What is a Symbol?

Valerie Allen

Abstract Focusing my discussion on the sixteenth and seventeenth centuries, I argue that the symbolic notation under development at the time reveals connections with rhetorical and poetic aesthetics. In the first section, I show how the mathematical strategies that notation facilitated rely on prudential rhetoric's sense of the opportune moment, also known as *kairos* or decorum. In the second, I show how the necessary balance within notation between compression and lucidity similarly relies on an aesthetic judgment that is essentially prudential. In the last section, I show how notation's ability to accommodate disjunctive values within a general symbol corresponds to similar capabilities in the poetic symbol. Through etymological analysis of the polysemy of its terminology, I demonstrate the philological orientation of early modern algebra.

Keywords Symbol • Algebra • Notation • Aesthetics

1 Introduction

By "symbol" in this paper I refer to "symbolic notation," and there is long established precedent to do so, as Joseph Moxon's textbook from 1700 demonstrates: "*Symboles*, are Letters used for Numbers in Algebra" (Moxon 1700, p. 167). The quotation appears in the *Oxford English Dictionary* as one of the earliest attested usages for the definition of symbol as "a written character or mark used to represent something; a letter, figure, or sign conventionally standing for some object, process, etc." (Oxford English Dictionary Online 2017, symbol n.1, 3). Changing my essential question, however, to "What is Symbolic Notation?" subtly narrows the discussion by eliminating any seeming connection between mathematics and the aesthetics of rhetoric or poetry. Yet, as I will argue, those connections persist even when delimiting discussion to notation *stricto sensu*. Although notation

V. Allen (✉)
John Jay College CUNY, 524 West 59th Street #7.63.04NB, New York, NY, 10019, USA
e-mail: vallen@jjay.cuny.edu

M. Zack, D. Schlimm (eds.), *Research in History and Philosophy of Mathematics*, Proceedings of the Canadian Society for History and Philosophy of Mathematics
La Société Canadienne d'Histoire et de Philosophie des Mathématiques,
DOI 10.1007/978-3-319-64551-3_4

generally avoids what Leibniz calls the "countless ambiguities" of ordinary language (Bochenski 1961, pp. 274–275), an etymological examination of the English word "notation" and its associated terms reveals semantic entanglements with rhetoric that call into question the referential purity of mathematical terminology.

The correspondence itself between early modern mathematics and rhetoric is by now well established. Giovanna Cifoletti argues that "the reform of mathematics and science in the sixteenth century was first conceived of as a reform of the [rhetorical] 'art of thinking'" (Cifoletti 2006, p. 376). For her, the origins of the new algebra were "essentially philological" (Cifoletti 2006, p. 388), while for G.E.R. Lloyd, mathematical *apodeixis* developed in part from rhetorical *epideixis* (Lloyd 1990, pp. 77–78). These studies trace the shared conceptual architecture of rhetoric and mathematics. My own analysis builds on that recently made by Travis Williams, who argues that mathematical notation is "enargetic," that is, that in naming its objects, it renders them visually vivid just as does rhetorical *enargeia* (Williams 2016). A general term in classical rhetoric, *enargeia* refers to descriptions so evocative that they appear to convert the reader or listener into an eyewitness. The orator Lysias allegedly could deliver his material so graphically that the listener felt he could "see the actions which are being described going on and that he is meeting face-to-face the characters in the orator's story" (Lanham 1991, p. 64). In so achieving that effect, *enargeia* blurs fundamental category distinctions between tell and show, diegesis and mimesis, description and naming, mention and use, even rhetoric and dialectic. Rhetoric aims to persuade by any available means rather than to argue by means of logic, yet *enargeia* causes one to see what one can only hear about, consequently to know that about which one only has an opinion.

The claim that notation describes even as it names is consequential because it suggests that notation constructs rather than merely labels mathematical ideas in the same way that rhetorical expression forms thought rather than supervenes a pre-existent idea. A growing body of recent work already claims notation to be less the mere bearer of content than the means by which new knowledge or content is generated (Heeffer 2009; Heeffer and Van Dyck 2010; Rotman 2000; Serfati 2010). Major insights from semiotics, cognition theory, cognitive science, and historical epistemology together demonstrate the cognitive agency of notation (Heeffer 2015). Drawing in the following three sections from prudential rhetoric, aesthetic theory, and etymological analysis, I textualize early modern symbolic notation, thereby arguing for its creative and mediatory function.

2 "Rhetorical" Algebra

The term conventionally refers to mathematical work that is expressed in ordinary language rather than symbols. By this definition, rhetorical algebra has to be notation-free (Heeffer 2009). I argue, however, for an underlying kinship between the methods of rhetorical expression and those of calculations performed in algebraic notation, by consequence claiming that algebra can be fully notational and at the same time deeply rhetorical.

