# Chapter 1
# Schrödinger's Color Theory and Its Background

**Abstract** Translations of Schrödinger's articles on color theory show us the continuing importance of his colorimetry. Schrödinger's color theory develops a tradition which begins with Newton, and which was developed by Helmholtz and by Grassmann. Schrödinger also wrote at a time when Fechner's influence on psychology was much stronger than it is now. Some colorimetric terms have changed since his articles were published: some are more precisely applied than general terms were in the 1920s. There have also been surprises since, such as Wald's discovery of small-field tritanopia, and the discovery of four-cone color systems in some women. Generally Schrödinger's approach to color theory is sophisticated, comprehensive, and usefully didactic. His axiomatic approach to the geometry of color space permits a close examination of current assumptions about the treatment of data from color matching and color comparison.

**Keywords** Colorimetry • Translation • History of ideas • Newton's Opticks • Grassmann • Helmholtz • Fechner's Law • Color space • Small-field tritanopia • Tetrachromacy • Schrödinger

## Section 1: About the Color Theory

The progress of science is not always smooth or easy. Sometimes a great scientist's effort is displaced: Newton spent a great deal of time writing religious tracts. Sometimes a scientists's effort is superseded or forgotten: Helmholtz is known for his work on the physiology of vision, not for his work in kinematics.[a] Then sometimes a scientist's effort in one domain is eclipsed by success in another: Schrödinger's work in theoretical physics is celebrated, but it may come as a surprise that he wrote extensively on color theory – meaning the theory of human color vision. His articles on color theory have been left uncollected, and much of that work has been left untranslated. Translated into English almost a hundred years after they were written, here are his principal articles on the subject.

Schrödinger published his articles on color theory in the 1920s. One article of nearly a hundred pages appeared in three sections, within the journal Annalen der Physik. He sought to interpret his work for the common scientific reader as well, in two popular articles and a textbook chapter. I became aware of



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Schrödinger's writings on color theory when I was a graduate student, leafing through older journals. At that time I believed that the existence and the importance of these articles were matters of common knowledge – at least among researchers in color science. I believe that his approach to colorimetry retains its fundamental importance. If nothing else, his colorimetry licenses a wider discussion of geometry applied to colorimetry. That is to say his colorimetry readies the mathematical foundations of advanced colorimetry. If some modern accounts of colorimetry crawl through masses of unconnected detail, Schrödinger's colorimetry soars in its formal sophistication. There have been translations of one or two of his articles (note the citations at the end of this volume), but this is the most comprehensive collection of his works on colorimetry in English to date.

One may ask the pedigree of Schrödinger's account of color theory, in other words its place in the history of ideas. (His biography has already been written, and larger overviews of the history of color systems are available elsewhere.)[(b)] His color theory has a strong lineage. His account of color space follows on a notion developed from Newton's Opticks, by way of Hermann von Helmholtz and Hermann Grassmann. His account responds directly to Helmholtz's hypothesis of a line element for color space, and it responds to Grassman's formalization of color theory as a vector space. Newton relies on an analogy from music, to arrange colors in a circle whose circumference is divided into seven parts. Though he does not insist on specific arclengths to represent all the different types of color, he considers the arrangement of spectral colors to be "proportional to the seven musical Tones of Intervals of the eight Sounds".[(c)] Though the musical analogy sets an initial arrangement of colors, Newton also introduces another formalism: a 'center of gravity' construction to model combinations of colors. Later authors will abandon the analogy of a musical scale, but the 'center of gravity' construction persists as a feature of color theory. Color mixture for Newton is a domain independent of other properties of physical optics, a domain tractable in a formal way. "And in this respect the Science of Colours becomes a Speculation as truly mathematical as any other part of Optiques."[(d)]

Hermann von Helmholtz sought to carry Newton's legacy forward, though Helmholtz leaves aside the analogy to tonal intervals. Helmholtz (1852) surmises that Newton's musical analogy may have been reinforced by his choice of sunlight as an illuminant, as well as his choices of crown glass or flint glass as prisms.[(e)] Those can distort intervals along the spectrum. Helmholtz promotes Thomas Young's (1802) three-color or **trichromatic** theory, though he considers Wollaston's (1802) work as its basis. Helmholtz also acknowledges limitations on Young's three-color theory, for example in its claim for the objectivity of three fixed primary colors. To extend the body of empirical evidence, Helmholtz (1855) devised a color-wheel apparatus, which he thought could be used to replicate earlier work with prisms – including that of Newton.

