler than the traveling salesman one, to which much attention has been devoted in quantum computing is the one of factorization of integers. Suppose you are given a number N which is a product of two prime numbers (numbers that are divisible only by 1 and by itself) and you are asked to find these numbers.[12] For example, $15 = 3 \times 5$ or $77 = 7 \times 11$. That's easy enough, and again, there is a "simple" method that solves this problem: go through the list of all prime numbers less than $\sqrt{N}$ and check if they divide N.[13]

But again, for products of large prime numbers, this method becomes terribly time consuming, and a lot of work has been devoted to finding more efficient algorithms.

There exist algorithms using quantum mechanics that, in principle, reduce spectacularly the time necessary to find the factors of a product of two prime numbers. The first "success" of this method was to factorize $15 = 3 \times 5$, which of course was not by itself a great revelation.

The way quantum mechanics enters here is via the superposition principle and interference, described in Chaps. 2–4 and illustrated by the double-slit experiment. Explaining how this works in any detail would again go beyond the scope of this book; roughly speaking, a quantum computer produces a superposition of several solutions to a problem and uses interference to select the correct one.

But this is more easily said than done and there is still a lot of work to do before quantum computers become part of our everyday reality.

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[11] For a general N, the number of lists of cities is $N! = N \times N - 1 \times N - 2 \ldots 3 \times 2 \times 1$.

[12] Solving this problem has applications in classical (i.e., non quantum) cryptography, but it would go beyond the scope of this book to explain that in detail.

[13] Obviously, if $N = p \times q$, either p or q must be less than or equal to $\sqrt{N}$.

7.7 The Trouble with Relativity*

The reader who has heard of the special theory of relativity may think that the latter implies that "nothing goes faster than light". But then, doesn't the effect discussed here, instantaneous action at a distance, contradict this statement?

Unfortunately, the answer is complicated and is, in a sense, both yes and no. Moreover, to explain why this is so in detail would go beyond the scope of this book. Or, at least, we would have to include a whole long chapter explaining the theory of special relativity.[14]

The only consequence of the theory of relativity that we need to explain is the *relativity of simultaneity.* This means that, while we naively think that there is a "now" that applies to the entire universe, i.e., it seems to make sense to say that an event here and an event on the moon happen at the same time, and while this was considered true in pre-relativistic physics, it is not the case in relativistic physics.

Briefly stated, the relativity of simultaneity simply says that, if someone passes my present location, but in a moving rocket, her present and my present will be different for distant events. Certain events that occur "now" for me will occur in the future for her and vice-versa. This is illustrated in Fig. 7.11.

This sounds fantastic at first sight and to justify it, one would have to explain the whole theory of relativity, which we shall not do. But there are many experimental situations where this relativity of simultaneity (or some similar property) can be checked, the most spectacular one being the GPS: if one did not take into account such relativistic effects (both those due to the special and to the general theory of relativity), all our indications of position would be wrong and planes, for example, would crash far away from their landing strip.

All experiments in high energy laboratories must also take the relativity of simultaneity into account.

One can also imagine two twins, one of which stays on Earth, and the other one travels in a very fast spaceship, goes far from the Earth, then makes a U-turn and comes back. When he finally comes back on Earth, he will find that he is younger than the twin that didn't travel. This thought experiment has obviously never been made with real twins, but it was invented to illustrate effects that are verified with clocks traveling in airplanes. Indeed, one can send identical and very precise clocks around the Earth in airplanes flying in opposite directions and note that they are no longer synchronized after going around the Earth.

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[14] See Taylor and Wheeler [186] for a rather elementary introduction to that theory and Maudlin [122] for a careful conceptual discussion.

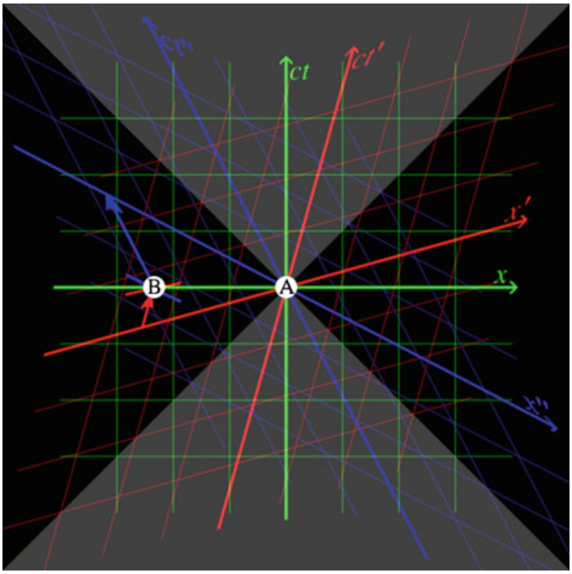

Fig. 7.11 Relativity of simultaneity: we indicate how things appear relative to three states of motion; one in *green*, one in *blue* and one in *red*. There are *green, blue* and *red lines* indicating sets of events that occur simultaneously relative to the state of motion corresponding to that color. In particular, the axis indicated x corresponds to all the events simultaneous with A relative to the state of motion indicated in *green*, the axis indicated x' corresponds to all the events simultaneous with A relative to the state of motion indicated in *red* and the axis indicated x'' corresponds to all the events simultaneous with A relative to the state of motion indicated in *blue* (the three different time axes, related to the three states of motion are denoted ct, ct', ct'', where c is the speed of light – this is a notation frequently used in relativity). So, event B is simultaneous with A relative to the state of motion indicated in *green*, but it occurred before A relative to the state of motion indicated in *blue*, and will occur after A relative to the state of motion indicated in *red* *Source* https://upload.wikimedia.org/wikipedia/commons/b/b1/RelativityofSimultaneity.svg

All these effects are not just qualitatively predicted by relativity, but are also quantitatively in very precise agreement with it.

To summarize, as surprising as the relativity of simultaneity may appear, it is a well established fact!

But what does it imply for EPR-Bell? The problem is that if simultaneity, or instantaneity, is relative to a state of motion, then with respect to which state of motion are the instantaneous actions at a distance, proven by EPR-Bell, instantaneous? Suppose that they occur simultaneously in the laboratory in which the experiment is made. Let's say that events that are simultaneous relative to the state of motion of the laboratory are represented by the green lines in Fig. 7.11, so that event A and B occur simultaneously relative to that state of motion. Relative to one state of motion different from the one of the laboratory (the blue lines in Fig. 7.11), one of these "simultaneous events" will occur *before* the other one (event B occurs before event A) and, relative to another state of motion different from the one of the laboratory (the red lines in Fig. 7.11), the same of these "simultaneous events" will occur *after* the other one (event B occurs after event A).

But that poses a serious problem for our notion of causality: indeed one would like to think that causes precede their effects in an absolute sense and one certainly would like to say that which event is a cause and which event is an effect does not depend on the state of motion relative to which those events are described.

Is there a solution to this problem? Unfortunately, not really. One possibility is to assume that there is a state of motion which is "privileged" in the sense that, relative to that state of motion, the real causes and effects occur and the causes precede their effects (for example, one could take that state of motion to be represented by the green lines in Fig. 7.11). One could consider that state of motion as one of absolute rest. This amounts to bringing back a sort of ether, which was thought, in the 19th century, to be a medium in which electromagnetic waves propagate.

The theory of relativity has not really refuted the existence of the ether, but it implies that this state of rest is not experimentally detectable, which has led to the abandonment of this notion.

Bringing back the ether does not lead to any contradiction but is somewhat unpleasant because it assumes the existence of some hidden, unobservable entity (the true state of motion in which causes and effects occur).

But giving up entirely the notion of causality is not an attractive idea either. The combination of nonlocal effects with the theory of relativity leaves us only the choice of our poison.[15]

What do "orthodox" quantum physicists say about this? In their language, as we saw in Sect. 7.4.2, it is the collapse of the wave function that is nonlocal. But the status of the wave function is ambiguous in ordinary quantum mechanics: many orthodox physicists view it as merely carrying "information" about the system, which means, if one tries to make this idea precise in our language, that particles do carry with them answers to the questions that will be asked later. And Bell has shown that this is impossible!

Alternatively, some orthodox physicists simply refuse to raise such questions, because they content themselves with "predicting results of observations", which of course they can do. But that does not remove the problem

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[15] See Maudlin [122] for an elaboration of this idea.

caused by the perfect correlations and the non-existence of a local explanation of those correlations, which implies that nonlocal actions are real.

Fortunately, things could be worse but they are not. What could be worse is that, if messages could be sent instantaneously, then one could send them into one's own past! Indeed one could send instantaneously a message to a person whose state of motion is such that what is simultaneous for him lies in our past. Then, that person could re-send the message instantaneously into our own past, since our past would just be his present. This is illustrated by Fig. 7.12: If one could send instantaneously a message, then A could send a message instantaneously to B, which moves relative to A but which, at time $t = 0$ for A, is in his present. But, since B is moving relative to A, his present is not the same as the one of A. The present of B is represented by the line $t_B = 0$ in Fig. 7.12 and includes events such as A' that are in the past of A. So, if one could send instantaneously a message, then B could send the message received from A to A', that is to A in his past. In that way, A ends up sending a message to his own past.

Of course, if one could send messages into one's own past, all kinds of paradoxes would occur: you could send a message telling yourself as a student what the questions are in a certain exam, or warn yourself not to take your car

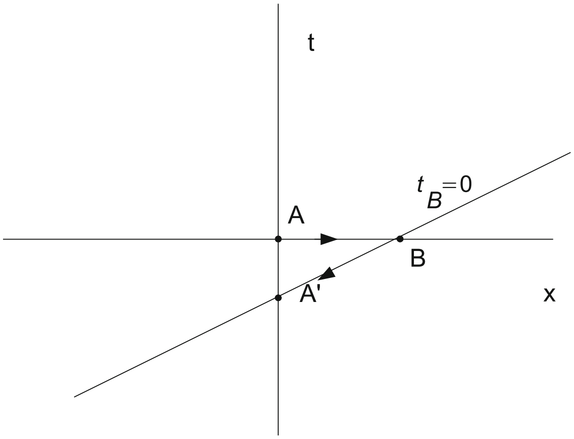

Fig. 7.12 If one could send instantaneously a message, then A could send a message instantaneously to B (indicated by the *arrow* on the x axis), which moves relative to A, but which, at time $t = 0$ (i.e., along the x axis) is in his present. But the present of B is represented by the *line* $t_B = 0$ and includes events such as A' that are in the past of A. The *line* $t_B = 0$ corresponds to one of the *red lines* in Fig. 7.11, while the axis $t = 0$ (the x axis) corresponds to a *green line* in Fig. 7.11. So, if one could send instantaneously messages, then B could send the message received from A to A' (indicated by the *arrow* on the $t_B = 0$ line), that is to A in his past. In that way, A ends up sending a message to his own past

on the day that you had an accident. Physics allow for many paradoxes and counterintuitive statements, but it cannot go that far![16]

So, to summarize: the answer to the question of the tension between relativity and nonlocality is both "no", if one means that this could allow the sending of messages into one's own past, and "yes": there is a serious problem if one wants to reconcile both ideas with a "causal" view of the world, where there are causes and effects and where the former precede the latter in a sense that is independent of the state of motion relative to which those causes and effects are described. In fact this is a major open problem in physics, although one that is not recognized as such by most physicists, because of their refusal to discuss the meaning of the wave function beyond being a tool to predict results of measurements.

7.8 Summary

In his chapter, we discussed the second and deepest mystery of quantum mechanics: nonlocality. Briefly put, nonlocality means that there are correlations between distant events that cannot be explained by antecedent causes. This implies that there must be some form of actions at a distance between the places where those correlated events occur.

We first illustrated the problem with the example of Einstein's boxes. Take a single particle in a box that is cut in two and let each half-box be sent far away from the other. When one opens one of the half-boxes, one always finds the particle in one of them. Then, one faces the following dilemma: either the particle is in one of the half-boxes before one opens one of them, but then the pure quantum mechanical description through the wave function is incomplete, or the particle is somehow created in one of the half-boxes when one opens one of them, but then some action at a distance takes place.

This action at a distance has to be instantaneous, its effect extends arbitrarily far, does not decrease with the distance between the two half-boxes, and is individuated in the sense that the action takes place only between the two half-boxes that came from the same box cut in two and containing one particle initially. Moreover, because of the random nature of the results, one cannot use this action (if it exists) to transmit messages.

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[16] As we explained in Sect. 7.3 the impossibility of sending a message instantaneously implies the impossibility of sending instantaneously either matter or energy.

Because of all those properties, the actions at a distance envisioned here are radically different from the ones of Newtonian physics discussed in Appendix 7.A.

However, one cannot solve the dilemma posed by the boxes in favor of the existence of actions at a distance. Indeed, one can constantly think that the particle is in one of the half-boxes before one opens one of them; we explain that in Chap. 8.

But the existence of actions at a distance can be proven directly as we saw in Sect. 7.4: one can devise an experiment, where two people who are far apart respond to three questions in a perfectly correlated way when the questions are the same. This again poses the dilemma: either the answers are coordinated in advance or there is some action at a distance between the two people. We call this dilemma the EPR part of the argument.

But the statistics of the answers when the questions are different (only one quarter of the same responses) rule out the possibility of such a coordination. That is the Bell part of the argument. So, we are left with only one possibility: actions at a distance!

At this stage, we do not know how such actions work or what causes them. But we did show that some escape strategies that have been proposed to maintain that the world is local after all do not work.

Moreover, quantum mechanics has applications in cryptography and teleportation of information that do depend on the nonlocal aspects of Nature revealed by the EPR-Bell reasoning.

Finally, we addressed the subtle issue of the tension between quantum nonlocality and relativity. The short answer is that, because of relativity, there is a tension between causality and quantum nonlocality, but not a sharp contradiction since that nonlocality does not allow the transmission of messages.

Appendices

7.A Nonlocality in Newton's Theory

Newton's theory of gravitation has also a nonlocal aspect, but which is different from the one discussed in Sect. 7.3.

Two of the most famous laws of classical physics are due to Newton. Suppose we have two bodies, labelled 1 and 2 whose masses are denoted M_1 and M_2. Then, they attract each other through a gravitational force proportional to the product $M_1 \times M_2$ of their masses and proportional to the inverse of the square of their distance d. The second law is that “the force is equal to the mass times the acceleration", the acceleration being the rate at which the velocity changes. So, if we change the position of body 2, we change the value of the distance d between the two bodies in that equation, and therefore the value of the acceleration of body 1. If the acceleration changes, this changes the velocity of that body, and if one changes its velocity, one changes the position of the body.[17]

This makes actions at a distance possible: since the gravitational force depends on the distribution of matter in the Universe, changing that distribution, say by waving my arm, instantaneously affects the motion of all other bodies in the Universe (of course, the effect is minuscule, but we give this simple example to illustrate the principle). That action at a distance has properties 1 and 2 of Sect. 7.3, since it is instantaneous and acts arbitrarily far.

But it does not have the other properties, 3, 4 and 5 of Sect. 7.3: it does not have property 3 because its effect decreases with the distance, since the effect is proportional to the inverse of the square of the distance d. Besides, it affects all bodies at a given distance equally: there is nothing special in body 2 except its distance with respect to body 1. In other words, unlike what happens with the boxes, it is *not* individuated, so it does not have property 4.

On the other hand, it can in principle be used to transmit messages (so it does not have property 5 of Sect. 7.3): if I decide to choose, at every minute, to wave my arm or not to wave it, one can use that choice of movements to encode a sequence of zeros and ones and, assuming that the gravitational effect due to that movement can be detected, one can transmit a message instantaneously and arbitrarily far (of course, the further away one tries to transmit it, the harder the detection).

---

[17] In formulas, the first law stated here says that

$$F = \frac{GM_1M_2}{d^2} \tag{7.1}$$

where F is the force exerted by one body on the other, G is a constant (called Newton's constant), which will not concern us here, and d is the distance between the two bodies. In reality, the force is a vector and we are indicating here only its length. The same is true for the acceleration below. This law is often called the "inverse square law", because of the factor d^2 in the denominator of the right hand side of (7.1).

The second law is:

$$F = Ma, \tag{7.2}$$

where F is the force exerted on the body of mass M, and a is the acceleration of that body.

Now, let us see what this implies for the acceleration of the body 1. We need to put an index 1 in (7.1) and in (7.2): $F_1 = \frac{GM_1M_2}{d^2}$ and $F_1 = M_1a_1$, where F_1 is the force exerted on body 1, of mass M_1, and a_1 is its acceleration. Inserting the first of these formulas into the second one, and dividing both sides by M_1, we get:

$$a_1 = \frac{GM_2}{d^2}, \tag{7.3}$$

namely the acceleration of body 1 depends instantaneously on its distance with respect to body 2.

Note that all this refers to Newton's *theory.* There have been no experiments performed in this framework that could *prove* that gravitational forces really act instantaneously or at least at speeds faster than the speed of light, and this is a major difference with respect to the situation in quantum mechanics.

7.B Proof of Bell's Theorem in Sect. 7.4

As we said in Sect. 7.4.1, there are three questions numbered 1, 2, and 3, and two answers Yes and No. If the answers are given in advance, there are $2^3 = 8$ possibilities:

1	2	3
Yes	*Yes*	*Yes*
Yes	*Yes*	*No*
Yes	*No*	*Yes*
Yes	*No*	*No*
No	*Yes*	*Yes*
No	*Yes*	*No*
No	*No*	*Yes*
No	*No*	*No*

So Alice and Bob could agree, for example to always answer "Yes" to the first question, "No" to the second one and also "No" to the third (let's call that the YNN strategy). Or they could follow each of the strategies YYN, NYN, and NNN one third of the time. Or they could choose their answers at random among the eight possibilities.

In any case, in *each situation* there are at least *two questions* with the same answer. Therefore,

$$\begin{aligned}&\text{Frequency (answer to 1 } = \text{answer to 2)}\\&+\text{Frequency (answer to 2 } = \text{answer to 3)}\\&+\text{Frequency (answer to 3 } = \text{answer to 1)} \geq 1\ ,\end{aligned} \tag{7.4}$$

since at least one of the identities: answer to 1 = answer to 2, answer to 2 = answer to 3, answer to 3 = answer to 1, holds in every run of the experiment.

But if

$$\begin{aligned}&\text{Frequency (answer to 1 } = \text{answer to 2)}\\&= \text{Frequency (answer to 2 } = \text{answer to 3)}\\&= \text{Frequency (answer to 3 } = \text{answer to 1)} = 1/4\ ,\end{aligned}$$

we get $\frac{3}{4} \geq 1$, which is a contradiction. ■

The inequality like (7.4), saying that a sum of frequencies are greater than or equal to 1, is an example of a *Bell inequality*, i.e., an inequality which is a logical consequence of the assumption of pre-existing values, but which is violated by quantum predictions.

7.C How to Encode Secret Messages?

Suppose that Alice and Bob possess a common key k, namely a random sequence of 0's and 1's, and that they want to use that sequence in order to encode a message m which is also a sequence of 0's and 1's, but a non random one (messages that have a meaning for us are not random) in such a way that the result looks random.

They can do that by adding "in binary addition" the message to be sent m and the sequence k; binary addition means that one adds each of the corresponding symbols in the two sequences according to the following rules:

- $0 + 0 = 0$,
- $0 + 1 = 1$,
- $1 + 0 = 1$,
- $1 + 1 = 0$.

It looks like those rules are the ordinary ones except for the last one where we seemingly made a mistake: $1 + 1 = 2$! But these are just rules that we define to be the valid ones if we deal with only two numbers, 0 and 1. There is no symbol 2 here, by definition.

For example, if the message to be sent is $m = 01101010$ and the common sequence or key is $k = 11011001$, we have $m + k = 10110010$ and that is the sequence which is sent by Alice to Bob through an open channel (and can in principle be obtained by the spy).

Since Bob also has the sequence k, it is very easy for him to obtain the original message m. Indeed, with our binary rules, we have $0 + 0 = 0$ and $1+1 = 0$. So, adding twice the same message amounts to adding 0 everywhere, which means that one does not change anything. So, one has $m + k + k = m$. Just to check, add $k = 11011001$ to $m + k = 10110010$, with the binary rules, and you will obtain back $m = 01101010$.

Now, if the sequence k is sufficiently long and sufficiently random one can use it to encode any given message (we just gave one with eight symbols to illustrate the method) and the regularities in the message m will disappear because of the randomness in k.

To check that the sequence $m + k$ will be as random as k, no matter what m is, just consider the least random m one can imagine: $m = 111111111\ldots$ that is every symbol of m is 1. Then, with the rules of binary addition, in the sequence $m + k$ one will simply have each symbol 0 in k replaced by 1 and each symbol 1 in k replaced by 0. But if the sequence k is random, the new sequence $m + k$ will also be random.

8

How to Do "The Impossible", a Quantum Mechanics Without Observers: The de Broglie–Bohm Theory

> But in 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the 'observer', could be eliminated.
>
> John Bell [14, p. 160]

8.1 Introduction

Let us see where we are: In Chap. 5, we learned that there is no easy way to understand what the wave function means. If we try to give a meaning by analyzing measurements within quantum mechanics, we only get unphysical macroscopic superpositions. If we try to give it a statistical meaning, we run into a contradiction, because of the no hidden variables theorems. In Chap. 7 we learned that there is something nonlocal going on in the world, but we do not know what.

What is needed is a theory that gives a meaning to the wave function, beyond being a tool for predicting the results of laboratory measurements. This theory will thus have to go beyond ordinary quantum mechanics, namely it will include "hidden variables", that give a more detailed description of a physical system than the one given by the wave function, but without being refuted by the no hidden variables theorem of Sect. 5.2. Because of the EPR-Bell argument, this theory will have to be nonlocal.

J. Bricmont, *Quantum Sense and Nonsense*, DOI 10.1007/978-3-319-65271-9_8

Surprisingly, not only does such a theory exist, but it was introduced at approximately the same time as the Copenhagen interpretation, even slightly before, in 1924–1927, by the French physicist Louis de Broglie. But, as we said in Chap. 1, that theory was rejected at the time of its introduction by a large majority of physicists, and ignored even by critics of the Copenhagen school, like Einstein and Schrödinger. The theory was even abandoned by its founder, only to be rediscovered and completed by the American physicist David Bohm in 1952, then further developed and advertised by John Bell.

We shall call this theory the de Broglie–Bohm theory, because although it was developed fully only by Bohm (who hadn't heard of de Broglie's work), it was introduced by de Broglie about 25 years before Bohm. Other people call that theory Bohm's theory or Bohmian mechanics [62]. We shall not argue about that.

Here is what it achieves:

- It is a "hidden variables" theory.
- Its "hidden variables" are not hidden at all (hence the expression "hidden variables" is quite a misnomer in this case).
- There is no fundamental role whatsoever for the "observer" in that theory.
- That theory is not contradicted by the no hidden variables theorems. It is a sort of statistical interpretation of quantum mechanics, but a consistent one.
- The de Broglie–Bohm theory is entirely deterministic.
- It accounts for all the observations used to justify the validity of ordinary quantum mechanics.
- It allows us to understand the "active role" of the measuring devices, meaning that a measurement in general does not record some pre-existing value of the system being "measured", as the "no hidden variables" theorems imply. But it does so without making it a philosophical a priori.
- It explains to some extent where the nonlocality of the world comes from.

It would seem that, given all the claims to the effect that such a theory is impossible, and given what it accomplishes, its mere existence should be a subject of considerable interest, but this is not the case. Although interest in the de Broglie–Bohm theory is probably increasing, it is still widely ignored or misrepresented, even by experts on foundations of quantum mechanics (we postpone the discussion of why this is so to Chap. 10).

From this comment, the reader can guess that the claims of this chapter are *not at all* universally accepted by physicists. In fact it represents a very minority view. We explain it because we think that the de Broglie–Bohm theory does solve the "mysteries" of quantum mechanics, but we emphasize that is not a standard opinion. We also think that this is the only existing way to solve those mysteries, but that is even less a generally accepted view, since there are a number of other solutions on the market. We shall discuss one of those alternative solutions in the next chapter and briefly mention the other ones.

As in the rest of this book, we shall avoid formulas and only rely on drawings to explain how the de Broglie–Bohm theory works.[1] We shall proceed slowly and step by step:

- Explain what the de Broglie–Bohm theory says about the world and how it deals with the double-slit experiment.
- What are "measurements" in that theory.
- The previous points concern the deterministic behavior of a given system. But one has also to explain where the statistical predictions of quantum mechanics come from in the de Broglie–Bohm theory.
- One has also to see what is the status, in the de Broglie–Bohm theory, of the reduction or collapse of the wave function.
- Finally, how the de Broglie–Bohm theory allows us to understand nonlocality.

But once this is done, we will have explained, within the de Broglie–Bohm theory, all the phenomena discussed in Chaps. 2 and 7 and we will have given a meaning to the formalism of Chap. 4.

Then, all the quantum mysteries will be clarified; in particular, the loose talk about the moon not being there when nobody looks at it or the cat being both alive and dead, will simply disappear. There will no longer be this incomprehensible duality of rules for the evolution of the wave function (depending on whether one measures something or not). Nonlocality of course will remain baffling but at least its origin will be clearer.

8.2 The de Broglie–Bohm Theory in a Nutshell

In the de Broglie–Bohm theory, particles have positions at all times, and therefore trajectories, and thus also velocities, independently of whether one measures them or not. The positions are, by convention, called the "hidden variables" of the theory, because they are not included in the purely quantum

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[1] See, e.g., [1, 190] for elementary introductions the de Broglie–Bohm theory and [7, 18, 27, 62, 93, 105, 189] for more advanced ones. There are also pedagogical videos made by students in Munich, available at: https://cast.itunes.uni-muenchen.de/vod/playlists/URqb5J7RBr.html.

description, given by the wave function Ψ. But here, the word "hidden" is silly since the positions are not hidden at all: they are the only things that are directly "seen". For example, in the double-slit experiment, one detects particle positions on the second screen in Fig. 2.6. Actually, as we shall explain in the next section and in Appendix 8.A, the particle positions are also the only things that are directly "seen" in *any* experiment.

In the de Broglie–Bohm theory, the *complete physical state* of a particle or a system of particles is given both by its wave function, which is the same as in ordinary quantum mechanics, *and* the positions of the particles.

They both change in time, in the following way:

1. The wave function evolves according to the usual laws, but *nothing special happens to it during measurements.*
2. The motion of the particles is guided by their wave function. This means that the velocity of a particle is a function of its wave function and its position, if we consider a single particle. If we consider a system composed of several particles, the velocity of each one of them is a function of the wave function of that system and of the positions of all the other particles. We will illustrate this motion below.

The de Broglie–Bohm theory is sometimes called the "pilot wave" theory, because the wave function tells the particle how to move.

Now, we *finally* have given a physical meaning to the wave function! It is not only something that allows to "predict results of measurements", but it has a clear physical role *outside the laboratories*: there are particles that move and the wave function guides the motion of the particles.

That's all! The de Broglie–Bohm theory is simply a theory of matter in motion, just like Newton's theory. Of course, the way particles move is different (the phenomena to be explained are radically different!) than in Newton's theory, but there is nothing philosophically new.

All we have to do now is to explain what this new kind of motion is and how it accounts for the strange quantum phenomena.

One should first stress an important fact about the motion of the particles, that will be used repeatedly in this chapter: if a wave function is composed of two non-overlapping parts, as in Figs. 4.5–4.7, then only the part of the wave function where the particle actually is matters as far as the guidance of the particle is concerned.[2] If the particle is in the left part of the wave function

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[2] When we say that a particle is "in" a part of the wave function, we mean that it is where that part of the wave function is non-zero. But we shall use the expression "in the wave function" as a shorthand.

of Fig. 4.5, then it will be guided only by that part of the wave function, and similarly if the particle is in the right part of that wave function. If the two parts are recombined and overlap again, as for example in Fig. 4.10, then the particle will be guided by this recombined wave function.

We shall now illustrate, via simple examples, how such guiding works, starting with the double-slit experiment, described in Sect. 2.1.

In Figs. 8.1, 8.2, 8.3 and 8.4, we show solutions of that experiment within the de Broglie–Bohm theory. Each (wavy) line represents the trajectory of a single particle (since particles are sent one by one, there is no interaction between the particles). Different lines correspond to different initial positions behind the two slits. Since we have many trajectories, those initial positions are not easy to distinguish visually, but one can think that each point behind the slits corresponds to the initial position of a trajectory.

In the de Broglie–Bohm theory, each particle goes through only one slit, but the wave function goes through both slits when they are both open (see Fig. 4.10), and this in turn affects the motion of the particle, since the wave function guides it. This is rather easy to understand intuitively: the wave function propagates like a wave. Obviously, a wave beyond the slits will be different

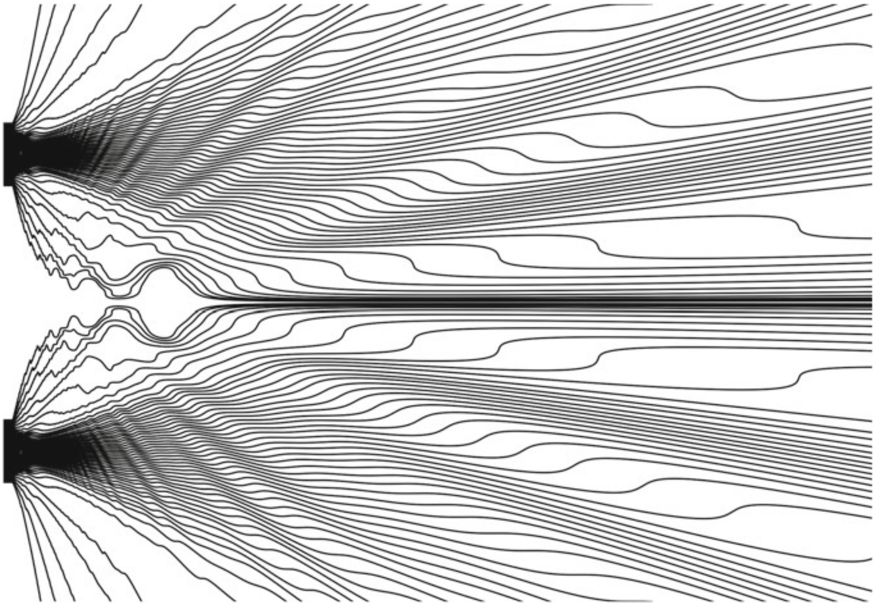

Fig. 8.1 De Broglie–Bohm trajectories computed for the double-slit experiment. Each (*wavy*) *line* represents the trajectory of a single particle. See Fig. 8.2 for another example of de Broglie–Bohm trajectories, with a different distribution of particles behind the slits (A. Gondran cc by-sa 4.0)

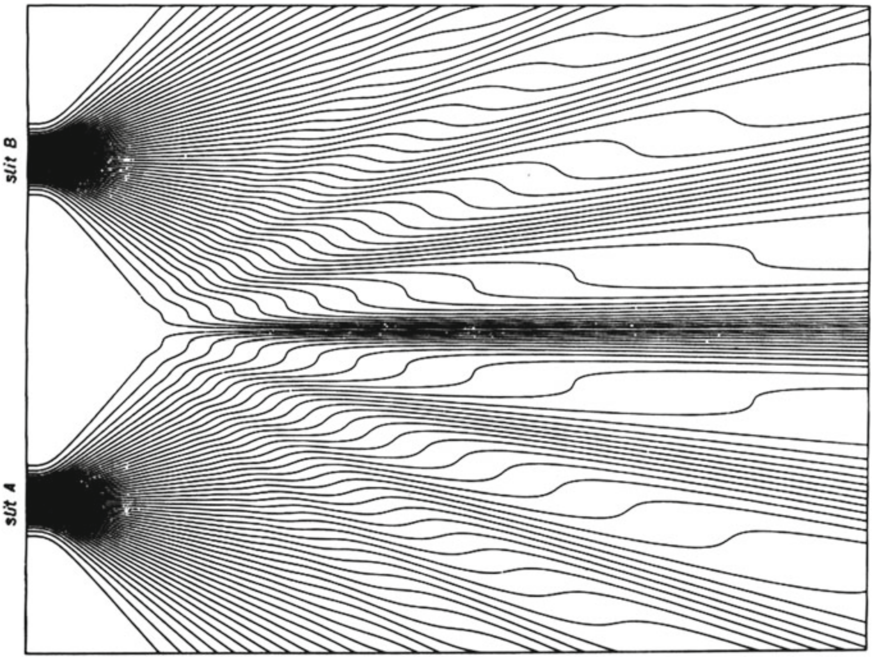

Fig. 8.2 An example of de Broglie–Bohm trajectories computed for the double-slit experiment, with a different distribution of particles behind the slits than in Fig. 8.1. Each (*wavy*) *line* represents the trajectory of a single particle. Reproduced with the kind permission of *Società Italiana di Fisica* and the authors, from C. Philippidis, C. Dewdney, B.J. Hiley: Quantum interference and the quantum potential, *Il Nuovo Cimento B* 52, 15–28 (1979)

if it has two sources (one for each slit) or one. Thus, the behavior of the particle (depicted in Figs. 8.1, 8.2, 8.3 and 8.4 when both slits are open) is affected by the fact that the slit *through which it does not go* is open or not.

As an analogy, imagine a water wave coming through the slits, and a small, light object being carried by that water wave; evidently, the form of the wave behind the slits will depend on whether one slit is open or both, and that will affect the motion of the object, even though the latter goes through only one slit.