Algebraic eloquence starts with the equation, which makes possible the composition of meaningful sentences, such as "x is equal to y," because, in grammatical terms, "is" serves as the copula between subject and complement. As the sequential logic of the medieval trivium demonstrates, grammar underlies rhetoric. At the heart of the equation lies the equals sign, the notation for which was introduced by Robert Recorde in 1557, although his parallel lines, originally about one cm long (depending on line length), have since been standardized and shortened to dash-length. Although his notation was neither immediately adopted as universal standard nor recognized at the time as an important advance in symbolic representation (Cajori 1928, pp. 297–298), the confusing variety at the time of various symbols for equality indexes the importance of his innovation. When Recorde introduced the notation, he valued it merely as a labor-saving device. "And to avoid the tedious repetition of these words 'is equal to' I will set as I do often in work use, a pair of parallels" (Recorde 2010, p. 242). He could just have easily written out in full "is equal to" (or Latin *æquatur*). But in retrospect, we appreciate the significance of the change. Michel Serfati observes how the equals sign "made it impossible to maintain the syntax of natural language in the text. In effect, a new symbolism expressing ideal interchangeability succeeded the predicate structure of rhetorical expression" (Serfati 2010, p. 108). (Gottlob Frege also noted how mathematical logic unyoked itself from the subject—predicate structure of traditional grammar (Frege 1970, p. 7).) There is reason to think that Thomas Harriot, who used Recorde's notation, understood its symbolic force in a way its inventor did not (Tanner 1962, p. 166). Recorde's example at the point when he introduces the equals sign, namely, "14 root plus 15 is equal to 71" (which we would write as $14x+15=71$), possesses "$14x+15$" as its subject in terms of traditional grammar and "is equal to 71" as its predicate. In terms of symbolic notation, the replacement of the words "is equal to" by the sign "$=$" flattens the ontological hierarchy of the sentence and bestows equal explanatory value on the phrases "$14x+15$" and "71." Thus flattened, the statement now runs in either direction, where traditional grammar allows us to move only from left to right, from subject to predicate. In *Alice's Adventures in Wonderland*, Lewis Carroll draws attention to this distinction between the grammars of ordinary and symbolic language when the Mad Hatter objects to Alice's claim that meaning what she says is the same thing as saying what she means (Carroll 2013, p. 53).

> "Not the same thing a bit!" said the Hatter. "You might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"

By commuting in either direction, the equation also undoes the logical antecedence of the grammatical subject to its predicate. Prior to (and even after) Recorde's innovation, operations on a sum such as "14 root plus 15 is equal to 71" comprised a series of successive acts during which the "equation" was out of kilter and not properly an equation at all. Herein lies the difference between, on the one hand, moving 15 from the subject (leaving 14 root) to the predicate (leaving 56) and, on the other, adding negative 15 to both sides of the equation in a single operation. What the equals symbol enables is an idealization of the equality relation, and hence the creation of a conceptual object that by definition is always in balance, despite the fact that while we manipulate equations we do so in a succession of operations distributed across real time. Notation invites us to separate the conceptual from the temporal, experiential, and mechanical domains, perhaps making it all the harder to think of the ideas it represents as being constituted in and by language.

In the equation proper, then, the terms of the mathematical sentence interchange with each other, something that could not happen in traditional subject—predicate grammar. Temporarily substituting a simpler term for a compound concept (as in u-substitution in calculus) "remained an inconceivable operation for the medieval rhetorical writing of mathematics" (Serfati 2010, p. 117). Yet even as notation appears to distance mathematical reasoning from verbal discourse, its kinship with rhetoric emerges in the following ways.

Symbolic notation enables the solution of mathematical problems by using strategies that are in essence rhetorical. It goes almost without saying that a solution often depends on a trivial manipulation that nonetheless generates non-trivial results, as, for example, when we multiply $\lim x\ 0[(1 - \cos x)/x]$ by its conjugate to determine the limit at 0. The value of the expression does not alter and the dilated language adds nothing but tautology and circumlocution yet it changes the form in such a way that new meaning emerges. Rhetorical manipulation of language, especially in the domain of forensic oratory, achieves similar ends. Rhetorical tropes subtly "turn" (as the etymology of "trope" implies) the significance of words while rhetorical figures change the form of words in order to emphasize them (Lanham 1991, p. 155). Both rhetorician and mathematician alike recognize the formal rules of the game by which the lexical and notational manipulations meaningfully inflect the argument.

In the five parts of classical rhetoric—invention, arrangement, style, memory, and delivery—were two techniques of style (Lat. *elocutio*, Gr. *lexis*) so fundamental that they affected the text's conceptual arrangement (Lat. *dispositio*, Gr. *taxis*) because of how substantively they manipulated content (Baltzell 1967; Faral 1924, p. 61). Amplification and abbreviation comprise categories of stylistic device rather than individual devices, and, of the two, amplification is generally the more important for early modern rhetoric because it is associated with grandeur and the high style. The importance of amplification or "surplusage" is evident throughout George Puttenham's treatise on poetic style, published as Thomas Harriot was writing his mathematical papers. One figure "fit for amplification," antitheton, restates every idea in the negative (a strategy well known to mathematicians): "Ready to *ioy* your gaines, your losses to bemone, ... Who onely bred your blisse, and neuer causd your care" (Puttenham 1589, p. 175).