Grassmann (1853) responds to Helmholtz in vindication of Newton. Grassmann does seek to show that Helmholtz's results coincide with Newton's for the most part. In the same text Grassmann introduces important new concepts to the study of color. Those concepts inform Schrödinger's later work. Grassmann introduces the notion of a measure of hue, and the axiomatization of operations on colors. Colors can be represented as line lengths with direction, and combinations of colors can be represented as geometric vector sums. Grassmann refers to his own earlier work on vector spaces, but he also introduces Möbius's barycentric coordinates and barycentric calculus.[(f)] He introduces them as part of the task of revising Newton's arrangement of colors about a circle by a centre-of-mass calculation. Grassmann introduces the formal machinery of affine geometry or projective geometry to the problem of characterizing color space; Helmholtz seems not to appreciate the full import of Grassmann's gambit. Helmholtz's response to Grassmann is once more a defense and re-interpretation of Newton's Opticks. Helmholtz (1855) seeks to explain Newton's color circle results in terms of the spectral sensitivity of the eye, and the optical properties of refractive materials. He attributes some variance to the Purkinje effect, as well. Newton's spacing of colors is said to need revision, and Newton's color circle needs to include purples as mixtures of red and violet (Newton did acknowledge the latter possibility.). Helmholtz claims that Newton's theory of color mixture is expressed by the color circle. Moreover, he claims that Newton's essential contribution to color theory is just the center-of-mass construction for the combination of colors in the color circle. It is noteworthy that Helmholtz interprets Grassmann's 'sum of colors' in the center-of-mass construction in a narrow way rather than as a vector sum. (As a consequence, he believes that Grassman is committed to a circular form as the boundary of color space.)

Helmholtz (1891) continues his formal development of color theory, still aiming to characterize Newton's laws of color combination. Helmholtz's aim is to develop a Riemannian 3-manifold for color space. His methods include the determination of intervals of *just-noticeable difference* or JND, following methods set out by Weber and Fechner earlier in the nineteenth century. (Note that Helmholtz uses 'difference in sensation' – Empfindungsunterschied – almost interchangeably with 'sensation of difference' – Unterschiedsempfindung.) Helmholtz includes comparisons of brightness, not just comparisons of hue among these differences. That is to say, comparisons of heterochromatic brightness also count as intervals or steps of just-noticeable difference. Lines in color space which prove to be lines of smallest color difference are taken as geodesic, in other words as shortest lines between colors as points in the color field. Helmholtz (1892b) generalizes his theory of color further. He assumes that the perception of differences in color originates with the perception of differences in brightness. (Helmholtz does recognize departures from Fechner's Law of just-noticeable differences, for color mixtures which include colors of low saturation.) Consequently differences in brightness and differences in color both contribute to a geometric representation of color. One may continue by characterizing a system of color for dichromats, and then extending the dichromatic system to trichromats. Newton's laws of color mixture are more easily seen to apply to color comparisons by dichromats. Helmholtz's (1892b) color system makes reference to three primary colors as reference points, and it places colors in a frame of positive rectilinear coordinates. Helmholtz's (1892) color system is the principal foil, and the main historical reference for Schrödinger's narrative on color theory. In that system any color can be expressed as a point in terms of three values: three positive rectangular coordinates in $x$, $y$, and $z$. Helmholtz's system is Riemannian in the sense that any distance between two neighbouring points is given by a differential expression of coordinates. That expression for color sensations in coordinates plays the role of an expression for the lengths of line elements. Helmholtz's system is three-dimensional including brightness, though he claims that any plane section of the color system is a color table in the sense given in Newton's Opticks. In contrast Schrödinger's colorimetry leaves us with a programmatic sketch: his colorimetry sets out the geometric framework for color space, but it does not complete the structure. It establishes an affine geometry to suit the basic evidence of color matching in colorimetry, but it stops short of specifying the Riemannian structure of advanced colorimetry for color similarity. The affine geometry is a default structure, sufficient only until a few problems in advanced colorimetry may have been solved. A geometry of advanced colorimetry waits on two things: a fuller corpus of empirical data, and a decision about the validity of the Weber-Fechner law. What is to be done about Fechner? One can – to repeat Stevens's phrase – honor Fechner and repeal his law.[(g)] Otherwise one is faced with the task of reconciling Fechner's Law for color space with legitimate competing interests. In either case Schrödinger's work – equally the work of color theory – remains unfinished.

