General Relativity, Black Holes and Planck Stars

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After the formulation of Special Relativity in 1905, Albert Einstein began looking for an extension of the theory, since Special Relativity has two limits: it favors a particular class of reference frames, the inertial frames (reference frames without acceleration, for which the law of inertia holds) and it cannot describe gravitational effects. Einstein worked 10 years, from 1905 to 1916, in order to obtain a satisfying solution which was able to solve these problems and this led to the formulation of General Relativity.

There is a link between these two limits of special relativity: in fact it is very hard to describe the effects of gravity without considering accelerated reference frames. This connection is one of the foundations of the theory; physically it corresponds to the equivalence between inertial mass from the Second Law of dynamics:

$$\vec{F} = m\vec{a} \tag{1}$$

and the gravitational mass, from the Universal Law of Gravitation

$$\vec{F} = G\frac{mM}{r^2}\hat{r} \tag{2}$$

which causes, such as Galileo Galilei noted in XVII century, that all bodies fall with the same acceleration. Einstein's argument was that this equality between the two forms in Newtonian dynamincs, was lacking theoretical justification, while one of the main goals of General Relativity is to explain this equivalence between inertia and gravitation, unifying the two concepts.

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Another important element at the base of General Relativity is the study of the relationship between physics and geometry: from the traditional idea of a separate geometry from any physical content, used as a mere mathematical tool, we arrive at the concept of geometry closely linked to physics: the geometry becomes, in fact, a physical object.

The Universe is not only composed by matter but is both matter and geometry. From this statement follows that the geometry, or spacetime, must have its own dynamics described by a field; in the language of General Relativity, this translates to the metric tensor $g_{\mu\nu}(x)$ which interacts with both matter and energy.

These are the concepts of General Relativity. As in the case of Special Relativity, we can appreciate Einstein's genius[1]: with great simplicity, unification was managed since general theory of spacetime became the theory of gravitation itself.

As in any physical theory, General Relativity is described by a fundamental equation, the Einstein Field Equation, which is one principle of the theory itself. Formulated by Einstein himself, the theory is based on the ideas that we have discussed previously. The equation reads:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{3}$$

where

- $R_{\mu\nu}$ is the Ricci tensor: it is one of the possible contraction of a more general tensor, Riemann tensor $R^{\sigma}_{\rho\mu\nu}$; it contains the dynamical effects of the gravitational field.
- $g_{\mu\nu}$ is the metric tensor: it is the fundamental object of theory, as it has the role of the "potential" in General Relativity.
- R is the scalar curvature; simply put, it describes the trace of the Ricci tensor (in General Relativity it is the lagrangian density for the gravitational field).
- $T_{\mu\nu}$ is the stress−energy tensor which contains the momentum and energy density of the gravitational field; basically put, it is our source for gravity.

This equation, despite appearing very compact and elegant, is not so simple to understand let alone to solve. In a few words, with this equation you can deduct the geometric structure of spacetime ($g_{\mu\nu}$) once the distribution tensor of matter and energy is known($T_{\mu\nu}$): the term $R_{\mu\nu} - 1/2 g_{\mu\nu} R$ contains information about the curvature of the spacetime.

An object such like our sun curves spacetime around itself; this means that the Earth isn't so much attracted by a central force (as Newton stated), as it is forced to follow the curvature imposed by the presence of the sun: we can understand this through an analogy, for example by placing an apple on top of a stretched out bedsheet (so as to represent a plain); experience tells us that the bedsheet will curve because of it's presence: the apple warps our plain therefore any other small object, placed

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[1] It is important to say that Einstein was not the only one working on these ideas, other contributions from others were crucial for General Relativity.

near the region of the bedsheet deformed by the apple, would be inevitably attracted toward the it.

We can conclude that not only space is subject to curvature, but also spacetime. This led to Einstein's prediction that time flows slower as we approach a source of a gravitational field: suppose we examin the case of two twin brothers, one who lives on Earth while the other one on a space station; the one that lives on Earth ages slower with respect to his brother.

Perhaps the most fascinating objects of General Relativity, which have always intrigued human imagination, are black holes. So, what is a black hole exactly? Massive stars, in the last stage of their evolution, stop producing energy and then collapse because of their self-attractive gravity. The radius of a star with mass M that is collapsing progressively decreases until it reaches the value of the so called Schwarzshild Radius

$$R_S = \frac{2GM}{c^2} \tag{4}$$

and the star becomes a black hole. Gravity, due to enormously concentrated mass, forces any object to move toward the black hole until the object meets a surface called event horizon of the black hole. This surface is characterized precisely by the Schwarzshild Radius of the black hole. An object that passes through the event horizon cannot go back, not even light can escape. We know that the escape velocity v_f from a mass M is

$$v_f = \sqrt{\frac{GM}{R}} \tag{5}$$

so if we were to cross the event horizon, then $R < R_S$; this means that the escape velocity under these conditions would have to be greater than the speed of light: $v_f > c$; so on an experimental level, direct measurements are fairly difficult to obtain.