Less cherished than the copiousness of amplification, rhetorical abbreviation nonetheless plays a valuable part in directing thought. Indeed, one of its main devices declares its intention in its very name: in Greek, *emphasis*, in Latin, *significatio*, which "leaves more to be suspected than has actually been asserted" (Rhetorica ad Herennium 1954, pp. 400–401). Abbreviation deploys other clipping devices such as the ablative absolute, which has been likened to algebraic brackets in the way it sets apart an idea from the main sentence into an inner clause (Harris n.d.), or zeugma, where one verb does the work of many in a sentence, Puttenham's example being, "Fellowes and friends and kinne forsooke me quite" (Puttenham 1589, p. 136). The density of the abbreviated statement defamiliarizes its conventional appearance, bracketing off the expression from prosaic fullness, attracting the mind to make sense of the compressed words, and thereby requiring one to "see" the idea anew and recognize different aspects of its structure. In the same way in mathematics, an unexpected compression arrests the reader and requires an effort to achieve recognition.

Sometimes the pith of a simplification provides the mot juste. Sometimes it is periphrastic expansion of the expression that enables the reader to proceed to a solution of the problem. The eloquent mathematician knows which algebraic tropes to use and when in order to lead the reader to that moment when persuasion meets conviction, and opinion converges on knowledge. The efficacy of notation emerges in the ability of the mathematical sentence to expand and contract as the occasion requires, the virtuosity of the mathematician in gauging the right moment to substitute. In prudential rhetoric that feel for the occasion goes by various names: *kairos*, *to prepon*, *opportunitas*, decorum (Baumlin 2002). Often mischaracterized as a preoccupation with style rather than substance, rhetoric fundamentally concerns itself with fitness, with finding the right expression for the right moment.

3 Beautiful Notation

In the sixteenth, more explicitly in the seventeenth century, symbolic notation offered a new kind of visual experience that made one read and think differently. Just as perspective painting geometrized and thus rationalized the viewing subject's position and relationship with the art object, so symbolic notation made mathematicians reassess how graphically (that is, by writing rather than by diagram) to represent conceptual objects. In an apt analogy Frege noted that as his notation is to ordinary language so the microscope (invented late C16th) is to the eye (Frege 1970, p. 6). His claim applies also to early modern symbolic notation. When Thomas Harriot, in 1609, became the first person known to view a celestial object through a telescope, he understood for himself that this "Dutch trunke" not only enabled him to do better and faster what he already could do; it also generated new images and hence enabled him to ask questions not possible hitherto (Royal Astronomical Society 1996). Symbolic notation similarly amplified his horizon of possibility. One could not simply think *with* the "great invention" of algebra, to use Cardano's phrase, but *through* it.

If notation amplifies, it also abbreviates. In the Preface to the Reader of his *Key of the Mathematics*, William Oughtred praised notation for its compressive power (Oughtred 1647; see also Pycior 1997, pp. 43–45; Williams 2016, pp. 191–192).

> Which Treatise being not written in the usuall syntheticall manner, nor with verbous expressions, but in the inventive way of Analitice, and with symboles or notes of things instead of words, seemed unto many very hard; though indeed it was but their owne diffidence, being scared by the newnesse of the delivery; and not any difficulty in the thing it selfe. For this specious and symbolical manner, neither racketh the memory with multiplicity of words, nor chargeth the phantasie with comparing and laying things together; but plainly presenteth to the eye the whole course and processe of every operation and argumentation.

Like the density of gold, the compressive power of algebraic notation raises the value of conceptual labor and makes mathematical thought precious. Thus, Oughtred, when speaking of reading the Ancient mathematicians, described a quasi-alchemical process by which he transformed base language into symbolic notation, “uncasing the Propositions and Demonstrations out of their covert of words, designed them in notes and species appearing to the very eye” (Oughtred 1647; Pycior 1997, p. 46). For a culture trained to associate eloquence with copiousness, the marvel of notation lay in its power to contract thought. Where to admirers of notation, its density increased its preciousness, to its detractors, the compression rendered notation doubly inefficient because it required the mind to work twice as hard, as Thomas Hobbes observed at the end of his fifth lesson to the Savilian Professors (Hobbes 1656, p. 54):

> I shall also add, that Symboles, though they shorten the writing, yet they do not make the Reader understand it sooner than if it were written in words. For the conception of the Lines and Figures (without which a man learneth nothing) must proceed from words, either spoken or thought upon. So that there is a double labour of the mind, one to reduce your Symboles to words (which are also Symboles) another to attend to the Ideas which they signifie.... And thus having examined your panier of Mathematiques, and finding in it no knowledge neither of Quantity, nor of Measure, nor of Proportion, nor of Time, nor of Motion, nor of any thing, but only of certain Characters, as if a Hen had been scraping there...

Precisely because notation adds another degree of separation from the concept under discussion, Hobbes, in his third lesson, found it to be no more than an arid formalism, not beautiful at all (Hobbes 1656, p. 23).

> Symboles are poor unhandsome (though necessary) scaffolds of Demonstration; and ought no more to appear in publique, than the most deformed necessary business which you do in your Chambers.