There is a leitmotiv in psychology, beginning with Weber and continuing with Fechner,[(h)] which maintains there is a determinate relation between changes in physical magnitudes and perceived changes in those magnitudes. Discussion of its psychological validity often dominates discussion in the experimental psychology of the time, as in Meinong's writing and influence.[(i)] That determinate relation has been claimed as a relation between physical magnitudes and qualities perceived in a sense modality. Some would go so far as to call it a relation between physical magnitudes and psychological qualities. The estimation of lifted weights provides an example. Over a large range of weights that can be lifted with one hand, a person may be asked to judge a just-noticeable difference in weight, or else a constant difference in weight over many trials. At least for the estimation of weights, a comparison weight which is noticeably heavier is one that adds a constant positive fraction of weight to the weight which serves as a standard. That constant fraction is maintained across a large range of weights, that is, for a variety of standard weights. Something similar occurs in the brightness of white lights: lights that are seen to exhibit equal steps of brightness will each add a constant fraction of intensity to the previous step. (Note that just-noticeable differences and equal steps are not the same here, though they are related.) That constant fraction for the brightness of white lights need not be the same fraction as the fraction for the heaviness of weights. We may go on to speak of just-noticeable differences or of equal steps for many qualities and modalities. The associated fractions are known as Weber fractions. Fechner surmised that these observations provided evidence for a logarithmic relation between psychological quantities and physical changes for many modalities. Perhaps the elision from the estimation of physical changes to changes in psychological quantities is unwarranted, but it seldom delays anyone in this discussion.

Most often force of will supervenes over logic in discussion of Fechner's Law. Fechner recognized criticisms of his logarithmic relation, but considered them a nuisance.[(j)] Later, Stevens recognized a heuristic value for Fechner's Law, but he did not stop to consider the assumptions which underlie its establishment. Stevens is well-known for supplanting Fechner's Law with a broader power-law relation. Yet there have been close rational and historical considerations of Fechner's Law which illuminate the assumptions involved. That is to say we know what sorts of experimental evidence might be brought in its favour. Still, in our century as previously, we are blithe in our approach to psychological 'laws' based on just-noticeable differences. Blithe, or else simply undisposed to close examination of the reasonable implications of such assumptions. We seem to have accepted the general tenor of the conclusions of such arguments, while we have – for the most part – jettisoned the premises.

One could say that a central question for Schrödinger's colorimetry is whether Weber fractions (i.e., Fechner intervals) remain constant for the brightnesses of many differently colored lights, or not. Better, one can say that Schrödinger realizes the centrality of Fechner's Law to contemporary accounts of color space. By his (Schrödinger's) own deliberations, he was disposed not to accept the assumptions of Fechner's Law. Acceptance of that Law he sees to be incompatible with the specification of a Riemannian line element for color space, in a colorimetry that would unify an account of large differences in color with the account of small differences.