Looking at Figs. 8.1, 8.2, 8.3 and 8.4 should dispel the "mystery" of the double-slit experiment, if we accept the idea that a particle can be guided by a wave. There is no sense in which the particle "goes through both slits" if they are both open, as one says too easily. The particle, *being a particle, always goes through only one slit.* The wave that guides its motion, *being a wave, goes through*

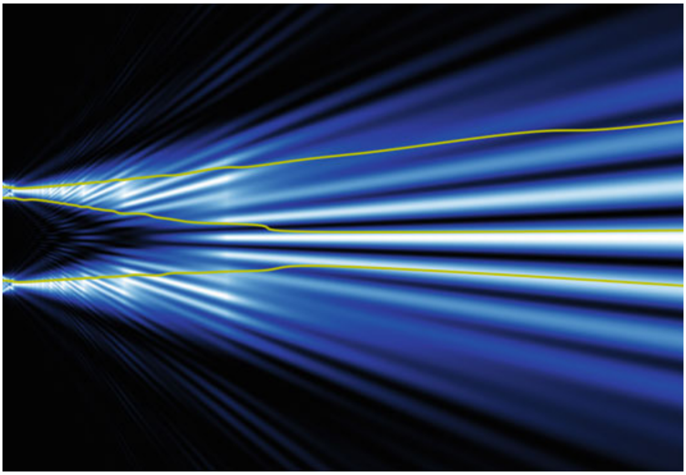

Fig. 8.3 De Broglie–Bohm trajectories computed for the double-slit experiment. The *white* and *blue* areas indicate places where the wave function is non zero and their intensity is proportional to the square of the wave function (*white* more intense, *blue* less intense). The *yellow lines* are three particular trajectories (A. Gondran cc by-sa 4.0)

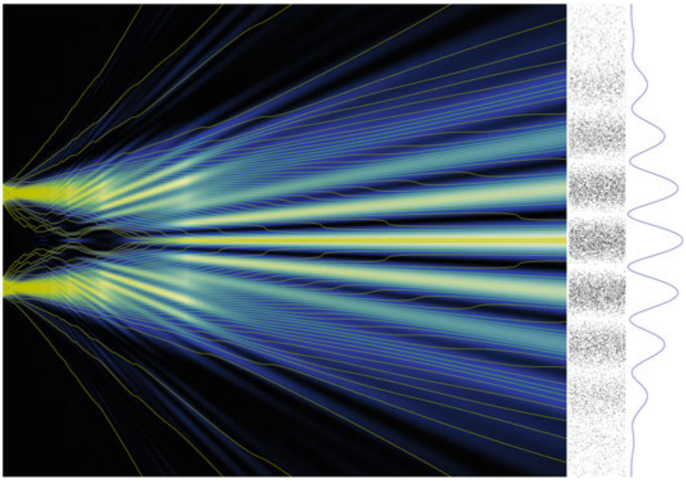

Fig. 8.4 De Broglie–Bohm trajectories computed for the double-slit experiment. The *white* and *blue* areas indicate places where the wave function is non zero and their intensity is proportional to the square of the wave function (*white* more intense, *blue* less intense). There are one hundred *yellow lines* indicating particular trajectories. The *blue curve* on the right of the figure indicates the density of particles detected on the second screen (A. Gondran cc by-sa 4.0)

both slits if they are both open, and that affects the motion of the particle behind the slits. What is surprising here?[3]

Note also that the particles do not cross the line in the middle of Figs. 8.1 and 8.2 (this is a property of the de Broglie–Bohm equations in this situation, but we shall not prove it). As a consequence, one can determine a posteriori through which slit the particle went, by detecting the particle on the screen, since the particle ends up in the upper part of the picture if and only if it goes through the upper slit. Note that this knowledge is obtained without "looking" directly through which slit the particle goes.

Now, if one puts a detector behind one of the slits, say the lower one as in Fig. 2.7, that allows us to record through which slit the particle went, this changes the part of the wave function going through that slit in such a way that this part no longer guides the motion of the particle that goes through the upper slit.[4] Then, if one considers only the events *where the detector does not detect the particle* and where one therefore knows that the particle goes through the upper slit, the pattern on the second screen, where the landing of the particle is detected, will be as if the lower slit was closed, i.e., as in part (a) of Fig. 2.6.

But there is nothing related to our knowledge in itself that influences the behavior of the particle; the detector interacts with the part of the wave function that guides the particles and that changes the future motion of the particle. Of course, that interaction also allows us *to know* which slit the particle went through, but the behavior of the particle is entirely guided by physical laws and does not depend at all on an "observer" looking at it.

Finally, if one considers the delayed-choice experiment of Sect. 2.2, there is again nothing surprising from the point of view of the de Broglie–Bohm theory and there is no action whatsoever of the future on the past! The particle always goes through one slit and is simply guided by the part of the wave function in which it finds itself. Because of the lenses in Figs. 2.9 and 2.10, the wave function behaves in a somewhat different way than in the usual double-slit experiment, but that's all: the region where the plate P may or may not be inserted is the region where the two wave functions, coming from the upper and the lower slits overlap and that gives rise to the interference pattern (Fig. 2.10). If one does not insert the plate P, those two wave functions continue their

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[3] The interested reader may look at [111], where an indirect measurement of trajectories of particles leads to a picture qualitatively similar to Figs. 8.1, 8.2, 8.3 and 8.4.

[4] In the usual approach, one says that the effect of the detector is to collapse the wave function, see Sect. 4.2. In the de Broglie–Bohm theory, there is never a real collapse but there is something playing a similar role, which is explained in Sect. 8.4.3.

propagation towards the counters C_1 and C_2, but there, they do not overlap any more and thus no interference pattern is observed (Fig. 2.9).

Actually, there is one small surprise in the delayed-choice experiment: if one computes the trajectories of the particles in the de Broglie–Bohm theory, one finds that the particles detected by the counter C_1 in Fig. 2.9 went through the lower slit while those detected by counter C_2 went through the upper one, contrary to what one might naively expect and what is usually said in presentations of the delayed-choice experiment (and that we followed in Sect. 2.2). The reason for that somewhat strange behavior is a property of the de Broglie–Bohm dynamics (not proven here): the particles cannot cross the horizontal line in the middle of Fig. 2.9, just as they could not do it in Figs. 8.1 and 8.2[5]; so they bounce back when they hit that middle line and go to the counter that is on the same side of that middle line as the slit that they went through.

Remember that in ordinary quantum mechanics particles do not follow any paths whatsoever, so, if one adopts the orthodox viewpoint, one should not be surprised by this counterintuitive behavior.

Only in a more complete theory, like the de Broglie–Bohm one, where one assigns trajectories to particles, can one meaningfully ask and answer questions such as: through which slit did the particle go? That the answer to that question is counterintuitive is no argument against the de Broglie–Bohm theory: why should physics in the microscopic scale satisfy our intuitions?

Here is how John Bell summarized the de Broglie–Bohm theory in the case of the double-slit experiment:

> Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave–particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.
>
> John Bell [14, p. 191]

It is interesting to compare this statement with what is claimed in a standard quantum mechanical textbook, which we already quoted in Chap. 2:

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[5] We did not explicitly draw that line in Fig. 2.9, but it is clear that there is an horizontal line in the middle of that figure, with respect to which the parts of the figure above that line and below it are symmetric images of each other.

> It is clear that [the results of the double-slit experiment] can in no way be reconciled with the idea that electrons move in paths. [...] In quantum mechanics there is no such concept as the path of a particle.
>
> Lev Landau and Evgeny Lifshitz [114, p. 2]

And, after describing the double-slit phenomenon, Feynman wrote:

> Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it.
>
> Richard Feynman, [79, p. 145]

What is surprising is the dogmatic assurance of those statements: how does one know that an experiment *can in no way* be reconciled with a concept or that no "deeper explanation" of a phenomenon than its description can be given?

We will now turn to natural questions that the reader may ask about the de Broglie–Bohm Theory. We will separate those questions into those whose answers are relatively simple, discussed below, and those whose answers are relatively more complicated, discussed in the Appendices. Sometimes we will invoke facts, about quantum mechanics or the de Broglie–Bohm theory, that we cannot prove without getting into the mathematical formalism.

8.3 How Do "Measurements" Work in the de Broglie–Bohm Theory?

We have seen in Sect. 5.2 that measurements in general cannot possibly reveal pre-existing properties of quantum systems. In particular, one cannot assign values to both the positions and the velocities of individual particles in such a way that their statistical distribution agrees with the quantum mechanical predictions.

But in the de Broglie–Bohm theory, particles do have a position and a velocity at each instant! Isn't that a plain contradiction?

No, because the no hidden variables theorem refer to *results of measurements* and the positions and the velocities in the de Broglie–Bohm theory refer to *properties of particles independently of measurements.*

To understand what is going on, we need to analyze how measurements work in the de Broglie–Bohm theory.

We will discuss here what "measurements" of velocities mean in the de Broglie–Bohm theory and we will discuss "measurements" of spin in

Appendix 8.A. We will come back to the relation between the de Broglie–Bohm theory and the no hidden variables theorem in Sect. 8.4.1.

8.3.1 "Measurements" of Velocities in the de Broglie–Bohm Theory

The simplest example is the measurement of velocities: how does one do that? One measures the difference of the positions at two different times and one divides by the length of the time interval.[6]

So, measurements of velocities are in the end dependent on measurements of positions. Now consider a particle in a box, like the one we introduced in Sect. 7.2. It turns out that, in the de Broglie–Bohm theory, for many wave functions associated to a particle in a box, the particle is actually at rest: it has a well-defined velocity, but which is equal to zero! Yet, quantum mechanics predicts that results of measurements of velocities will have a probability distribution which is quite different from zero (another quantum fact that one has to accept without proof).

But how does one measure the velocity of the particle in the box? One cannot just look at it with God's eye so to speak and *see* that it is at rest. One way to measure this velocity is to open the box, let the particle move and detect its position after some time. Then, one obtains its velocity, as we said, by computing the difference between the positions at the initial time and the final one and dividing by the length of the time interval.[7]

But, in the de Broglie–Bohm theory, opening the box changes the wave function of the particles in the box; that in turn causes the particles in the box to start to move (remember: that's because the wave function guides the particles), and they move in such a way that, if we measure the positions of the particles after some time, and compute their velocity with the above method, we obtain results whose statistical distribution agrees with the quantum mechanical predictions.[8]

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[6] In formulas, if t_1 and t_2 are the two times, we get $v = \frac{x(t_2)-x(t_1)}{t_2-t_1}$. To be precise, this formula defines an average velocity. Below, we will let $t_2 \to \infty$.

[7] The reader might worry that we do not know the initial position with infinite precision and that measuring it might disturb the wave function of the particle. That is true, but we can assume that the box is relatively small and that if one measures the later position after a long enough time, the uncertainty about the initial position within the box will not affect very much the final result for the velocity: if t_1 is the initial time, t_2 the final one and $x(t_1)$ the initial position, $\lim_{t_2\to\infty} \frac{x(t_1)}{t_2-t_1} = 0$, so that $v = \lim_{t_2\to\infty} \frac{x(t_2)-x(t_1)}{t_2-t_1} = \lim_{t_2\to\infty} \frac{x(t_2)}{t_2-t_1}$ does not depend on $x(t_1)$.

[8] However, describing how this "measurement of velocities" works in detail would take us beyond the scope of this book. See [27, Sect. 6.3], [36, Sect. 5.1.4] for more details.

But this means that our "measurement of velocity" did *not* measure the initial velocity of the particle (which was zero!).

Note also that, unlike what Heisenberg's inequality is often taken to mean, not only do particles have both a position and a velocity at all times, but one can *know* both with arbitrary precision, at least in the example of the particle in the box: indeed, we know from the de Broglie–Bohm theory, that the velocity is zero and we can, in principle, measure independently its position with arbitrary precision.

But if we "measure its velocity" by the above procedure (opening the box and measuring the position later), then we obtain something entirely different, and if we take the size of the box as related to the "spread" of the initial positions (see Sect. 4.4) and consider the "spread" of the statistical distribution of results of what are called the "measurements of the velocities", those two "spreads" will satisfy Heisenberg's inequality, simply because the "measured" statistical distributions agree with the ones predicted by the quantum mechanical formalism and the Heisenberg inequality is just a mathematical consequence of that formalism.

But in the de Broglie–Bohm theory, particles do have positions and velocities at all times, and the values of those variables, being well-defined, would *not* satisfy Heisenberg's uncertainty relations. The latter are satisfied by the results of measurements of positions and velocities, but at least for the velocities, these measurements results from interactions with the particle being measured (letting it move) and do not reveal the true velocity of the particle (here equal to zero).

8.4 Things Not Discussed in Detail

There are several natural questions that the reader may raise about the de Broglie–Bohm theory: what is the relationship of that theory with the nonlocality discussed in Chap. 7? How does a deterministic theory, like the de Broglie–Bohm one, account for the statistical predictions of quantum mechanics? If the wave function does never collapses in the de Broglie–Bohm theory, what happens with that rule?

The first question will be answered in Appendix 8.B, and the two others below; but we will start with the relationship between the de Broglie–Bohm theory and the no hidden variables theorem.

8.4.1 Why Isn't the de Broglie–Bohm Theory Refuted by the No Hidden Variables Theorem?

We just answered that question in the previous Sect. 8.3.1: the no hidden variables theorem of Sect. 5.2 says that one cannot introduce "hidden variables" simultaneously for both positions and velocities *in such a way that their statistical distribution coincides with the quantum mechanical predictions*.

In the de Broglie–Bohm theory, we do introduce both positions and velocities, but, and that is the important point, *their statistical distribution does not coincide with the quantum mechanical predictions*. Indeed, we just saw that in the example of the particles in a box, we can have particles that are at rest (hence, their velocity is zero), while the quantum mechanical prediction for the *"measurements" of velocities* is not zero! The crucial word here is "measurements" (with scare quotes). Those "measurements" do not measure any pre-existing property of the particles but are the results of interactions with those particles, as we explained in Sect. 8.3.1. And the de Broglie–Bohm theory does predict correctly the statistical results of those measurements.

So, there is no contradiction with the no hidden variables theorems. As we explain in Appendix 8.A, the same thing is true for the "measurements" of the spin (they are the results of interactions and do not reveal a pre-existing property of the particle).

Here is how Bell summarized the situation:

> [...] the word [measurement] comes loaded with meaning from everyday life, meaning which is entirely inappropriate in the quantum context. When it is said that something is 'measured' it is difficult not to think of the result as referring to some pre-existing property of the object in question. This is to disregard Bohr's insistence that in quantum phenomena the apparatus as well as the system is essentially involved.
>
> John Bell [12, p. 34]

Bell is here referring to statements of Bohr such as:

> [...] the *impossibility of any sharp distinction between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.*
>
> Niels Bohr [31, p. 210], quoted in [14, p. 2] (italics in the original)

However, in the de Broglie–Bohm theory, the fact that measurements do not in general measure an intrinsic property of the particle follows from the equations of the theory and not from some more or less a priori notion or some "intuition" suggested by the strange behavior of quantum particles.

What precedes cannot be emphasized strongly enough: *the de Broglie–Bohm theory is a "hidden variables" theory where the hidden variables (the positions) are not hidden (they are the only thing that we ever detect) and that is not refuted by the no hidden variables theorems.*

8.4.2 Where Does "Randomness" Come from in the de Broglie–Bohm Theory?*

So far, we have discussed how various individual trajectories behave in the de Broglie–Bohm theory, but what about the statistics of the results? How can a deterministic theory reproduce apparently random results?

To start, think of coin tossing: each toss of a coin is a perfectly deterministic process (if you are worried about the free will of the tosser, let us replace him by a machine) and is entirely determined by the initial properties of the coin, namely its position, velocity, the way it rotates etc. When we want to explain why the results of tossing several coins look random (random was defined in Sect. 3.1.1), we have to say that these initial properties are also random; one of the reasons that they are random is that a slight change in those initial properties (a little more velocity, a faster rate of rotation) will make the coin fall heads instead of tails and vice-versa; therefore, one cannot control those initial properties with the precision that would be needed in order to obtain a definite result.

Something similar happens in the de Broglie–Bohm theory. Let us consider, for simplicity, systems composed of a single particle, like in the double-slit experiment; the extension to systems with many particles is rather easy, but will not be discussed in detail here. Let us also first discuss the quantum mechanical predictions for measurements of positions.

Consider a large number of independent particles, as we did in the double-slit experiment. And assume that each particle has the same wave function as the others. The word "independent" means here that they are sent one by one so that they cannot interact with each other.

If one has a large number of particles, distributed in some random fashion, one can define the statistical distribution of that set of particles, as we did in Sect. 3.4.1. Now, consider Fig. 8.5 in which only a few points are indicated on the left (for better visibility) but where the continuous curve is supposed to represent the statistical distribution of the particles that would be obtained if one had a large number of particles.

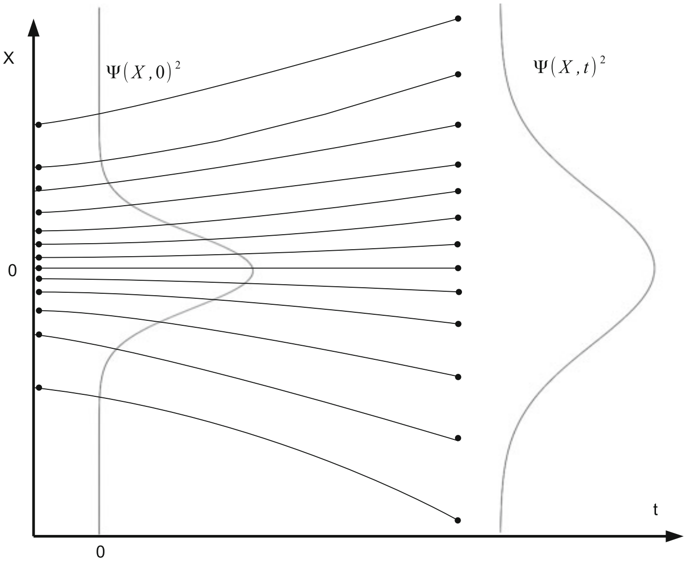

Fig. 8.5 Illustration of properties of the $\Psi(x,t)^2$ distribution, in one dimension, for a Gaussian Ψ. Each *dot* represents the position of a particle, both at time 0 and at time t, connected by trajectories. The statistical distribution of particles is given by $\Psi(x,0)^2$ on the left of the picture and by $\Psi(x,t)^2$ on the right

Of course, this notion becomes precise only in the limit where the number of particles tends to infinity, but we can use that concept "approximately" when the number of particles is large enough. After all, remember that the number of particles in a small quantity of matter is on the order of Avogadro's number, namely on the order of 10^{23} (1 followed by 23 zeros), so that we shall always feel free to assume that the number of particles is so large that the approximation we make by "taking it to infinity" does not matter.

Now suppose that we have a large number of particles, whose initial statistical distribution is given. Each initial position $X(0)$ of a particle gives rise to a unique trajectory, hence to a unique position $X(t)$ at any given later time t. This is illustrated in Fig. 8.5, where the lines going from left to right correspond to trajectories and the curve on the right gives the distribution of the positions $X(t)$ at the later time t.

Hence, if we start with an initial statistical distribution of the particles,[9] we shall have a well-defined statistical distribution of the particles at all later times:

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[9] See Fig. 3.2 for an illustration of that concept.

each value of $X(0)$ gives rise to a unique value of $X(t)$; so, if we have a large number of values of $X(0)$, with a certain statistical distribution, we shall have a corresponding set of values of $X(t)$, which will also have certain statistical distribution. This statistical distribution is uniquely defined, once we know the initial statistical distribution of the particles and the way particles move.

Now, an important property of the de Broglie–Bohm theory (which we shall not prove) is that, if we have a large set of particles, having an initial statistical distribution equal to the square of the wave function at time 0, $\Psi(x, 0)^2$, then, at any later time t, the statistical distribution of the particles that we just defined will be also equal to the square of the wave function, but at time t, $\Psi(x, t)^2$. This is illustrated in Fig. 8.5.

This property of the de Broglie–Bohm theory can also be illustrated by Figs. 8.1 and 8.2. There, one assumes a statistical distribution of particles (i.e., of initial positions just behind the slits, which are not easily visible in Figs. 8.1 and 8.2, because of the large number of lines starting behind the slits) given by $\Psi(x, 0)^2$, 0 denoting the time of passage through the slits. Then, the right side of the figure indicates the place where the particle lands on the screen and these dots have a statistical distribution given by $\Psi(x, t)^2$, t being the time of arrival on the screen. This is similar to what one saw in Chaps. 2 and 4, for example in Fig. 4.10.

So if we assume that the statistical distribution of the particles in some specific situation is determined at some initial time by $\Psi(x, 0)^2$, this will be true at all later times and therefore coincide with the usual quantum predictions.

Now, consider measurements of other quantities than positions, for example velocities. As we already said, when we "measure" the velocity, we do it, indirectly, by measuring positions. So, if one predicts correctly (meaning, in agreement with the usual quantum predictions) the statistics of the positions of particles, we automatically predict the correct statistics for the results of velocity measurements and in fact also for any other measurement.[10]

Now comes a deeper question: if assuming that, at some initial time, the distribution of the particles being determined by $\Psi(x, 0)^2$ implies the correct quantum prediction at later times, what justifies that assumption about the statistical distribution of the particles at that "initial" time?

Well, we can simply assume that this assumption held also at some earlier time t, say $t = -1$: at that time, the distribution of particles would be given by

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[10] The same thing is true for the "measurement" of the spin (see Appendix 8.A): what we see directly is only the particle going out of a box through the upward hole or the downward one. But that means that the only thing that we directly observe are *positions*. So, if we correctly predict the results of detection of positions (and the de Broglie–Bohm theory does predict correctly the positions), we also correctly predict the results of "measurements" of spin.

$\Psi(x, -1)^2$. Then, the previous reasoning would then imply that this assumption at time -1 implies that the statistical distribution of the particles at time 0 is given by $\Psi(x, 0)^2$.

But it is obvious that we then get into a "chicken and egg" problem, since the assumption at time $t = -1$ will be justified by a similar assumption at time $t = -2$, etc., and, ultimately, we have to make assumptions that go back to the beginning of the Universe. And making assumptions about the beginning of the Universe is something that always makes some people (including this author) uneasy.

But since the de Broglie–Bohm theory is deterministic, "randomness" can only come from assumptions about initial conditions and the latter always ultimately refer to those of the Universe.

This argument may seem to some readers to be an instance of what is called "GIGO", or "garbage in, garbage out", namely that we just assume what is to be proven. But that is not quite true; our argument relies on a non-evident property of the de Broglie–Bohm theory, namely that assuming that the statistical distribution of the particles at some "initial" time is given by $\Psi(x, 0)^2$ guarantees that this statistical distribution will be given by $\Psi(x, t)^2$ at all later times.

Let us come back to our initial example of coin tossing. When we explained why the results of tossing several coins look random we said that the initial conditions of each coin that determine its motion are also random.

But our Universe could be different: certain machines tossing coins could be so finely tuned that they would produce far more heads than tails for example. So, if we want to explain why the machines tossing the coins produce random results, we have to go back in time and examine how they were built. And then, how whatever was used to build them was built etc. But this is also a "chicken and egg" problem going back further and further in time.

Thus, even to explain the simplest random results, such as those of coin tossing, we are logically brought back to the initial properties of the Universe!

It is no different in the de Broglie–Bohm theory, except that the "random" initial distribution of particle positions is assumed to be given by $\Psi(x, 0)^2$.

Unfortunately, going further into this discussion will be too technical for this book. Let us simply conclude by saying that the properties of the de Broglie–Bohm theory give a perfectly coherent understanding of the random nature of the quantum predictions simply by making assumptions on the initial distribution of the positions of the particles whenever one repeats many times the same experiment, with the same initial wave function.

In practice, when one explains the random nature of the results of coin tossing, one does not go back to the origin of the Universe, but one is happy to refer to the uncontrollable initial conditions when coins are tossed, and we can take a similar attitude with respect to the de Broglie–Bohm theory.

8.4.3 What About the Collapse of the Wave Function?*

The short answer, which we shall elaborate in this section, is that there is never any collapse of the wave function in the de Broglie–Bohm theory, but there is a collapse "in practice", which coincides with the one in ordinary quantum mechanics, at least when the latter is unambiguous. This collapse is often called an "*effective collapse*", which just means in practice but not in principle.

As we saw, if a wave function is in a superposed state, namely a sum of two (or more) terms corresponding to different physical situations, like going through one slit or the other in the double-slit experiment, then one has to keep both terms in order to predict the future behavior of the system correctly.

Indeed, even when the two parts of the wave function are initially far apart from each other, like when one part goes through one slit and the other part through the other slit in the double-slit experiment (see Fig. 4.10, just after the slits), they may recombine later, in the sense that the two parts of the wave function will overlap (see Fig. 4.10, further to the right), and then both terms will affect the behavior of the particle.[11]

This is what happens in the double-slit experiment and gives rise to the interference pattern on the second screen (see Fig. 4.10 and Figs. 8.1, 8.2, 8.3 and 8.4 for the behavior of the particles in the de Broglie–Bohm theory).

But what happens when we "observe" a quantum system? We already described what happens in the quantum formalism in Sect. 5.1: in order to observe something, we need that the particle interacts with a macroscopic system, because that is the only sort of thing that we can directly perceive. Such a system could be any detector in a laboratory, a pointer pointing up or down, or a cat that can be alive or dead, but must be composed of a large number of particles.

Suppose that we have two terms, each of which corresponds to macroscopically distinct situations, for example the wave function (5.5) of Sect. 5.1 which is:

$$\text{a superposition of the state } \varphi^{\uparrow}\Psi_1 \text{ and of the state } \varphi^{\downarrow}\Psi_2\ , \tag{8.1}$$

where Ψ_1 and Ψ_2 correspond to the wave functions localized near the upper and the lower slit respectively and $\varphi^{\uparrow}$ and $\varphi^{\downarrow}$ are the wave functions associated

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[11] See the Glossary for a formal definition of the notion of overlap of two wave functions.

to the last two pictures in Fig. 5.3, namely to macroscopic bodies (the pointers) detecting through which slit the particle goes.

It is important to notice that the symbols $\varphi^{\uparrow}$ and $\varphi^{\downarrow}$ refer to systems having a large number of particles on the order of Avogadro's number $\sim 10^{23}$, and that each particle of those bodies has its own wave function, so that each of the symbols $\varphi^{\uparrow}$ and $\varphi^{\downarrow}$ actually corresponds to a large number of wave functions for individual particles and is an aggregate of all those wave functions.[12]

The particle will be "in" only one of these terms (meaning that only one of the two terms in (8.1) will be non-zero where the particle is): either the particle goes through the upper slit and the term $\varphi^{\uparrow}\Psi_1$ is the one which is non-zero where the particle is, or it goes through the lower slit and the term $\varphi^{\downarrow}\Psi_2$ is the one which is non-zero where the particle is.

As we said, in principle, we must keep both terms because they may overlap later and produce interference effects as in Fig. 4.10. Thus, keeping only one term (the one in which the particle is) could lead to different predictions.

The interference pattern would be destroyed if the two wave functions did not overlap as they do in that figure. However, when one considers *macroscopic* systems, namely systems composed of a large number of particles, like a pointer or a cat, in a superposed state like (8.1), we need the overlap to occur for *each wave function of each particle* in the system represented by $\varphi^{\uparrow}$ or $\varphi^{\downarrow}$. But, and that is another fact that we will not prove, making the two parts of the wave function of each particle overlap is *in practice* impossible if the number of particles is very large. As an analogy, suppose that you try to control the tossing of a coin so that it falls heads. If you are clever enough you might be able to do it once, twice, maybe ten times, but to do it on the order of 10^{23} times would be in practice impossible.

Thus, if we can be sure that no overlap will occur in the future between the two terms in a wave function like (8.1), because they refer to macroscopic objects, we can simply keep the term in which the particle happens to be (and we know which one it is because of the coupling between the particle and the macroscopic device, by simply looking at the latter: the pointer is up or down), as far as the predictions for the future behavior of the system are concerned.

Reduction or collapse of the wave function in the de Broglie–Bohm theory is as simple as that. It is just a practical impossibility, not an "in principle" one. This reduction is a matter of degree: as the number of particles increases it becomes more and more difficult to make the two parts of the wave function overlap in the future, but there is no fixed number for which there would be a sharp jump from a non-reduced wave function to a reduced one.

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[12] This is a simplification, but it will be adequate for our argument.

So, in some sense, we do “collapse” the wave function when we look at the result of an experiment. But this is only a practical matter. We can still consider that the true wave function is and remains forever given by the time evolution of the full wave function (8.1). It is simply that one of the terms of the wave function no longer guides the motion of the particle, either now or at any time in the future and it would just be cumbersome to keep it in our calculations, but the results would be the same if we did.

The measuring process here is an entirely physical process, with no role whatsoever left to the observer. And the latter only uses the reduction of the wave function as a practical tool for further calculations on the system.

Finally, let us stress that there is a rather common misconception about this “in practice” reduction of the wave function,[13] namely that this phenomenon not only allows us to make sense of the effective collapse within the de Broglie–Bohm theory, but that it is also, on its own, sufficient to account for the collapse rule, within ordinary quantum mechanics. The idea is roughly that, if the different terms in the sum involving a macroscopic object like (8.1) do not overlap, then we just pick up the one we see at the end of the experiment in order to predict the future behavior of the system.

The crucial difference between that view and the de Broglie–Bohm theory is that, in the latter, there is a fact of the matter as to where the particle *is* and as to whether the pointer *points* up or down. Then, we learn where the particle is by looking at the macroscopic measuring device, and use that information to predict the future behavior of the particle in a simpler way than if we kept the whole wave function. But we learn something that exists in the world, independently of whether we look at it or not.

But if we do not reason within the de Broglie–Bohm theory and remain within ordinary quantum mechanics, there is no fact whatsoever that distinguishes one term from the other in a sum like (8.1), except our *observations*.

So, we then go back to square one: putting our observations in the very formulation of our physical theories, which is exactly what we have been trying to avoid all along and that the de Broglie–Bohm theory manages to do.

There are more sophisticated ways to try to make the practical impossibility of interference between macroscopic wave functions the cornerstone of a solution to the measurement problem, for example, the many-worlds interpretation of quantum mechanics, but this will be discussed in Chap. 9.

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[13] This is related to what is called *decoherence* in the literature.

8.5 Is It that Simple?*

By simply assuming that particles have positions (hardly a revolutionary idea, although not a generally accepted one) and that their motion is guided by the wave function (an idea which is also not really revolutionary, but also not generally accepted), we have accounted for the interference phenomena in the double-slit experiment. By doing so, we have completely eliminated the role of the observer and we have done that within a deterministic theory.

Could it be that the solution to all the conceptual problems of quantum mechanics is that simple? The answer is again yes and no. If one is interested in what is called non relativistic quantum mechanics (namely the part of quantum mechanics that leaves aside the theory of relativity), which basically covers most of physics, like atomic, molecular and solid state physics, the foundations of chemistry and all applications to modern electronics, then the answer is yes.

But there is a part of physics dealing with waves rather than particles, like the electromagnetic waves. And there is a quantum theory for those electromagnetic waves, related to high energy physics, the sort of physics tested in accelerators such as the ones at CERN. This is a spectacularly successful part of physics, with a correspondence between experimental observations and theoretical predictions superior to anything else in science.

Moreover, it is crucial for that part of physics to take into account the theory of relativity. Therefore, a natural question for the de Broglie–Bohm theory (and a frequent objection raised against it) is whether there is an extension of de Broglie–Bohm theory to the quantum theory of the electromagnetic waves, and whether this extension incorporates the theory of relativity.

A detailed answer to that question is unfortunately too complicated to be given here. The brief answer is that, yes there is a way to extend the de Broglie–Bohm theory to quantum electromagnetic waves, but there is no unique way to do that and it is not clear which extension is the best.

As for relativity, the problem is the same as the one discussed in Sect. 7.7, namely the nonlocal effects whose reality is proven by the EPR-Bell argument, irrespective of one's views on quantum mechanics.

If we consider our discussion of nonlocality in Appendix 8.B, we see that, in the situation illustrated in Figs. 8.8 and 8.9, one measures the spin of the A particle before one measures the one of the B particle. But, because of the relativity of simultaneity, "before and after" are relative to the state of motion of the system in which the system is described, see Sect. 7.7. And that is a serous problem if we want to have a causal view of the world, where causes precede their effects in an absolute sense.

But the situation is not any better in ordinary quantum mechanics. There, the nonlocal effects are due to the collapse of the wave function, which in the EPR-Bell situation, is nonlocal, since a measurement on the A particle collapses the wave function of both the A and the B particles, no matter how far apart those particles are. Of course, since the status of the wave function is unclear, that difficulty can be swept under the rug, which is often done, as we explained in Sect. 7.5.

If one looks at books on quantum field theory or relativistic quantum mechanics, the collapse rule is almost never mentioned, although it is supposed to be a basic tenet of any quantum theory. The reason is that the collapse rule cannot be treated in a relativistic fashion, precisely because, in the EPR-Bell situation, it is a nonlocal operation. Indeed, as explained in Sect. 7.7, relative to one state of motion, the measurement of the A particle will occur before the one of the B particle and it is that measurement that will induce the collapse of the wave function of both the A and the B particles. Relative to another state of motion, the measurement of the B particle will occur before the one of the A particle and it is that measurement that will induce the collapse of the wave function of both the A and the B particles. So, if collapses of wave functions are real physical operations, it is not at all clear how to reconcile causality with the fact that the chronological order of those operations depends on the state of motion relative to which they are described.

So, the problem of combining causality, nonlocality and relativity is not just a defect of the de Broglie–Bohm theory, since nonlocality is an unavoidable feature of Nature. How to fully reconcile quantum nonlocality and the theory of relativity is an open problem, but for everyone, not just for defenders of the de Broglie–Bohm theory, although most physicists refuse to admit that this problem is a real one.

8.6 A Last Look at Traditional Questions

At the risk of repeating ourselves (the impatient reader may skip this section) we want to discuss again two of the main questions that we have raised in this book: does quantum mechanics imply the "death of determinism" in physics and is quantum mechanics a complete theory?

8.6.1 So, Does God Play Dice After All?

If there is one sentence of Einstein that anybody who has an interest in quantum mechanics must have heard, it is: "God does not play dice" [35, p. 91]. We will discuss that sentence in its historical context in Sect. 10.1, but here we want to reformulate that question: does quantum mechanics prove that the Universe is indeterministic? In Chap. 3, we claimed that it is not easy to prove such a statement, because apparent indeterminism can always be due to an incomplete description of physical systems. One way to "prove" indeterminism is to claim that quantum mechanics is both intrinsically indeterministic and complete, but its completeness is precisely what has to be demonstrated.