Love it or hate it, notation indisputably possesses brevity as its chief characteristic. Its density must thus be offset with precision or what Frege calls “perspicuity” (Übersichtlichkeit) (Frege 1879, pp. vii, 9; Frege 1970, pp. 8, 17; Schlimm 2017, p. 29). Notation must be abbreviated enough to see the statement as a complete and meaningful expression yet amplified enough to avoid ambiguity and to make demonstrable the sequence of its logical steps.

Finding the optimal point between brevity and perspicuity is not a requirement exclusive to notation; it is fundamental to aesthetics, as Aristotle observed in his *Poetics* when considering the ideal duration of a tragedy. The right duration should be long enough for the full sequence of causation to emerge yet short enough to apprehend as a single, complex action, as an aesthetic totality. Thus, to take *Oedipus the King* as the *locus classicus* of this principle (Aristotle being a great admirer of Sophocles), Oedipus's tragedy carries the impact it does because the action of the play is, on the one hand, sustained enough for him to see how discrete facts of his past are in fact causally connected yet, on the other, compressed enough to comprehend the truth in one terrible moment of recognition (*anagnoresis*). Recognition is a cognitive act that depends on the play's optimal duration, on criteria that are essentially aesthetic. To use a scholastic distinction, where the play's brevity enables understanding of the action's quiddity or "whatness," its magnitude enables understanding of its haecceity or "thisness." Sufficient distance enables one to apprehend what this action *is*, sufficient proximity to apprehend what *this* action is.

Aristotle's word for what I have termed "duration" is *megethos* [magnitude, size, greatness] (Aristotle 1995, 1449b, l. 24). Of course, there is no quantitative formula that could determine *megethos* in this context because the duration of the play time is neither exact nor constant and requires the same kind of qualitative judgment needed in prudential rhetoric, which estimates the right word for the right occasion. Ancient Greek often used different words to distinguish between the qualitative and quantitative aspects of a thing. Thus, although *kairos* and *chronos* both mean "time," the former refers to the opportune moment while the latter refers to sequential, measurable time. Compare *bios* and *zoe*, both meaning life, where *bios* means purposive existence and *zoe* bare existence (Kerenyi 1962, pp. 12–13). Unlike these linguistic couplets, the single word *megethos* does double duty to refer to qualitative and quantitative size. In Aristotle's *Poetics* it refers to qualitative judgment; in his *Metaphysics*, *megethos* means measurable quantity, whether length (*mekos*) in one dimension, breadth (*planos*) in two, or depth (*bathos*) in three (Aristotle 1933, 1020a, ll. 10–13). Early English shares with Greek this fluid relation between measurement and ethos. Middle English *mesure* means "calculation," "just enough," and "moderation"; that is, it encompasses in one word the purely quantitative, the capacity for sufficiency to the occasion, and the ability to maintain balance and observe restraint (Oxford English Dictionary Online 2017, measure (n.); Middle English Dictionary 2001, mesure (n.)). To *mesure* a thing meant to capture its ethical nature just as surely as it meant to take its vital statistics.

In the classical and medieval lexicon of quantification, the measurable and the beautiful (or in Aristotle's case, the tragic) mutually constitute each other. The three conditions of beauty, claims Aquinas are *integritas* (totality, wholeness), *proportio* (internal harmony between the parts), and *claritas* (radiance, brightness, luminosity) (Aquinas 1964, Ia 39.8, ad 1). In the correspondences I have been tracking, rhetorical abbreviation and notational brevity enable apprehension of a thing in its totality, of its *integritas* and *quidditas*; rhetorical amplification and notational perspicuity enable apprehension of a thing's internal organization, of its *proportio* and *haecceitas*. As for *claritas*, Aquinas's third condition of beauty, it relies on an epistemology, even a theology, of light that derives from Neoplatonist thought. As light emits energy, it diffuses itself and is radiant (Tatarkiewicz 1970, p. 29). Like light, beauty is self-evident (note the verb *videre* in "evident"). It is not something one needs to be persuaded about in the same way that no one argues that two and two equal four; they just do and either you “see” it or you don’t. The *quod erat demonstrandum* of a proof shows that it has reached a point when inferences are no longer needed and the truth of the proposition proclaims itself. In the same way radiant beauty is deictic. It shows itself as an objective reality, it “evidences” itself, and the only proper response is, “Yes, I see” (Allen 2016, p. 151). To its admirers, notation’s density and ductility made it shine like bright gold, beautiful indeed.