Schrödinger hints at a greater mathematical sophistication in colorimetry.[(k)] In introducing basic colorimetry, he sets out a condition to determine whether or not a definition of brightness is possible in colorimetry – which is his initial motivating question for colorimetry. That stipulation is a condition on a set of partial differential equations, known as a Pfaffian system (see the relevant parts of Chap. 5 here, and also footnote 3 to Chap. 4). Basic colorimetry must satisfy this condition if the notion of heterochromatic brightness is to make sense. Schrödinger also specifies a provisional line element for the Riemannian geometry of the color manifold. He specifies a line element which improves on the one Helmholtz had defined.[(l)] Both line elements are then placed in doubt. Schrödinger leaves the business of a line element for the color manifold unspecified and unfinished. Two aims of his colorimetry collide in the specification of a line element. One aim is to reconcile small-scale differences with large-scale ones, meaning that there should be a meshing of gears between strongly heterochromatic colorimetry and the colorimetry of adjacent colors in the manifold. Another aim is to adhere to Fechnerian proportion for equal increments of intensity across the color manifold. Perhaps the two constraints could even have been reconciled, but for the intrusion of the Bezold-Brücke effect.[(m)] The discriminability of hues changes appreciably across the spectrum as light intensity is varied. Though the Bezold-Brücke pattern of changes is known by experiment, that pattern does not scale with the neat proportions of a colorimetry based on Fechner's Law. In short: specification of the line element remains an open problem. Is there even a line element tractable in mathematical terms, which is subject to all these influences? Subsequent authors quote (and criticize) Schrödinger's line element for color space as if its form had been settled as definite rather than left uncertain, as it is. Here Schrödinger's very hesitation shows the sophistication of his approach to colorimetry. (One might also ask what a suitable form might be, for a line element free of the Fechnerian constraint.)

As with any historical translation, the issue of change in language may be raised for this collection of Schrödinger's papers. Have the technical terms of color science changed so much as to be unrecognizable? I think not. Both in his training (he was Franz Exner's assistant) and in his recognition of Helmholtz's contributions, Schrödinger was in the mainstream of color theory. Some contemporary terms of art have fallen into disuse, such as 'alychne' (it indicates the locus of colors in a color diagram which have an ideal but fictional property of zero brightness). Some other terms have been sharpened over time – for example, 'luminous efficiency' has a more specific role in color theory than does 'brightness'. Yet none seem irretrievably unfamiliar.

I do not wish to say that these texts lack any decent translation at all. David MacAdam's translations provide something of a counterexample, though he published only short fragments of the work in translation.[(n)] There is a particular problem, though, for which I would like to make my stance clear. There is a cluster of words used in the development of what are known today as color-matching functions and fundamental response curves. The associated terms have changed through time, as can be known even by comparing W.D. Wright's (1947) uses of the terms with Schrödinger's. Of course it might also be that the terms have been made obscure in translation: MacAdam (1970) uses 'calibration' as an adjective to cover terms ranging from chromaticity coordinates to fundamental color stimuli, to an extent that make the modern interpretations of color-matching functions and fundamental stimuli barely recognizable. I take that gloss in translation to pose a greater danger to these texts than change in language; my own glossary for these terms is given as a footnote to this introduction.[(o)]

There are two more factors which may have clouded other translations. One is appreciation of the subject matter. Translation preserves meaning: a text which lacks meaning is not susceptible to translation. A translator also needs to be comfortable with the meaning of the original text. Schrödinger's writing is clear – evidently – but not everyone will be comfortable with its depth. The second factor concerns a translator's linguistic skill. I mean more by that than education in the technical arts of translation. Style may also cloud the result of translation. Some may consider colloquial style to be offhand for the translation of *Buchwurmsprache*. Others may lament the gap between North American usage and the Queen's English, and so forth. One particular point of style is important here: where translation is less than perfect, translation into one's first language is likely happier – as a matter of familiarity in style – than translation in another direction. Few translations achieve a beautiful balance of meaning and style.

Whatever the faults of the present translation, my earnest wish is that the original meaning may shine through.

There have also been some surprises in color science over the last century, as one might hope. Schrödinger maintained that no authentic cases of tritanopia had been reported – that is, no 'blue-blind' observers had been documented whose condition did not involve severe ocular trauma. Though rare, such individuals are now known to exist. He might have been surprised by discovery of small-field tritanopia – that most of us are tritanopes when the field of view is restricted to a very small angular extent under fixed viewing conditions.[(p)] He might also have been surprised by the efficient manufacture of fluorescent pigments that borrow energy from non-visible regions of the spectrum, to display brighter visible colors. Similarly, he maintained that the color space of his basic colorimetry is three-dimensional, in the sense that triplets of spectral colors are linearly independent. Combinations of four spectral colors – so he supposed – would produce nothing new. He took pains to arbitrate between a color manifold of dimensionality three, and a manifold of dimensionality four. In other words he thought that the manifold of colors is covered completely and exhaustively by pure spectral colors and their binary mixtures. He may also have been surprised in that much: there are indications that a fair proportion of women possess four chromatic systems rather than three, meaning that they possess four distinct populations of cone cells, and that they discriminate color reliably and functionally better as a consequence.[(q)] Then it may well be the case that we need a four-dimensional manifold of color space, to describe the ability these women have to judge differences in color.