But now, we can say more: we have a theory that does complete quantum mechanics and that is deterministic, so that the claim that quantum mechanics proves indeterminism is surely false. However, determinism in the de Broglie–Bohm theory is a special sort and has two properties that make it somewhat different from what one might expect from a deterministic theory in the setting of classical physics:

(1) First of all, the de Broglie–Bohm theory is nonlocal. This means that, even if one wants to determine the future of what happens only in a given region of space, denoted A, one has in principle to specify the physical state of the entire Universe, since events in regions that are arbitrarily far from region A might influence instantaneously what happens in the latter.
This does not contradict the deterministic nature of the theory, but one would naively expect that, in a deterministic theory, it would be sufficient to know the initial conditions in a neighborhood of region A in order to predict the future in that region, at least for short times. But that is not true in the de Broglie–Bohm theory.
Of course the same thing happens in Newton's theory, since gravitational forces also act arbitrarily far and in principle instantaneously; but at least their effects decrease with distance, which is not true for the EPR-Bell nonlocal effects.
What remains true is that the correlations between distant particles that give rise to nonlocality are difficult to maintain in practice over large distances, so that, again in practice, the determinism of the theory would hold even if one forgot about events very distant from region A in our specification of the initial conditions of the Universe. But that is an "in practice" statement not an "in principle" one.

(2) Secondly, the de Broglie–Bohm theory contains in its very formulation an element of radical uncertainty that one might not expect in a deterministic theory. Indeed, the best analogy is to think of the initial conditions of quantum systems as being like the ones of a large number of coins that are being tossed.

> Although, in principle, the end result of each coin tossing can be determined if one knew the initial conditions with sufficient precision, in practice it is impossible to do. For quantum systems, this impossibility is even more an "in principle" one, but the simplest way to explain the situation is through this analogy.

So, coming back to Einstein's famous quote, no, God does not play dice or at least there is no argument based on quantum mechanics that indicates that he does. The idea of determinism can be maintained, thanks to the de Broglie–Bohm theory, but it is of rather special type.

8.6.2 Is Quantum Mechanics Complete?

We have repeatedly asked that question in this book, but now we can give it a clear answer: no, ordinary quantum mechanics does not give a complete description of physical systems, and one can give a more complete description of them than the one given by the wave function, in which the "observer" looses entirely its special status. Moreover, because that more complete theory, the de Broglie–Bohm one, introduces only the particle positions as "hidden variables" and accounts for the measurements of everything else in terms of interactions between the particle and some apparatus, it avoids being refuted by the no hidden variables theorems of Sect. 5.2.

But there is a weaker sense in which ordinary quantum mechanics is complete, namely as far as empirical predictions are concerned; one might call it "predictively complete". That is simply because, in the de Broglie–Bohm theory, one cannot control the initial conditions of the particles well enough to be able to make more precise statistical predictions than the usual ones. We have sketched an explanation of why this is so in Sect. 8.4.2. The simplest way to understand this situation is by analogy with a set of tossed coins whose initial conditions could not be controlled sufficiently well, so as to produce different statistics than the usual ones (half heads, half tails).

But the value of the de Broglie–Bohm theory lies in its explanatory power, not in its predictions.

8.7 Conclusion: The Merits of the de Broglie–Bohm Theory

First of all, let's ask: what is the relationship between the de Broglie–Bohm theory and ordinary quantum mechanics? The quick answer to this question is that it is *not a different theory*! More precisely, the de Broglie–Bohm theory *is* a theory, while ordinary quantum mechanics is not. Indeed, quantum mechanics doesn't even pretend to be a theory, but rather claims to be an algorithm allowing us to compute "results of measurements".

Another way to say this is that ordinary quantum mechanics is the algorithm used to compute results of measurements that can be *derived* from the de Broglie–Bohm theory: in that theory, measurements do not really measure anything (except for detections of positions) but are interactions between a macroscopic system and a microscopic one. Once one understands that, the mystery of the ever present "observer" of standard quantum mechanics disappears.

One might also say that ordinary quantum mechanics is simply a truncated version of the de Broglie–Bohm theory or that the de Broglie–Bohm theory is a completion of ordinary quantum mechanics: in the latter, one ignores the particle trajectories, but since the empirical predictions of the de Broglie–Bohm theory are statistical, and are the same as those of ordinary quantum mechanics, there are no practical consequences of that omission.

Thus, ordinary quantum mechanics is sufficient "for all practical purposes" to use Bell's expression [12], for which he even invented an acronym: FAPP. But it is the de Broglie–Bohm theory that *explains* why ordinary quantum mechanics is sufficient FAPP, something that is true but mysterious without de Broglie–Bohm.

These remarks also provide a reply to a frequent objection raised against the de Broglie–Bohm theory: what are the new predictions made by that theory compared to ordinary quantum mechanics? Once we understand that the de Broglie–Bohm theory is just a way to make sense of ordinary quantum mechanics, which, on its own, does *not* make sense as a theory about the world outside the laboratories, that objection collapses.

In fact, it is excellent news that the de Broglie–Bohm theory does *not* make new predictions with respect to those of ordinary quantum mechanics or, at least, that it does not make predictions at variance with those of the latter. Otherwise, it would simply be refuted by experiments, given the incredible empirical success of ordinary quantum mechanics. What the de Broglie–Bohm theory does is to explain what goes on in the world that makes ordinary quantum mechanics successful, but not contradict or complement the latter's predictions.

To physicists making the "no new predictions" objection to the de Broglie–Bohm theory, one should retort: what does ordinary quantum mechanics say about the world outside the laboratories? The answer is likely to run into difficulties for the reasons discussed in Chap. 5: either because of the existence of macroscopic superpositions or because of the no hidden variables theorems. And if the answer is that ordinary quantum mechanics does not say anything about the world outside the laboratories, the next question should be: "Are you satisfied with that state of affairs? And, if yes, why do you build laboratories then, if they do not lead to any knowledge of the world outside the laboratories?".

If people reply that the many extremely successful technological applications of quantum mechanics show that no question should be asked about the latter, the answer that we have already given is that, the more "it works", the more it is natural to ask ourselves "why does it work so well?".

As we saw, the de Broglie–Bohm theory eliminates the dual nature of the time evolution in quantum mechanics: one between observations and one during observations. It also explains in a natural way why, as the no hidden variables theorems show, one cannot introduce hidden variables for both positions and velocities (or for the spin values, see Appendix 8.A). Finally, even the strangest aspect of all of quantum mechanics, nonlocality, is made more understandable thanks to the de Broglie–Bohm theory (see Appendix 8.B).

Moreover, since it is a truism that a single counterexample is enough to refute a general claim, the de Broglie–Bohm theory is a counterexample to three claims that have been almost universally accepted by physicists, commented by philosophers, taught in classes, and sold to the general public:

1. That quantum mechanics signals the end of determinism in physics.
2. That quantum mechanics assigns a special role, in its very formulation, to the "observer". There has been quite some debate as to whether this "observer" is a set of laboratory instruments or a human consciousness, but the debate would never have got under way if the central role of observations in quantum mechanics had not been accepted to start with.
3. That quantum mechanics is something that "nobody understands", to quote Richard Feynman [79]; that quantum mechanics is mysterious and requires a far more drastic revision in our ways of thinking than any previous scientific revolution.

By simply existing, being deterministic, and describing the "measurements" as purely physical processes, the de Broglie–Bohm theory constitutes a refutation of claims 1 and 2.

A further quality of de Broglie–Bohm theory is its perfect clarity, which refutes claim 3. In that theory, we just deal with matter in motion, just as in classical physics, but of course with very different laws of motion than the classical ones, which is to be expected, since the phenomena to be explained (like interference) are radically different from the classical ones.

Given that there are endless bookshelves of confused talk about the role of the "observer" in physics or about the death of determinism, or about the radical incomprehensibility of the quantum world, this is no small feat, especially given the number of times that this accomplishment has been declared impossible.

Finally, coming back to the three fundamental questions raised in the beginning of this book (indeterminism, the role of the observer and nonlocality), we have already answered how the de Broglie–Bohm theory answers the first two. As for nonlocality, the merit of the de Broglie–Bohm theory is to make it explicit: when the wave function of a pair of particles is like the one described in Sect. 7.4, the motion of these particles is coordinated in such a way that, acting on the wave function of the pair near where one particle is, may affect the behavior of the other particle, even if that particle is arbitrarily far away from where the action takes place.

This is also often considered an objection to the de Broglie–Bohm theory, but since Bell has shown that nonlocality is here to stay, even if quantum mechanics was superseded some day by another theory, far from being a defect, the natural account of nonlocality within the de Broglie–Bohm theory is one of its greatest merits.

Let us leave the last word to John Bell:

> Bohm's 1952 papers on quantum mechanics were for me a revelation. The elimination of indeterminism was very striking. But more important, it seemed to me, was the elimination of any need for a vague division of the world into "system" on the one hand, and "apparatus" or "observer" on the other. I have always felt since that people who have not grasped the ideas of those papers ... and unfortunately they remain the majority ... are handicapped in any discussion of the meaning of quantum mechanics.
>
> [...]
>
> Why is the pilot wave picture ignored in textbooks? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism are not forced on us by experimental facts, but by deliberate theoretical choice?
>
> John Bell [14, pp. 173, 160]

One possible answer to that last question could be that "the pilot wave picture" is just one "interpretation" of quantum mechanics among many, so why pay attention to that one alone? The next chapter will deal with that objection.

8.8 Summary

The de Broglie–Bohm theory is a theory of matter in motion, just like the whole of "classical" physics is (meaning the whole pre-quantum physics, including the theories of relativity). In the latter, particle move under the influence of forces or of electromagnetic waves or because of the structure of space-time. In the de Broglie–Bohm theory, there is an object, the wave function, that guides the motion of the particles. That motion is illustrated in Figs. 8.1, 8.2, 8.3 and 8.4 and is very non classical but it accounts for the observations.

One of the most important aspect of the de Broglie–Bohm theory is that it explains what happens in what are called "measurements": first of all, the latter are always in the end measurements of positions. This is both true for measurements of velocities, that depend on measuring the distance between two positions at different times or the measurement of spin, which depends through which hole a particle exits from a box.

In the de Broglie–Bohm theory, measurements of velocities do not reveal a value that is "already there": for velocities, in some situations, the true pre-measurement velocity is zero, but the "measured" one is not.[14] Measurements are interactions between particles and some macroscopic objects and that interaction is described by the de Broglie–Bohm theory.

That measurements are interactions and not just passive observations is what one would expect on the basis of the no hidden variables theorems of Sect. 5.2, and is also one way to understand the Copenhagen view, but here this fact is shown to be a consequence of the theory, not an a priori claim.

One might wonder how does one recover the statistical quantum predictions in a deterministic system such as the de Broglie–Bohm theory. This can only be done via suitable assumptions on the initial conditions of the system. It turns out that those assumptions are rather natural: one has only to assume that the initial positions of the particles of any system are distributed according to the quantum mechanical statistics. This may seem like assuming what has to be proven, but it is not, since the validity of this statement depends on a non

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[14] And for the spin, there is simply no pre-existing value of the spin being "measured", see Appendix 8.A.

obvious property of the de Broglie–Bohm theory: if the initial positions of the particles are distributed according to the square of the wave function $\Psi(x, 0)^2$ at some "initial" time, they will be distributed at a later time according to the square of the wave function at that time $\Psi(x, t)^2$.

In the de Broglie–Bohm theory, the wave function always evolves according to the usual Shrödinger's equation and never collapses. But then, how does one explain the practical necessity to use the collapse rule in ordinary quantum mechanics? The answer is that, when a particle interacts with a measuring device, namely a big system, the wave function of the latter becomes coupled to the one of the particle: this leads to macroscopic superpositions, where one part of the wave function of the pointer is up and the other down.

But then, because macroscopic objects contain many particles, in order for the two parts of the wave function (up and down) to interfere with each other, the wave function of each particle of the up part of the pointer would have to overlap with the corresponding wave function of the same particle in the down part of the pointer. However, this requires too many overlaps so that it becomes impossible in practice (even if not in theory) to make the up and down parts of the wave function of the pointer interfere with each other.

Then, one can just use for later purposes the wave function of the part of the pointer which we see – either up or down. But, unlike in ordinary quantum mechanics, here there is a fact of the matter as to whether the pointer is up or down at the end of the experiment and then, looking at the result has no physical significance whatsoever.

In Appendix 8.B, we will explain why the de Broglie–Bohm theory is nonlocal: in the situation discussed in Chap. 7, a single wave function may guide simultaneously two particles together. So that, acting on one part of the wave function, may influence the way both particles are guided, no matter how far apart those particles are.

Of course, because of the EPR-Bell result, this nonlocality is a quality rather than a defect.

Finally, we stressed that, even if the de Broglie–Bohm theory is not the final word on quantum mechanics, particularly when it comes to a quantum theory of waves and fields and the theory of relativity, it has the merit of completely eliminating the observer from quantum mechanics, clarifying the paradoxes surrounding the notion of measurement, restoring determinism and making the unavoidable nonlocality of the Universe somewhat more understandable.

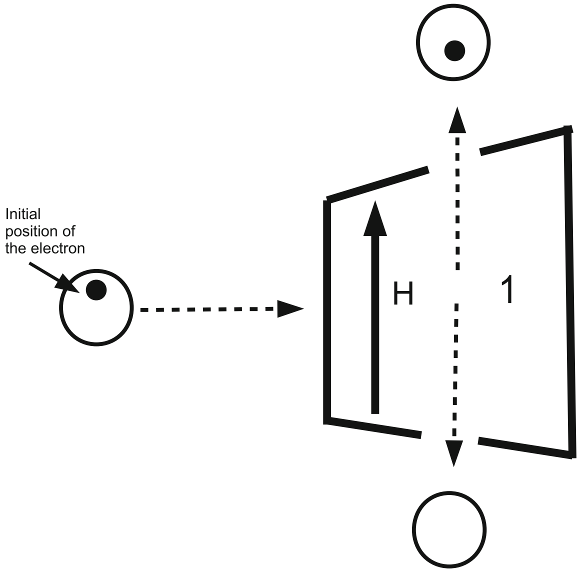

Fig. 8.6 An idealized spin measurement: A particle is sent towards a *box*, which is perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1. The wave function associated to the particle is represented by a disk. In the *box*, the wave function splits into two parts, one going upward in the direction of the field, the other going downward, in the direction opposite to the one of the field. The particle position is indicated by a *dark dot*. In the de Broglie–Bohm theory, if the particle starts initially above the horizontal line in the middle of the figure (at the level of the rightward pointing *arrow*), it will always go in the upward direction, namely here in the direction of the field. This figure corresponds to the situation described in Fig. 7.3, but within the de Broglie–Bohm theory

Appendices

8.A "Measurements" of the Spin in the de Broglie–Bohm Theory

We will describe here how a spin measurement works in the de Broglie–Bohm theory.[15] In Fig. 8.6, we show the wave function, represented by a disk moving towards the box with a magnetic field H in it. As far as the wave function is concerned, it splits itself in two parts, one going in the direction of the field,

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[15] It should be stressed that all the "experiments" are only meant to illustrate the theory, not to explain how real experiments are performed. Some ideas of this appendix come from Chap. 7 of David Albert's book *Quantum Mechanics and Experience* [1].

the other in the direction opposite to the one of the field. Those two parts are represented by two disks.[16]

But all we directly observe is the final position of the particle. As shown in Figs. 7.3 and 7.4, it will either go in the direction of the field, or in the direction opposite to the one of the field. One can show that, in the de Broglie–Bohm theory, if the particle starts initially above the horizontal line in the middle of the Fig. 8.6 (at the level of the rightward pointing arrow), it will always go in the upward direction, namely in the direction of the field in this figure (again, a property of the de Broglie–Bohm theory that we shall not prove).

Now, here is something a priori surprising, but which is fundamental if one wants to understand the de Broglie–Bohm theory. Suppose that we reverse the direction of the magnetic field, relative to its direction in Fig. 8.6, as is done in Fig. 8.7.

And let us start with *exactly* the same wave function and *exactly* the same particle position, as in Fig. 8.6.

One can show that the particle will again go in the upward direction, see Fig. 8.7. But now, this is the direction *opposite to the one of the field.*

In the situation of Fig. 8.6, one would say that the spin is "up", in Fig. 8.7 that it is "down" (up just means "in the direction of the field", down means "in the direction opposite to the field"). But the only difference between the two figures comes from the orientation of the field. As far as the particle is concerned, its complete physical state, namely its wave function and its position are exactly the same in both situations.

In other words, the value up or down of the spin that actually results from the "measurement" does not depend only on the wave function and the initial position of the particle (which, remember, in the de Broglie–Bohm theory, is the *complete* description of the physical state of any system), but on the concrete arrangement of the "measuring" device. Here the scare quotes that we used all along when speaking of measurements are finally understandable: there is no intrinsic property of the particle that is being "measured", in general, in a "measurement", except for measurements of positions.

Of course, since the system is deterministic, once we fix the full initial state (the wave function and the position) of the particle *and* the experimental device, the result of the experiment is pre-determined. But that does not mean that the spin value that we "observe" is pre-determined, because, as we saw, we can measure the spin by orienting the magnetic field in one direction or

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[16] At least for some wave functions, which we will assume are those associated with the particles here. With some simplification, one may assume that each part of the wave function takes a constant value on those disk and vanishes elsewhere.

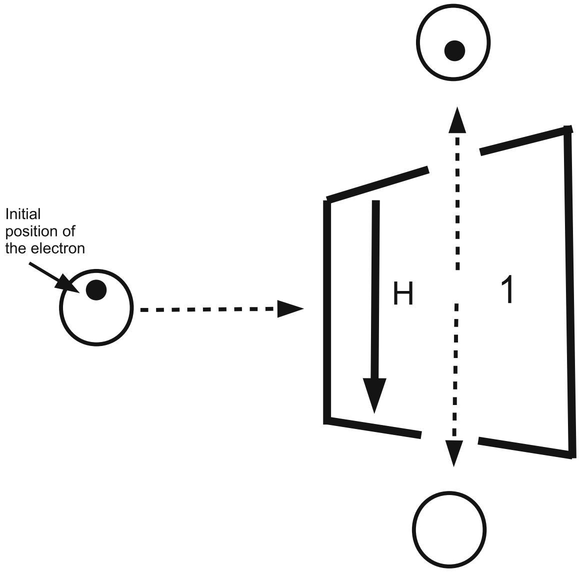

Fig. 8.7 An idealized spin measurement with the field reversed relative to Fig. 8.6: A particle is sent towards a *box*, which is perpendicular to the plane of the figure, and in which there is a magnetic field H oriented downwards along the vertical axis, denoted 1. The wave function associated to the particle is represented by a disk. In the *box*, that wave function splits into two parts, one going downward in the direction of the field, the other going upward, in the direction opposite to the one of the field. The particle position is indicated by a *dark dot*. In the de Broglie–Bohm theory, if the particle starts initially above the horizontal line in the middle of the figure (at the level of the rightward pointing *arrow*), it will always go in the upward direction, namely in the direction opposite to the one of the field in this figure, as opposed to what happens in Fig. 8.6

the opposite one. So the value of the "spin" of the particle that results from a measurement depends on our conventions, which means that it does not exist as an intrinsic property of the particle.

8.B How Does the de Broglie–Bohm Theory Account for Nonlocality?

As we saw in Chap. 7 there exist nonlocal effects in Nature.[17] But we do not know what these effects are, because in ordinary quantum mechanics nonlocality manifests itself through the "collapse rule", and the meaning of

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[17] Some ideas of this appendix come from Chap. 7 of David Albert's book *Quantum Mechanics and Experience* [1].

that rule depends on the meaning of the wave function, which itself is unclear. But in the de Broglie–Bohm theory, the wave function has a clear meaning: it guides the motion of the particles.

The de Broglie–Bohm theory for a single particle is essentially local: the particle is always guided by the part of the wave function in which it finds itself. There is some weak form of nonlocality, if one wants to use that term here, because the motion of a particle going through one slit in the double-slit experiment may be affected by the part of its wave function going through the other slit, as we saw in Figs. 8.1, 8.2, 8.3 and 8.4.

But, when one discusses one particle, everything is still *local*, in the sense that those effects are felt only when one part of the wave function comes back and becomes superposed with the other part, the one in which the particle is. This leads to interference phenomena, but there is no instantaneous action at a distance here, since the effect will take the time needed for the two wave functions to be recombined.

The same holds for Einstein's boxes in the de Broglie–Bohm theory: the particle is always in one of the half-boxes and we simply *learn* in which box it is by opening one of them. In that situation, the wave function is partly in each of the boxes, and that might have an effect if, instead of opening the boxes far away from each other, one were to bring them together again and then recombine those two parts of the wave function.

So, in the thought experiment of Einstein's boxes, there is no action at a distance whatsoever, from the point of view of the de Broglie–Bohm theory. But, of course, coming back to the dilemma of Sect. 7.2 (either there are actions at a distance or quantum mechanics is incomplete), the de Broglie–Bohm theory is based on the idea that quantum mechanics *is incomplete*!

But we learned in Chap. 7 that there are nonlocal effects when we deal with at least two particles.

It would go far beyond the scope of this book to really explain how nonlocality appears in the de Broglie–Bohm theory, but we will sketch what happens in the EPR-Bell situation discussed in Chap. 7.

We will first describe what happens when one measures the spin of two particles far away from each other, when the wave function is as in Sect. 7.4.2, and then explain what is nonlocal in those experiments.

Consider the left part of Fig. 8.8. If we measure first[18] the spin of particle A (the box in which it is measured is closer to where the particles came from than the one measuring the spin of particle B), we shall get the up result, since, again, there is an horizontal line in the middle of Fig. 8.8 (at the level

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[18] We discussed in Sect. 8.5 the problems that this notion of "first" implies if we take into account the theory of relativity.

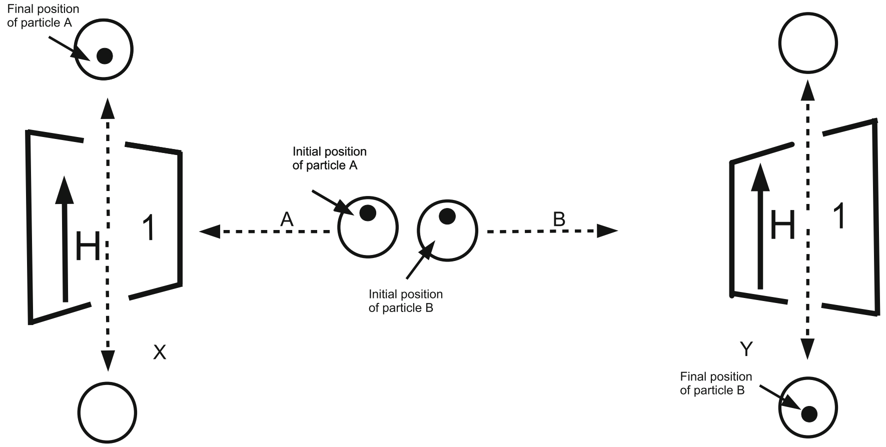

Fig. 8.8 Two particles, A and B are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1. The wave functions associated to the particles are represented by disks. In the *boxes*, the wave functions split into two parts, one going upward in the direction of the field, the other going downward, in the direction opposite to the one of the field. The particle positions are indicated by *dark dots*. Suppose we measure the spin of the A particle first (the *box* in which it is measured being closer to where the particles came from than the one measuring the spin of particle B). In the de Broglie–Bohm theory, if the A particle starts initially above the horizontal line in the middle of the figure (at the level of the two *arrows*), it will always go in the upward direction, namely in the direction of the field. But then, since the wave functions of the two particles are such that they are (anti)-correlated, the B particle will have to go in the direction opposite to the one of the field namely downwards. This figure corresponds to the situation described in Figs. 7.5 and 7.6, but within the de Broglie–Bohm theory

of the two arrows) that particles cannot cross, as was the case in Figs. 8.6 and 8.7. Since the A particle starts above that line, it will have to go up. So the A particle will go in the direction of the magnetic field. By definition, its spin will be "up".

But then, since particle B always goes in the direction of the field opposite to the one taken by the A particle, it will have to go down, that is in the direction opposite to the magnetic field, since the field is oriented in the same way in the boxes at X and at Y, see the right part of Fig. 8.8.

Note that this behavior of the B particle is independent of where it starts: above (as in Fig. 8.8) or below the horizontal line in the middle of Fig. 8.8 (at the level of the two arrows). That is because, once the spin of particle A has been measured, there is no symmetry any more between the top and bottom halves of the figure and that implies that the line in the middle can be crossed

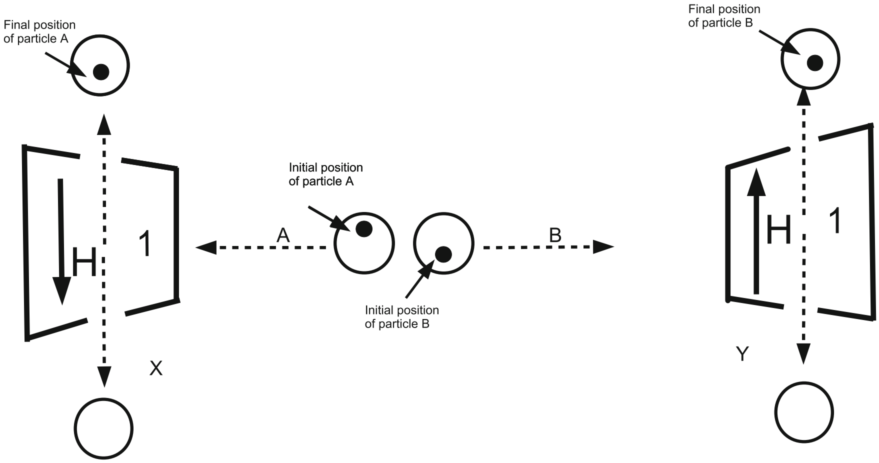

Fig. 8.9 Measurement of the spin on the left first, with the field reversed on the left hand side relative to the one of Fig. 8.8: two particles, A and B are sent towards *boxes*, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H oriented upwards along the vertical axis, denoted 1 on the right and downwards on the left. The wave functions associated to the particles are represented by disks. In the *boxes*, the wave functions split into two parts, one going in the direction of the field, the other going in the direction opposite to the one of the field. The particle positions are indicated by *dark dots*. Suppose we measure the spin of the A particle first (the *box* in which it is measured being closer to where the particles came from than the one measuring the spin of particle B). In the de Broglie–Bohm theory, if the A particle starts initially above the horizontal line in the middle of the figure (at the level of the two *arrows*), it will always go in the upward direction, namely in the direction opposite to the one of the field. But then, since the wave functions of the two particles are such that they are (anti)-correlated, the B particle will have to go in the direction of the field, namely upwards

by the B particle (again, a fact about the de Broglie–Bohm theory that we cannot explain in detail).

Now, suppose that one reverses the direction of the field on the left side of Fig. 8.8, but that one does not reverse it on its right side, which measures the spin of particle B, see Fig. 8.9. Then, let us measure first the spin of particle A, as in Fig. 8.8. Since there is an horizontal line in the middle of Fig. 8.9 (at the level of the two arrows) that particles cannot cross, if the A particle starts above that line, as in Fig. 8.9, it will have to go up.

Thus the A particle goes now *in the direction opposite to the magnetic field*, and, by definition, its spin will be "down". But then, in that situation, particle B must go in the direction of the field, since the two particles are (anti)-correlated.

But that means that the B particle must now go up, see Fig. 8.9, instead of down, as it did in Fig. 8.8, and its spin will be "up".[19]

So by changing the orientation of the field on the left of Fig. 8.9, relative to Fig. 8.8, while doing nothing whatsoever on the right of Fig. 8.9, we affect the trajectory of particle B (in one situation, it goes up, in the other one it goes down) which may be arbitrarily far away from the A particle. This is the way the action at a distance manifests itself in the de Broglie–Bohm theory.

This action does not allow the transmission of messages, because, in the situation of Figs. 8.8 and 8.9, if one repeats the experiment many times, the A particle will start half of the time above the horizontal line in the middle of Fig. 8.8 (at the level of the two arrows) and half of the time below it. When it is above the middle line, it will go up in Fig. 8.8 and particle B will go down. If the A particle starts below the middle line, it will go down and the B particle will go up. If one reverses the field on the left, as in Fig. 8.9, and if the A particle starts above the middle line, both particles will go up. If the A particle starts below the middle line, both particles will go down.

So, there is a genuine action at a distance here, since acting on the A particle (by choosing how to measure its spin) instantly affects the behavior of particle B.

However, since there is no way to control whether the A particle will start above or below the middle line in Figs. 8.8 and 8.9, there is no way to control whether changing the orientation of the magnetic field at X will make particle B go up or down at Y. So there is no way, by playing with the orientation of the magnetic field at X, to send a message at Y (which one could of course do if one could decide, by choosing the orientation of the field at X, to make particle B go up or down).

The fact that the de Broglie–Bohm theory is nonlocal is a quality rather than a defect, since Bell showed that any theory accounting for the quantum phenomena must be nonlocal. Moreover, the nonlocality is of the right type, i.e., just what is needed because Bell's results, but not more, where "more" might be a nonlocal theory allowing the instantaneous transmission of messages.

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[19] As in Fig. 8.8, this holds irrespectively of the initial position of the B particle, since, once the spin of the A particle has been measured, there is no longer a line in the middle of the figure that the B particle cannot cross.

9

Many Worlds?

9.1 Alternatives to the de Broglie–Bohm Theory

A reader who would search the literature will find many different "interpretations" of quantum mechanics. For example, there is a "bibliographic guide" by the Spanish physicist Adán Cabello [37], discussing only "foundations of quantum mechanics" that contains more than 10.000 references!

Obviously we cannot discuss all of them. But they can be roughly divided into four categories, which are pretty much logical alternatives:

(1) The wave function is a complete description of any physical system and it evolves according to the usual laws, the linear evolution between observations and the collapse rule during observations. One then tries to "make sense" of this framework or to "interpret" it.
(2) One completes the wave function by adding variables (called "hidden") to the usual formalism.
(3) The wave function evolves according to laws different from the usual ones: for example, it may collapse spontaneously even when no observations are made.
(4) The wave function is the complete description of any physical system, but it always evolves according to the linear Schrödinger evolution, and never collapses.

For the reasons explained in Chap. 5, we do not think that alternative (1) can work: the linear evolution leads to macroscopic superpositions and one can

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only recover the usual description of the macroscopic reality by introducing a collapse of the wave function in an arbitrary fashion.

There are also versions of (1) that are essentially philosophical: we are urged, in order to understand quantum mechanics, to give up notions such as “objective reality”. We have explained our objections to such a philosophical escape route in Chap. 6.

The de Broglie–Bohm theory is an instance of alternative (2), but there are not many other such instances: indeed, the no hidden variables theorem of Sect. 5.2 practically prevents the introduction of “hidden variables” other than positions.[1]

In option (3), one modifies the usual time evolution, outside of measurements: the wave function spontaneously collapses but in such a way that collapses are very rare for a single particle, but very frequent for macroscopic systems, such as measuring devices.[2] Then, the pointer may spontaneously go up or down, and the cat stays alive or dies spontaneously, sparing us the embarrassment of macroscopic superpositions.

But then, one necessarily obtains empirical predictions that differ in principle from those of ordinary quantum mechanics, for systems with one or few particles. Various theories of this sort have been proposed, but it should be stressed than they are not “interpretations” of quantum mechanics, but alternatives to it and that they can only be true if quantum mechanics itself is false. The theories that have been proposed depend on parameters that are chosen so that the differences between their empirical predictions for microscopic systems and those of quantum mechanics are so small that they cannot be detected with current technologies. This arbitrary choice of parameters that protects the theory against empirical refutation is not an appealing move, to say the least. But one should keep one’s mind open – after all, how can we be certain that the predictions of ordinary quantum mechanics will always be correct?

Finally, one can try option (4), namely ordinary quantum mechanics, but without the collapse rule. But then, how can one reconcile this with our familiar experience of the world, where pointers are either up or down and cats are either alive or dead?

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[1] Let us mention, for the sake of completeness but without discussing them in detail, that there exists versions of alternative (2) based on “consistent histories” or “decoherent histories” that attempt to give an “objective” account of quantum mechanics, but without introducing particle trajectories, as in the de Broglie-Bohm theory. These ideas have been proposed by Murray Gell-Mann and Jim Hartle [85], Robert Griffiths [96] and Roland Omnès [136]. However, those approaches run into difficulties because of the no hidden variables theorems (see e.g. [36, Sect. 6.3] and references therein).

[2] See for example [8] for a review of such models and [36], Sect. 6.2, for a critique of them.

The answer to that question is called the many-worlds interpretation; it was introduced in 1957, in his PhD thesis, by Hugh Everett III, who was then working at Princeton University under the supervision of John Wheeler [74].

The many-worlds interpretation is nowadays a rather popular view among some physicists, particularly cosmologists, and it has a science fiction sort of attractiveness, which is why we shall discuss it in some detail.

9.2 The Many-Worlds Interpretation

The many-worlds interpretation (MWI) claims that, when the proverbial cat (or any other macroscopic body) finds itself in a superposed state, such as those discussed in Chap. 5, then, instead of having a collapse by fiat as in ordinary quantum mechanics or a collapse "in practice" as in the de Broglie–Bohm theory, both terms simply continue to exist. But how can that be possible? We always see the cat alive *or* dead but not both! The short answer is that they both exist, but in different "universes" or "worlds".[3]

Hence, whenever an experiment leads to a macroscopic superposition, the universe splits into two worlds, one for each possible result.[4] But why do I always perceive only one of the results? The answer is simple: I, meaning my body, my brain (and thus also my consciousness) becomes correlated with the states of the cat, so there are two or more copies of me also, one seeing the dead cat in one world, another seeing the live cat in another world. And that, of course, is also true for everything else: every molecule in the entire universe gets to be copied twice (maybe not instantaneously, but that is a separate question), since the two different copies of me, or of the cats, could in principle interact later with those molecules and that might affect their state.

The Israeli physicist and proponent of the many-worlds interpretation, Lev Vaidman, describes this multiplication of worlds in vivid terms:

> "I" am an object, such as the Earth, a cat, etc. "I" is defined at a particular time by a complete (classical) description of the state of my body and of my brain. "I" and "Lev" do not refer to the same things (even though my name is Lev). At the present moment there are many different "Lev"s in different worlds (not more than one in each world), but it is meaningless to say that now there is another "I". I have a particular, well defined past: I correspond to a particular "Lev" in

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[3] The reader can watch the 2012 popular presentation of the many-worlds interpretation by David Wallace called *The Long Earth: Multiverse Physics* (available at `www.youtube.com/watch?v=GRJT9qY21nA`), to see that this is indeed the way this theory is presented to a general audience.