Despite its kinship with rhetoric, algebraic notation developed in early modern England as a form of writing that eschewed, as Seth Ward put it, the “confusion or perturbatio[n] of the fancy made by words” (Pycior 1997, p. 115). Time and again, its virtue was trumpeted as the ability to communicate without the equivocations and amphibologies of ordinary language. The measure of such confidence in notation’s promise is given by John Wilkins’s attempt to bypass the constraints of all dialects through devising a concept-script in which symbols stood not for words but for “*things* and *notions*” (Wilkins 1668, p. 13). Yet, at the same time, thinkers of the day preoccupied themselves equally with rationalizing, methodizing, and “polishing” the English language into an idiom apt for scientific inquiry (Sprat 1667, p. 41). As interested in the logical systematicity of ordinary language as of mathematics, John Wallis, a talented cryptographer (Pycior 1997, p. 105), wrote a *Grammar of the English Language*. The capacity of English to name without dissembling and to stay close to concrete things made it the language of choice for empirical pursuit. By virtue of its plainness, English served as the matrix or mother tongue in which notation’s precious density of thought could gestate. All the more reason, then, to pay attention to the words they used to describe this great invention of algebra.

4 The Words They Used

In this last section, I analyze the semantic histories of the English keywords used by the early modern algebraists to develop a specialized terminology for their mathematical pasigraphy: *specious*, *symbole*, and *note* or *notation*. My etymological analysis does not aim to recover some originary meaning and enthrone it as the word’s proper signification. Nor does it offer a selection of definitions from which any can be chosen at will to change the meaning of a word. Rather it troubles the assumption that notation merely behaves like a well-behaved tool: “By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems” (Whitehead 1948, p. 39). Etymology makes opaque the transparency of a linguistic sign that appears to adequate one word with one thing. It sounds out the multiple, sometimes disjunctive meanings that reverberate in an apparently univocal word.

Specious: Oughtred, quoted above, refers to the “specious and symbolicall manner” of notation. The word today usually has pejorative meaning, so it jars to encounter it used in a positive sense. By it, Oughtred means the habit of using letters rather than numbers: “literal” quantities as distinct from “numerical” quantities (Oxford English Dictionary Online 2017, literal (adj.), I.4). In this context, “literal” and “specious” both refer to algebraic notation. The distinctive occurrence of “specious” originates with François Viète, who uses it in its logical sense to distinguish between species and individual (Oxford English Dictionary Online 2017, species (n.) II.8.b). Early in his introduction to the *Analytic Art*, Viète describes “symbolic logistic as [a logistic] that can be expressed through species or forms” (*Speciosa [logistice] quae per species seu formas exhibitur*) (Viète 2006, p. 17). The logical meaning of *species* as “type” is clear from his context but the word had a broad associative range in Latin. More generally *speciosa* meant “beautiful” and *species* “appearance” or “outward form.” It was within this non-logical semantic field that the words *specious* and *species* first existed in English. Oughtred took a word that in English basically meant “beautiful” and gave it technical application.

Deriving from Latin *species* (appearance), which itself derived from *specere* (to behold), the English word “specious,” first attested in the early fifteenth century, meant “pleasing appearance” (Oxford English Dictionary Online 2017), specious (adj.) 1.a; *Middle English Dictionary* specious (adj.)). Within a century, it meant bright or showy appearance (as in a bird’s plumage); within another, it had distinguished itself from the “real,” in a contrast between appearance and reality (Oxford English Dictionary Online 2017, specious (adj.) 1.b and 2). Shortly thereafter, within the seventeenth century, it could imply fair-seemingness in the pejorative sense, hence fallaciousness (Oxford English Dictionary Online 2017, specious (adj.) 3, 4). The algebraic sense, obsolete today, constitutes the adjectival form of the logical term *species*, and it is arresting that the first attested use of the word to mean “insincere” or “fallacious” should occur in the *Leviathan* of philosopher Thomas Hobbes (Oxford English Dictionary Online 2017, specious (adj.) 3a, 3b; Hobbes 1651, pp. 73, 110). Hobbes’ opinion of symbolic notation was famously low, and it must have seemed to him a most felicitous double entendre to extend the meaning of the word “specious” from “apparent” to “falsely plausible.” In one of his many sideswipes at the mathematician John Wallis, who was a lover of symbolic notation, Hobbes wrote: “And for that [treatise] of your Conic Sections, it is so covered over with the scab of symbols, that I had not the patience to examine whether it be well or ill demonstrated” (Hobbes 1656, p. 316).

Yet to Wallis and other fellow-admirers, Viète’s choice of *species* (meaning “type”) to refer to general number offered an incisive way to combine “within the symbolic system two concepts hitherto considered as opposites, the arbitrary and the fixed, or the one and the multiple or even, maybe more significantly, the unspecified and the singular” (Serfati 2010, p. 109). They intuited that notation could hold together within its tensile economy ideas that, when unfolded into ordinary language, seemed to contradict each other. Such notation must indeed have seemed *speciosa* (beautiful) and pleasing to the eye.