Neither discovery – small-field tritanopia or tetrachromacy in some observers – affects Schrödinger's color theory in a fundamental way. On these counts we can tell how his color theory is extensible – how it may be extended to subsume these new findings. His color theory is still fresh in that much. I hope the reader finds a full and reasonable exposition of the theory of colorimetry in these pages, one that ignites our modern imagination about the geometric nature of color space.

## Notes

a. This includes [***Helmholtz***, H. L.F. von. The origin and meaning of geometric axioms I. Mind, **1**(3), 301 – 321 (1876). & ***Helmholtz***, H. L.F. von. The origin and meaning of geometric axioms II. Mind, **3**(10), 212 – 225 (1878).]. Sophus ***Lie*** revealed the lacuna in ***Helmholtz***'s account: [***Lie***, S. Bemerkungen zu von Helmholtzs Arbeit: Ueber die Tatsachen, die der Geometrie zu Grunde liegen. In S. Lie, Gesammelte Abhandlungen, 2. Band, 1. Teil. Leipzig: B.G. Teubner, 374 – 379 (1886/1935).]

b. ***Schrödinger***'s biography can be found as: [***Moore***, W. Schrödinger: Life and thought. Cambridge: Cambridge University Press (1989).] A more comprehensive account of color systems, including non-metric systems, is contained in: [***Wyszecki***, G. Farbsysteme. 2d. ed. Göttingen: Musterschmidt-Verlag (1962).] or else [***Wyszecki***, G. and ***Stiles***, W.S. Color science. 2d. ed. New York: John Wiley & Sons (1982).]

c. Their arrangement is "proportional to the seven musical Tones of Intervals of the eight Sounds, *Sol*, *la*, *fa*, *sol*, *la*, *mi*, *fa*, *sol* contained in an Eight, that is, proportional to the numbers $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{16}$, $\frac{1}{9}$ " [***Newton***, p.114, The first book of Opticks, part II (1704)].
d. He continues: "I mean so far as they depend on the nature of Light, and are not produced or altered by the power of imagination, or by striking or pressing the Eyes." [***Newton***, p. 48, The second book of Opticks, part II (1704)].
e. Citations for the articles discussed in this section are:
   ***Grassmann***, H.G. Zur Theorie der Farbenmischung. Annalen der Physik und Chemie (J.C. Poggendorff's Annalen), **89** (Dritte Reihe **29**), 69 – 84 (1853).;
   ***Helmholtz***, H.L.F. von. IV. Ueber die Theorie der zusammengesetzten Farben. Annalen der Physik und Chemie (J.C. Poggendorff's Annalen), **87** (Dritte Reihe **27**), 45 – 66 (1852).; ***Helmholtz***, H.L.F. von. Ueber die Zusammensetzung von Spectralfarben. Annalen der Physik und Chemie (J.C. Poggendorff's Annalen), **94**(1) (Vierte Reihe **4**), 1 – 28 (1855).; ***Helmholtz***, H.L.F. von. Versuch einer erweiterten Anwendung des Fechnerschen Gesetzes im Farbensystem. Zeitschrift für Psychologie und Physiologie der Sinnesorgane, **2**, 1 – 30 (1891).; ***Helmholtz***, H.L.F. von. Kürzeste Linien im Farbensystem: Auszug aus einer Abhandlung gleichen Titels in Sitzungsberichte der Akademie zu Berlin, 17. Dezember 1891. Zeitschrift für Psychologie und Physiologie der Sinnesorgane, **3**, 108 – 122. (1892).; ***Helmholtz***, H.L.F. von. Versuch, das psychophysische Gesetz auf die Farbenunterschiede trichromatischer Augen anzuwenden. Zeitschrift für Psychologie und Physiologie der Sinnesorgane, **3**, 1 – 20 (1892b).; ***Wollaston***, W.H. A method of examining refractive and dispersive powers, by prismatic reflection. Philosophical Transactions of the Royal Society, 1 January, **92**, 365 – 380 (1802).; ***Young***, T. The Bakerian lecture: On the theory of light and colors. Philosophical Transactions of the Royal Society of London, January 1, **92**, 12 - 48 (1802).
f. In other words the use of a non-Euclidean coordinate system for color space, as in a system of barycentric coordinates [***Möbius***, A. F. Der barycentrische Calcul 1827. In: Gesammelte Werke, Band 1. Leipzig: Salomon Hirzel, 1 – 389 (1885).], was carried on by [***Grassmann***, H. G. Zur Theorie der Farbenmischung. Annalen der Physik und Chemie (J.C. Poggendorff's Annalen), **89** (Dritte Reihe **29**), 69 – 84 (1853). See page 83.] ***Grassmann***'s earlier work on vector spaces, the Ausdehnungslehre, can be found as [ ***Lewis***, A.C. (Ed.) Landmark writings in western mathematics 1640 – 1940. Chapter 32 – Hermann G. ***Grassmann***, Ausdehnungslehre, first edition (1844), pp. 431 – 440 (2005).].
g. [***Stevens***, S.S. On the psychophysical law. Psychological Review, **64**(3), 153 – 181 (1957)]. ***Stevens*** proposed a power-law relation whose exponent varied by modality, to supplant ***Fechner***'s 'Law'.
h. Beginning with ***Weber*** [***Weber***, E.H. De pulsu, resorptione, auditu et tactu. Leipzig: C.F. Koehler (1834). & ***Weber***, E.H. Ueber den Raumsinn und die Empfindungskreise in der Haut und im Auge. Berichte über die Verhandlungen der königlich sächsischen Gesellschaft der Wissenschaften zu Leipzig,