[4] The splitting can be in more than two worlds if there are more than two possible results of the experiment: the universe splits into as many worlds as there are possible results of the experiment.

> 2012, but not to a particular "Lev" in the future: I correspond to a multitude of "Lev"s in 2022. In the framework of the MWI it is meaningless to ask: Which Lev in 2022 will I be? I will correspond to them all. Every time I perform a quantum experiment (with several possible results) it only seems to me that I obtain a single definite result. Indeed, Lev who obtains this particular result thinks this way. However, this Lev cannot be identified as the only Lev after the experiment. Lev before the experiment corresponds to all "Lev"s obtaining all possible results.
>
> Lev Vaidman [191]

Bryce DeWitt, another proponent of the many-worlds interpretation, also stresses that this multiplication of worlds has to be taken literally and not metaphorically:

> This universe is constantly splitting into a stupendous number of branches,[5] all resulting from the measurement like interactions between its myriad components. Moreover, every quantum transition taking place on every star, in every galaxy, in every remote corner of the universe is splitting our local world on earth into myriads of copies of itself.
>
> Bryce S. DeWitt [56], reprinted in [57, p. 161]

To say that this view is weird is accepted by all sides. Indeed, after the passage quoted here, Bryce DeWitt also writes:

> I still recall vividly the shock I experienced on first encountering this multiworld concept. The idea of 10^{100+} slightly imperfect copies of oneself all constantly splitting into further copies, which ultimately become unrecognizable is not so easy to reconcile with common sense.
>
> Bryce S. DeWitt [56], reprinted in [57, p. 161]

That last phrase might qualify as being the understatement of the century.

Note that even in his original "many-worlds" paper [75] (reprinted in [57, p. 146]), Everett stressed that "*all* elements of a superposition (all 'branches') are 'actual', none any more 'real' than the rest." Everett felt obliged to write this because "some correspondents" had written to him saying that, since we experience only one branch, we have only to assume the existence of that unique branch. This shows that some early readers of Everett were already baffled by the radical nature of the "many-worlds" proposal.

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[5] The word "branches" refers to the different worlds. (Note by J.B.).

But why am I not conscious of this proliferation of copies of myself? To explain that, one appeals to the fact, roughly explained in Sect. 8.4.3, that for macroscopic objects, wave functions corresponding to different macroscopic states do not interfere with each other, in practice at least, so that the different copies of me do not "see" each other and do not interact with each other. They could pass through the same place without having any effect on each other, since the wave functions associated to the different worlds evolve independently of each other.

Defenders of the many-worlds interpretation argue that a similar objection (about me not being conscious of those copies of myself) was raised against Copernicus and Galileo (why don't we feel the motion of the Earth around the Sun?), against Darwin (how come we are so different from apes or other animals if we have common ancestors?), and against the atomic theory of matter (how come things look full when they are mostly empty, since atoms occupy a very small part of space?), and that, in each case, science answers those objections by explaining why things look to us in certain ways, while in reality they are not as they seem. The absence, in practice, of interference between wave functions of macroscopic objects, explained in Sect. 8.4.3, is supposed to be a similar answer for this multiplication of unobservable worlds and identities.

But there are several differences between the many-worlds interpretation and those historical examples. The atomic theory made many novel predictions, and Galileo and Darwin's theories had great explanatory power (and led also later to many confirmed predictions). The many-worlds interpretation provides no new prediction and we shall argue below that, far from having great explanatory power, it has not yet been formulated in a consistent way.

The many-worlds interpretation has a sort of charm that can excite our imagination: I can imagine another world where I would be married with the woman that left me for another man, yet another world where I would have had different questions at a difficult entrance exam and would have passed it, still another one in which I would not have taken my car on the day that I had a terrible accident etc.

But these are just dreams: reality is much more complicated. The many-worlds interpretation does not give us any idea whatsoever of how many worlds there are, or what they look like. In principle, "all one has to do" is to take the wave function of the entire Universe and find how it evolves under the usual linear evolution. To put it very mildly, this is easier said than done, since we have no idea whatsoever of what the wave function of the entire Universe is.

9.3 Critique of the Many-Worlds Interpretation

Even putting aside the weirdness of the multiplication of "worlds", one must ask whether the scheme of the many-worlds interpretation is coherent. One of the main problems concerns the quantum mechanical statistical predictions. Let us make the following thought experiment. "I" decide to repeat many times a similar experiment. For example, one could send a particle towards a wall with two slits, as in the double-slit experiment, and see whether it goes through the upper or the lower slit. In the many-worlds interpretation both results (the particle goes through the upper slit and it goes through the lower one) are realized, but in different worlds. One can then repeat the same experiment with a lot of similarly prepared particles.

One may also couple these results to the proverbial cat that is alive and dead, as we did in Chap. 5. Then, in the many-worlds interpretation, after one experiment, there will be one world where the cat is alive and another one where she is dead. We putted "I" above in quotation marks, like Lev Vaidman did, because after one experiment, there will be one copy of me in one world, seeing the cat alive and another copy of me in the other world, seeing the cat dead. Let us call these two copies of me my "descendants".

Now, since I have decided to repeat the experiment, each copy of me in each world will redo it and there will be two new worlds coming out in each of the worlds already created. We shall have four worlds after two experiments, with, in each of them, a copy of myself, so that, altogether, I shall have four "descendants". And we shall have eight worlds after three experiments etc.

What will my descendants see, lets say after three experiments? One of them will see three dead cats and another one will see three cats alive. But three of them will see one cat alive and two dead cats and another three of them will see one cat dead and two alive cats.[6] That account for all eight of my descendants.

The first point to notice is that all possibilities occur. If I repeat the experiment n times (with n large, say one hundred) one of my descendants will see n dead cats and another one will see n live cats.

Suppose that the probability of having the cat alive is $\frac{1}{2}$ and of course also $\frac{1}{2}$ for the probability of having the cat dead. Now quantum mechanics predicts that if one repeats that same experiment a large number of times, one will see the cat alive approximately half of the time and one will see her dead approximately half of the time.

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[6] If there is one live cat and two dead cats, then the cat alive may occur in any one of the three successive experiments. Hence, there are three possibilities.

But then, my descendants that see the cats always alive or always dead will stop believing in quantum mechanics after a while, since their observations will be radically different from what quantum mechanics predicts.[7]

Moreover, there are also many sequences of worlds where the cats are alive approximately one quarter of the time and dead approximately three quarters of the time, and that is true for any statistics different from the cat being alive approximately half of the time and the cat being dead approximately half of the time.

So that we can be certain that many of our descendants will *not* observe the quantum predictions in their worlds.

But one could argue, on the basis of the law of large numbers (see Sect. 3.4.1) that, at least in the vast majority of worlds, the quantum predictions will be obeyed, since, in the vast majority of worlds, the frequencies will be close to $(\frac{1}{2}, \frac{1}{2})$. It is just like with coin tossing: each world splits into two worlds, one for each possible outcome, so that, after n experiments, there will be, in the vast majority of worlds, approximately $\frac{n}{2}$ worlds where the cats end up alive and approximately $\frac{n}{2}$ worlds where they end up dead.

So, in that situation, although many of my descendants will not believe in quantum mechanics after a while, because what they see contradict the quantum mechanical predictions, the vast majority of my descendants will continue to observe approximately what quantum mechanics predicts.

But what happens if, instead of being $\frac{1}{2}$ for each outcome of a single experiment, the probabilities predicted by quantum mechanics are, say, $\frac{1}{4}$ for one outcome and $\frac{3}{4}$ for the other? Then, the same use of the law of large numbers leads to the conclusion that, in the vast majority of worlds, the quantum predictions will *not* be observed, since, in those worlds, our descendants will see cats ending up alive approximately half of the time and dead also approximately half of the time, instead of the frequencies $\frac{1}{4}$ and $\frac{3}{4}$ that quantum mechanics predicts.

Another way to visualize the problem is to imagine a biased coin, with probability $\frac{3}{4}$ of falling heads and $\frac{1}{4}$ of falling tails. What sense could we make of those probabilities if we were told that both results, heads and tails, happen, but in "different worlds", each of which includes of course observers that "see" the coin being heads in one world and being tails in another world? Upon repetition of the coin tossing in each successor world, most of our descendants

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[7] The quantum probability of seeing n cats alive or n cats dead in a row is $(\frac{1}{2})^n$, since in each experiment each outcome has probability $\frac{1}{2}$. This is just like having a coin tossed n times and falling always heads or always tails.

would see the coin falling heads approximately half of the time and tails the other half.

Of course, this is also true for many people whom one might call our "cousins", namely descendants, like us, of some of our ancestors, if many identical experiments have been made in the past in each world. After all, many similar experiments are made in our world and some of them have two outcomes that do not have the same probability of one-half. So, when experiments with two outcomes whose probabilities are not both equal to one-half are performed repeatedly, the sequence of worlds in which the quantum probabilities are observed are statistically rare. So, why should I expect to belong to such a sequence of worlds?

This is *prima facie* a serious problem for the many-worlds interpretation. There have been many proposals to solve this problem but it would be too long and too technical to discuss all of them here.

One such proposal is to say that the worlds in which the quantum predictions are statistically violated have less "reality" or "intensity" or have a lower "degree of existence" than the ones in which those predictions are realized.

But although one can imagine oneself living in "another world" than the one we live in (for example, if some event that did occur in the past had not occurred), it is difficult to see what it could mean to "exist" in another world, but with a different "intensity" than the one we live in.

We already noted that, in his original paper, Everett stressed that "*all* elements of a superposition (all 'branches') are 'actual', none any more 'real' than the rest." ([75], reprinted in [57, p. 146]). But if all worlds are equally real, how come that some are less real than others?

Another proposal to solve the above-mentioned problem (a solution also going back to Everett) is to define a probability distribution on the set of worlds that is constructed so that the worlds in which the quantum mechanical predictions are violated are improbable. But that probability distribution is a pure mathematical construction that has nothing to do with the observed frequencies in the many worlds that are supposedly created.

Moreover, in all what we said above, we left out the most serious problem facing the many-worlds interpretation, namely the problem of "ontology", to use a term introduced in Sect. 5.3. Indeed we have been talking as if the worlds that multiply themselves are real worlds in our familiar three-dimensional space. But one of the selling points used by the defenders of the many-worlds interpretation is what they call the "pure wave function ontology". For them, the only thing that exists is the wave function. The motivation for adopting this idea is that it makes the many-worlds interpretation supposedly parsimonious in its ontology. Indeed, in the de Broglie–Bohm theory for example, we have both a wave function and particles whose motion is guided by the wave function, so that there are more "beings" in that world than in a world where the only being would be the wave function. Everything else being equal, ontological parsimony is obviously a virtue.[8]

But here, everything else is *not* equal. After all, the wave function is a function,[9] and a function is simply a mathematical object that associates a number to each of its arguments. But the arguments or variables that it depends on are also mathematical objects. Even if those variables characterize a cat or a pointer, saying that the wave function and only the wave function exists does not mean that cats or pointers exist in our familiar three-dimensional world. In fact, it means exactly the opposite, since the defenders of the many-worlds interpretation do *not* postulate that some of those points are material points located in our three-dimensional world, and they regard them as being just mathematical symbols.

All the wave function does is to associate a number to a set of points that would correspond to a cat or a pointer if one assumes that those points are material points, not just mathematical objects. And, if one does assume that there are material points, then the ontology would include both the wave function *and* those material points.

If one wants to make sense of the intuitive picture of the many-worlds interpretation, with actual worlds proliferating in ordinary space, then one has to add something to the ontology, besides the wave function. But then, the "ontological parsimony" argument collapses, and it is not even clear what is to be added.[10]

The many-worlds interpretation is popular among some physicists, as long as it is formulated verbally and rather loosely (All quantum possibilities coexist! There is no collapse of the wave function!). However, it remains to be seen how to formulate it in a clear way (i.e. with an explicit definition of what exists besides the wave function) and to make the quantum mechanical predictions likely to be true in the world we live in.

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[8] This is sometimes called the principle of Ockham's razor, which goes back to the scholastic philosopher William of Ockham, and according to which one should not introduce in our explanations more entities than what is necessary.

[9] As we explained in Sect. 5.3, that function depends in general on several parameters. If one considers the wave function of the Universe, that function is defined on an incredibly large number of parameters, and, if one includes fields in our description, that number is infinite.

[10] See [2] for a clear addition of an explicit ontology that gives a meaning to the many-worlds interpretation, but done by people who are not supporters of the many-worlds interpretation.

9.4 Summary

Among the multitude of “interpretations” of quantum mechanics (most of which however fall in one of the four categories of Sect. 9.1), we chose to discuss a particularly popular one: the many-worlds interpretation. In that interpretation, there is never any collapse of the wave function; every outcome of every experiment is realized, but in parallel universes, that are endlessly proliferating, with everything, including us humans, being copied in each such “world”.

This theory has a certain appeal because it seems to eliminate the measurement problem without modifying the ordinary quantum evolution of the wave function and without introducing “hidden variables”.

But we showed in the previous section that it cannot account in a natural way for the observed quantum statistics, for two reasons: first, all possible outcomes of all experiments ever done are realized in some worlds so that, there will always be many sequences of worlds in which the quantum predictions are not observed.

Moreover, in many situations (whenever an experiment has two outcomes that do not have the same probabilities), it is in the vast majority of worlds that the quantum predictions will not be observed. So that the quantum mechanical predictions become extremely improbable and therefore the many-worlds interpretation does not offer a plausible explanation of why they are observed in the world we live in, which is the only one we are conscious of.

Finally, the deeper problem of the many-worlds interpretation is the one of “ontology”: the defenders of that interpretation like to say that all there is in the world is the wave function (of the Universe). But if that was the case, then there would be no pointers, no cats, no humans, since these “objects” are situated in our familiar three dimensional space and, at best, the wave function would induce a certain probability for those objects to be here or there, but, in an ontology where there is only the wave function, these objects simply do not exist and therefore attributing a probability to their (non-existing) positions does not make sense.

10

A Revised History of Quantum Mechanics

As we already mentioned, there were heated debates about the meaning of quantum mechanics at the time of its inception, in the late 1920s, mostly between Bohr and Einstein. Those debates continue till the present, even though the majority of physicists think that this issue has been settled.

Indeed, the orthodox view of the history of quantum mechanics is that "Copenhagen" won. That is, Niels Bohr got the better of Albert Einstein in his many debates with him and, after that, no serious objections to the standard interpretation of quantum mechanics were raised. The 1964 John Bell discovery of his inequalities and their subsequent experimental confirmations further vindicated Bohr's viewpoint. In particular Bell showed that "hidden variables theories" had to be nonlocal and therefore physically unacceptable. Sometimes people will add that David Bohm, reviving old ideas of Louis de Broglie, tried to give a hidden variables alternative to quantum mechanics, but they will emphasize that he failed.

In other words, history has proven another version of Margaret Thatcher's TINA: There Is No Alternative (to ordinary quantum mechanics).

If the reader has gotten this far, he or she may guess that we do not agree with this view of history. That is correct (and is quite an understatement).

In this chapter, we will explain why this view is indeed wrong. Moreover, the fact that it persists in the mind of many physicists is a major obstacle to a better understanding of quantum mechanics.

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We claim that the ideas of Einstein, Schrödinger, and Bell were widely misunderstood, while those of de Broglie and Bohm were simply ignored most of the time.[1]

In fact, it would suffice to *read carefully* what de Broglie, Einstein, Schrödinger, Bohm and Bell actually said to realize that their views were ignored or misunderstood, irrespective of whether one agrees with them or not.

We will divide our little rewriting of history into four parts: the debate between Niels Bohr and Albert Einstein; what were the opposing attitudes of Max Born and Erwin Schrödinger in regard to that debate; how John Bell was misunderstood and finally how Louis de Broglie and David Bohm were ignored.

10.1 The Bohr–Einstein Debate

That debate is one of the most famous intellectual debates of the 20th century, and certainly the most famous one between physicists. There are several episodes in this debate. The first one took place during the fifth Solvay Conference in 1927, which is the founding event of the "Copenhagen" interpretation of quantum mechanics. Another less well known one occurred during the sixth Solvay Conference in 1930. The most famous discussion surrounded the EPR paper of 1935. In 1949, there was a collective volume devoted to Einstein "philosopher–scientist" [169], where all aspects of Einstein's thoughts were discussed, including those about quantum mechanics, and where both Bohr and Einstein expressed their opinions on the subject.

We first need to outline the main positions of Einstein and Bohr and understand what their disagreement was about. The difficulty is that while I find Einstein's ideas clear enough, I am not quite sure what Bohr's views were. What is curious is that most physicists will claim that they agree with Bohr, but do not always agree with each other about what he really meant.

10.1.1 What Was the Debate Really About?

We already mentioned in Sect. 8.6.1 the most famous sentence of Einstein about quantum mechanics: "God does not play dice". Einstein did write that, in a letter to Max Born [35, p. 91] and apparently repeated that sentence several

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[1] For other books giving heterodox accounts of the history of quantum mechanics, see Bacciagaluppi and Valentini [7], Beller [17], Cushing [44], Freire [84], and Wick [204]. The book by Jammer [108] is a comprehensive overview of the history of quantum mechanics, from a more or less orthodox viewpoint.

times. He even said, that, if God did play dice, he would rather “be a cobbler, or even an employee in a gaming-house, than a physicist”.[2]

The first thing to emphasize, as Einstein himself did many times, is that his “God” had nothing to do with the personal gods of the “revealed” religions. God, for Einstein, is not very different from “the laws of Nature” (this is sometimes called the god of Spinoza).

The usual interpretation of that sentence is that Einstein was an old-fashioned guy who wanted strict deterministic laws and could not reconcile himself with the intrinsic randomness of quantum mechanics.

But if one reads Einstein beyond this one sentence, one realizes that he was far more preoccupied with nonlocality than with determinism. In 1942, he wrote in another letter that “I cannot believe for a single moment’ that “God plays dice and uses “telepathic” methods (as the present quantum theory requires of him).” ([69] quoted in [60, p. 68]) The “telepathic methods” being the actions at a distance that, as shown by the EPR argument discussed in Chap. 7, are implied by the idea that ordinary quantum mechanics gives a complete description of physical systems.

Besides, Wolfgang Pauli testified in a letter to Max Born in 1954 that Einstein had told him “emphatically many times” that “he disputes that he uses as a criterion for the admissibility of a theory the question: ‘Is it rigorously deterministic?”’ [35, p. 221].

The dilemma for Einstein was: either quantum mechanics is complete, in which case it is nonlocal, or it is local but incomplete, and hidden variableshave to be added to it. In his contribution to the 1949 collective volume devoted to himself and entitled *Reply to criticisms*, Einstein did pose again the question in the form of a dilemma, implied by the EPR paradox, dilemma that we have emphasized several times, in Sect. 4.3 and in Chap. 7:

> [...] the paradox forces us to relinquish one of the following two assertions:
>
> (1) the description by means of the ψ-function is complete,
>
> (2) the real states of spatially separated objects are independent of each other.
>
> Albert Einstein [169, p. 682]

Let us now try to formulate Bohr’s viewpoint. This is not something everybody agrees on, but, from what I understand, he insisted that quantum mechanics, and in particular the uncertainty relations, impose limitations to how much we can know about the microscopic world. If I want to know something about a quantum object, I must interact with it through measuring devices. But, if

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[2] Albert Einstein, letter to Hedwig Born, 29 April 1924 [35, p. 82].

the latter exert an uncontrollable action on the object being measured, which is what Bohr took quantum mechanics to mean, how can I know what properties the object had before being interacted with?

So, says Bohr, the quantum object and its measuring device form an "inseparable whole". One cannot assign separate properties to the object and to the measuring device.

That of course has radical implications: we can never *know* what properties the quantum object would have had if we had not interacted with it. So, an objective description of the physical world is simply impossible, if "objective" means a description independent of our means of observations.

To contrast the situation here with the pre-quantum one, consider celestial mechanics: Newton's laws allow us to predict *where* the moon will be at a given time. They do not tell us that we will *see* the moon at a given place if we care to look. That last statement requires other assumptions: that the sky is clear, that our eyes or telescopes function properly etc. These assumptions have nothing to do with celestial mechanics per se. Of course, looking at the moon has no influence on where the moon is, so we can check by direct observation the location of the moon at a given time, assuming what we said about the sky, our eyes etc.

But suppose you were told that looking where the moon is would affect its position in an unavoidable and uncontrollable fashion. Then of course, it would be difficult to say where the moon was before one looked at it.

And that is basically what Bohr said, not about the moon of course, but about atomic objects. Note that there is nothing a priori irrational in his attitude. There is no talk about the (real) moon not being there when nobody looks, or about the mind interacting directly with matter. But Bohr had to think that there was a quantum realm, the one of atoms, photons and electrons, and a classical one, in which measuring devices indicate unambiguous answers. The problem with that view is that, as Schrödinger's cat shows, this approach is not consistent with the universal validity of quantum mechanics and, if one has to put an artificial dividing line between a "quantum world", in which objects do not have definite properties, and a "classical one", in which they do, where do we put that line? And how are we supposed to understand the "ontological" jump between a world where objects do not have any definite properties and one where they do?

The problem with Bohr's writings, apart from their obscurity (a fact admitted by most people), is that his position on quantum mechanics was presented, not just as the best one can do in a bad situation, at least for the time being, but as some final eternal limitation on our understanding of the world.

The big catchword in Bohr's writings is "complementarity". We already explained the concept in Sect. 2.1, and emphasized that it should not be understood in the ordinary sense of the word. Two complimentary pictures of an object in the usual sense of he world, could give a more exact or more complete description of that object. But in Bohr's sense, complimentary pictures *exclude* each other: if we set up one type of experiment, we can talk about, for example, positions, and if we set up another type of experiment, we can talk about velocities; or we could talk about spins in different directions, one for each experimental setup (see Sect. 7.4.2 and Appendix 8.A for the notion of spin). In each setup, we can have well-defined answers, but it is pointless to try to combine these different "complementary" aspects of reality into a coherent mental image.

The basic misunderstanding between Bohr and Einstein is that Einstein did not dispute Bohr's emphasis on the limitations of our (current) measurements, but wanted to show, through indirect arguments based on locality, that quantum mechanics could not possibly be a complete theory. And, therefore, that a future more complete theory was in principle possible; moreover, in that theory, the centrality of measurements and (maybe) also indeterminism might be bypassed. Remember that, as we saw in Sect. 6.2, for Einstein, observation was a derived notion, not at all a primitive one and it was "the theory that tells us what can be observed".

The root of the difference between Einstein and Bohr is that Einstein was arguing at the level of *what there is*, insisting that a complete description of physical systems *must* go beyond the description given by ordinary quantum mechanics, if the world is local. Bohr, on the other hand, was answering systematically at the level of *what we can know*. Bohr was not answering Einstein, he was simply not really listening to his objections.

We already saw an example of that misunderstanding in Sect. 4.3 in which we discussed Einstein's reaction at the 1927 Solvay Conference, where he explained that if a wave function spreads itself over a half circle, while one always detects the particle in one piece at a given point, then one has two options: the first one is that the square of the wave function expresses the probability that the particle *is* somewhere, in which case it is not a complete description of the system (the particle has a position besides its wave function). So this is a "statistical" view of the wave function; it does not describe entirely an individual system, but rather a set of similar systems.

The second option is to insist that the wave function is a complete description, but then the detection of the particle at a point implies a sort of collapse of the spread out wave function, which, as Einstein said: "assumes an entirely peculiar mechanism of action at a distance, which prevents the wave continuously distributed in space from producing an action in *two* places on the screen." [7, p. 441].

In that debate, Bohr admitted not to see what Einstein meant, since he replied: "I don't understand what precisely is the point which Einstein wants to [make]." [7, p. 442].

However, in the volume dedicated to Einstein, Bohr did come back to this 1927 discussion in some detail. But his arguments there were centered on whether one could make *observations* that go beyond those allowed by quantum mechanics – for example, to control where the particle will end up on the screen in Fig. 4.11 or to detect, in the double-slit experiment, through which slit the particle went, without destroying the interference pattern. Bohr did show that this was impossible, but that did not answer Einstein's objection, which had to do with the "peculiar mechanism of action at a distance". Bohr did not address that issue at all.

The debate went on, first at the sixth Solvay meeting, held in Brussels in October 1930, which we will not discuss,[3] then around the famous EPR paper of 1935.

10.1.2 The "Bolt from the Blue": The Einstein–Podolsky–Rosen Argument

We explained the EPR argument in Chap. 7, first through an anthropomorphic analogy (Sect. 7.4.1), and then using spin variables in a version of the argument due to Bohm [23] (Sect. 7.4.2). The original EPR article did not use spin, but position and velocity.

The authors considered two particles starting from the same place and moving in opposite directions, in such a way that their velocities were equal in size but with opposite directions. Of course, since ordinary quantum mechanics does not admit that particles have trajectories, what this means in the "orthodox" language, is that, if we measure the velocities of the two particles, the results will be equal, but with opposite signs and likewise for the positions. This is similar to the perfect correlations for the spin measurements discussed in Sect. 7.4.

Thus, argued EPR, by measuring the velocity of one particle, one could know the velocity of the other particle. But if one measured instead the position of that particle, one would know the position of the other particle (since they move at the same speed in opposite directions). However, if the two particles are far apart, and if there are no actions at a distance of any sort, then the

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[3] See e.g. [36, Sect. 7.1.3] for a discussion of this encounter.

choice that we make of the quantity to measure on one particle cannot affect the physical state of the other particle. Thus, that second particle must possess a well-defined velocity *and* a well-defined position, and this shows that quantum mechanics is incomplete, since the wave function does not assign any precise value to those variables.

However, the EPR article was not as clearly written as one might have liked it to be. In particular, there was no need to invoke *both* position and velocity. The fact that one of them, say position, could be determined for one particle by a measurement on the other particle, arbitrarily far away, is enough to prove incompleteness of quantum mechanics (since the wave function does not in general assign a determinate position to either particle), if one takes no action at a distance for granted.

The article, according to Einstein, was written by Podolsky "for reasons of language", because Einstein's English was far from perfect. But Einstein complained in a letter to Schrödinger on 19 June 1935 [66] that "it has not come out as well as I really wanted" and that "the main point was, so to speak, buried by the erudition".

One rather common misconception is to think that the goal of EPR was to beat the uncertainty principle (see Sect. 4.4) and to show that one could *measure simultaneously* with an arbitrary precision the position and the velocity of both particles.

But the main point of the EPR paper was not to claim that one could *measure* quantities that are impossible to measure simultaneously according to quantum mechanics, but rather that, if one can learn something about a physical system by making a measurement on a distant system, then, barring actions at a distance, that "something" must already be there before the measurement is made on the distant system.

Actually, Einstein explicitly denied that his goal was to refute the uncertainty principle in a letter to Schrödinger of 19 June 1935. It was in this same letter that Einstein gave the "boxes" argument discussed in Chap. 7 (in a slightly different form), and that argument alone proves the incompleteness of quantum mechanics, if Nature is local, without getting into the more complex EPR argument.

The "onslaught" of EPR "came down upon us as a bolt from the blue", wrote the Belgian physicist Léon Rosenfeld, who was in Copenhagen at that time and who reported EPR's argument to Bohr. The latter then worked intensely "week after week" on a reply [201, p. 142]. There is a widespread belief among physicists that Bohr came up with an adequate answer to the EPR paper [30]. But, as we will try to show, Bohr's answer is hard to understand, despite the fact that, according to Rosenfeld, one of his favorite aphorisms was a line from the poet Friedrich von Schiller [201, p. 143]: "Only fullness leads to clarity, and truth dwells in the depths."

So what was Bohr's reply? Here is his crucial point. EPR had written that:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity,
>
> Albert Einstein, Boris Podolsky, Nathan Rosen [65], reprinted in [201, pp. 138–141]

This again means that if, by doing something on particle A, we can learn something about the situation of particle B, located far away from particle A, then, barring actions at a distance, what we learn must exist before we learn it.

Bohr replied:

> [. . .] the wording of the above-mentioned criterion [. . .] contains an ambiguity as regards the meaning of the expression "without in any way disturbing a system". Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system* [. . .] their argumentation does not justify their conclusion that quantum mechanical description is essentially incomplete [. . .] This description may be characterized as a rational utilization of all possibilities of unambiguous interpretation of measurements, compatible with the finite and uncontrollable interaction between the objects and the measuring instruments in the field of quantum theory.
>
> Niels Bohr [30], quoted in [14, p. 155] (italics in the original)

Rather than dissecting ourselves the part of Bohr's answer quoted above, we will leave that task to John Bell who performs it remarkably well:

> Indeed I have very little idea what this means. I do not understand in what sense the word 'mechanical' is used, in characterizing the disturbances which Bohr does not contemplate, as distinct from those which he does. I do not know what the italicized passage means — 'an influence on the very conditions [. . .]'. Could it mean just that different experiments on the first system give different kinds of information about the second? But this was just one of the main points of EPR, who observed that one could learn *either* the position *or* the [velocity][4] of the second system. And then I do not understand the final

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[4] Here, we replaced the word momentum in the text by velocity, to be consistent with the rest of our text.

> reference to 'uncontrollable interactions between measuring instruments and objects', it seems just to ignore the essential point of EPR that in the absence of action at a distance, only the first system could be supposed disturbed by the first measurement and yet definite predictions become possible for the second system. Is Bohr just rejecting the premise — 'no action at a distance' — rather than refuting the argument?
>
> John Bell [14, pp. 155–156] (italics in the original)

Bell's judgment on the EPR-Bohr debate is quite trenchant:

> I felt that Einstein's intellectual superiority over Bohr, in this instance, was enormous; a vast gulf between the man who saw clearly what was needed, and the obscurantist.
>
> John Bell in [19, p. 84]

"What was needed" was of course a more complete description of the quantum systems than the usual one, if locality had to be maintained.

It is interesting to note that, in 1949, when Bohr reviewed his debates with Einstein in the volume devoted to the latter [169], he apologizes for his own "inefficiency of expression which must have made it very difficult to appreciate the trend of the argumentation [...]" [31, p. 234]. However, Bohr did not seize this opportunity to make himself clearer and basically repeated what he had said in his previous debates with Einstein.

On the other hand, Einstein, in that same volume, summarizes Bohr's position as accepting that nonlocality is a fact implied by quantum mechanics. However, this was never said very clearly by Bohr.

Finally, one must say that there is a sense in which Einstein was wrong: Nature is not local, as we saw in Chap. 7. In that sense, as Bell says, "the EPR paradox is resolved in the way which Einstein would have liked least." [14, p. 11].

But that was proven long after the EPR argument, by Bell in 1964. To quote Bell again: "It would be wrong to say that 'Bohr wins again'; the argument [of Bell] was not known to the opponents of Einstein, Podolsky and Rosen." [14, p. 150].

10.2 Born and Schrödinger

Born and Einstein were lifelong friends; they both had to leave Germany, to avoid persecution, Born settling in Edinburgh, Einstein in Princeton. They exchanged a long correspondence, about physics, personal matters, and politics, which was edited by Born in 1971, long after Einstein's death in 1955. In that correspondence, they also discussed quantum mechanics. It is rather fascinating to read their exchange of letters and to observe the systematic misunderstanding of Einstein's point of view by Born.

For example, Einstein wrote an article in 1948 and sent it to Born (it is reproduced in the correspondence), begging him to read it as if he had just arrived "as a visitor from Mars". Einstein hoped that the article would help Born to "understand my principal motives far better than anything else of mine you know" [35, p. 168].

In that article, Einstein restated his unchanging criticisms. He explained that "An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects 'are situated in different parts of space'."

Einstein called A and B these different parts of space and characterized this independence by saying: "external influence on A has no direct influence on B.", if one assumes no action at a distance.

Here is Born's reaction to that very same article:

> The root of the difference between Einstein and me was the axiom that events which happens in different places A and B are independent of one another, in the sense that an observation on the states of affairs at B cannot teach us anything about the state of affairs at A.
>
> Max Born [35, p. 176]

What Born said was that making an experiment in one place *teaches* us something about what is happening at another place, which is unsurprising. If, in the anthropomorphic example given in Sect. 7.4.1, both people had agreed on a common strategy, one would learn what B would answer to question 1, 2, or 3, by asking A that same question. But, and that was Einstein's point, it would mean that the answers were predetermined.

As Bell comments: "Misunderstanding could hardly be more complete. Einstein had no difficulty accepting that affairs in different places could be correlated. What he could not accept was that an intervention at one place could *influence*, immediately, affairs at the other." [14, p. 144].

This suggests that Born thought, in our language, that Alice and Bob have pre-determined their answers to the three possible questions that may be asked to them, or, in the language of Sect. 7.4.2, that the two particles have a spin that is up or down in all directions before their measurements, or that the two particles flying apart in the EPR example have correlated positions and velocities (see Sect. 10.1.2). In other words, it seems that Born did in fact agree with Einstein that quantum mechanics is incomplete, but simply did not understand what Einstein meant by that expression.

Lest the reader believe that everybody misunderstood Einstein and that there must have been something wrong with his way to express himself, we want to contrast the reactions of Bohr and Born with the one of Schrödinger.

Schrödinger was, together with Einstein, one of the two major critics of the Copenhagen interpretation of quantum mechanics. His most well-known contribution to the discussions about quantum mechanics is his cat, which is supposed to be "both alive and dead". This example occupies only a few lines in his 1935 paper on "The present situation in quantum mechanics" [174].

The problem that bothered Schrödinger and that led to his cat example was basically the same as the one that Einstein mentioned at the 1927 Solvay Conference: the wave function gets spread out as time goes by (see Fig. 4.11). He summarized the situation as follows: "The emerging particle is described, if one wants to explain intuitively, as a spherical wave that continuously emanates in all directions and that impinges continuously on a surrounding luminescent screen over its full expanse. The screen, however, does not show a more or less constant uniform glow, but rather lights up at *one* instant at *one* spot […]" ([174], reprinted in [201, pp. 156–157]).

Schrödinger realized that this spreading also affects macroscopic objects, if one applies the quantum formalism all the way to them, as we explained in Sect. 5.1. He illustrated this with the cat example:

> One can even set up quite ridiculous[5] cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.
>
> It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be *resolved* by direct observation.
>
> Erwin Schrödinger [174], reprinted in [201, p. 157]

---

[5] In German, the word is "burlesque". (Note by J.B.).