Today's Physicists try to study the possible effects of a black hole on its own surrounding environment. Recently the existence of gravitational waves was confirmed, another wonderful result of General Relativity. Basically, as electromagnetic waves are emitted by electrically charged sources, also matter itself can emit radiation, known now as gravitational radiation, in a very subtle way; In analogy to electromagnetic radiation, which is an electric or magnetic (or both) field that propagates in spacetime, gravitational radiation is a perturbation of the spacetime that propagates in itself; we can imagine this through a simple example: a pebble falling the surface of a body of water (spacetime) causes a series of the ripples (gravitational waves). In february 2015, the LIGO experiment confirmed the existence of gravitational waves, where the source consists of two black holes that merged.

Black holes, by a theoretical point of view, are very fascinating objects. They allow us to understand better gravity and they constitute a challenge to our ideas about the description of the fundamental laws of physics. One of the principles of the quantum mechanics is the assumption that in a system the information is conserved i.e. in our system all the information that we need to describe the initial state of the system, are contained in the system itself (in the language of quantum mechanics, physicists say that the time evolution of a system is described by a unitary transformation). During the formation of a black hole, the information contained in the collapsing matter is confined inside its event horizon. A black hole seen from the outside can be born by the union of stars or it can be formed by an aggregation with of much smaller objects, such as atoms or molecules. The two situations are completely indistinguishable: the information about the initial state of the black hole is not known for an observer outside the horizon.

By a classical point of view it is not a contradictory fact, the black hole lives forever and all the information contained in it are preserved but are also completely inaccessible. The situation is quite different if we consider quantum effects. As Stephen Hawking showed, a black hole can "evaporate" through the emission of thermal radiation from the event horizon. In proximity to the event horizon, quantum fluctuations can create a pair of particles and one of these is capable of escaping the black hole. It can then be revealed as radiation, while the other one is captured by the black hole. Thanks to this mechanism, a black hole loses mass which is released as radiation and and for this reason a black hole will evaporate. The final result of this process is the complete disappearance of the black hole but also consequently all the information contained in the black hole are destined to disappear, in complete contradiction with the principles of quantum mechanics which we discussed earlier.

One important thing to take into account is that Hawking Radiation is an effect of quantum mechanics in a classical gravitational field (described by General Relativity), therefore it is an approximation known as semi-classical: this approximation studies a physical problem partially in a classical way using gravitational interaction (a classical spacetime background) while on the hand, in a quantum mechanical manner since pair production due to the quantum fluctuations is obviously a quantum effect.

Hawking Radiation gives us a clear indication that general relativity is a model that cannot work at a fundamental level. We need a much more fundamental theory capable of overcoming the formal difficulties of General Relativity and be able to reconcile gravitation (the theory of spacetime) with quantum mechanics: a quantum theory of gravitation. Over the years, theoretical physicists proposed several theories (String Theory, Loop Quantum Gravity,Asymptotically Safe Gravity, Causal Sets, etc.) and each of these theories has some very interesting aspects but also all of them have unclear problems to solve.

An interesting answer to this information paradox is proposed by Loop Quantum Gravity. Black Holes could reach a new evolutional state of their life where a "quantum gravitational" pressure generated by quantum fluctuations of spacetime counterbalances the collapse of matter and causing it to become a new astrophysical object called "Planck Star"[2] When a massive star, as we said early, stops producing energy and becomes a black hole inside the horizon, matter keeps to collapse until it reaches the state of a Planck star. How big is this hypothetical astrophysical object? It would be so small that it would be odd to classify it as a star; in comparison, our sun would be as large as an atom if it were to become a planck star. Common sense

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[2] C. Rovelli, F. Vidotto "Planck Stars" https://arXiv.org/pdf/1401.6562v4.pdf.

dictates this is a quirky way to imagine this a atom sized star, since very compact objects such as neutron stars have a radius of roughly about a few kilometers, in this case we talk about 10^{-10} m. The example of the sun allows us to understand that the matter inside a planck star would have an extreme mass density. Furthermore the authors show that a planck star is not stable but after reaching a maximum compression, it subsequently will start an expansion mechanism which leads to the explosion of the black hole and all of the information it once held would be expelled. How long does this process last? It depends. As we have seen in the example of the two brothers, for an observer placed inside the event horizon of the black hole the process is of short duration, it is instantaneous; instead for an observer outside the horizon the process is extremely long because of time dilatation effects due to the presence of a very strong gravitational field. For this reason, we will never realize that this process is taking place and the black hole would seem to remain the same.

In the hypothesis of the existence of black holes old as the Universe (13 billion of years) and if they are exploding now we could measure some signals due to their explosion and it would for the first time be a quantum gravity effect.

A possible candidate for this type of signals could be a Fast Radio Burst. What we know about fast radio bursts is that they are intense radio signals isolated with a duration of milliseconds. The frequency of these signals is about 1.3 GHz (hence a wavelength of 20 cm). Fast Radio Burst are still a mystery in astronomy and their origin is unknown. These signals are, in theory, of extragalactic origin. They are extremely energetic signals since the total energy emitted in the radio length by a source is estimated to be of the order 10^{38} erg. The wavelength of the signal emitted during the explosion of the black hole was estimated by the authors and does not appear to be in perfect agreement with experimental data but the result could be improved using a more accurate model, for example, as the authors suggest, rotational effects of the black hole could be give greater wavelengths.