mathematisch-physische Classe, **2**, 85 – 164 (1852).] and continuing with ***Fechner*** [***Fechner***, G. T. Ueber ein psychophysisches Grundgesetz und dessen Beziehung zur Schätzung der Sterngrössen. Berichte über die Verhandlungen der königlich sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physische Classe, **4**, 457 – 532 (1859). & ***Fechner***, G. T. Revision der Hauptpuncte der Psychophysik. Leipzig: Breitkopf und Härtel (1882). & ***Fechner***, G. T. Über die psychischen Maβprinicpien und das ***Weber***'sche Gesetz. Philosophische Studien, **4**, 161 – 230 (1888).]. Discussion of the 'Law' had a central place in nineteenth-century discussion of experimental psychology (then 'experimental philosophy').

i. ***Meinong*** elaborated ***Fechner***'s arguments at length [eg., ***Meinong***, A. Ueber Sinnesermüdung im Bereiche des Weber'schen Gesetzes. Vierteljahrsschrift für wissenschaftliche Philosophie, **12**(1), 1 – 31 (1888). & ***Meinong***, A. Über die Bedeutung des ***Weber***schen Gesetzes. Beiträge zur Psychologie des Vergleichens und Messens. Erster Abschnitt: Von Grössengedanken und dessen Anwendungsgebiet. Zeitschrift für Psychologie und Physiologie der Sinnesorgane, **11**, 81 – 99 (1896).]. As Chair of Philosophy at Graz, ***Meinong*** had great influence on contemporary psychology. See Bertrand ***Russell*** [***Russell***, B. Review of Alexius ***Meinong***'s Ueber die Bedeutung des Weberschen Gesetzes. Mind (New Series), **8**, 251 – 256 (1899)] for an argument against ***Meinong***'s position.

j. Note especially the 'quality objection' put forward by Johannes von ***Kries*** [***Kries***, J. von. Ueber die Messung intensiver Grössen und über das sogenannte psychophysische Gesetz. Vierteljahrsschrift für wissenschaftliche Philosophie, **4** (3), 257 – 294 (1882b)]. ***Niall*** provides an English translation: [***Kries***, J. von. Conventions of measurement in psychophysics: von ***Kries*** on the so-called psychophysical law. Spatial Vision, **9**(3), 275 – 305 (1882/1995). ] See ***Michell*** [Michell, J. Measurement in psychology: critical history of a methodological concept. New York: Cambridge University Press (1999).] for a thorough and critical account of assumptions in Fechnerian psychophysics.