Of course, Schrödinger saw that as a *reductio ad absurdum* of the idea that quantum mechanics is complete, since an indeterminacy that could be acceptable in the microscopic domain to which we have no direct access (all sides agree on that, even the "realists") "becomes transformed into macroscopic indeterminacy" with the cat example.

In other words, for the cat, the description "both alive and dead" is certainly not complete. Now, some people seem to think that quantum mechanics predicts that the cat is both alive and dead and that, since quantum mechanics is complete, the cat must be in such a state. This, to put it mildly, is a huge misunderstanding of Schrödinger's views. And of course, a mistreatment of logic as well: we have no evidence whatsoever that the cat is both alive and dead.

Moreover, since we never see such a cat, we have to assume that looking at the cat "collapses" its wave function into one of the two possibilities. But again, this is a pure *deux ex machina*, since "looking" is not associated with any precise physical interaction. It is far more sensible to consider, with Schrödinger, that the cat example shows that there is a problem with ordinary quantum mechanics.

Actually, for the sake of historical accuracy, it should be mentioned that the cat argument was suggested to Schrödinger by Einstein, in a slightly different form. In a letter to Schrödinger, Einstein considered a "charge of gunpowder", that would be in the situation where it could explode or not explode and whose wave function would then describe "a sort of blend of not-yet and already-exploded systems."[6]

But both in his "cat" paper and in two others [175, 176] published in 1935–1936, Schrödinger also discussed and extended the EPR idea.

Schrödinger compares the EPR situation with the one of schoolchildren who can be asked two questions, but which of the two questions is asked *first* is chosen at random. It happens that the schoolchildren always answer the first question correctly; how can that happen (systematically) if they do not know the answer to *both* questions? This is like asking either the position *or* the velocity of the particle A at X. By doing the corresponding measurement on particle B, far away from A, we know what the answer will be. A later measurement on particle A will always give the correct answer for the position or the velocity of the particle, where "correct" means the one inferred from the measurement on particle B. If there is no influence whatsoever from the measurement on particle A from the one done on particle B, how come that the measurement done on particle A gives that correct answer, if it does not

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[6] Einstein, letter to Schrödinger, 8 August 1935 [80, p. 78].

"know" the answer to both questions (meaning if both its position and velocity are not predetermined)?

Schrödinger saw a deep mystery behind the findings of EPR and did not think that ordinary quantum mechanics offered an answer to them. We leave it to the reader to appreciate the enormous difference between Schrödinger's and Bohr's understanding of the problem raised in the EPR article, both papers, the ones of Bohr [30] and of Schrödinger [174], being published during the same year (1935).

10.3 Misunderstandings of Bell

It is easy to understand why people misunderstood Bell. There was an almost 30 years time gap between the EPR argument (1935) and Bell's paper (1964). By the time the latter appeared, the physics community was convinced that Bohr had answered adequately EPR and in any case, the idea that there could be conceptual problems with quantum mechanics was almost totally absent from people's minds.

In order to explain the misunderstandings of Bell's results, let us first summarize what we said in Sect. 7.5: if one takes Bell's result discussed in Sect. 7.4 alone, and if one forgets about the EPR argument, that result can be stated as a "no hidden variables theorem": Bell showed that the mere supposition that the measured values of the spin pre-exist to their "measurement", and are perfectly (anti)-correlated, combined with the 1/4 frequency for the (anti)-correlation of measurements along different axes, leads to a contradiction. Since the perfect (anti)-correlation and the 1/4 frequency are quantum mechanical predictions that have been amply verified experimentally, this means that these hidden variables or pre-existing values cannot exist.

But Bell almost always presented his result *in combination with* the EPR argument, which shows that the mere assumption of locality, combined with the perfect correlation when the directions of measurement (or questions) are the same, implies the existence of the supposedly "impossible" hidden variables. So for Bell, his result, combined with the EPR result, was not a "no hidden variables theorem", but a nonlocality theorem, the result on the impossibility of hidden variables being only one step in a two-step argument.

Viewing Bell's argument as a refutation of hidden variables theories (an almost universal reaction, as we will see) is doubly mistaken: first because, combined with EPR, Bell proves nonlocality not merely a "no hidden variables theorem". Moreover Bell explained and defended all his life the de Broglie–Bohm theory, which is a "hidden variables" theory, but a consistent one.

How could Bell both refute hidden variables theories and defend them? Although nobody seems to think that Bell was crazy, the misunderstanders of Bell have never asked themselves that elementary question. As an aside, I may add that, when I read Bell's book [14], I was absolutely fascinated by the fact that, throughout the book, he explains and defends the de Broglie–Bohm theory, while most physicists seem to think that he had refuted that theory.

One problem with Bell is that, although, unlike Bohr, his writings are very clear and to the point, Bell tends to be concise. So, if you miss the gist of what he says somewhere, you may miss it for good.

Before giving examples of misunderstandings, let us show that Bell was perfectly clear about what his theorem implied:

> Let me summarize once again the logic that leads to the impasse. The EPRB correlations[7] are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting.[8] But this has implications for non-parallel settings which conflict with those of quantum mechanics. So we *cannot* dismiss intervention on one side as a causal influence on the other.
>
> John Bell [14, pp. 149–150] (italics in the original)

He was also conscious of the misunderstandings of his results:

> It is important to note that to the limited degree to which *determinism*[9] plays a role in the EPR argument, it is not assumed but *inferred*. What is held sacred is the principle of "local causality" — or "no action at a distance". [...]
>
> It is remarkably difficult to get this point across, that determinism is not a *presupposition* of the analysis.
>
> John Bell [14, p. 143] (italics in the original)

And he added, unfortunately only in a footnote:

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[7] Here EPRB means EPR and Bohm, who reformulated the EPR argument in term of spins [23]. (Note by J.B.).

[8] "Local magnet setting" means, in our language, the direction in which the spin is measured. (Note by J.B.).

[9] Here, "determinism" refers to the idea of pre-existing values. (Note by J.B.).

> My own first paper on this subject (*Physics* **1**, 195 (1965))[10] starts with a summary of the EPR argument *from locality to* deterministic hidden variables. But the commentators have almost universally reported that it begins with deterministic hidden variables.
>
> John Bell [14, p. 157, footnote 10] (italics in the original)

Yet, the great American physicist Murray Gell-Mann (Physics Nobel Prize in 1969 for his work on the theory of elementary particles), wrote: "Some theoretical work of John Bell revealed that the EPRB experimental setup could be used to distinguish quantum mechanics from hypothetical hidden variable theories". He added that quantum mechanics was vindicated by the outcome of various experiments [86, p. 172].

This is of course, completely forgetting the EPR part of the argument: the only hidden variables that Bell considered were precisely those that were needed, because of the EPR argument, in order to "save" locality. So if there is a contradiction between the existence of those hidden variables and experiments, it is not just quantum mechanics that is vindicated, but locality that is refuted.

Eugene Wigner also saw Bell's result solely as a no hidden variablesresult and praised Bell for giving "the most convincing argument against the theory of hidden variables" [207, p. 291].[11]

To give yet another example of misunderstanding of Bell, coming from a well-known defender of the Copenhagen interpretation, the German-born British physicist Rudolf Peierls, who made major contributions to nuclear physics, attributed to Bell the merit of having shown that: "there is no sensible view of hidden variables which doesn't conflict with these experimental results", where by "these results" he meant those verifying Bell's inequalities [47, p. 77].

Basically the same mistake was made in 1999 by one of the most famous physicists of our time, Stephen Hawking:

> Einstein's view was what would now be called a hidden variables theory. […] [Hidden variables theories] form the basis of the mental picture of the universe, held by many scientists, and almost all philosophers of science. But these hidden variable theories are wrong. The British physicist, John Bell, who died recently, devised an experimental test that would distinguish hidden variable theories. When the experiment was carried out carefully, the results were inconsistent with hidden variables.
>
> Stephen Hawking [99]

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[10] Reference [9], reprinted as Chap. 2 in [14]. (Note by J.B.).

[11] See Goldstein [91] for a further discussion of Wigner's views.

Note that neither Gell-Mann nor Wigner, Peierls or Hawking mention the word "nonlocality", which was central to all of Bell's writings.

We could continue and fill several pages of such quotes,[12] but we will rather turn to an amusing summary of the situation by David Mermin:

> Contemporary physicists come in two varieties.
>
> Type 1 physicists are bothered by EPR and Bell's theorem.
>
> Type 2 (the majority) are not, but one has to distinguish two subvarieties.
>
> Type 2a physicists explain why they are not bothered. Their explanations tend either to miss the point entirely (like Born's to Einstein)[13] or to contain physical assertions that can be shown to be false.
>
> Type 2b are not bothered and refuse to explain why. Their position is unassailable. (There is a variant of type 2b who say that Bohr straightened out the whole business, but refuse to explain how.)
>
> David Mermin [125]

However, the same David Mermin also wrote: "To those for whom nonlocality is anathema, Bell's Theorem finally spells the death of the hidden-variables program." [127, p. 814]. But, Mermin should know that Bell's theorem, combined with EPR, shows that nonlocality, whether we consider it anathema or not, is an unavoidable feature of the world.

To conclude, let us mention some more positive reactions to Bell's result. The American physicist Henry Stapp, wrote that "Bell's theorem is the most profound discovery of science" [183, p. 271]. David Mermin also mentions an unnamed "distinguished Princeton physicist" who told him [125]: "Anybody who's not bothered by Bell's theorem has to have rocks in his head."

Finally, Feynman asked himself, in a 1982 paper, whether quantum mechanics can be "imitable by a local classical computer", namely by any physical system that works mechanically and whose operations are local, as they are in ordinary computers. Stated in our language, Feynman asked whether quantum mechanics is local.

He then derived an inequality similar to Bell's (no reference to Bell's work is made in his paper, which is probably due to the fact that Bell was almost universally ignored back then), from which he drew the following conclusion:

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[12] The interested reader can find some of them in [36, pp. 258–263].

[13] See Sect. 10.2. (Note by J.B.).

> I've entertained myself always by squeezing the difficulty of quantum mechanics into a smaller and smaller place, so as to get more and more worried about this particular item. It seems to be almost ridiculous that you can squeeze it to a numerical question that one thing is bigger than another.
>
> Richard Feynman [78, p. 485]

When Feynman writes that "one thing is bigger than another", he refers to inequalities like $\frac{3}{4} \leq 1$, which is obviously correct, but is the opposite of a Bell inequality (which we defined in Appendix. 7.B).

Finally, let us mention that the article by Feynman from which this quote is taken is very often cited nowadays (but not so much when it was published), because it is one of the founding paper of the field of "quantum computing" (see Sect. 7.6.3).

But since this article, being similar to the earlier one of Bell [9], shows that there are nonlocal effects in Nature (although Feynman did not emphasize this point), one may hope that the conclusion of the EPR-Bell argument will be better appreciated in the future.

10.4 The Non-reception of de Broglie's and Bohm's Ideas

10.4.1 The Tragic History of de Broglie

The French aristocrat Prince Louis de Broglie is one of the most singular, original, and, in a sense, tragic figures in the history of science.

In several notes to the French *Académie des Sciences* written in 1924–1927, and in his Ph.D. thesis, which pre-date all other major works on quantum physics, de Broglie not only associated waves with particles, but also introduced the idea that the motion of particles may be guided by waves. The French physicist Paul Langevin sent a copy of de Broglie's thesis to Einstein, who replied that de Broglie had "lifted a corner of the great veil" [64] and Einstein circulated the thesis in Germany.

De Broglie was invited to present a report at the famous 5th Solvay Conference in Brussels, in October 1927. There, he presented what we call the de Broglie–Bohm theory, but without explaining how measurements work in that theory (here, we simplify somewhat).

Einstein thought that [7, p. 441] "Mr de Broglie is right to search in this direction", i.e., to try to associate particle trajectories to the quantum waves.

Some other people supported de Broglie at that Conference, but the general reaction was negative. In particular, Pauli raised an objection that de Broglie could not answer. Actually, a complete answer to Pauli's objection was only given in 1952 by Bohm.

In any case, de Broglie left the Solvay Conference discouraged. Not only because of Pauli's criticisms, or because of the generally negative reactions, but also because he realized that his theory was, in our language, nonlocal and, since he could not believe that nonlocality was real (note that this was 37 years before Bell's 1964 paper), he did not take his own theory seriously.

He actually searched for an alternative theory, but since he could not overcome the mathematical difficulties associated with that alternative, he gave up his search and followed the Copenhagen orthodoxy for the next twenty five years. That is, until he received a manuscript from a "young American physicist" [48, p. 67], David Bohm. This paper rekindled his interest in foundational problems in quantum mechanics, but de Broglie spent most of his time afterwards trying to revive his unsuccessful alternative solution.

One might wonder why people like Schrödinger and Einstein, who did not agree with the orthodoxy, either in 1927 or later on, did not pursue de Broglie's ideas. This is probably due once again to the nonlocality inherent in the de Broglie approach. And this is something that nobody before Bohm was willing to accept; as we saw, Einstein and Schrödinger, disliked nonlocality so much that they called it "telepathy" or "magic".

De Broglie is a tragic figure in the history of physics. He was undoubtedly a precursor and he did influence his successors in an indirect way. A case in point is Schrödinger, who wrote [172, p. 46]: "My method was inspired by L. de Broglie and by brief, yet infinitely far-seeing remarks of Einstein." However, his 1927 theory is to a large extent forgotten, and he has been rather neglected, even nowadays, in his own country, despite his 1929 Physics Nobel Prize and his having been the "perpetual secretary" for the mathematical and physical sciences section of the French *Académie des Sciences* from 1942 until 1975.

It should be emphasized that de Broglie's theory had nothing to do with "solving the measurement problem", unlike almost all the subsequent discussions about quantum mechanics. It was meant to be a physical theory, albeit a provisional one. Of course, de Broglie did not really believe in it. But who can blame him? Even the standard theory of quantum measurements explained in Chap. 5 was not fully developed then and nobody could accept the most revolutionary feature of the de Broglie theory, namely its nonlocality.

The tragedy of de Broglie is that he did "lift a corner of the great veil" and saw far beyond anybody else at the time; so far, indeed, that he could not see clearly himself.

10.4.2 David Bohm: Dissident and Outcast

Louis de Broglie and David Bohm were about as different as two physicists can be. The first was a French aristocrat, a Catholic and a conservative. Although his views on quantum mechanics were ignored, he was at the height of institutional recognition. The second was Jewish and an American leftist, indicted at the time of McCarthyism, who was expelled from Princeton University and had to go in exile to make a living, even losing his U.S. citizenship.[14]

Indeed, in Bohm's case, the little history of science got entangled with the big history of political events, in particular those of the Cold War. Bohm had been a member of the American Communist Party for a few months in 1942–1943, at a time when the United States and the Soviet Union were allied in World War II. He left the party because he found the meetings boring [146, p. 58], but kept a lifelong interest in "Marxist" philosophy, and later in Hegelianism, and in particular the notion of dialectics (we will come back to that in Sect. 11.6.1). But Bohm had also been working for his Ph.D. at the Radiation Laboratory of the University of Berkeley under the supervision of Robert Oppenheimer, the head of the Manhattan Project (the project that built the atomic bomb), and that, combined with his communist sympathies, led later to Bohm's troubles.

In 1946, after receiving his Ph.D., he was hired by Princeton University, where he taught, in particular, quantum mechanics, and did research on plasma physics. In May 1949, during the McCarthyite witch hunt, he was called by the House Committee on Un-American Activities to testify against some of his former colleagues who were suspected of pro-Soviet espionage. He refused to testify, encouraged to do so by Einstein, who nevertheless warned him that he might be jailed, and, to justify himself, Bohm invoked the Fifth Amendment of the United States Constitution, against self-incrimination.[15] Because of that, he was indicted in 1950 for "contempt of Congress", a charge of which he was eventually cleared in May 1951. However, Princeton University, upon hearing of the indictment, suspended Bohm from all his teaching duties and forbade him to set foot on campus (although his salary was still paid).

Even though Bohm was cleared of all charges, the university refused to renew his contract; the President of Princeton University even claimed that this was due to scientific and not political reasons, a transparent lie, and, as a result, Bohm was unable to find employment anywhere else in the United States. He left for Brazil, where the University of São Paulo offered him a job. He was

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[14] For a biography of Bohm, see the work of David Peat [146], that we follow here.

[15] That amendment says, among other things, that: "No person shall be compelled in any criminal case to be a witness against himself."

then deprived of his United States passport, which he could get back only if he returned to the United States, and had to obtain Brazilian citizenship in order to travel (and to obtain that citizenship, Bohm had to renounce his American one, which he was only able to retrieve decades later, in 1986). This allowed him to move to Technion in Israel, in 1955. In 1957, he went to England, where he was eventually made Professor of Theoretical Physics at the University of London's Birkbeck College. He was elected Fellow of the Royal Society only in 1990, at the age of 72, and died two years later.

In 1951, Bohm published *Quantum Theory*, a book based on his courses at Princeton, which is often considered as one of the best "orthodox" textbooks on quantum mechanics [23], and where he emphasized the need to understand quantum mechanics, beyond the mathematical formalism. In this book, Bohm reformulates the EPR argument in terms of spin variables, the formulation used in Sect. 7.4.2, but he sides with Bohr in using this argument against the idea of "hidden variables". However, according to Murray Gell-Mann, he discussed his book with Einstein and came back saying [86, p. 170]: "He [Einstein] talked me out of it. I'm back where I was before I wrote the book."

In 1952, David Bohm published two papers in *Physical Review* that give in complete detail what we call the de Broglie–Bohm theory. He was not aware of the previous work by de Broglie, who, however, quickly reacted (after receiving a preprint of Bohm's work and before its publication) in order to establish his priority.

Bohm did remark that he had been influenced by two Soviet physicists, Blokhintsev and Terletskii, who had started a criticism of the interpretation of Bohr and Heisenberg, and viewed quantum mechanics as a statistical theory, but without proposing any theory for the behavior of individual systems, as well as by "the stimulus of discussion with Dr. Einstein" [26, p. 110].

The main novelty with respect to de Broglie is that Bohm gave a complete analysis of the measurement process within the de Broglie–Bohm theory (see Sect. 8.3), which allowed him to answer Pauli's objections to de Broglie at the 1927 Solvay Conference.

The first person to express a negative reaction to Bohm's papers, months before their publication, was, of all people, de Broglie. He not only established his priority, but also criticized Bohm's theory. Bohm's reaction was that de Broglie had not "really read my article, but simply reiterated Pauli's criticisms, which led him to abandon the theory, but did not point out my conclusion that these objections are not valid" [25].

Another somewhat surprising negative reaction to Bohm came from Einstein, who wrote to Born:

> Have you noticed that Bohm believes (as de Broglie did, by the way 25 years ago) that he is able to interpret the quantum theory in deterministic terms? That way seems too cheap to me.
>
> Albert Einstein, letter to Max Born, 12 May 1952 [35, p. 192]

Later, Einstein even raised technical objections to Bohm, to which the latter replied, but that are too mathematical to be discussed here (see e.g. [36, Sect. 5.D.2] or [105, Sect. 6.5]).

The deep reason for Einstein's hostility toward de Broglie and Bohm is not known, but can perhaps be understood if one remembers that, for Einstein, the main problem was nonlocality and that, of course, the de Broglie–Bohm theory did nothing to eliminate nonlocality.

In the mainstream physics community, pro-Copenhagen so to speak, there were two kind of reactions to Bohm: emotional ones that were often quite spectacular, and scientific or philosophical ones.

Concerning the emotional reactions, David Peat, who was both a friend and a biographer of Bohm, interviewed Max Dresden, a physicist who had read Bohm's papers and who visited Oppenheimer's group at the Princeton Institute for Advanced Study, where he gave a seminar explaining Bohm's work. Here are the reactions of certain people at the Institute for Advanced Study in Princeton, according to Max Dresden[16]:

> "We consider it juvenile deviationism," Oppenheimer replied. No, no one had actually read the paper — "we don't waste our time." […]
>
> Reactions to the theory were based less on scientific grounds than on accusations that Bohm was a fellow traveller, a Trotskyite, and a traitor.[17] It was suggested that Dresden himself was stupid to take Bohm's ideas seriously. […] the overall reaction was that the scientific community should "pay no attention to Bohm's work". As Dresden recalled, Abraham Pais also used the term "juvenile deviationism". Another physicist said Bohm was a "public nuisance". Oppenheimer went so far as to suggest that "if we cannot disprove Bohm, then we must agree to ignore him."
>
> David Peat [146, p. 133]

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[16] This is of course based only on Dresden's recollections. But we will quote in the next chapter similar recollections of John Clauser. According to the biographer of Bohm, David Peat, Dresden made those remarks also on the floor at the American Physical Society meeting in Washington in May 1989; see [146, p. 340, note 51].

[17] "Fellow traveller" referred to people close to the Communist Party (Note by J.B.).

Hints of the Princeton reaction reached Bohm in exile in Brazil. He wrote to his friend Miriam Yevick [146, pp. 133–134]: "As for Pais and the rest of the 'Princetitute',[18] what those little farts think is of no consequence to me. In the past 6 years, almost no work at all has come out of that place. [. . .] I am convinced that I am on the right track."

Not all was quiet on the physics front!

Let us now consider the more serious reactions to Bohm by some of the most famous people associated with the Copenhagen interpretation, Pauli, Heisenberg, and Rosenfeld.

As we have seen, the de Broglie–Bohm theory does not make any new predictions compared to ordinary quantum mechanics. It is simply a theory about the world as opposed to a recipe allowing us to predict results of measurements. Said otherwise, it is a rational completion of ordinary quantum mechanics.

But given the general tendency of the "Copenhagen" people to consider as "metaphysical" all questions related to anything other than measurements, it was to be expected that their rejection of Bohm's ideas would be based on this anti-metaphysical attitude.

In 1951, Pauli wrote to Bohm, agreeing that he did not see in Bohm's paper ([142], quoted in [146, p. 127]): "any longer the possibility of any logical contradiction", but that, since no new predictions were made, his theory was still "a check, which cannot be cashed."

But he also wrote, in 1951, to his friend Markus Fierz ([143], quoted in [146, p. 128]): "Bohm keeps writing me letters 'such as might have come from a sectarian cleric trying to convert me particularly to de Broglie's old theory of the pilot wave.' In the last analysis Bohm's whole approach is 'foolish simplicity', which 'is of course beyond all help'."

Heisenberg wrote a rather detailed defense of the orthodoxy in *Physics and Philosophy* [100], where he discusses Bohm's ideas. Apart from objections too technical to be discussed here, Heisenberg fell back on the familiar idea that Bohm's theory is just like the Copenhagen interpretation, but expressed in a different language, which "from a strictly positivistic standpoint" amounts to an "exact repetition" and not a counterproposal to the Copenhagen interpretation [100, p. 115].

This does not square with the fact that Heisenberg also argued that quantum mechanics had put an end to the "ontology of materialism", namely "to the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them" [100, p. 115]. One cannot have it both ways: saying that Bohm's theory

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[18] Here Bohm refers of course to the Princeton Institute for Advanced Study. (Note by J.B.).

reintroduces an “objective ontology” and, at the same time, saying that it is just the same theory (expressed in a different language) as one that denies the possibility of such an ontology.

The most vehement critique of Bohm was probably the close friend of Bohr, Léon Rosenfeld who considered that Bohm’s ideas were [162, p. 495]: “a short-lived decay product of the mechanistic philosophy of the nineteenth century” and suggested that the introduction of “hidden variables” would be “empty talk”.

He wrote to Bohm:

> I shall certainly not enter into any controversy with you or anybody else on the subject of complementarity, for the simple reason that there is not the slightest controversial point about it. [...] I am inclined to return that it is just among your Parisian admirers that I notice some disquieting signs of primitive mentality.[19]
>
> Léon Rosenfeld, letter to Bohm, 20 May 1952, quoted in [146, p. 130]

While Bohm was marginalized, partly for political reasons,[20] but also because of his heretical views on quantum mechanics, Rosenfeld served, according to the historian of science A.S. Jacobsen [107, p. 23]: “as consultant or referee in matters of epistemology of physics and the like at several well-reputed publishing houses and at the influential journal *Nature*. In this capacity he used his influence effectively, and several books and papers, among them some by Frenkel, Bohm, and de Broglie, were rejected on this account.”

Finally, one of the biggest ironies of the history of quantum mechanics is that Bell’s result is regularly invoked against the de Broglie–Bohm theory, since Bell is “understood” as having refuted hidden variables theories or, at least, to have shown that any hidden variables theory must be nonlocal, hence physically unacceptable. We have already explained several times, in particular in Sect. 10.3, why this is both a huge logical mistake (forgetting EPR) and a misreading of Bell himself who always defended the de Broglie–Bohm theory. Bell even says that it was the obvious nonlocality of that theory that led him to his inequality: he realized that the de Broglie–Bohm theory eliminated the central role of the observer, but wondered whether one could “do better” so to speak, namely eliminate that role in a local theory, and he then showed that it was impossible.

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[19] He was probably referring to people around de Broglie, such as Jean-Pierre Vigier, who were working on the de Broglie–Bohm theory (Note by J.B.).

[20] See [135] for more details.

10.5 Summary and Conclusions

At the end of Chap. 8, we faced a paradox: if the de Broglie–Bohm theory has all the qualities that we attribute to it, why is it so little appreciated among physicists? Part of our answer is based on the view of the history of foundations of quantum mechanics that we presented here. To say that this view is not generally accepted among physicists is quite an understatement, otherwise they would probably also view the de Broglie–Bohm theory differently.

So, the reader will have to decide for herself or himself whether we argued our case successfully. We think that, in the debate between Bohr and Einstein, which was the most important one between the pro and the anti Copenhagen people, Bohr did not adequately answer Einstein's ideas because he never really grasped the latter's objections. Einstein used the notion of locality to argue that quantum mechanics is incomplete and Bohr replied that no measurements could be made that would be more precise than what is allowed by the quantum formalism. This limitation on our capabilities to make predictions remains true in the more complete de Broglie–Bohm theory, because of the randomness in the initial conditions, as we explained in Sect. 8.4.2.

That is why Bohr did not really refute the EPR paper, unless one interprets his answer as conceding the existence of actions at a distance. But that, if it had been said clearly, would have been quite a shock to the physics community.

The dominant reaction was that Bohr had replied adequately to EPR and their argument was forgotten. So that, when Bell showed in 1964 that the assumption of "hidden variables" that EPR made in order to save locality could not be maintained, few people paid any attention to what he really said. In fact, Bell's result was almost universally taken to mean that "hidden variables" theories were impossible. This was all the more bizarre that Bell himself always defended the 1952 Bohm hidden variables theory.

So, the ignorance and misunderstanding of EPR led to the ignorance and misunderstanding of Bell.

Concerning the de Broglie–Bohm theory, the first version, the one presented by de Broglie at the 1927 Solvay Conference, mostly fell on deaf ears. Moreover, his proponent abandoned his own theory shortly after that Conference, for various reasons, but among which its nonlocal character was an important one.

Then came Bohm in 1952. He gave the full-fledged version of the de Broglie–Bohm theory, including the analysis of the measurement process and answered all the previous objections.

But the times of this discovery could hardly have been worse. On the personal side, Bohm was victim of the McCarthyite witch hunts, was expelled from

Princeton University and had to find employment in Brazil. On the scientific and philosophical side, almost everybody was won over to the Copenhagen orthodoxy and did not want to have anything to do with conceptual questions that did not yield new experimental predictions.

Moreover, the theory was nonlocal, and was formulated 12 years before Bell showed that nonlocality is an unavoidable aspect of reality. That nonlocality actually did not bother Bohm (who always liked "holism", meaning the interconnectedness of everything in the Universe), but was unthinkable for almost everyone else, including Einstein and de Broglie, who might otherwise have been more sympathetic to Bohm.

The nonlocality inherent in Nature, not just in the de Broglie–Bohm theory, is slowly but increasingly being appreciated and accepted. Note however that the issue was raised by EPR in 1935. Maybe the 1952 papers by Bohm will eventually be appreciated with a similar time delay.

11

The Cultural Impact of Quantum Mechanics

11.1 Introduction

The mysterious character of quantum mechanics has led to numerous abuses, misinterpretations, speculations and extrapolations, perhaps more than any other scientific theory. It would take an encyclopedia to cover all of them, so we limit ourselves to an overview of some of the interactions between quantum mechanics and our non-scientific culture.

We have seen that the two "mysteries" of quantum mechanics concern the role of the observer and actions at a distance. A third alleged "novelty" supposedly introduced by quantum mechanics is the death of determinism. Almost all the abuses or invalid extrapolations of quantum mechanics rely on one or more of those ideas.

It must be recognized that a number of famous physicists share responsibility for such abuses or extrapolations. That should be kept in mind before putting all the blame on philosophy, the social sciences or literary criticism for fostering confusion.[1]

Links have been claimed between quantum mechanics and various pseudosciences, Western and Eastern religions, and certain social sciences. Moreover, interactions of various sorts have been proposed between quantum mechanics and philosophy and even politics. We shall discuss each of these topics.

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[1] In our jointly authored book, *Fashionable Nonsense* [180], Alan Sokal and I explicitly avoided criticizing non-scientists for the nonsense they might have said about quantum mechanics (otherwise the book would have been much longer), precisely because we were aware of the confusion spread by physicists on that topic.

J. Bricmont, *Quantum Sense and Nonsense*, DOI 10.1007/978-3-319-65271-9_11

11.2 Quantum Mechanics and Pseudo-science

The general issue of pseudo-science is far too vast to be discussed here, and we shall limit ourselves to the ways in which quantum mechanics is sometimes used to support various pseudo-scientific claims.

Pseudo-scientific references to quantum mechanics usually come down to attempts to confirm miraculous discoveries, unprovable assertions, implausible cause-effect relationships, and other counterintuitive mysteries by pretending to link them to the mysteries of quantum mechanics. Inasmuch as the "discoveries" of pseudo-science greatly resemble the "miracles" of the past, they should be regarded with the same skepticism.

In the chapter on miracles of his "Enquiry Concerning Human Understanding" [106] the 18th century Scottish philosopher David Hume pinpointed the crucial idea: if you yourself witness a miracle, or some extraordinary phenomenon, you have to decide whether or not you are victim of a delusion. However, most people who believe in miracles do not directly witness extraordinary phenomena, but hear reports by others who claim to have witnessed such phenomena.

But why should one believe those reports? Everybody has experiences of people who are deceived or deceive others.

Therefore it is always more rational, when one hears reports of extraordinary phenomena to believe that someone has been deceived or has deceived others, something everyone has experienced directly, than to believe in the reality of the extraordinary claims of which we have no direct experience.

Hume used the example of the resurrection of the dead to illustrate this idea:

> When anyone tells me that he saw a dead man restored to life, I immediately ask myself whether it is more probable that this person either deceives or has been deceived or that what he reports really has happened. I weigh one miracle against the other, and according to the superiority which I discover I pronounce my decision and always reject the greater miracle. If the falsehood of his testimony would be more miraculous than the event that he relates, then he can claim to command my belief or opinion, but not otherwise.
>
> David Hume [106, p.59]

This argument has a far more general application: one should ask the same questions to journalists, politicians, bankers, scientists, psycho-analysts, and priests as well as pseudo-scientists: what reason do you give me to believe what you say rather than believe that you (or other people on which you rely) deceived themselves or deceive others?

If one limits that question to scientists and pseudo-scientists, the difference between the two is that scientists can use two sorts of arguments to reply to that question: first, there is the extraordinary coincidence between theoretical predictions and experiments; but if a layperson does not appreciate that argument, due to lack of access or of knowledge of what is actually done in scientific laboratories, there are all the modern technologies (electricity, planes, cars, telecommunications etc.), that would have looked like miracles in the 18th century, but, unlike the miracles that Hume doubts, are real and visible to everyone.

The main difference between science and pseudo-science, and which essentially defines the distinction between the two, is that the latter is not able to provide such answers to our question (why believe what you say rather than believe that some people deceive themselves or deceive others?)

One way for pseudo-scientists to weaken this argument is to argue that what counts as a miracle depends on our background knowledge or assumptions. Hume gives the example of an Indian prince who, living in a warm climate, could not believe that water can freeze and he considers that the prince's opinion was rational (even though it was wrong of course). If the prince had travelled to a Northern country, or had known what we know now about phase transitions he would have of course considered the freezing of water as real.

Similarly, if people had been told in the 18th century that planes can fly, they would have been rational in not believing that. Yet, planes do fly!

So, Hume's skepticism concerns the rules of rationality: how should we proceed to find what is true? But no such rule can guarantee that we will not make mistakes, because our access to truth depends on our information and the latter may always be insufficient, as the example of the Indian prince illustrates.

This example also illustrates the fact that what we consider as being miraculous depends on our previous experiences and that new experiences might modify our judgment.

And this is where quantum mechanics enters: people who believe in telepathy or in a direct intervention of consciousness on matter will claim that these effects are not so miraculous after all, since quantum mechanics has "proven" that the consciousness of the observer plays a role in reducing the wave function, or because nonlocality has "shown" that what we think as being the usual spacial separation between distant events (like those involved in telepathy) is an illusion.

But to refute that claim is easy and does not depend on one's understanding of quantum mechanics: even those who, like Wheeler or Wigner, consider that quantum mechanics gives a certain role to consciousness in physics do not think that our *conscious choices* affect Nature in any way. In other words, there is nothing in our thinking or our wishes that will make the pointer at the end of the experiment go up rather than down, or the cat end up being alive rather than dead, or vice versa.[2]

It is the same thing with nonlocality: as we emphasized in Chap. 7, quantum nonlocality, although it is a real form of action at a distance, does not allow anyone to send messages faster than the speed of light. So that refutes all claims that quantum nonlocality might make telepathy more plausible that it was before the advent of quantum mechanics.

Finally, there is no substitute for evidence: even if quantum mechanics made paranormal claims more plausible (which it does not), it would not make those claims true. One would still need reproducible experimental evidence for those claims and that is exactly what is lacking.