Symbol: In the passage quoted, Oughtred uses the terms “specious” and “symbolic(al)” interchangeably. Yet the meaning of “symbol” is by no means self-evident, the word’s semantic trajectory being fitful and indirect. Deriving from the Greek verb, *symballein*, “to throw together,” the *symbolon* functioned like a tally stick except that coins were broken into half. From this act of matching fragments to make a connection, the early Christian theologian Cyprian applied the word to the baptismal creed, a symbol that comprised the matching half and thus the distinguishing mark of a Christian. One of the word's earliest meanings in English was consequently "a formal authoritative statement or summary of the religious belief … of a particular church or sect; a creed or confession of faith," so one could speak of "the Symbole of the Apostles," meaning the Apostles' Creed (Oxford English Dictionary Online 2017, symbol (n.1) 1a). From the association with pithiness or succinctness developed in the sixteenth century a more general meaning of symbol as a formula, maxim, motto, or synopsis. By 1656, one could speak of a symbol as "a short and intricate riddle or sentence" (Oxford English Dictionary Online 2017, symbol (n.1) 1b). For Oughtred, then, writing in the 1640s, a symbol bore connotations of hermeneutic compression, and offered an apt enough name for notation.

A second sense of the word, independent but connected, emerged in the late sixteenth century (Oxford English Dictionary Online 2017, symbol (n.1) 2a):

> Something that stands for, represents, or denotes something else (not by exact resemblance, but by vague suggestion, or by some accidental or conventional relation); esp. a material object representing or taken to represent something immaterial or abstract, as a being, idea, quality, or condition; a representative or typical figure, sign, or token.

Key here is the idea of substitution of one entity for another. The first attested occurrence of "symbol" as an English word with substitutionary meaning comes in poetry, in Edmund Spenser's *Faerie Queene*. The knight Sir Guyon, who represents temperance in Spenser's elaborate allegory, encounters a dying woman, who has stabbed herself from grief at being betrayed, and who daubs her infant son in her blood as a "pledge" to her unblemished chastity (Spenser 2013, II.i.37). Guyon vows to avenge her death and tries to wash the infant in the nearby fountain but the stains remain. The Palmer (or pilgrim) who accompanies Guyon explains why. The waters are pure and will not permit admixture with the spilt blood, so the infant must remain with bloodied hands as a "Sacred Symbole" of his mother's innocence and reminder to avenge her (Spenser 2013, II.ii.10). Resonant with allusion to Pilate's hand-washing declaration of innocence, this enigmatic passage is difficult to interpret. Rejected by the pure water, the blood implies impurity, yet the Palmer explicitly interprets the blood as a mark of innocence, hence purity (as well as a goad to vengeance). As a symbol, the bloody hands become overdetermined, their multiple meanings—impurity, innocence, and a reminder of vengeance—potentially contradicting each other. "The symbol has condensed, collapsing within itself disjunctive and identificatory understandings of the sign, forcing the reader's attention onto its 'mystery'" (Tribble 1996, p. 32).

At this point algebraic symbolism, which deals in precision, seems to part company with poetic symbolism, which deals in polysemy. Yet Viète's "specious" notation shares with poetry the power to hold together seeming disjunctives in a "dialectic," as Serfati describes it. Values that are both arbitrary and fixed in notation "appear as contradictions in natural language" (Serfati 2010, p. 110). The generality of an algebraic expression enabled an equation's values to be true at all times, unlike the rule of false position, which muddled its way to a solution by means of determinate values that were technically incorrect. In algebra, "the variable is always correct until and through the moment when it can be replaced by a determinate number" (Williams 2016, p. 200). Just so, the poetic symbol holds together meanings that, in the contingent points of narrative, denote variously. In this way symbolic expression denotes differently from pedestrian language. Ideas that contradict in daily vernacular become coherent and fruitful within the symbol's referential universe.

Notation: The word "note" seemed to promise some freedom from complicity with poetics and aesthetics. Compact and cryptic, a note had the advantage of being distinct from letters, which mimicked sounds and thus individual languages, as John Wilkins observed: "Besides this common way of Writing by the ordinary *Letters*, the Ancients have sometimes used to communicate by other *Notes*, which were either for *Secrecy*, or *Brevity*" (Wilkins 1668, p. 12). (Wilkins, by the way, preferred the term "character" for the concept-script he developed.) Leibniz's Latin term for what we call symbol was *nota* (Bochenski 1961, pp. 274–275). Present as early as the period of Old English (Oxford English Dictionary Online 2017, note (n.2) I.1.a), "note" from the outset suggested a distinguishing mark other than an alphabetic one.

From "note" derives "notation." Its mathematical meaning, namely "the process or method of representing numbers, quantities, relations, etc., by a set or system of signs or symbols, for the purpose of record or analysis," only dates from the early eighteenth century, and applies subsequently to music, dance, chemistry, logic, chess, linguistics, and so on (Oxford English Dictionary Online 2017, notation (n.) 6). "Notation" as a mathematical term of art constitutes a second-order nominalization from the concrete to the abstract, as if to claim the term exclusively for a specialized concept-script, but the word has an older history in English, its earliest occurrence, from the mid-sixteenth century, coming in a treatise of grammar and rhetoric (Sherry 1555, pp. xxv–xxvi):

> By notacion, that is, when by certain markes, and signes we do describe any thing: as, if a man understandyng anger, wil saye it is the boylyng of the minde, which bringeth palenes unto the countenaunce, burning to the iyes, tre[m]bling to the partes of the body. Also, when for the proper name we put the country, the sect, or some great act: as. For Virgil, the poet of Mantua. For Aristotle, the prince of peripateticall schole. For Scipio, the destroyer of Carthage and Numance.