k. This sophistication is evident through his career, of course: cf. ***Schrödinger***, E. Expanding universes. Cambridge at the University Press (1956). Not that such sophistication was always lacking in later color theory: the affine-geometric account of basic colorimetry was taken up by [***Schelling***, H. von. Advanced color geometry. Journal of the Optical Society of America, **45**(12), 1072 – 1079 (1955). & ***Schelling***, H. von. Concept of distance in affine geometry and its applications in theories of vision. Journal of the Optical Society of America, **46** (5), 309 – 315 (1956).]

l. There is a long history of the line-element following Helmholtz and Schrödinger: cf. [***Stiles***, W.S. Line element in colour theory: A historical review. In: J.J. Vos, L.F.C. Friele & P.L. Walraven, Eds. Color metrics: Proceedings of the Helmholtz Memorial Symposium. Soesterberg, Netherlands: AIC/Holland & Institute for Perception TNO, 1 – 25 (1972).; ***Wyszecki***, G. Über die Metrik des visuell homogenen Farbenraumes. In: International discussion of problems in color metrics. Heidelberg: Die Farbe, 100 – 108 (1955).; ***Wyszecki***, G. Recent developments on color-difference evaluations. In: J.J. Vos, L.F.C. Friele & P.L. Walraven, Eds. Color metrics: Proceedings of the Helmholtz Memorial Symposium. Soesterberg, Netherlands: AIC/Holland & Institute for Perception TNO, 339 – 379 (1972).], up to more recent accounts such as [***Raj Pant***, D. & ***Farup***, I. Riemannian formulation and comparison of color-difference formulas. Color Research and Application, **37**(6), 429 – 440 (2012).; ***Raj Pant***, D. & ***Farup***, I. Geodesic calculation of color difference formulas and comparison with the Munsell color order system. Color Research and Application, **38**(4), 259 – 266 (2013).; ***Jain***, A.K. Color distance and geodesics in color 3 space. Journal of the Optical Society of America, 62(11). 1287 – 1291 (1972).]

m. Cf. ***Ejima***, Y. and ***Takahashi***, S. Bezold-Brücke shift and nonlinearity in opponent-color process. Vision Research, **24**, 1897 – 1904 (1984).

n. See the list of Translations at the end of this volume. ***MacAdam*** was also well aware of ***Schrödinger***'s work on the brightness of colored pigments: see [***MacAdam***, D.L. The theory of the maximum visual efficiency of colored materials. Journal of the Optical Society of America, **25**, 249 – 252 (1935). & ***MacAdam***, D.L. Maximum visual efficiency of colored materials. Journal of the Optical Society of America, **25**, 361 – 367 (1935b).]

o. [Trans.] I have tried to maintain the following glossary for ***Schrödinger***'s technical vocabulary of colorimetry:

*Aichkurven*: color-matching functions (in graphic form)

*Eichfarben*: calibration colors

*Eichfunktionen*: color-mixture functions (near in meaning to 'color-matching functions')

*Eichkurven*: color-mixture curves

*Eichlichter*: calibration lights

*Eichwerte*: trichromatic coefficients (near in meaning to 'chromaticity coordinates')

*Elementarempfindungskurven*: fundamental response curves

*Farbkoordinaten*: color coordinates

*Grundempfindungen*: fundamental stimuli

For ***Wright***'s terminology, see [Wright, W.D. Researches on normal and defective colour vision. St. Louis: The C.V. Mosby Company (1947)].

p. ***Wald***, G. Blue-blindness in the normal fovea. Journal of the Optical Society of America, **57**(11), 1289 – 1303 (1967).

q. ***Jameson***, K.A., ***Highnote***, S.M. & ***Wasserman***, L.M. Richer color experience in observers with multiple photopigment opsin genes. Psychonomic Bulletin & Review, **8**(2), June, 244 – 261 (2001).