Actually John Wheeler who, as we saw in Chap. 1, was quite interested in the role of consciousness in quantum mechanics, was indignant about the suggestion that this might have anything to do with parapsychology. After having been put together with Eugene Wigner on a panel with several parapsychologists at a meeting of the American Association for the Advancement of Science in 1979, he sent a letter of protest published in the *New York Review of Books*, where he wrote: "in the quantum theory of observation, my own present field of endeavor, I find honest work almost overwhelmed by the buzz of absolutely crazy ideas put forth with the aim of establishing a link between quantum mechanics and parapsychology as if there were any such thing as 'parapsychology'." [199]

On the other hand, a good example of ignoring Hume's advice, mixed up with quantum speculations is the story of Uri Geller: this Israeli magician claimed to be able to use supernatural powers in order to bend spoons at a distance. He was also supposedly endowed with the gift of remote viewing.

Two "researchers" at the private Stanford Research Institute (the SRI was initially part of the prestigious Stanford University, but is independent of it since 1970 and is now named SRI International), Harold E. Puthoff and Russell Targ, invited Uri Geller to their institute in order to test "scientifically" his extraordinary powers. The results were apparently positive and were even published in the prestigious scientific journal *Nature*, but with many reservations from the editors [156].

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[2] However, it should be mentioned that there is at least one famous physicist who mixes up homeopathy, parapsychology, mysticism and quantum mechanics: the Cambridge physics professor Brian Josephson who received the 1973 Nobel Prize in Physics, but for work unrelated to pseudo-science (see http://www.tcm.phy.cam.ac.uk/~bdj10/ for his work). He is usually regarded as a "crank" by other physicists.

This publication provoked quite a lot of interest among critics of parapsychology, in particular James Randi, the well-known magician who spends part of his time debunking pseudo-scientific claims. Randi showed how Uri Geller spoon bending was nothing more than conjurer's tricks and he pinpointed several flaws in the SRI's experiments (Randi refers to Puthoff and Targ as the Laurel and Hardy of "psi" research [158, p. 137]). Since the remote viewing of Uri Geller was never reproduced in controlled experiments, this story is now considered to be a classical example of the gullibility of alleged scientists, see [45, 158, 159].

What was special about Puthoff and Targ is that their "research" claimed to be linked to quantum nonlocality and the role of consciousness in physics, which probably made them more credulous vis-à-vis Geller's claims.

As an aside, let us mention that Uri Geller managed to persuade both the NASA and the CIA to fund his research![3]

The Geller episode is part of a larger story, recounted by David Kaiser in *How the Hippies Saved Physics* [110]. The title is deliberate hyperbole, but the story is quite interesting. During the 1970s, as an offspring of the Bay Area's counterculture, a group of Berkeley physicists founded the "Fundamental Fysiks Group" (sic). They had an intense interest in a strange mixture of quantum mechanics, particularly Bell's theorem and nonlocality, parapsychology, telepathy and mysticism.

Although the positive role attributed by Kaiser to that group may be debated, it is true that they helped keep alive foundational questions about quantum mechanics that were almost taboo in the physics community during the 1970s (see Sect. 11.6.2 below).

Parallel to quantum-based claims of parapsychologists, Deepak Chopra [42] introduced a new field of "alternative medicine" which he called "quantum healing". But here, as Chopra himself admits, the word "quantum" is used as a metaphor[4] and quantum healing is simply a new instance of long-standing claims that the state of the mind directly influences the well-being of the body or, as Chopra puts it, "changes the biology".

The connection between the state of mind and the body is a complicated subject that has to be investigated by the usual methods of science, but quantum mechanics plays no role here since, as we already emphasized, even according to textbook quantum mechanics (with its ambiguity about the role of the observer), our conscious choices have no effect on the outcomes of experiments.

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[3] See [3] and https://www.cia.gov/library/readingroom/document/cia-rdp96-00791r000100480003-3.

[4] But he also says that all of science is a metaphor, so it is not clear what this admission really means.

In 1998 Chopra was awarded the Ig Nobel Prize (a parody of the Nobel Prize also called Ignobel) in the physics category for "his unique interpretation of quantum physics as it applies to life, liberty, and the pursuit of economic happiness".

To summarize, given the absence of direct evidence and of plausibility for the claims of quantum "miracles" within quantum theory, it is fair to say that invoking quantum mechanics to support "unconventional" science is a further proof of the charlatanism of those who invoke it!

11.3 Quantum Mechanics and Eastern Mysticism

Frijtof Capra wrote one of the most influential books on the alleged implications of quantum mechanics, *The Tao of Physics* [39]; afterwards, Gary Zukav wrote another book, in a similar spirit, *The Dancing Wu-Li Masters* [211]. We will discuss here the first author. The main claim of Capra is that the "New Physics", mostly quantum mechanics, but also the theory of relativity brings our "Western" science closer to the thought of Eastern philosophies or religions, Hinduism, Buddhism, Chinese Thought(Confucius), Taoism, Zen.[5]

When one encounters claims about modern physics having been foreshadowed by ancient religious texts, whether it be the Bible, the Koran or those of the Eastern traditions, the first question to ask, is: where are the equations and where is or was the technology? Of course, they are nowhere to be found in those texts. But this obvious remark is of some importance, because physical theories such as quantum mechanics do of course include equations and have many applications.

So that the relation between modern physics and ancient texts is at best at the level of an analogy or a metaphor. But even those analogies can be questioned in the case of Eastern mysticism and quantum mechanics.

Capra summarizes the Eastern spiritual traditions by characterizing them as based on a "mystical experience – on a direct non-intellectual experience of reality" (p. 45). He adds:

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[5] For a (small) selection of other books written in a similar spirit, see [94, 160, 167, 185, 195, 208]. The Indian "eco-feminist" and anti-GMO activist Vandana Shiva manages to mix up global warming, quantum non-locality and indeterminism, and the butterfly effect (which is purely classical) with traditional ways of thinking in her video: https://www.youtube.com/watch?v=4cdFXKDAaQw (after min. 11).

> [...] the basic elements of the world view which has been developed in all these traditions are the same. These elements also seem to be the fundamental features of the world view emerging from modern physics. The most important characteristic of the Eastern world view – one could almost say the essence of it – is the awareness of the unity and mutual interrelation of all things and events, the experience of all phenomena in the world as manifestations of a basic oneness. All things are seen as interdependent and inseparable parts of this cosmic whole; as different manifestations of the same ultimate reality. The Eastern traditions constantly refer to this ultimate, indivisible reality which manifests itself in all things, and of which all things are parts.
>
> Frijtof Capra [39, p 45]

The links that Capra finds between quantum mechanics and Eastern thought are based on the usual suspects: the role of consciousness and nonlocality. The latter is supposed to justify the notion that "all phenomena in the world are manifestations of a basic oneness". But that is true only in a very metaphorical sense. Systems that have interacted at some time remain linked to each other, as we saw in Chap. 7, provided they remain isolated from the rest of the Universe, which is difficult to achieve over long periods of time or over long distances. More to the point, this has nothing whatsoever to do with human sensibility, which is what Capra speaks about.

Indeed, there is no evidence that quantum mechanical nonlocality has any effects at the macroscopic level, except for some very special experiments, those that are designed to test Bell's inequalities for example. If particles interact at some time with each other, they may become correlated in some way and remain so in principle for an arbitrarily long time. But the key term here is "in principle". In practice, they quickly become correlated with many other particles as well and the original effect becomes wiped out so to speak, unless one isolates them from their environment.

There are many things that happen "in principle" in physics but that are negligible in practice. If I move my body, I change the distribution of mass in the universe and that affects the motion of all other bodies in the universe.[6] But nobody studying celestial mechanics is going to take such effects into account (otherwise, all celestial calculations would become impossible). One could also use Newton's theory of gravitation to claim that there is a "basic oneness" in Nature, and that we are all connected to each other by gravitational forces.

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[6] This is because the attraction between two bodies depend on their distance (it decreases like the square of the inverse of this distance) and moving my arm changes that distance. Moreover, in Newton's theory of gravitation, this effect is "in principle" instantaneous, see Appendix 7.A.

But, probably because gravitation is less mysterious than quantum mechanics, this is not the sort of example that is used by Capra and others, although it would be just as illegitimate as the ones that are used and for the same reason: trying to find practical and concrete implications of physical effects that exist, but only "in principle".

However, the worst aspect of Capra's approach is the opposition between an "Eastern" and a "Western" way of thinking, where the Eastern one is associated with mysticism and the predominant role of intuition, while the Western one is associated with "classical" rationality. After all, both intuition and rationality are human faculties, East and West, and both are present in mental activities from scientific investigation to theology. What is specific about modern physics is mathematical reasoning, method and experimentation. These are the same in East and West.

Indeed, if a "Western" scientist goes, say, to China to give a seminar, he can use almost the same notations as the ones he uses at home. Moreover, the background knowledge that he will assume from his audience is the same as in the West, and he does not have to possess the faintest idea about, for example, the Yin and the Yang in order to communicate with his Chinese colleagues.

When Chinese scientists use quantum mechanics to create super-secure cryptography or to develop quantum computers, as they now do, they use the same concepts and equations as Westerners do.

The development of exactly the same "Western" science in Asia and elsewhere gives the lie to the very notion of science being particularly Western. Sometimes this universality of science is considered to be a leftover from Western colonialism, but that is obviously not the case: however eagerly they may embrace our supposedly Western science, non Western peoples have kept their eating habits, most of their customs, their specific superstitions and are often politically opposed to the "West".

Moreover, even a cursory look at the stacks labelled "spirituality", "meditation" or "alternative medicine" in a major Western bookstore will convince anybody that there is a lot of mysticism and "intuitive thinking" on this side of the world (and not necessarily inspired by Eastern "ways of thinking"). The true opposition is not between "East" and "West", but between a rational, scientific worldview and one based on a "direct non-intellectual experience of reality".

11.4 Quantum Mechanics and God

> […] [philosophers who make logical analysis the main business of philosophy] confess frankly that the human intellect is unable to find conclusive answers to many questions of profound importance to mankind, but they refuse to believe that there is some "higher" way of knowing, by which we can discover truths hidden from science and the intellect.
>
> Bertrand Russell [165, p. 824]

Typing "quantum mechanics and God" in Google returns more than a million results. This may not be very significant, but it is still a large number for two topics that, as we shall try to argue, have nothing whatsoever in common.[7]

In fact, the topic "quantum mechanics and God" is a sub-topic of a larger one that has become extremely fashionable nowadays: Science and Religion.[8]

In order to discuss this issue, one must first distinguish between two radically different notions of God or of religion: the one that the immense majority of believers actually believe in and the one that a few philosophers and theologians like to discuss. We will call the first God the superstitious one and the second the

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[7] A poll published in August 2010, conducted with 16 members of the prestigious Collège de France in Paris showed that 85% thought that one could "reconcile" science and faith, although 75% of them did not believe in God.

[8] Here are a few examples of this "interaction" between science and religion:

There exists a large annual prize (worth $1,100,000), the Templeton prize, that "celebrates no particular faith tradition or notion of God, but rather the quest for progress in humanity's efforts to comprehend the many and diverse manifestations of the Divine." (http://www.templetonprize.org/purpose.html). This prize has been given to several scientists interested in "science and religion", in particular to the physicist Paul Davies (in 1995), who wrote the influential book *God and the New Physics* [46].

The physicist Frank Tipler wrote books on "The physics of immortality and "of Christianity" [187, 188].

One of the most famous scientists linking science and religion is the English theoretical physicist and Anglican priest John Charlton Polkinghorne. On the back cover of his book, *Quantum Physics and Theology: An Unexpected Kinship* [154], one reads:

> Among the many parallels he identifies are patterns of historical development in quantum physics and in Christology; wrestling with perplexities such as quantum interpretation and the problem of evil; and the drive for an overarching view in the Grand Unified Theories of physics and in Trinitarian theology. Both theology and science are propelled by a desire to understand the world through experienced reality, and Polkinghorne explains that their viewpoints are by no means mutually exclusive.

The Vatican Observatory and the Center for Theology and the Natural Sciences of Berkeley organized a Conference exploring "the creative interaction among quantum physics, philosophy and theology" [166].

On the Muslim side, one finds for example *Islam's Quantum Question: Reconciling Muslim Tradition and Modern Science* [97].

metaphysical God. We will start by explaining the meaning of the superstitious God, even though it is only the metaphysical one that has a connection to science and to quantum mechanics.

If a Westerner goes to a temple in India, he will see people offering flowers to stone statues. Or, at least, that is how it generally looks to Western eyes and most Westerners, whether they are Christians, Muslims, Jews or non-believers, dismiss that behavior as superstitious (or at least, that is what they used to say, before it became politically incorrect).

But what is so strange about that, compared to candles burnt in churches, flowers in cemeteries, or prayers in front of statues of saints (if God is everywhere, as theologians maintain, why bother to pray to Him at special places of worship?).

This shows that the God or Gods that people really believe in is a sort of person, although a very powerful one, who cares about human beings, answers our prayers, helps us when in need, and rewards or punishes us in this life or in the afterlife.

To believe in that sort of God(s) is very hard indeed for a scientifically minded person. First of all, is there one God or several? If there is one, which one is it? Some Gods are very upset if one eats pork or beef or drinks wine, others don't care. One can hardly speak of the "same God".

Difficulties of this sort multiply ad infinitum once one thinks about the various interpretations of the same "sacred texts" within the same religion, say Christianity with its division between Catholics, Orthodox and Protestants, themselves divided into myriads of churches. The same divisions exist in Islam (between Shias and Sunnis among others) or between liberal, conservative and orthodox Jews.

Despite centuries of debates, theologians of various faiths not only do not agree with each other about what God really says, they do not even agree on a method that would allow them to figure out what God really said.

This is in sharp contrast with scientists, but also with historians, detectives or technicians, who may disagree among themselves within each profession on many issues but tend to agree on what a valid argument can be or what evidence means. There is no indication that, say, Muslim and Christian theologians have agreed on a method to decide whether the Koran or the Bible is the "true" message of God.

Another problem concerns miracles (putting even aside Hume's argument against such beliefs): if one believes that one God makes miracles, what about the miracles made by the other Gods? Or those that were made by Gods that nobody believes in anymore (the Greek or Roman Gods for example)?

Moreover, whenever there has been an attempt to test empirically the efficacy of prayers, the results have generally been negative (see e.g. [120]).

Even assuming that miracles occur, why aren't they more obvious? After seeing the objects cast off by visitors to Lourdes, the French writer Anatole France supposedly remarked: "All those canes, braces and crutches, and not a single glass eye, wooden leg, or toupee!" Why is God impotent before missing eyes and limbs?[9]

Finally, most religious people in the West believe in a Revelation, but which one? And how does one explain that these revelations took place at a certain time and place in history, although they were supposed to benefit all of mankind? As the 18th century French free-thinker Baron d'Holbach put it ironically:

> Should not the fears and uncertainties of one who honestly examines the revelation adopted by Christians redouble when he sees that his god claimed to make himself known to only a few favored beings, while remaining hidden to all other mortals, for whom that revelation was equally necessary? […]
>
> What can this person think of a god who punishes millions of human beings for ignoring secret laws that he stealthily divulged in some obscure and remote corner of Asia?
>
> Philippe Henri d'Holbach, [104]

So, there is a straightforward opposition between science and religion if religion is understood as it is by most believers, as a sort of superstition (divine intervention and answers to prayers) or a set of arbitrary and untestable beliefs (in a given Revelation and its "correct" interpretation).

It is important to notice that this opposition between science and religion does not stem from the content of scientific theories (such as the theory of

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[9] Quoted in https://newrepublic.com/article/63388/seeing-and-believing. Actually, the reaction of Anatole France was closer to Hume's argument against belief in miracles:

> Happening to be in Lourdes one month of August, I visited the grotto where countless crutches were hung as evidence of miraculous healing. My companion pointed to these trophies from some infirmary and whispered in my ear:
>
> "A single wooden leg would be much more convincing."
>
> That makes sense. But philosophically speaking, the wooden leg would be worth no more than a crutch. If a truly scientific spirit were called upon to observe that a man's amputated leg was suddenly reconstituted in a pool or elsewhere, he would not exclaim: "It's a miracle!" Rather, he would say: "A single observation suggests that in as yet undetermined circumstances the tissues of a human leg have the property of reconstituting themselves like lobster claws, crayfish legs and certain lizard tails, but much faster."
>
> Anatole France, [82]

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evolution), but from the spirit of science, which refuses to take anything on trust and demands arguments and empirical support before accepting any proposition as true.

The growing influence of the scientific spirit explains why Christianity has increasingly evolved towards a belief in a less specific, less anthropomorphic, less interventionist God. This started with Protestants, but then, after the Council of Vatican II, has been spreading throughout most of the Catholic world.

And this brings us to the other notion of God, the impersonal God of some theologians and philosophers, the one that the "dialogue" between science and religion is supposed to be all about. This is what we call the metaphysical God, the God that "created the Universe", the one behind the Anthropic Principle[10] or the Intelligent Design supposedly guiding evolution. This is where quantum mechanics sometimes enters, not that it provides a direct argument in favor of the existence of God, but, since it supposedly renders reality "mysterious" or maybe even "subjective", it also supposedly makes room for some equally mysterious supernatural entity.

A famous physicist who leaned towards the idea of establishing a connection between quantum mechanics and an impersonal God is Wolfgang Pauli[11]:

> [...] science and religion *must* have something to do with each other. (I do *not* mean "religion within physics", nor do I mean "physics inside religion", since either one would certainly be "one-sided", but rather I mean the placing of both of them within a whole.) I would like to make an attempt to give a name to that which the new idea of reality brings to my mind: the idea of reality of the symbol. [...] It contains something of the old concept of God as well as the old concept of matter (an example from physics: the atom. The primary qualities of filling space have been lost. If it were not a symbol how could it be "both wave and particle"?). The symbol is symmetrical with respect to "this side' and 'beyond" [...]. The symbol is like a god that exerts an influence on man but which also demands from man that he have a back effect on him (the God symbol).
>
> Wolfgang Pauli [141, pp. 193–194] (italics in the original)

The impersonal God is the one whose existence has been "proven" by the argument of the first cause, by the argument from design, by the ontological

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[10] This is the notion that, if certain physical constants were slightly different from what they are, life would be impossible in our Universe. Hence, the argument goes, those constants must have been fine-tuned by a higher intelligence who was interested in our being able to exist.

[11] This comes from a private letter, but we will return to Pauli's ideas in Sect. 11.7.

argument, by St Anselm, St Thomas, Descartes and even the 20th century logician Kurt Gödel (see [137]) and many others.[12]

There are two problems with that God: first, it is neither the one real worshippers believe in nor is it a God that one has any reason to believe in.

The first problem is often ignored because of the use of the same word, "God". But imagine that the metaphysical God did exist. Why would He be in the least concerned about our well-being? Or answer our prayers? Or put us in Heaven or Hell in the afterlife? Why, if He built this gigantic universe where we seem to be just somewhat evolved creatures living on a planet near a star like billions of others, are we so important for Him? Why didn't He create us from scratch instead of letting a long evolution produce such far from perfect humans?

Nobody has ever given an argument showing a link between the metaphysical God and the ones that religious people really believe in. Unfortunately, few people, even among non-believers, notice the need for such an argument.

But what about the existence of this metaphysical God? There are two sorts of arguments that are traditionally given in its favor, a priori and a posteriori ones. The a priori arguments, like the first cause or the ontological argument, need not concern us, because they are not related to science, and also because, assuming that they do prove the existence of something, that something is as removed as one can be from the personal human-centered God that people believe in.

The a posteriori arguments consist in saying that we need God to explain the Big Bang, or what happened before that, or some aspects evolution, or the appearance of life, or the fine-tuning of physical constants that made life possible. God could also be the universal "observer" that randomly collapses the wave function. It is here that science and religion supposedly enter in "dialogue". But that God is always purely and simply the "God of the gaps", which means that, whenever there is a gap in science, one can always invoke that God as the answer that fills the gap.

The reader may ask: why not? Why exclude a priori God as a scientific hypothesis? Why couldn't God be the answer to some factual questions that science is unable to answer?

The reason is that there is a crucial difference between "God" and any scientific hypothesis, even the most apparently far-fetched ones like the idea of

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[12] The ontological argument consists in defining God as a Being endowed with all qualities. Then one argues that existence is a quality and thus that God exists. Others argue that God is the cause of everything or that life, or human life, or consciousness, cannot be explained naturally and thus require a supernatural designer.

many-worlds or of space-time having more than four dimensions: scientific hypotheses are always defined by specific properties that should imply testable consequences, even if they are not testable at present or, if no testable consequences can be presented now, one at least hopes to be able to do so in the future.

But the "God of the gaps" has no such properties. It is simply what is "needed" to fill the gaps. To turn that God into something that has definite properties, one has to conflate that God with the one of some revealed religion, but then, again, which one? Suppose someone says that the answer to some open problem in science is that the Olympic Gods (or maybe Zeus alone) did it! Would anybody take that seriously? But why should it be different for any other God?

And, if we have to go back to the property-less metaphysical God, we realize that this is not a concept but a word and we might as well say: "We don't know". That is an honest answer, but ignorance of course does not prove the existence of anything.

Some scientists take the God of the gaps too seriously and feel that science needs to provide answers to every question, or otherwise the door might be open to a religious alternative. But once one understands that the God of the gaps is nothing but a way of saying "we don't know" (and, moreover that, even if it was more than that, it would have nothing to do with the Gods that people believe in), there is no religious alternative to be worried about.[13]

Actually, it is quite likely that there will always be gaps in our scientific world-view (so that the God of the gaps will always be invoked by people who do not understand the emptiness of that concept). Indeed, what are we, if not some products of natural evolution whose abilities are always limited in various ways, including our mental abilities? If one remembers this, there is no reason to suppose that the set of questions that we are able to ask will coincide with

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[13] For example, long before Darwin, the French thinker Denis Diderot answered in the following way the issue of the chicken and the egg:

> If you are troubled by the question of whether the egg came before the hen or the hen before the egg, that's because you assume that animals were originally what they are now. What foolishness! We don't know any more about what they were than we do about what they'll become. The imperceptible earthworm which wriggles in the mud is perhaps in the process of developing into a large animal, and an enormous animal, which astonishes us with its size, is perhaps in the process of developing into an earthworm and is perhaps a particular momentary production of this planet
>
> Denis Diderot [59]

The point of this quote is not that Diderot somehow anticipated the theory of evolution (he didn't get it quite right) but that one can answer the argument from design without knowing the theory of evolution, by simply saying: "we don't know", but one can always think of other answers than "God".

the set of testable and empirically verified answers that we are able to find.

A final remark: with the rise of liberal Christianity, religions that used to enforce rigid dogmatism have become some of the main advocates of philosophical relativism. Realizing that one cannot tell which God is the true one, and realizing that the property-less metaphysical God is not really worth bothering with, they have turned to "let everyone believe what he or she wants", not simply in the civil libertarian sense that people should be legally allowed to defend whatever opinion they have, no matter how crazy, but that it is intellectually respectable to hold religious views without ever giving any argument for them, just because they are "our views" or those of "our culture". Of course, those who believe that quantum mechanics has made everything dependent on personal "observations", and hence, in some sense, subjective, will probably find such arguments congenial.

But such a move destroys the most fundamental bedrock of rational enquiry: the idea of objective truth, independent of human desires and subjectivity.

Time will tell whether this degeneration into subjectivism will do even more harm than the dogmatisms of old.

11.5 Quantum Mechanics and Philosophy

Given its revolutionary character, it is not surprising that quantum mechanics has had an impact on several philosophical currents of the 20th century, both because physicists have claimed that quantum mechanics has philosophical implications and because philosophers have sometimes tried to see how quantum mechanics might affect their own views.

Unlike the alleged links between quantum mechanics and pseudoscience, mysticism or religion, the links between science and philosophy have a long respectable history and have to be taken seriously. Unfortunately, we cannot do justice to the richness of the philosophical debates that took place around quantum mechanics, nor to the subtleties of the all philosophical schools that we shall discuss.

11.5.1 Quantum Mechanics and the "Mind-Body Problem"

> As a rule, the materialistic dogma has not been set up by men who loved dogma, but by men who felt that nothing less definite would enable them to fight the dogmas they disliked. They were in the position of men who raise armies to

> enforce peace. Accordingly we find that, as ancient orthodoxies disintegrate, materialism more and more gives way to scepticism.
>
> Bertrand Russell, [115, p. xi]

One of the most debated issues in the philosophy of mind is the relation between the mind and the body. This question is as old as philosophy itself, but a good modern exposition of the issue was given by Thomas Nagel in an article entitled: "What is it like to be a bat?" [131]

Bats have a special sense, echolocation. One may thus ask: what do they *feel* when they use that sense? If one tries to answer that question, one realizes that, no matter how much one knows about the brain of the bat or its entire body, one will not have the slightest idea as to what the bat feels. One may know what happens in the body of the bat down to the last atom, possess all the computing power one wants, analyze the brain at whatever level of abstraction one can imagine, one still gets zero information on how it feels to be a bat.

The reason is simply that there is something qualitative about the feelings of the bat that is not included in any objective, quantitative, description of what goes on in the brain of the bat.

The bat is just chosen to make the problem obvious, but the same question can be asked about how can men know what women feel in childbirth. In fact, one can go further and ask how anybody knows how other people feel when they are in pain or experience any other sensations. One does not gain that knowledge by analyzing what happens physically inside other people's brains (again no matter how detailed that analysis may be) but by analogy with one's own sensations. If somebody tells you that he has a toothache and you have suffered from a similar pain, then you know what it feels like to suffer that pain. But you know it only "internally", from your own experience and not "externally", by analyzing what goes on the brain of the person who has a toothache.

The gap between this subjective knowledge of what "it feels like" to be in pain, and our external, objective, quantitative description of what goes on in the brain when pain occurs, is what is called the "mind-body problem", or at least is one aspect of it.

There exist many reactions to this problem, and we are not going to decide which is the correct one. One may claim that, some day, science will bridge the gap between the physical description of the brain and the subjective aspect of our experiences. Others think that, in order to bridge that gap, one needs a new kind of science. Finally some people argue that this gap is a "mystery" that lies beyond the limits of human understanding (see for example [123]).

But where does quantum mechanics come in? One possible link is through the claim that the collapse of the wave function is produced by a non physical consciousness. For example, there is rather popular movie, "What the Bleep Do We Know!?" (available online), that promotes the idea of a link between quantum mechanics and consciousness, along with quite a lot of pseudoscience.[14]

But as we already discussed, there is no direct evidence for this claim, and the (effective) collapse of the wave function is a purely physical effect in the de Broglie–Bohm theory. But even most of those who reject that theory (the majority of physicists) do not accept the notion of a consciousness totally independent of the brain. Besides, even if one were to accept the idea that a mind, independent of the body, intervenes in the collapse process, there is nothing whatsoever in quantum mechanics to suggest that our conscious choices affect the collapse of the wave function one way or another.

So, there is no reason to take seriously this sort of link between consciousness and quantum mechanics.

There is a totally different way to link consciousness and quantum mechanics, which is to suggest that the latter may provide a scientific way to understand this mysterious link between the body and the mind.

Let us consider first that claim from the point of view of the de Broglie–Bohm theory. Since it is just a theory of matter in motion, with the wave function guiding that motion, there is nothing radically new with respect to classical physics and it does not change anything as far as the link between the mind and the body is concerned.

What happens if one considers that link from a more standard point of view about quantum mechanics? It depends which one: if quantum mechanics is simply a tool to predict results of measurements, then obviously it does not deal with the mind-body problem, simply because it does not deal with the world outside the laboratories.

A totally different link has been suggested by the well-known physicist Roger Penrose (partly in collaboration with the anesthesiologist Stuart Hameroff) [147, 148]. Penrose believes that collapses of the wave function do occur, but are spontaneous physical effects, linked to gravitational forces, and he thinks that this would explain some non computational aspects of the mind. By that expression, Penrose means that the human mind is able to do reasonings that no machine, no matter how sophisticated, could possibly do. The arguments of Penrose to arrive at that latter conclusion are technical (they rely on Gödel's theorem, a famous result in logic) and controversial, so we will not discuss

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[14] One of the participants in the movie, the well-known philosopher of science David Albert, has bitterly complained about the fact that the film-makers have selected parts of his interview in order to misrepresent his views.

them. Their views on spontaneous collapses of the wave function are also completely speculative and, even if they were true, it is hard to see how they would help bridge the gap between our subjective sensations and our objective view of the world, since quantum mechanics in their view lies entirely on the "objective" side of the gap.

So, as far as we can see, a physics based on quantum mechanics is unlikely to change anything to the riddle posed by the link between the body and the conscious mind.

11.5.2 Quantum Mechanics and "Positivism"

By positivism, we do not refer to the ideas of the French 19th century thinker Auguste Comte, but to a philosophical current, often called logical positivism and developed after WWI mostly by Austrian and German thinkers, that was centered initially around the Vienna Circle.[15]

That current of thought tried to purge philosophy of "metaphysics" and to base our knowledge on secure foundations. This was certainly an admirable goal. Those secure foundations would be logic and mathematics on the one hand, and direct experience, or sense data, on the other, hence the name logical positivism. The positivists also tried to get rid of metaphysics by defining as meaningless sentences that could not be verified.

One of the main problems of logical positivism is that the notion of "sense data" is not as clear as they thought. Certainly science relies on observations based on our senses, but as discussed in Chap. 6, the notion of observation is not simple: does it refer only to our own subjective experience or to the perception of objects existing outside of us? If one adopts the first meaning, one easily falls into solipsism, but if one follows the second meaning, one loses "certainty" since our senses can always deceive us.

A related problem was connected with the rejection of "metaphysics". Indeed, what is metaphysics? If it is the belief in entities whose existence cannot be "verified by experience", one has again to say precisely what "verified by experience" means. It is true that, for example, the existence of angels cannot be verified, but how does one verify the existence of atoms?

There is of course a huge difference between the evidence for the existence of atoms and the total absence of evidence for the existence of angels, but, even

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[15] The Vienna Circle was inspired by the work of the Austrian physicist and philosopher Ernst Mach. The members of the Vienna Circle included Rudolf Carnap, Philipp Frank, Hans Hahn, Ernest Nagel, Otto Neurath, and, in a similar circle in Berlin, Carl Hempel and Hans Reichenbach. There was some proximity, at least in the minds of the logical positivists, between them and Albert Einstein, Kurt Gödel, Bertrand Russell, and the early Ludwig Wittgenstein.

in the case of atoms, the evidence is indirect, so that it is not easy to define in general and independently of the context what "verified by experience" means.

With the rise of Nazism, the logical positivists had to emigrate and became rather influential in the United States. Although they clearly had their hearts in the right place, trying to be empirical and scientific and rejecting confused and a priori thinking, there was a certain youthful enthusiasm and naiveté in their endeavor. And when it came to quantum mechanics, although there was not much of an explicit link between the positivists' movement and the "Copenhagen" school, their emphasis on "observations" as being the only thing one could speak about "meaningfully" or "scientifically" helped create a *Zeitgeist* that tended to strengthen the "Copenhagen" school's rejection of all questions about the meaning of quantum mechanical formalism as being "metaphysical". We will see in Sect. 11.6 that this attitude was also reinforced by the post-WWII political climate.

11.5.3 Quantum Mechanics and "Postmodernism"

It is somewhat difficult to provide a precise definition for an intellectual current that prides itself in avoiding precision and definitions. That is the problem with "Postmodernism". In our joint book *Fashionable Nonsense*, Alan Sokal and I wrote that postmodernism is "characterized by the more-or-less explicit rejection of the rationalist tradition of the Enlightenment, by theoretical discourses disconnected from any empirical test, and by a cognitive and cultural relativism that regards science as nothing more than a "narration", a "myth" or a social construction among many others." [180, p.1].

Of course, postmodernism is much more than that, and has also impacts on visual arts, literature, music etc. It is a *Zeitgeist* rather than a well-defined philosophy and it is pretty much the one we live in.

In a sense, postmodernism can be understood as a revolt against logical positivism, viewed as stifling, as inhibiting creativity, and suppressing sentiments. This is one aspect of the perpetual swing of the pendulum, going from the Enlightenment to Romanticism and *Lebensphilosophie* (Life-philosophy) in the 18th and 19th centuries, or from pre-WWI scientism in Germany to a post-WWI anti-scientific climate during the Weimar Republic (and also after that, of course...).[16]

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[16] The historian of science Paul Forman argues in [81] that the hope of a German victory in WWI was based in part on the ingenuity of German science, and that hope was still alive at the beginning of the summer of 1918. When everything collapsed, including the Empire, in the fall of 1918, the shock was unprecedented and its effect on the period after the Great War produced a general anti-scientific climate during the Weimar republic. Forman sees the anti-causal approach in quantum mechanics as an adaptation of scientists to this hostile cultural climate.

Logical positivism relied on sharp distinctions between facts and values, observations and theory, statements that refer to the world (called synthetic) and statement that are mere definitions or tautologies (called analytic). They also had great faith in the scientific method as the only way to arrive at true statements about the world. Although none of what they said was completely wrong, the distinctions that they were relying upon were not as sharp as they thought, and the scientific method is not so easy to define independently of the context in which it is applied.

We already explained, through Hume's example of the Indian prince who did not believe that water can freeze, that what is rational to believe or not depends on our background information; therefore, what is scientific and what is not depends too much on the context to be subject to a full general definition a priori.

These difficulties were underlined by philosophers and historians of science, such as Willard von Quine [157], Paul Feyerabend [76] and Thomas Kuhn [112].[17]

A further step was taken by some sociologists and historians of science who argued that if there is no objectively definable scientific method, then the latter must necessarily be the result of a consensus internal to the scientific community. From that, one easily arrives at complete relativism: the scientific community, or rather the scientific communities, establish their own rules of validation for what they will decide to be "true" which is not essentially different from the practices of traditional societies, religious groups or political movements.

Such an attitude towards science is only a small part of the postmodernist *Zeitgeist* that celebrates the diversity of viewpoints, the lack of clarity, the indefinite, and the victory of subjectivity over objectivity.

Although postmodernists are in general hostile to science, with its "pretension" of objectivity, there is one scientific argument that they tend to adore: the "Copenhagen" interpretation of quantum mechanics, at least as it is popularly presented: particles follow different trajectories at the same time! The observer is necessary to collapse the wave function! The universe is an inseparable whole! Randomness is everywhere! This is perfectly congenial to the postmodernist *Zeitgeist* and is the antithesis of everything the postmodernists dislike about science: determinism, materialism, objectivity, etc. The many-worlds interpretations (all possibilities are realized simultaneously!) fits even better into that *Zeitgeist*.