The word began its life in English as a rhetorical trope, a name for the kind of circumlocution or "speaking around" that captured in description the nature of a thing or person. In the rhetorical treatises, Latin *notatio* went hand in glove with *effictio*, together constituting a complete *descriptio* of persons. Where *effictio* dwelt on the corporeal features of the person described, *notatio* sketched the quintessential character of a person, and was a kind of *ethopoeia* (character description). It delineated a person in pithy, well-chosen details, omitting anything extraneous. All of these rhetorical terms, *notatio* included, classify as kinds of *enargeia* that selectively represent the object of description. The more vividly they do their job, the more they unravel the distinction between description and naming. Notation as a rhetorical term already came associated in the minds of these classically trained, early modern mathematicians with a descriptive act that distilled the very essence of the object of reference into the mark of signification. It is ironic then that when John Wallis refers to the "Notes or *Symbols*" of algebra saving one from a "multitude of Words, and long Periphrases" (Pycior 1997, p. 113), he had already mentally sundered algebraic notation from rhetorical *notatio* and forgotten their shared genealogy.

5 Conclusion

Like Theophrastus's character sketches, notation functions as a kind of outline or diagram of the concept. Where Hobbes saw only random hen scrapings, these early modern mathematicians saw marks that could illustrate rather than merely denote a concept. When Recorde introduces his equals sign to save himself tedium he describes his parallels "as gemowe [twin, from Latin *gemini*] lines of one length, thus = because no 2 things can be more equal" (Recorde 2010, p. 242). Thomas Harriot appears to have built his notation for inequality on the foundation of Recorde's parallels (Tanner 1962, pp. 166–167). Although not all mathematical *notes* were motivated in this way, their creators held these symbols to standards of decorum and beauty just as poets and rhetoricians did their words. Compact where words tended to be copious, early modern mathematical notation observed its own aesthetic criteria, paying attention to the spatial management and visual logic of the page. In doing so, symbolic notation positioned mathematical objects and operations in a textual practice as richly polysemous as poetry, even if its squiggles only looked, to Hobbes and his ilk, like hen scrapings.

References

Allen, Valerie (2016) "Chaucer and the Poetics of Gold", in Beatrice Fannon (ed.) *New Casebooks: Reading Medieval Literature*, London: Palgrave, pp. 144—159.

Aquinas, Thomas, St. (1964-1975) *Summa Theologiae*, ed. and trans. Thomas Gilby, Oxford: Blackfriars.

Aristotle (1933, repr. 1989) *Metaphysics, Books I—IX*, ed. and trans. Hugh Tredennick, Cambridge, MA: Harvard University Press.

Aristotle (1995) *Poetics*, ed. and trans. Stephen Halliwell, Cambridge, MA: Harvard University Press.

Baltzell, Jane (April 1967) "Rhetorical 'Amplification' and 'Abbreviation' and the Structure of Medieval Narrative" *Pacific Coast Philology* 2, pp. 32—39.

Baumlin, James S. (2002) "Ciceronian Decorum and the Temporalities of Renaissance Rhetoric", in Phillip Sipiora and James S. Baumlin (eds) *Rhetoric and Kairos: Essays in History, Theory, and Praxis*, Albany: State University of New York Press, pp. 138-164.

Bochenski, I.M. (1961) *A History of Formal Logic*, rev. and trans. Ivo Thomas, Indiana: Notre Dame University Press.

Cajori, Florian (1928—1929), *A History of Mathematical Notations*. 2 vols. Chicago: Open Court, repr. in one vol. (2012) Mineola, NY: Dover.

Carroll, Lewis (2013) *Alice in Wonderland*, ed. Donald J. Gray, 3rd edn, New York and London: Norton.

Cifoletti, Giovanna (2006) "Mathematics and Rhetoric. Introduction", *Early Science and Medicine* 11.4, 369—389.

Faral, Edmond (1924, repr. 1962) *Les arts poétiques du XIIe et du XIIIe siècle: recherches et documents sur la technique littéraire du moyen âge*, Paris: Champion.

Frege, Gottlob (1879) *Begriffsschrift eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Halle: Louis Nebert.

Frege, Gottlob (1970) *Begriffsschrift* (English), in *Frege and Gödel: Two Fundamental Texts in Mathematical Logic*, ed. Jean van Heijenoort, Cambridge MA: Harvard University Press.

Harris, William. http://www.orbilat.com/Languages/Latin/Alternative_Grammars/Harris_Grammar/Latin-Harris_17.html

Heeffer, Albrecht (2009) "On the Nature and Origin of Algebraic Symbolism", in Bart van Kerkhove (ed.), *New Perspectives on Mathematical Practices: Essays in Philosophy and History of Mathematics*, Singapore, World Scientific Publishing, pp. 1—21.

Heeffer, Albrecht and Maarten Van Dyck, eds, (2010) *Philosophical Aspects of Symbolic Reasoning in Early Modern Mathematics*, Studies in Logic 26, London: College Publications, King's College.