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[17] We will not discuss the ideas and the nuances of these authors in detail, see e.g. Chap. 4 of *Fashionable Nonsense* [180] for a more detailed discussion.

Of course, it is mostly the subjective sounding statements of quantum physicists and their popularizers that are to the liking of the postmodernists, since the seem to license a relativist worldview.[18]

The main problem with the link between postmodernism and quantum mechanics, is that it is all a matter of analogies and metaphors. The postmodernists are usually interested in human affairs: history, politics, sociology, etc. But there is no logical link whatsoever between the quantum behavior of electrons, photons or atoms and human behavior. Of course, analogies can sometimes be useful when they relate an unfamiliar situation to a more familiar one. Here it works the other way around, and introduces unnecessary obscurity, since human affairs are far more familiar to most people than the highly mathematical way in which physicists deal with the quantum world.

11.6 Quantum Mechanics, Ideology and Politics

We have seen in Sect. 10.4.2 that Bohm was a victim of the witch hunts of the Cold War and that his work was derided rather than refuted. But things were much worse than that, because the discussions about quantum mechanics became entangled during that period with political issues that were irrelevant to science, but had a great psychological impact. This was true on *both sides* of the Cold War divide. We shall discuss first the "Marxist" side, then the "Western" side.

11.6.1 Quantum Mechanics and Marxism

Bohm and Rosenfeld, although polar opposites on the interpretation of quantum mechanics (Rosenfeld was a close friend of Bohr and an arch-critic of Bohm), were both philosophically "Marxists". We use the word "philosophically" here, because, although Bohm was briefly communist during

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[18] The reader can find examples of that literature in Alan Sokal's article: "Transgressing the boundaries: Towards a transformative hermeneutics of quantum gravity", and in the references given there [179, 181]. This article was actually a parody of postmodernist nonsense that the American physicist Alan Sokal managed to publish in 1996 in the rather fashionable journal *Social Text*, but the quotes cited here are authentic. Moreover, the article starts by praising some of the most subjectivist sounding quotes of Bohr and Heisenberg.

Sokal made an analysis of the links between postmodernism and pseudoscience in [181, Chap. 8].

For a critique of the postmodernist use of quantum mechanical antirealist rhetoric by a philosopher who used to be sympathetic to postmodernism, see Norris [133].

WWII and Pauli jokingly wrote a letter to Rosenfeld starting with "Dear ($\sqrt{\text{Trotsky} \times \text{Bohr}}$ = Rosenfeld)" [107, p. 4], neither of them was very much engaged in political parties as such (even though they were both on the left). So their interest was mainly in Marxist philosophy. However, what this means is ambiguous as the example of Bohm and Rosenfeld will illustrate.

On the one hand, Marxists claim to be materialist. Classical eighteenth century materialism was a combination of realism (in the sense used in Chap. 6), an empirical approach to the world (rejection of metaphysics and a priori knowledge, similar to the 20th century logical positivists), and the idea that the mind is in some sense an emanation of the body. While not directly rejecting any of these ideas, Marxists have added two important adjectives: their materialism is supposed to be "historical" and "dialectical".

Because of the first adjective, Marxists tend to view all ideas as being produced by "social structures" or by the "ruling classes". To the extent that this applies to the natural sciences, it cuts science from its link to the natural world, since, in the end, it is society that imposes the frame of reference through which the scientists "view" the world. For example, in 1935, the Soviet historian of science Boris Hessen, suggested that the "mechanical world view" of Newtonian physics was linked to the rise of the bourgeoisie [103]. This approach can be seen as a predecessor of the social constructivism which is at the root of postmodernism, as discussed in the previous subsection.

The second adjective, "dialectical", is hard to define. The Marxist idea of dialectic is borrowed from the German philosopher Georg Hegel, who was a mentor of both Marx and Engels and who influenced either positively or negatively much of German philosophy in the 19th century. But Marx wanted to turn dialectics upside down. He wrote [121]: "With [Hegel, the dialectics] is standing on its head. It must be turned right side up again, if you would discover the rational kernel within the mystical shell." But what this means is not at all clear.

Sometimes dialectics is presented as a set of "laws of thought", for example, trying to find a synthesis between opposite ideas, sometimes as a "law of society", showing how human societies develop historically from one "mode of production" to another, through "contradictions", and sometimes, especially in Engels' *Dialectics of Nature* [71][19] as "a law of Nature", showing that the latter is subjected, just as human societies are, to perpetual change and, supposedly, to progress. It also refers at times to holistic ways of thinking and to the alleged interconnectedness of everything in the universe.

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[19] This book was just a collection of personal notes that Engels never published. They were only published later in the Soviet Union, and sometimes worshipped by Communists.

Marxists have tended to lambast non-dialectical materialists as being "mechanical", or "metaphysical". Yet, it is fair to say that what Marxists called "mechanical materialism", i.e., eighteenth century materialism, is in fact closer to the views of most practicing scientists (who do not use that expression of course), except, again, some of those involved in quantum philosophy.

In that sense, Rosenfeld and Bohm had views at variance with those of most physicists. Indeed, the part of Marxism that they both liked was the "dialectical" aspect. But this word can mean very different things. For Rosenfeld, it meant the need to abandon determinism (which was supposed to be a leftover from "mechanical materialism") and complementarity was of course viewed as a typically dialectical notion (indeed both complementarity and dialectics are rather ill-defined notions and thus able to fit any reader's favorite interpretation). For Bohm, it was the holistic aspect of quantum mechanics, due to its nonlocal character, that sounded dialectical.

Of course, the association of Bohm with Marxism did not help his scientific reputation, at least in predominantly anti-Marxist circles in the West, and his later rapprochement with a figure like the somewhat mystical Indian thinker Jiddu Krishnamurti only made matters worse.[20].

Rosenfeld, on the other hand, was, as he said himself, fighting on two fronts [161, p. 482], because he was also critical of the idealism of some of the orthodox "Copenhagen" people, particularly Heisenberg.

Indeed, although prominent Soviet physicists, such as Lev Landau, were staunch defenders of the Copenhagen interpretation, there was also a current in Soviet science and philosophy that considered those views "idealist" because of the constant reference to the "observer". In 1908, Lenin had published a book entitled *Materialism and Empirio-criticism*, where the word "materialism" refers more or less to what we call realism in Chap. 6 and "empirio-criticism" was a version of idealism that was fashionable at that time [118]. The book was a radical critique of idealism, which Lenin saw as a danger, because it encouraged various forms of subjectivism among some Marxists.

During the Soviet period, but also among non-Soviet Marxists, Lenin's book was referred to by people who did not like what they considered idealist tendencies in science or philosophy, and that often included the Copenhagen interpretation of quantum mechanics (but not by Rosenfeld himself, who regarded Lenin's book as a return to "mechanistic materialism" [161, p. 482]).

When Marxism fell out of fashion, its association with the critique of the Copenhagen school only made any such critique even more suspect, especially in places where Marxism had been influential, like France and, of course, the

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[20] See [146] for a detailed description of the relationship between Bohm and Krishnamurti

ex-Soviet bloc. So for a while, discussing problems of quantum mechanics with scientists from those countries became especially difficult.

All this illustrates what Steven Weinberg calls the "unreasonable ineffectiveness of philosophy" in the sciences [197, p. 169], if by "philosophy" one means the reliance on vague and a priori notions which seem to provide a solid foundation for one's views, while in reality each author interprets those notions according to what he or she already believes for other, non-philosophical, reasons. Using this "method" one can arrive at radically different conclusions, as Rosenfeld and Bohm show, starting from the same "philosophy". On the other hand, one must admit that even this kind of "philosophy" can be inspirational, as it certainly was the case for Bohm. The mistake, however, is to support one or another scientific view because of one's commitment to such a philosophy.

11.6.2 Quantum Mechanics and the Cold War Mentality

The association between politics and quantum mechanics also existed in the post-World War II period on the "Western" side of the Cold War divide. We saw in Sect. 11.5.2 that the dominant philosophy at that time in the United States was logical positivism and that its emphasis on "observations" as being the only thing that could be "meaningful" helped to strengthen the dogmatism of the "Copenhagen" school.

But that philosophical baggage was not the main obstacle to an open discussion of the problems of quantum mechanics. Indeed there was in fact very little open-mindedness on these issues, but this was mostly for socio-psychological and even political reasons. The American physicist John Clauser, who made important contributions, both theoretical and experimental, to the first tests of Bell's inequalities, describes in vivid terms the situation when he was a student.[21] After characterizing adhesion to the standard views on quantum mechanics as a sort of religion, Clauser wrote:

> During the post-war years, the United States quickly became embroiled both in the cold war and in an internal anti-communist frenzy. Driven by Sen. Joe McCarthy, stigmas then came into vogue. [...] Unfortunately, acceptance of stigmas by the populace appears to foster the malignant creation of additional stigmas, specially in an environment that is dominated by religious fanaticism. Thus, in keeping with the times, a very powerful secondary stigma began to develop within the physics community towards anyone who sacrilegiously was critical of

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[21] Clauser's experience is similar to mine, but in a very different place. See also Gisin's book [90] for several examples of how, even not so long ago, people who raised questions about the foundations of quantum mechanics were silenced.

> quantum theory's fundamentals. The stigma long outlived the McCarthy era and persisted well into the 1970s and 1980s. Sadly, it effectively kept buried most of the untidiness left behind by quantum theory's founders, and physicists went on about their business in other areas. The net impact of this stigma was that any physicist who openly criticized or even seriously questioned those foundations (or predictions) was immediately branded as a "quack". Quacks naturally found it difficult to find decent jobs within the profession.
>
> John F. Clauser [43, pp. 71–72]

Clauser mentions Einstein, Schrödinger, and de Broglie as well-known critics who could hardly be dismissed as "quacks". But:

> Instead, gossip among physicists branded these men "senile". This is not a joke. On many occasions, I was personally told as a student that these men had become senile, and that clearly their opinions could no longer be trusted in this regard. This gossip was repeated to me by a large number of well-known physicists from many different prestigious institutions. Given this branding, their leadership role in charting the course of progress in physics was thereby severely limited. Under the stigma's unspoken "rules", the worst sin that one might commit was to follow Einstein's teaching and to search for an explanation of quantum mechanics in terms of hidden variables, as Bohm and de Broglie did.
>
> John F. Clauser [43, p. 72]

As an example of the consequence of this stigma, Clauser mentions a policy of the American Physical Society, editing *The Physical Review* and *Physical Review Letters* according to which referees should reject any paper on foundations of quantum mechanics that was not "mathematically based *and* gave new quantitative experimental predictions." As Clauser observes, under those rules, "Bohr's response to EPR certainly could not have been published" [43, p. 72], and the same is true for the 1952 articles by Bohm (the rules were formulated in the 1970s). Clauser continues:

> Religious zeal among physicists prompted an associated powerful proselytism of students. As part of the "common wisdom" taught in typical undergraduate and graduate physics curricula, students were told simply that Bohr was right and Einstein was wrong. That was the end of the story and the end of the discussion. Of course, it was the end, because the concluding chapters of the story were not yet written. Bohm's and de Broglie's alternative [...] were neither thought, nor even cited. Any student who questioned the theory's foundations or, God forbid, considered studying the associated problems as a legitimate pursuit in physics was sternly advised that he would ruin his career by doing so. I was given

> this advice as a student on many occasions by many famous physicists on my faculty at Columbia and Dick Holt's faculty at Harvard gave him similar advice.
>
> John F. Clauser [43, pp. 72–73]

Clauser then discusses Bohm, de Broglie, and Bell. For Bohm, he writes: "Given the era, no one dared trust his opinion since he was openly communist." De Broglie was more prominent, because he was both a Physics Nobel Prize winner (in 1929) and the perpetual secretary for the mathematical and physical sciences section of the French *Académie des Sciences*, but "his publications went largely unread, since, of course, everyone already 'knew' that he was senile."

As for Bell, Clauser explains that he was very careful: Bell avoided attending an awards ceremony for a prize given to him by Charles Brandon, a founder of *Federal Express*, in order not to appear linked to a non-scientific foundation. He never openly discussed his study of the foundations of quantum mechanics with his colleagues at CERN and thus led a sort of double life. Finally, when Alain Aspect, who was preparing his own experiments [6], came to visit him, "Bell's first question to him was 'Do you have a permanent position?' " [43, pp. 73–74].

Clauser also speaks of "evangelical theoreticians" who were not even willing to consider experimental tests of Bell's inequalities, because they were so convinced that quantum mechanics must be correct in all situations (and maybe also because they did not appreciate how radical the conclusions of the EPR–Bell argument were). Even the great Richard Feynman, when Clauser told him of his project to test Bell's inequalities, "immediately threw me out of his office saying 'Well, when you have found an error in quantum-theory experimental predictions, come back then, and we can discuss *your* problem with it' " [43, p. 71].

As a student, Clauser himself "had difficulty understanding the Copenhagen interpretation". And he "found Bohm's and de Broglie's work refreshing, since they do give real physical-space models of what is happening" [43, p. 78]. When he carried out his experiments, he "believed that hidden variables may indeed exist". By that time, the McCarthy era was "in the distant past" and:

> Instead the Vietnam war dominated the political agenda of my generation. Being a young student living in this era of revolutionary thinking, I naturally wanted to "shake the world".
>
> John F. Clauser [43, p. 80]

Thus the testing of Bell's inequalities may have been a collateral benefit of opposition to the Vietnam war. But the upshot of the experiment was not what Clauser expected, and did not prove quantum mechanics wrong, but rather established the reality of nonlocality, something that probably "shakes the world" even more (provided that the world is willing to listen!).

The story recounted by Clauser illustrates the fact, which has been frequently observed, that in formally free societies, unpopular views and heterodox opinions can be silenced as effectively and sometimes more effectively than in dictatorships. Instead of putting people in jails or camps, it is enough, in order to impose the dominant views, to stigmatize the free thinkers, to censor unorthodox publications, and to refuse to give jobs to those who do not toe the line. Of course, such methods are less unpleasant for the victims than the more brutal ones inflicted by dictatorships, but that does not mean that they are less effective. Given the relative comfort of modern life in developed societies, who will risk marginalization and a ruined career simply for expressing publicly his or her thoughts?

Yet, as John Stuart Mill famously said, in any case, a great harm is done:

> But the peculiar evil of silencing the expression of an opinion is that it is robbing the human race; posterity as well as the existing generation; those who dissent from the opinion, still more than those who hold it. If the opinion is right, they are deprived of the opportunity of exchanging error for truth: if wrong, they lose, what is almost as great a benefit, the clearer perception and livelier impression of truth, produced by its collision with error.
>
> John S. Mill [128, p. 76]

11.7 'Abuses' of Quantum Mechanics in the Human Sciences

Some of the greatest physicists of the past managed to come up with unfounded "extensions" or "extrapolations" of quantum mechanics to the human sciences. Not mentioning them would be unfair to the non-physicists whose confusions we criticize. We should first keep our own house in order. Here, then, are some examples.[22]

Consider what Niels Bohr once said about psychology:

> Above all, however, this domain [psychology], as already mentioned, is distinguished by reciprocal relationships which depend upon the unity of our consciousness and which exhibit a striking similarity with the physical consequences

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[22] This subsection is largely based on an article by the historian of science Mara Beller: *The Sokal hoax: at whom are we laughing?* [16]. We will limit ourselves to examples of extrapolations due to famous physicists, because they are sufficient to illustrate what are the problems with these extrapolations.

> of the quantum of action. We are thinking here of well-known characteristics of emotion and volition which are quite incapable of being represented by visualizable pictures. In particular, the apparent contrast between the continuous onward flow of associative thinking and the preservation of the unity of the personality exhibits a suggestive analogy with the relation between the wave description of the motions of material particles, governed by the superposition principle, and their indestructible individuality. The unavoidable influence on atomic phenomena caused by observing them here corresponds to the well-known change of the tinge of the psychological experiences which accompanies any direction of the attention to one of their various elements.
>
> Niels Bohr [28, p. 99]

As is often the case with Bohr's statements, this one is not entirely clear. But it is an instance of the wrong kind of analogy that we discussed in Sect. 11.5.3: whatever one thinks about "emotion and volition" or "the continuous onward flow of associative thinking and the preservation of the unity of the personality", what is gained, for non-physicists, by comparing them with "the wave description of the motions of material particles, governed by the superposition principle, and their indestructible individuality"?

This amounts to comparing something vague but relatively intuitive (the human psyche) with something precise but totally unintuitive (the superposition principle).

Finally, according to his friend, Léon Rosenfeld, Bohr seems to have believed in the universal applicability of his ideas:

> On one of those unforgettable strolls during which Bohr would so openly disclose his innermost thoughts, we came to consider that what many people nowadays sought in religion was a guidance and consolation that science could not offer. Thereupon Bohr declared, with intense conviction, that he saw the day when complementarity would be taught in the schools and become part of general education; and better than any religion, he added, a sense of complementarity would afford people the guidance they needed.
>
> Léon Rosenfeld [163]. Reprinted in [164, p. 535]

Next, consider the following "application" of quantum mechanics to politics by Max Born:

> The thesis 'light consists of particles' and the antithesis 'light consists of waves' fought with one another until they were united in the synthesis of quantum mechanics. [...] Only why not apply it to the thesis Liberalism (or Capitalism), the antithesis Communism, and expect a synthesis, instead of a complete and permanent victory for the antithesis? There seems to be some inconsistency.

> But the idea of complementarity goes deeper. In fact, this thesis and antithesis represent two psychological motives and economic forces, both justified in themselves, but, in their extremes, mutually exclusive. [...] there must exist a relation between the latitudes of freedom Δf and of regulation Δr, of the type $\Delta f \Delta r \sim p$ which allows a reasonable compromise. But what is the'political constant' p? I must leave this to a future quantum theory of human affairs.
>
> Max Born [33, pp. 107–108]

This is reasoning by analogy and extrapolation from physics to politics at its worst. The Heisenberg inequality (see Sect. 4.4) is a well established relation between results of measurements of physical quantities that *can* be expressed by single numbers. Here the relation is "applied" to totally ill-defined and probably undefinable quantities.

Indeed, whatever one thinks of the need for economic freedom and regulation, it seems rather naive to believe that such complex concepts could be expressed by single numbers Δf and Δr, and even more naive to think that those numbers are related by a simple inequality. This way of speaking gives a veneer of science to a non-scientific approach of human affairs and, given the (well deserved) reputation of Max Born as a major physicist, pushes the argument based on authority to its maximum effect.

Then there is the case of Wolfgang Pauli, who is well-known for his friendship with the psychoanalyst Carl Gustav Jung.[23] Pauli's main concern was to obtain a unified view of the world, including both mind and matter. In other words, he was seeking ways to bridge the gap between the body and the mind discussed in Sect. 11.5.1.

Alluding to Einstein's claim of an alleged incompleteness of quantum mechanics, he wrote: "However, this does not indicate an incompleteness of quantum theory within physics, but an incompleteness of physics within the totality of life." ([140], quoted by [4, p. 123]).

Pauli was careful about not making too direct a link between quantum mechanics and his speculations about Jungian psychoanalysis or the unity of mind and body, but he was clearly dissatisfied with the fact that ordinary quantum mechanics does not make an explicit reference to the consciousness of the observer. In a letter to his friend Markus Fierz he expressed doubts that matter is always treated correctly, "if we observe it, as we do in quantum mechanics, namely leaving the internal state of the observer totally out of consideration."([144], quoted by [4, p. 121]).

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[23] The reader can find several favorable references to Jungian psychoanalysis in Chaps. 17 and 21 of Pauli's *Writings on Physics and Philosophy* [145]. See also [4, 117] for Pauli's views on religion and "deep psychology".

As an example of his attitude, Pauli allowed himself to be guided outside his field by "just that agreement on the *meaning* of ideas coming up more or less simultaneously in different branches of knowledge [...]: "Correspondence (Entsprechung)", "complementary pairs of opposite" and "wholeness" occur independently both in physics and in the ideas of the unconscious. The "unconscious" itself has a certain analogy with the "field" in physics, and both are brought into the realm of the unrepresentable (Unanschauliche) and paradoxical through the problem of observation." ([145, p. 164]).

The problem is that he saw analogies between physics and (Jungian) psychoanalysis that may have led him to think that this latter field was more scientific than it actually is.

John Bell's opinion was that:

> In later years, Pauli seems to have decided that Bohr himself was not a complete supporter of the Copenhagen interpretation. He reproached Bohr according to the following lines: Bohr insisted that there was this division between the quantum mechanical system and the classical apparatus. He explicitly repudiated the idea that the human mind was somehow an important element in quantum mechanics. [...] But Pauli was attracted to that idea, and at the end of his life he became increasingly religious. He felt that it was wrong to separate science from religion; it was wrong to separate psychology from physics. He felt that the *real* Copenhagen interpretation did insist that the mind was something that you could not avoid referring to in formulating quantum mechanics. Pauli thought, as far as I can judge, that the division between system and apparatus was ultimately between mind and matter.
>
> John Bell [19, p. 53]

None of this can excuse the social scientists who "apply" quantum mechanics to their domain, since such applications simply do not exist, but it shows that the responsibility for this sort of "abuse" is at least shared by some prominent physicists.

11.8 A Plea for Modesty and for a Separation of Domains

> The concept of "truth" as something dependent upon facts largely outside human control has been one of the ways in which philosophy hitherto has inculcated the necessary element of humility. When this check upon pride is removed, a further step is taken on the road towards a certain kind of madness — the intoxication

> of power […] to which modern men, whether philosophers or not, are prone. I am persuaded that this intoxication is the greatest danger of our time, and that any philosophy which, however unintentionally, contributes to it is increasing the danger of vast social disaster.
>
> Bertrand Russell [165, p. 782]

It was probably unavoidable that an intellectual revolution of the magnitude of the quantum one would provoke reactions in every possible domain: art, philosophy, politics, social sciences, religions, superstitions.

Moreover, the two scientific revolutions of the beginning of the 20th century, the theories of relativity and quantum mechanics, seemed to have dealt a final blow to all previous certainties: relativity showed, among other things, that the notion of simultaneity is relative to our state of motion, and quantum mechanics seemed to have made every possible intuitive image of the microscopic world impossible, and to have shown that Nature is intrinsically random.

It is understandable that such revolutions would produce a great deal of skepticism with respect to our ability to understand the world.

If one combines this skepticism with the widespread human tendency to extrapolate scientific results outside of their sphere of application, one obtains a perfect recipe for the explosion of irrationalism and subjectivism to which our present time bears witness.

But the announcement of the death of rationalism may have been premature. First of all, as we tried to argue throughout this book, a rational understanding of quantum mechanics is possible, although it requires a departure from the traditional textbook approaches to the subject. The departure isn't that radical: all we have to admit is that particles do follow trajectories, and they do so by being guided by their wave function. There is nothing extraordinary in that idea, but it does go against "conventional wisdom".

But whatever one thinks of this way to solve the conceptual problems of quantum mechanics, one should at a minimum emphasize that quantum mechanics (even understood in the traditional way) does not imply anything whatsoever concerning the validity of telepathy, alternative medicines, Eastern philosophies, or the existence of God, and has no consequences whatsoever in regard to human affairs.

Quantum mechanics is of course interesting for philosophers, but only provided it is properly understood, which is not an easy task, partly because of the sophisticated nature of the mathematical formalism of quantum mechanics, but mostly due to the great lack of clarity and consensus among physicists themselves as to what it all means. The first error to avoid is to "choose" a certain view of quantum mechanics on the basis of one's preference for a particular philosophy, whether it be materialism, or dialectics, or positivism or one form or another of idealism and subjectivism.

Finally, there is something specific about quantum mechanics: it is not just that the science has been abused by ignoramuses, but that the scientists themselves, at least a good fraction of them, have contributed to that "abuse". And they have done it in many ways: by claiming that one has put back an unspecified observer at the center of science (and, then, who could be blamed for identifying this observer with the human subject?); or by claiming that intelligibility is unattainable in the microscopic realm, and that this is so *ad vitam aeternam*. They have behaved like people celebrating defeats as if they were victories.

But this was, in general, more due to *hubris* than mysticism: it would have been more honest, but more humble too, to admit that quantum theory did have shortcomings and to look for a more detailed theory. The real trouble with the dominant discourse about quantum mechanics is that the *hubris* of the scientists, claiming that their understanding of quantum mechanics was somehow an ultimate understanding, provided ammunition to the mystics and to all the defenders of anti-scientific world views.

Another problem, beyond *hubris*, was the respect for authority, as illustrated by the quotes of John Clauser in Sect. 11.6.2. Mara Beller gave the following example of the authority of Bohr:

> Carl von Weizsäcker's testimony is a striking example of the overpowering, almost disabling, impact of Bohr's authority. After meeting with Bohr, von Weizsäcker asked himself [194]: "What had Bohr meant? What must I understand to be able to tell what he meant and why he was right? I tortured myself on endless solitary walks." Note that von Weizsäcker did not ask, "Was Bohr right?" or "To what extent, or on what issue, was Bohr right?" or "on what issues was Bohr right?", but, quite incredibly, he wondered what must one assume and in what way must one argue in order to render Bohr right?
>
> Mara Beller [16]

A more extreme example of Bohr worship has been exemplified by John Wheeler, who compared Bohr's wisdom to nothing less than "that of Confucius and Buddha, Jesus and Pericles, Erasmus and Lincoln [202, p. 226]" [84, p. 82].

To conclude this brief tour of non-scientific extrapolations and exploitations of quantum mechanics, one should not minimize the role played by those scientists who have emphasized the "disappearance of objectivity" or even of "reality" supposedly implied by the quantum discoveries.

It is the scientific spirit, more than philosophy, which has been the main defender of the notion of objective truth in the modern world and which has thereby, to quote Russell, “inculcated the necessary element of humility” in human thought, as opposed to the wishful thinking of political, religious, and pseudo-scientific ideologies. If science itself gives up, or seems to give up that notion, then indeed a “check upon pride is removed”, with unpredictable consequences.

12

Summary of the Main Theses of This Book

Throughout this book, we have considered three questions:

(1) Does quantum mechanics put an "observer" (human or not) at the center of its description of the world?
(2) Does quantum mechanics imply the "death of determinism"?
(3) Does quantum mechanics mean that there exist instantaneous actions at a distance?

We first explained in Chap. 2, using the double-slit experiment, why there appears to be support for these three ideas: whether one slit or both slits are open influences the behavior of the particles, as shown by the interference patterns. And the same thing happens if we check a posteriori through which slit the particle goes. That gives credence to (1): "knowing" through which slit the particle goes seems to affect its behavior. Assertion (2) is supported by the fact that one cannot control or predict where the particle will land on the second screen and assertion (3) by the fact that the two slits can be far apart, although the interference effect does decrease with the distance between the slits, or if the second wall is close to the first one. This leads to paradoxes: in the delayed-choice experiment, it seems that one can determine by the choices we make now, what happened in the past, even billions of years ago.

In Chap. 3, we emphasized that the mere fact that one cannot predict or control certain events does not mean that they do not obey deterministic laws. Indeed those laws might be unknown to us. We also explained that events that do obey deterministic laws, like throwing coins or dice, may appear "random" if one does not describe their behavior in sufficient detail.

J. Bricmont, *Quantum Sense and Nonsense*, DOI 10.1007/978-3-319-65271-9_12

In Chap. 4, we introduced the fundamental concept on which all quantum predictions are based: the wave function. We emphasized that the latter gives a very efficient way to predict the *results of measurements* carried out in the laboratory, but is not supposed to mean anything outside of that framework. Indeed, the rules of quantum mechanics assume that the wave function behaves differently when the quantum object is being measured or observed than when it is not. Outside of measurements, the wave function evolves deterministically, while, during measurements, it collapses in a random fashion.

With the wave function concept, one can predict the observed behavior in the double-slit experiment and the one of many other more complicated situations.

But, of course, this formalism seems to justify assertion (1), since measurements are given a special role in it, and it seems to justify assertion (2), since the formalism assigns probabilities to results of measurements but those probabilities cannot be derived from a deterministic dynamics, at least not in ordinary quantum mechanics.

In Chap. 5, we tried to go further by giving a meaning to the wave function that would go beyond the recipes outlined in Chap. 4. If we want to understand the collapse rule by analyzing the measurement process using the quantum mechanical formalism, we run into the problem of macroscopic superpositions, which are illustrated by Schrödinger's cat.

One can also try to give a statistical interpretation to the wave function, namely consider that it determines the probability of a particle to have certain properties, like a position and a velocity, whether we measure them or not, and then, consider that measurements simply reveal those properties. Introducing those properties means that one introduces what are called hidden variables, namely variables that complete the usual quantum description. But we then have to face the no hidden variables theorems: in particular, it is simply impossible to assign to particles a statistical distribution of positions and velocities, independently of measurements, that would coincide with the quantum mechanical predictions for those measurements.

This sort of results, maybe even more than the usual quantum formalism, lead some people to a form of despair or renunciation: one cannot understand what is going on in the microscopic world and we have to limit ourselves to "predict the results of experiments".

In Chap. 6 we discussed the opposite attitude: that everything is all right after all, since quantum mechanics predicts what one can observe and since everything we know about the world derives from our observations. But that attitude reflects a misunderstanding of the problem: the observations that one is talking about in quantum mechanics are observations made thanks to certain instruments in laboratories and they are treated as a *deus ex machina* by the usual theory. There is nothing remotely similar in any other scientific theory, where observations are explained as resulting from certain interactions between the physical world and our instruments. In physical theories other than quantum mechanics, these interactions are described by the theory and are not given a special status.

In Chap. 7, we turned to issue (3): we did show that there are perfect correlations in the world that cannot be explained by local causal mechanisms, where "local" means that those causal mechanisms propagate at a finite speed. This is certainly the most surprising consequence of quantum mechanics, but it is based on a simple logical argument and on well verified experiments. The argument consists in two steps: one is that the perfect correlations in certain spin experiments require that spin values be pre-determined before their measurements, if we rule out any nonlocal causal effects, or, in the analogical story with Alice and Bob, that both Alice and Bob have pre-determined and coinciding answers to the questions that they are asked when they arrive at X and Y. That part is due to Einstein Podolsky and Rosen and means that one needs "hidden variables", since those spin values are not included in the pure wave function description. The second step is due to Bell and is a no hidden variables theorem: assuming the mere existence of those pre-determined values leads to a contradiction. If one consider only one of those two steps, as is often done, nothing dramatic follows. But if one combine both, a local view of the world becomes untenable.

Chapter 8 is the heart of this book: we discussed there the de Broglie–Bohm theory, which is simply a completion of ordinary quantum mechanics. In that theory, particles have positions at all times, whether they are observed or not, and therefore also trajectories and velocities, and the motion of the particles is guided by their wave function. We showed that this accounts in the simplest possible way for the results of the double-slit experiment: the particle, being always localized somewhere, as particles should be, goes through only one slit. But the wave, being delocalized, as waves should be, goes through both slits and guides the particles in a way that leads to the interference effects.

An obvious objection to the de Broglie–Bohm theory is that, if the no hidden variables theorem of Chap. 5 does not permit an assignment of values to both positions and velocities, how come that in the de Broglie–Bohm theory, particles do possess both of those properties? The answer is simply that the "measured" values of the velocity in that theory do not coincide with the actual values of the velocity of the particles. We considered the simple example of a particle at rest in a box, namely with zero actual velocity, but whose "measurements of velocity" will agree with the quantum predictions that give a well-defined statistical distribution of velocities, which are in general not equal to zero.

This leads us to the deepest lesson of the de Broglie–Bohm theory: measurements other than measurements of positions (which simply record where the particle is) do not, in general measure any intrinsic property of the particles, but are genuine interactions between the latter and the measuring device.

So, not only does the de Broglie–Bohm theory give a clear physical meaning to the wave function, it also explains how measurements work, without making them a *deus ex machina*, as in ordinary quantum mechanics.

This also applies to the measurements of spin, introduced in Chap. 7. But applied to the EPR-Bell situation, it implies that the interactions between the particles and the measuring devices has a nonlocal character. Because of the results of Chap. 7, showing that there are nonlocal effects in Nature, this is a quality and not a defect of the de Broglie–Bohm theory.

Furthermore, one accounts in the de Broglie–Bohm theory for the randomness of the results of quantum experiments by suitable assumptions of the randomness of the initial positions of the particles. This point was not developed in detail, because it would be too technical to do so.

Finally, reconciling the nonlocality of the de Broglie–Bohm theory with relativity is an open question, but it is a problem caused by the nonlocality of the world, shown by EPR-Bell, not by the de Broglie–Bohm theory in particular.

In Chap. 9, we discuss a rather popular "interpretation" of quantum mechanics: the many-worlds one, where the wave function never collapses and the universe constantly splits into zillions of parallel universes in which copies of ourselves live different lives, totally unaware of each other. This interpretation has an obvious science fiction character, which can be appealing or not. But we argued that it cannot be formulated in a way that would make most copies of ourselves observe the quantum predictions.

The main claim of Chap. 10 is that critics of the orthodoxy, Einstein, Schrödinger, de Broglie, Bohm and Bell, far from being refuted, were generally ignored and misunderstood.

Bohr did not understand Einstein's objections, which were based on the implicit nonlocality of quantum mechanics if the latter is supposed to be complete, and not simply on the lack of determinism or on Heisenberg's uncertainty principle. Schrödinger's cat was a *reductio ad absurdum* of the usual quantum formalism but is nowadays often interpreted as showing that the unfortunate cat is both alive and dead before anybody looks at it.

De Broglie is a tragic figure in the sense that he did initiate the de Broglie–Bohm theory, but did not really believe in it (partly because of its nonlocal character) and was not supported in his endeavor by other critics, such as Einstein and Schrödinger.

When Bohm rediscovered and completed de Broglie's theory, in 1952, the times were as bad as they could be for the reception of that theory: first of all, after the war, people were convinced that the pre-war "philosophical" debates about quantum mechanics were either futile (the slogan being "shut up and calculate") or had been won by the "Copenhagen" side. Besides, Bohm was victim of the McCarthyite witch hunts, which forced him to leave the United States; this drastically limited his possibility of having his views given a fair hearing.

Finally, Bell's result is almost universally viewed as a no hidden variables result, which it is, but, since it rules out hidden variables that would be necessary to save locality (as shown by EPR), his result is, in reality, a nonlocality result. This produced a sort of comedy of errors: while Bell has always defended the de Broglie–Bohm theory, he is often supposed to have proven that hidden variables theories, such as the de Broglie–Bohm one, are refuted by experiments!

In Chap. 11 we reviewed various extrapolations and abuses of quantum mechanics in the pseudo-sciences, religions, philosophies, politics and human sciences. We argued that those abuses are illegitimate irrespective of one's views of the foundations of quantum mechanics, but that they have sometimes been encouraged by famous physicists.

In a nutshell, the message of this book is that:

(1) Ordinary quantum mechanics predicts results of all sorts of measurements with remarkable precision, but one cannot understand it as a theory about the world outside the laboratories. That is why the observer or observations play a central role in that theory.
(2) One can use experiments whose results are predicted by quantum mechanics in order to show, indirectly, that there are some nonlocal effects in the world.
(3) There is a way to complete quantum mechanics into a theory about the world, where experiments are accounted for by the theory and do not enjoy a special status. That theory is nonlocal, but deterministic and the quantum randomness comes only from randomness in the initial conditions of the particles.

Erratum to: Quantum Sense and Nonsense

Erratum to:
J. Bricmont, *Quantum Sense and Nonsense*,
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In the original version of the book, the sentence "Copernicus Books is a brand of Springer" has to be included at the bottom of copyright page after "Printed on acid-free paper" line in frontmatter. The erratum book has been updated with the change.

The updated online version of this book can be found at
https://doi.org/10.1007/978-3-319-65271-9

J. Bricmont, *Quantum Sense and Nonsense*,
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Glossary

Prominent Scientists

We will give some information on the main scientists referred to in the book and indicate where they are mentioned;

Alain Aspect (1947–) French physicist, mostly known for his experiments, with J. Dalibard and G. Roger, verifying the violation of Bell's inequalities that are discussed in Chap. 7; see [6].

John Stuart Bell (1928–1990) Irish physicist who discovered in 1964, Bell's inequality that, combined with the EPR argument, establishes the existence of a subtle form of actions at a distance, see Chap. 7. He was also one of the main critique of the "Copenhagen" interpretation of quantum mechanics and a defender of the de Broglie–Bohm theory, see [14]. For the misunderstandings of Bell's results, see Sect. 10.3.

David Bohm (1917–1992) American physicist who rediscovered in 1952 the theory proposed in the 1920s by de Broglie, completed it and proved its consistency, see Chap. 8 and [24]. Bohm was marginalized partly for political and partly for ideological reasons, see Sects. 10.4.2–11.6.2. Indeed, Bohm was briefly a Communist during World War II and was victim of the McCarthyite witch hunts after the war; he was always interested in Marxist philosophy, but in later years he became close to the Indian thinker Jiddu Krishnamurti.

Niels Bohr (1885–1962) Nobel Prize in Physics, 1922. Danish physicist who invented in 1913 a model of the atom, still used as an intuitive approximation, and according to which the atom is a miniature solar system, with the nucleus in place of the sun and the electrons circling around it like the planets, but

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on a well-defined set of orbits, having different energies. He is also known for being the father of the Copenhagen interpretation of quantum mechanics (Bohr lived in Copenhagen during most of his life) and for his discussions with Einstein, see Sect. 10.1 and [28, 30, 31].

Max Born (1882–1970) Nobel Prize in Physics, 1954. German physicist who, together with Werner Heisenberg and Pascual Jordan, developed one version of the formalism of quantum mechanics (Schrödinger developed the other). He is also known for having given a statistical meaning to the wave function, concerning the results of measurements. He wrote several popular essays on physics and also on politics [32–34], and maintained a lifelong correspondence with Einstein, see Sects. 10.2–11.7 and [35]. Politically, Born was the antithesis of his former collaborator Pascual Jordan, since Born was very much opposed to nuclear armament and to the arms race.

John Clauser (1942–) American physicist who did early theoretical and experimental work on Bell's inequality. He described vividly the climate of intimidation reigning during the Cold War about the foundations of quantum mechanics, see Sect. 11.6.2.

Louis de Broglie (1892–1987) Nobel Prize in Physics, 1929. French physicist, known for having associated waves to particles; de Broglie actually developed a first version of the de Broglie-Bohm theory, see Chap. 8, but he did not really believe in it, for various reasons, in particular because of its nonlocality. We discuss the non reception of his views in Sect. 10.4.1; see [50, 87].

Albert Einstein (1879–1955) Nobel Prize in Physics, 1921. German-born, later U.S. citizen, he is the most well-known physicist of the 20th century, with the two theories of relativity (the special one, reconciling mechanics and electromagnetism, in 1905, and the general one, dealing with gravitation, in 1915), but also the photoelectric effect (for which he received the Nobel Prize in Physics), the proof of the existence of atoms and many other contributions. Here, we were mostly interested in his lifelong critique of the Copenhagen interpretation of quantum mechanics, in particular his 1935 argument done in collaboration with Boris Podolsky and Nathan Rosen [65], and proving that either the quantum mechanical description is incomplete or nonlocal effects do exists in Nature. See Chap. 7 for that argument, Sect. 10.1 for his debate with Bohr and Sect. 6.2, [35, 67, 70], for his more general views on philosophy and physics.

Richard Feynman (1918–1988) Nobel Prize in Physics, 1965. American physicist known for his work on quantum electromagnetism and particle physics. He is also the author of the famous *Feynman Lectures on Physics*, whose third volume is devoted to quantum mechanics [78] and in which the double-slit experiment is put forward as the hallmark of quantum behavior, see Chap. 2. One of Feynman's most famous sayings is that "nobody understands quantum mechanics" [79].

Murray Gell-Mann (1929–) Nobel Prize in Physics, 1969. American physicist known for his work on particle physics (discover of the quarks). Together with Jim Hartle he has developed a "decoherent history" interpretation of quantum mechanics [85]. His views on science are developed in *The Quark and the Jaguar*, [86].

Werner Heisenberg (1901–1976) Nobel Prize in Physics, 1932. A German physicist, one of the founder of modern quantum mechanics, which he developed with Max Born and Pascual Jordan. He is also known for his famous inequality, sketched in Sect. 4.4. Heisenberg is considered as one of the main defenders of the "Copenhagen" interpretation of quantum mechanics, although it is not clear that his views coincide with those of Bohr, see Sect. 10.4.2 and [100–102].

Pascual Jordan (1902–1980) A German physicist, one of the founder of modern quantum mechanics, with Max Born and Werner Heisenberg. He contributed also to the development of quantum field theory and even of pure mathematics. He was quite engaged politically, first a member of the Nazi party between 1933 and 1945, then as a Christian Democratic member of the Bundestag after the war, supporting the nuclear armament of Germany. His political activities may have contributed to a lesser recognition of his work. He was a great defender of the "Copenhagen" interpretation of quantum mechanics, mixing it with his views on positivism, vitalism and politics, see [209].

Wolfgang Pauli (1900–1958) Nobel Prize in Physics, 1945. Austrian born, later Swiss and American citizen. Using Heisenberg's newly found quantum mechanics, Pauli computed exactly the quantum properties of the hydrogen atom. He made many other contributions to quantum mechanics, in particular showing that two electrons cannot have exactly the same wave function.[1] He was a staunch defender of the "Copenhagen" interpretation of quantum mechanics, although it is not clear that his views coincide with those of Bohr or of Heisenberg, see Sect. 10.4.2. He was a friend of the psychoanalyst Carl Gustav Jung, see Sect. 11.7 and [4, 117, 145].

Léon Rosenfeld (1904–1974) A Belgian physicist who contributed in many ways to the development of quantum mechanics; in particular by early work in quantum electrodynamics. He was a close friend of Bohr and one of the most

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[1] To be precise, one should say "not the same quantum state", but, in this book, we did not distinguish between the wave function and the quantum state.

vehement defender of the "Copenhagen" interpretation of quantum mechanics and opponent of de Broglie and, specially, of Bohm. While both Rosenfeld and Bohm were interested in Marxist philosophy (Pauli jokingly called Rosenfeld the $\sqrt{\text{Trotsky} \times \text{Bohr}}$), they were polar opposite when it came to the understanding of quantum mechanics, see Sect. 11.6.1 and [107]; see [164] for Rosenfeld's papers on physics and philosophy.

Erwin Schrödinger (1887–1961) Nobel Prize in Physics, 1933. The Austrian born, later Irish, physicist is famous for his equation that governs the behavior of the wave function outside of measurements, see Chap. 4. His formulation of quantum mechanics is equivalent to the one of Born, Heisenberg and Jordan, but much more used nowadays. He was always very critical of the "Copenhagen" interpretation of quantum mechanics, and in 1935 introduced the example of the cat that is both alive and dead was meant to be a *reductio ad absurdum* of the claim that quantum mechanics provides a complete description of physical systems, see Chap. 5 and Sect. 10.2. As an aside, Schrödinger is also notorious for his *ménage à trois*, living with his wife and another woman married to one of his colleagues. This may sound less paradoxical than his cat, but was the cause of much public opprobrium at the time. Schrödinger had, unlike most other physicists, an acute understanding of the problem raised by Einstein, Podolsky and Rosen in [65] and invented the word entanglement to describe the wave function of the system considered by EPR, see Sect. 10.2. His writings on life inspired James D. Watson, one the discoverer of the structure of the DNA. Schrödinger had also a lifelong interest in the Vedanta philosophy of Hinduism.

John von Neumann (1903–1957). A Hungarian born, later American, mathematician, one of the most famous of the 20th century, with discoveries ranging from pure logic to computers, economics and nuclear weapons. His main contribution to quantum mechanics is contained in his book *Mathematical Foundations of Quantum Mechanics* [193], which is the first mathematical formulation of quantum mechanics. He also developed a "quantum logic", with the American mathematician Garrett Birkhoff [21], and tried to prove, via a no hidden variables theorem (see Sect. 5.2), that the quantum mechanical description is complete and indeterministic. The de Broglie–Bohm theory provides a counterexample to that claim. Von Neumann's proof of his no hidden variables theorem was correct, but the assumptions that he made to prove it were too restrictive (see [10]).

John Archibald Wheeler (1911–2008). An American physicist who made several contributions to nuclear physics and general relativity. He introduced the notion of "delayed-choice" double slit experiment, which led him to conclude that "the past is not really the past until it has been registered" [47, p. 68], see Sect. 2.2. In the de Broglie-Bohm theory, the "delayed-choice" experiment does not imply anything strange, see Sect. 8.2. Despite making somewhat weird claims, Wheeler was totally opposed to any pseudo-scientific exploitation of quantum mechanics, see Sect. 11.2 and [199].

Eugene Wigner (1902–1995) Nobel Prize in Physics, 1963. A Hungarian born, later American, mathematical physicist, Wigner contributed to nuclear physics, quantum mechanics and mathematics. He was sometimes critical of the "Copenhagen" interpretation of quantum mechanics, but partly because it did not go far enough, in a sense, since it did not explicitly include the role of consciousness in the theory. Indeed, Wigner wrote that it was "not possible to formulate the laws of quantum mechanics in a fully consistent way without reference to the consciousness." [205, p. 169].

Concepts

We will give definitions of some of the main concepts used in the book and indicate where they are discussed.

Action at a Distance. This means that, by acting at some place A, one can instantaneously affect the physical situation at some place B, no matter how far A and B are from each other. See Chap. 7.

Avogadro's number. It is more or less the number of atoms in one gram of hydrogen, and equals approximately 6×10^{23} namely 6 followed by 23 zeros.

Beables. A term introduced by John Bell to refer to what exists, or what a given theory postulates as existing, as opposed to what is observable, see Sect. 5.3. In the de Broglie–Bohm theory, the beables are the positions of the particles and the wave function, see Sect. 8.2.

Bell Inequality. An inequality which is implied by certain assumptions about the existence of hidden variables and contradicted by quantum mechanical predictions (that have been verified). However, this statement can be misleading if one does not add that, under an assumption of locality, Einstein, Podolsky, and Rosen have shown that those hidden variables necessarily exist. So Bell's result, combined with the EPR argument is a nonlocality result rather than a mere no hidden variables theorem. This is explained in Chap. 7, and, for the word "inequality", see Appendix 7.B. For the misunderstandings of this result, see Sect. 10.3.

Collapse (or Reduction) of the Wave Function. The time evolution of the wave function consists of two different (and incompatible) rules: Schrödinger's evolution between measurements and the collapse rule during measurements. If one measures a system whose wave function is a superposition of two different wave functions, that superposition suddenly jumps and collapses during measurements to one of the two terms, depending on which one is observed. See Sect. 8.4.3 for the corresponding notion ("effective collapse") in the de Broglie–Bohm theory.

Complementarity. A concept central to Bohr's thinking, referring to the fact that there exist pairs of quantities that one cannot measure simultaneously. Indeed, because of the collapse rule, measuring one quantity will, in general, affect the result of the measurement of the other quantity. For example, in the double-slit experiment, we can either check through which slit the particle went, when both slits are open, but then the particle behaves as a particle, or we can ignore which slit the particle went through, but then the particle behaves as a wave (see Sect. 2.1 and in particular part (c) of Fig. 2.6). What one cannot do is, at the same time, see through which slit the particle goes *and* observe the interference pattern of part (c) of Fig. 2.6. The word "complementary" is used by Bohr here in a non-habitual fashion: the wave description and the particle one are "complementary" in the sense that they *exclude* each other. The de Broglie-Bohm theory explains how to combine both aspects: the particles move as particles, but they are guided by their wave function.

Consistent Histories. Also known as decoherent histories. Those terms refer to interpretations of quantum mechanics, proposed by Murray Gell-Mann and Jim Hartle [85], Robert Griffiths [96] and Roland Omnès [136], that try to give an "objective" account of quantum mechanics, but without introducing particle trajectories, as in the de Broglie-Bohm theory. Those approaches run into difficulties because of the no hidden variables theorems (see e.g. [36, Sect. 6.3]).

Copenhagen Interpretation. This refers to a set of ideas, whose main defenders were Bohr, Born, Heisenberg, and Pauli, along with Jordan, Rosenfeld, von Neumann, Wigner, and Wheeler. These ideas emphasize the role of observations in the very formulation of the quantum mechanical laws. For some authors, the "observer" is an inanimate macroscopic object, for others a conscious subject, but in any case, it is introduced as a *deus ex machina* in the quantum formalism. This formulation lingers in most textbooks or popular presentations of quantum mechanics and has given rise to many philosophical commentaries. We introduce the role of the observer in Chap. 4, discuss the Copenhagen position in Sect. 5.4 and criticize its philosophy in Chap. 6. The main alternative to the Copenhagen interpretation is given in Chap. 8.

Determinism. A system is deterministic if, once the state of the system at a given time is specified, the dynamics of the system determines a unique state for all later times, see Chap. 3.

Entanglement. This refers to a situation comprising two particles that are separated, possibly far apart; the behavior of each particle is undetermined according to ordinary quantum mechanics but both particles have nevertheless a perfectly correlated behavior. We introduced that notion in Sect. 7.4.2 and discuss how it works in the de Broglie-Bohm theory in Appendix 8.B.

EPR. This acronym refers to Einstein, Podolsky, and Rosen who showed that, in certain situations, assuming that the world is local implies that quantum mechanics is incomplete, in the sense that certain "hidden variables" must have definite values independently of any measurement. This is discussed in detail in Chap. 7. For misunderstandings of this result, see Sect. 10.1.

Heisenberg's Inequality or "Uncertainty Principle". An inequality showing that the less "spread out" are the results of a position measurement of a particle, then the more "spread out" are results of a velocity measurement of that same particle, and vice-versa, see Sect. 4.4. This inequality refers to results of measurement and does not imply anything, by itself, on what happens outside the laboratories. See Sect. 8.3.1 for the meaning of this inequality in the de Broglie-Bohm theory.

Hidden Variables. Any variables that characterize the state of an individual system beyond its wave function. In the de Broglie–Bohm theory, the hidden variables are the positions of the particles (see Sect. 8.2), and nothing else. Introducing hidden variables other than the positions of the particles is delicate because of the no hidden variables theorem (see Sect. 5.2).

Idealism. We use the word as being related to an attitude with respect to knowledge, not in a moral or political sense. Idealists ascribe most, if not all, of our discourses about the world as coming from within ourselves and not talking really about the world "out there", see Sect. 6.1. An extreme form of idealism is solipsism.

Initial Conditions. The precise quantitative description of the state of a system at a given "initial" time, such that, once those conditions are specified, the law of physics determine that state for all later times. Examples of initial conditions are the positions and the velocities of a system obeying Newton's laws of motion, or the wave function for a quantum system, as long as it obeys Schrödinger's equation, namely as long as no measurements are made on that system. In the de Broglie Bohm theory, the initial conditions of a system are its wave function and the positions of the particles.

Interference. This is illustrated by the double-slit experiment: when both slits are open, the density of arrivals of the particles on the second screen is different from the sum of the densities produced by each slit when only that slit is open, see Sect. 2.1. Since particles are sent one by one, they seem to interfere with themselves, or to go through both slits when the two slits are open, and through one slit otherwise. The solution to this mystery within the de Broglie-Bohm theory is given in Sect. 8.2.

Law of Large Numbers. The law of large numbers means that, when one repeats many times the same experiment whose outcome is random, certain regularities occur. For example, if one tosses a fair coin a large number of times, one will get approximately the same number of heads as tails, each number being almost equal to half the total number of tosses. One also gets that each consecutive pair of results, HH, HT, TH, TT occurs about one quarter of the time. It also implies that statistical distributions of results such as those shown in Chap. 2 do not depend on where and when those experiments are made, even though each individual experiment will give a particular result, which is in general different from the results of all other similar experiments, see Sect. 3.4.1.

Measurement. Measurements play a central role in ordinary quantum mechanics, since the wave function acquires a physical meaning only through them. However, the expression is misleading, since it gives the impression that some pre-existing property of the quantum system is being revealed by the "measurement". This is discussed in Sect. 5.2, where this naive view of measurements is shown to be untenable because of the no hidden variables theorem. In the de Broglie–Bohm theory, measurements are interactions between a microscopic system and a piece of apparatus, interactions whose statistical results coincide with the quantum predictions (see Sect. 8.3 and Appendix 8.A).

No Hidden Variables Theorems. A set of theorems that show that one cannot assign values to certain quantities that would pre-exist to their "measurement" and that the latter would simply reveal. In Sect. 5.2 we state such a no hidden variables theorem for positions and velocities. Bell's result in Sect. 7.4 is also a no hidden variables theorem, but, combined with the EPR argument, it is actually a nonlocality theorem, see Sect. 7.5.

Nonlocality. A theory is nonlocal if it allows actions at a distance. We saw in Chap. 7 that any theory describing the world must be nonlocal and we explained in Appendix 8.B how nonlocality appears in the de Broglie-Bohm theory.

Overlap (of Wave Functions). When a wave function is a sum of two other wave functions:

$$\Psi(x) = \Psi_1(x) + \Psi_2(x)$$

we say that they overlap when there are values of x such that both $\Psi_1(x)$ and $\Psi_2(x)$ are different from zero. Figures 4.5–4.7 are examples of non-overlapping wave functions, while in Fig. 4.10, the two parts of the wave function emanating from the two slits initially do not overlap, but do overlap after some time, and that overlap produces the interference effects. We explain in Sect. 8.4.3 why having the many different wave functions associated to a macroscopic body overlap with each other is impossible in practice and why that implies the "in practice" reduction of the wave function in the de Broglie-Bohm theory.

Positivism. This refers to logical positivism, or logical empiricism, a philosophical school of thought originally centered around the post-World War I Vienna Circle. The aim of that school was to base all our knowledge on direct observations and logical deductions, see Sect. 11.5.2.

Randomness. We defined in Sect. 3.1.1 a sequence of symbols, say of 0's and 1's, to be apparently random if every finite series of symbols appear in it with a frequency that depends only on its length. For this definition to make sense, we idealize the sequence of symbols to have an infinite length. In practice, the length of any sequence is finite and the definition has to be understood in an approximate way. We distinguished between sequences that are truly random and those that are merely apparently random. An apparently random sequence is truly random if no conceivable deterministic mechanism can generate it. The results of a sequence of coin tosses gives an example of an apparently random sequence, since each tossing obeys deterministic laws. We emphasized that one cannot give a criterion showing that a sequence is truly random. The role of randomness in the de Broglie–Bohm theory is discussed in Sect. 8.4.2.

Realism. We use the word as being related to an attitude with respect to knowledge, not in a moral or political sense. Realists think that there is a world outside our consciousness, that propositions are true or false depending on whether they adequately reflect or not properties of that world and that a partial knowledge of those properties is possible, see Sect. 6.1.

Schrödinger's Equation. This equation governs the time evolution of the wave function outside of measurements, see Sects. 4.1 and 8.2 for its role in the de Broglie–Bohm theory.

Solipsism. The idea that everything outside my consciousness is an illusion, or that I am constantly dreaming, see Sect. 6.1. This is a radical version of idealism, but idealists can easily fall into that position.

Spin. An important concept in quantum mechanics, but that we treat only descriptively, since we do not want to use the mathematical formalism of quantum mechanics. The fact that electrons have a property called spin only means that, if those particles are sent in a box with a magnetic field in it, they will either go in the direction of the field or in the direction opposite to the field. That is all we use concerning the notion of spin, see Sect. 7.4.2. See Appendices 8.A and 8.B for the notion of spin in the de Broglie-Bohm theory.

Statistical Distribution. Consider a sequence of results of a large number of repetitions of the same experiment, for example, the results of a large number of coin tosses. One obtains the statistical distribution of such results by counting, in each sequence of results, the frequencies with which a given event occurs. For example, for coin tossing, the events would be a H or a T, a pair HH, HT, TH, TT, a triple HHH, HHT, HTH, etc., see Sect. 3.4.1 and Fig. 3.2 for more general experiments. When a suitable probability distribution is associated to an experiment, the law of large numbers implies that, in the vast majority of sequences of results obtained by repeating that experiment, the frequency with which a given event occurs coincides with the probability assigned to that event.

Statistical Interpretation. A statistical interpretation of quantum mechanics relates the wave function to a probability distribution of certain events outside the laboratories. We discuss these interpretations in Sect. 5.2 and show that, in general, they run into difficulties because of the no hidden variables theorems. The de Broglie-Bohm theory is a consistent statistical interpretation of quantum mechanics, see Chap. 8 and, in particular Sect. 8.4.1.

Superposition. We say that a wave function is in a superposition if it is the sum of two wave functions corresponding to two different physical situations, see Sect. 4.1, where we introduce this notion for microscopic systems. In Sect. 5.1, we explain that the formalism of quantum mechanics leads to macroscopic superpositions, that are hard to make sense of.

Theories of Relativity. There are two theories of relativity, the special and the general one. The special theory of relativity was developed around 1905 and is due mostly to the work of Lorentz, Poincaré and Einstein. It changed Newton's laws of motion in order to make them compatible with the newly discovered laws of electromagnetism. The general theory of relativity was developed around 1915 and is due mostly to the work of Einstein and Hilbert. It replaced and extended Newton's laws of gravitation and is often summarized by saying that the geometry of space-time is curved under the influence of matter and energy. In this book, we were mostly interested in the relativity of simultaneity implied by the special theory of relativity, see Sect. 7.7. For the treatment of that problem in the de Broglie-Bohm theory, see Sect. 8.5 and Appendix 8.B.

Wave Function. The basic object characterizing the state of one or several particles in quantum mechanics, see Sect. 4.1. The problem is that the meaning of the wave function is intrinsically linked to the notion of measurements, as explained in Chaps. 4 and 5. In the de Broglie–Bohm theory, the wave function has a clear physical meaning outside of measurements: it both guides the motion of the particles and its square $\Psi(x, t)^2$ is related to the probability distribution of where the particle is located, see Sects. 8.2 and 8.4.2.

Further Reading

We give here a selection of works, some popular and some not, some with the same perspective as the one of this book and some not. We give brief comments to tell the reader the category in which the various books fits. Obviously, this selection is partial (the bibliography by Cabello contains more than 10.000 items [37]!) and reflects to some extent my interests, but it includes works defending the Copenhagen interpretation, including recently revived forms of it, as well as the many-worlds interpretation. We put a star (*) next to the books that are rather technical.

Works Related to the de Broglie-Bohm Theory

(1) D. Albert: *Quantum Mechanics and Experience*, [1]. A very pedagogical and elementary introduction to the quantum formalism and to several of its "interpretations": the de Broglie-Bohm theory, but also spontaneous collapses of the wave function and a version of many-worlds interpretation.

(2) J.S. Bell: *Speakable and Unspeakable in Quantum Mechanics. Collected Papers on Quantum Philosophy*, [14]*. The collection of papers by John Bell. Very profound, but not elementary.

The next two books give the views of de Broglie, after the 1952 papers by Bohm:

(3) L. de Broglie: *Non-linear Wave Mechanics: A Causal Interpretation** [49].

(4) L. de Broglie: *The Current Interpretation of Wave Mechanics: A Critical Study** [50].

Works that give a more detailed discussion of the de Broglie-Bohm theory:

J. Bricmont, *Quantum Sense and Nonsense*, DOI 10.1007/978-3-319-65271-9

(5) D. Bohm and B.J. Hiley: *The Undivided Universe** [27]. The book presenting Bohm's perspective.
(6) J. Bricmont: *Making Sense of Quantum Mechanics** [36]. More advanced than this book, but still rather elementary.
(7) D. Dürr and S. Teufel, *Bohmian Mechanics. The Physics and Mathematics of Quantum Theory** [62]. A mathematically and conceptually rigorous treatment.
(8) S. Goldstein: Bohmian mechanics* [93]. A careful review article.
(9) P. Holland: *The Quantum Theory of Motion. An Account of the de Broglie–Bohm Causal Interpretation of Quantum Mechanics** [105]. A more physics oriented approach.

Bell's Theorem and Nonlocality

(1) M. Bell and S. Gao (ed.): *Quantum Nonlocality and Reality** [15]. A collection of viewpoints on Bell's theorem and nonlocality.
(2) N. Gisin: *Quantum Chance. Nonlocality, Teleportation and Other Quantum Marvels. Foreword by Alain Aspect* [90]. A popular book on nonlocality, by a leading experimentalist on the subject, from a perspective different from ours: Gisin argues that there is an intrinsic randomness in Nature.
(3) S. Goldstein, T. Norsen, D.V. Tausk and N. Zanghì: Bell's theorem* [92]. A careful review article.
(4) T. Maudlin: *Quantum Nonlocality and Relativity* [122]. A careful discussion of Bell's theorem and what it implies concerning relativity.
(5) G. Musser: *Spooky Action At a Distance: The Phenomenon That Reimagines Space and Time–and What It Means for Black Holes, the Big Bang, and Theories of Everything* [130]. A popular science book on nonlocality, and several other topics.

Classical Texts from the "Copenhagen" School

(1) N. Bohr, *Atomic Theory and the Description of Nature* [29]. A collection of articles by Bohr exposing his views on quantum mechanics.
(2) M. Born: *Natural Philosophy of Cause and Chance* [32]. The text of popular lectures by Born given at Oxford in 1948, where he explains his views on quantum mechanics and determinism.
(3) W. Heisenberg: *Physics and Philosophy. The Revolution in Modern Science* [100]. Here and in the two books below, Heisenberg explains his views on science, philosophy and society, as well as his conversations with Einstein. In this book, he defends the Copenhagen interpretation against its critics.

(4) W. Heisenberg: *The Physicist's Conception of Nature* [101].
(5) W. Heisenberg: *Physics and Beyond. Encounters and Conversations* [102].
(6) W. Pauli: *Writings on Physics and Philosophy* [145]. A collection or articles by Pauli on physics and its history and philosophy.

The Many-Worlds Interpretation

(1) D. Deutsch: *The Fabric of Reality* [55]. A popular presentation of the many-worlds interpretation.
(2) B. DeWitt, B. and R.N. Graham (eds): *The Many-Worlds Interpretation of Quantum Mechanics** [57]. A collection of early articles on the many-worlds interpretation, including the original ones by Hugh Everett.
(3) S. Saunders, J. Barrett, A. Kent and D. Wallace (eds): *Many Worlds? Everett, Quantum Theory, and Reality** [168]. A collection of articles on the many-worlds interpretation, some favorable, some critical.
(4) D. Wallace: *The Emergent Multiverse: Quantum Theory According to the Everett Interpretation** [196]. An exposition and defense of a version of the many-worlds interpretation.

History of Quantum Mechanics

(1) G. Bacciagaluppi and A. Valentini: *Quantum Mechanics at the Crossroads. Reconsidering the 1927 Solvay Conference** [7]. An English translation of the Proceedings of the 1927 Solvay Conference, with a detailed introduction about the history of quantum mechanics, the de Broglie-Bohm theory and the misunderstandings concerning de Broglie.
(2) M. Beller: *Quantum Dialogue: The Making of a Revolution* [17]. A history of quantum mechanics that emphasizes the role of dialogues between scientists and of their doubts in the development of science; it also analyzes the rhetoric that led the Copenhagen views to become dominant.
(3) M. Born (ed.): *The Born–Einstein Letters* [35]. The correspondence between Born and Einstein, about life, politics but also quantum mechanics.
(4) J.T. Cushing: *Quantum Mechanics. Historical Contingency and the Copenhagen Hegemony** [44]. A view of the history of quantum mechanics arguing that the victory of the Copenhagen interpretation was historically contingent and not necessary.
(5) A. Einstein and L. Infeld: *The Evolution of Physics: From Early Concepts to Relativity and Quanta* [67]. The history of physics from an Einsteinian perspective.

(6) O. Freire: *The Quantum Dissidents: Rebuilding the Foundations of Quantum Mechanics (1950–1990)* [84]. A history of quantum mechanics focusing on the fate of the "heterodox" or "dissidents": Bohm, Everett and others.

(7) L. Gilder: *The Age of Entanglement. When Quantum Physics Was Reborn* [89]. A history of quantum mechanics based on dialogues between scientists, that did not literally occur, but that are based on their writings.

(8) M. Jammer: *The Philosophy of Quantum Mechanics. The Interpretation of Quantum Mechanics in Historical Perspective* [108]. A rather complete history of all the interpretations of quantum mechanics, explained in a neutral fashion.

(9) D. Kaiser: *How the Hippies Saved Physics: Science, Counterculture, and the Quantum Revival* [110]. The story of how fringe physicists, coming from the counterculture and located around Berkeley, kept alive the interest in Bell's theorem and nonlocality, during the late 1970s.

(10) W. Moore: *Schrödinger: Life and Thought* [129]. A sympathetic biography of Schrödinger.

(11) A. Pais: *Niels Bohr's Times: In Physics, Philosophy, and Polity* [139]. A sympathetic biography of Bohr.

(12) D. Peat: *Infinite Potential: The Life and Times of David Bohm* [146]. A sympathetic biography of Bohm.

(13) M. Schlosshauer (ed.): *Elegance and Enigma. The Quantum Interviews* [170]. Interviews of 17 physicists and philosophers about all the issues related to foundations of quantum mechanics and representing a variety of points of view.

(14) D. Wick: *The Infamous Boundary: Seven Decades of Controversy in Quantum Physics* [204]. A history of quantum mechanics sympathetic to Bell and Bohm.

Other Works

(1) D.I. Blokhintsev: *The Philosophy of Quantum Mechanics** [22]. A Soviet-era book offering a statistical interpretation of quantum mechanics that wants to be realist (but that runs into difficulties because of the no hidden variables theorems of Sect. 5.2).

(2) D. Bohm: *Causality and Chance in Modern Physics* [26]. Bohm's views on physics and philosophy as they were shortly after he published his 1952 theory.

(3) P.C.W. Davies and J.R. Brown (eds): *The Ghost in the Atom: A Discussion of the Mysteries of Quantum Physics* [47]. A series of interviews for the

BBC with Alain Aspect, John Bell, John Wheeler, Rudolf Peierls, David Deutsch, John Taylor, David Bohm, Basil Hiley that cover a variety of views on the foundations of quantum mechanics.

(4) B. d'Espagnat: *Conceptual Foundations Of Quantum Mechanics** [52]. One of the early work of the foundational problems of quantum mechanics, including a discussion of Bell's theorem.

(5) B. d'Espagnat: *On Physics and Philosophy* [53]. A comprehensive discussion of the problems of quantum mechanics and of various philosophical responses to them, from the viewpoint of d'Espagnat for whom quantum mechanics means that reality is "veiled", that is in some sense unknowable.

(6) R. Feynman: *The Character of Physical Law* [79]. A very pedagogical series of lectures by Feynman, with a chapter on quantum mechanics.

(7) G.C. Ghirardi: *Sneaking a Look at God's Cards — Unraveling the Mysteries of Quantum Mechanics** [88]. A popular book on all foundational issues of quantum mechanics, very clear on Bell's theorem, by one of the founder of the idea of spontaneous collapse of the wave function [8].

(8) R.B. Griffiths: *Consistent Quantum Theory** [96]. A modern version of the Copenhagen view, similar to the one of Omnès.

(9) F. Laloë: *Do We Really Understand Quantum Mechanics?** [113]. A book that covers in detail all the foundational issues and interpretations of quantum mechanics.

(10) R. Omnès: *Understanding Quantum Mechanics** [136]. A modern version of the Copenhagen view, similar to the one of Griffiths.

(11) R. Penrose: *Fashion, Faith, and Fantasy in the New Physics of the Universe** [149]. A book critical of fashionable trends in physics, including string theory and cosmology, but also of ordinary quantum mechanics (the part called "faith").

(12) P.A. Schilpp (ed.): *Albert Einstein, Philosopher-Scientist* [169]. A volume devoted to all the aspects of Einstein's work, with an autobiography of Einstein, including his objections to orthodox quantum mechanics, and a chapter where Bohr recounts his discussions with him.

(13) S. Weinberg: *Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature* [197]. A philosophical reflection on the search for fundamental theories by one of the greatest contemporary physicists, with a chapter explaining his views on the problems of quantum mechanics.

(14) J.A. Wheeler and W.H. Zurek (eds): *Quantum Theory and Measurement** [201]. Collection of all the main classical articles on foundations of quantum mechanics.

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