Heeffer, Albrecht (2015) "On the Historiography of Mathematical Notations", in Douglas R. Furman and Albrecht Heeffer (eds), *Writing the History of Mathematical Notation 1483-1700* by Sister Mark Leontius Schulte, Boston, MA: Docent Press, pp. 3—43.

Hobbes, Thomas (1651) *Leviathan, or The Matter, Form, and Power of a Common-Wealth Ecclesiastical and Civil*, London: printed for Andrew Crooke, at the Green Dragon in St. Paul's Church-yard.

Hobbes, Thomas (1656) *Six Lessons to the Professors of the Mathematiques*, London: printed by J.M. for Andrew Crook, at the Green-Dragon in Paul's Church-yard.

Kerenyi, C. (1962) *The Religion of the Greeks and Romans*, trans. Christopher Holme, New York: Dutton.

Lanham, Richard A. (1991) *A Handlist of Rhetorical Terms*, 2nd edn, Berkeley: University of California Press.

Lloyd, G.E.R. (1990) *Demystifying Mentalities*, Cambridge: Cambridge University Press.

Middle English Dictionary (2001), University of Michigan http://quod.lib.umich.edu/m/med/

Moxon, Joseph (1700) *Mathematicks made easie. or, A mathematical dictionary explaining the terms of art and difficult phrases used in arithmetick, geometry, astronomy, astrology, and other mathematical sciences*, 3rd edn, London: printed for J. Moxon at the *Atlas* in *Warwick-Lane* and Thomas Tuttell at the *King's Arms* and *Globe* at *Charing-Cross*.

Oughtred, William (1647) *The Key of the Mathematicks New Forged and Filed: Together with a Treatise of the Resolution of all kinde of Affected Æquations in Numbers*, London: printed by Tho. Harper for Rich. Whitaker.

Oxford English Dictionary Online (2017), Oxford: Oxford University Press.

Puttenham, George (1589) *The Arte of English Poesie*, London: printed by Richard Field, dwelling in the black-Friers, neere Ludgate.

Pycior, Helena M. (1997) *Symbols, Impossible Numbers, and Geometric Entanglements: British Algebra through the Commentaries on Newton's Universal Arithmetick*, Cambridge: Cambridge University Press.

Recorde, Robert (2010) *The Whetstone of Witte, A Facsimile*, ed. Thomas Gordon Roberts, Mickleover, Derby: Renascent Books.

Rhetorica ad Herennium (1954) ed. and trans. Harry Caplan, Cambridge, MA: Harvard University Press.

Rotman, Brian (2000) *Mathematics as Sign: Writing, Imagining, Counting*, Stanford: Stanford University Press.

Royal Astronomical Society (1996-2015) https://www.ras.org.uk/news-and-press/news-archive/68-news2009/1641-celebrating-thomas-harriot-the-worlds-first-telescopic-astronomer-ras-pn-0947

Schlimm, Dirk (Published online. 2017) "On Frege's *Begriffsschrift* Notation for Propositional Logic: Design Principles and Trade-offs", *History and Philosophy of Logic*, 1—34.

Serfati, Michel (2010) "Symbolic revolution, scientific revolution: mathematical and philosophical aspects", in Heeffer and Van Dyck, eds, *Philosophical Aspects*, pp. 103—22.

Sprat, Thomas (1667) *The History of the Royal-Society of London for the Improving of Natural Knowledge*, London: printed by T.R. for J. Martyn and J. Allestry.

Spenser, Edmund (2013) *The Faerie Queene*, rev. 2nd edn, ed. A.C. Hamilton, Hiroshi Yamashita, and Toshiyuki Suzuki, London and New York: Routledge.

Sherry, Richard (1555) *A Treatise of the figures of Grammer and Rhetorike*, profitable for al that be studious of eloquence, and in especiall for suche as in grammer scholes doe reade moste eloquente poetes and oratours. London: in aedibus Ricardi Totelli.

Tanner, R.C.H. (1962 Dec.) "On the Role of Equality and Inequality in the History of Mathematics" *British Journal for the History of Science* 1.2, pp. 159—69.

Tatarkiewicz, Wladyslaw (1970) *History of Aesthetics, Vol. II: Medieval Aesthetics*, The Hague and Paris: Mouton.

Tribble, Evelyn B. (1996) "The Partial Sign: Spenser and the Sixteenth-Century Crisis of Semiotics," in Douglas F. Rutledge (ed.), *Ceremony and Text in the Renaissance* (Cranbury, NJ: Associated University Presses), pp. 23—34.

Viète, François (2006) *The Analytic Art*, trans. T. Richard Witmer, Mineola, NY: Dover.

Whitehead, Alfred North (1948, repr. 1972) *An Introduction to Mathematics*, Oxford: Oxford University Press.

Wilkins, John (1668) *An Essay towards a Real Character, and a Philosophical Language*, London: printed for Sa. Gellibrand and John Martyn.

Williams, Travis D. (2016) "Mathematical *Enargeia*: The Rhetoric of Early Modern Mathematical Notation", *Rhetorica* 34(2), 163—211.