Approximate Quality Criteria for Difficult Multi-Objective Optimization Problems

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Abstract. This paper introduces approximate analytic quality criteria useful in assessing the efficiency of evolutionary multi-objective optimization (EMO) procedures. We present a summary of extensive research into computing. In the performed comparative study we take into account the various approaches of the state-of-the-art, in order to objectively assess the EMO performance in highly dimensional spaces; where some executive criteria, such as those based on the true Pareto front, are difficult to calculate. Whereas, on the other hand, the proposed approximated quality criteria are easy to implement, computationally inexpensive, and sufficiently effective.

Keywords: Multi-objective optimization · Genetic algorithms · Evolutionary computations · Pareto-optimality · Quality criteria · Approximation

1 Introduction

In the recent decades a great number of various multi-objective genetic algorithms (MOGA), also referred to as evolutionary multi-objective optimization (EMOO) methods [1,3–6,8–11,22,28,29,32,36–38] have been proposed for solving multi-objective problems in multi-dimensional spaces.

A great number of various mechanisms for generating new solutions and decision-making processes have been proposed and implemented in genetic and evolutionary algorithms. On the other hand, there are only a few isolated attempts of applying some sexual categories in the genetic reproduction mechanisms known from the literature [12,14–16,19,21,25–27,30,31,33,35].

There are a number of standard performance indices for the Evolutionary Multi-Objective Optimization (EMOO) that can be used to evaluate the results of the Multi-Objective Genetic Algorithms (MOGA): to mention Hyper Volume (HV), Maximum Spread (MS), Generational Distance (GD), Spacing (SP), according to [1,3,34], or Global Optimality Level (GOL) proposed by [13,14,16–19]. Nevertheless, the use of some of the above methods in multidimensional optimization tasks [19] can be problematic.

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Application of Hyper Volume requires huge computational cost for the on-line calculation of the hypervolume contribution, which makes such calculations completely impractical in the case of difficult multi-dimensional optimization problems [7,19,34]. In addition, many of the standard performance indicators (such as the Generational Distance) are based on the knowledge of the true Pareto front. In practical engineering optimization tasks, it can be very difficult, if not impossible, to determine the true Pareto front [19].

In this paper, we propose new approximate quality criteria useful in evaluating the effectiveness of the MOGA approach, that make an alternative to the standard performance indices used in EMOO.

The offered computational study embraces the SMS-EMOA [7], MOEA/D-DE [22] GDE3 [20], NSGA2 [29], and three gender-based (multi-sexual) algorithms: our gender mechanisms, i.e. Genetic Gender Approach (GGA), and Virtual Gender Algorithm (VGA) with HPR [2,13,14,16–19], and MSGA [21], most similar to GGA. The considered examples of the EMOO problems are the known multi-objective benchmark problems [37].

2 Evaluating MOGA

In order to measure the effectiveness of multi-objective optimization algorithms the following indicators: **Hyper Volume**, **Spacing** and **Generational Distance**, are often recommended to be used as standards [3]. Examples of their usage and some discussion can be found also in our recent paper [19].

Hyper Volume, HV, also know as an S-metric or Lebesgue measure [3], represents the n-dimensional objective space which contains the set of Pareto-optimal solutions and a reference point (for maximization tasks the origin is used for this purpose). Generally speaking, the greater HV, the better the Pareto set.

Generational Distance, GD, is a measure of the average overs between particular solutions of the obtained Pareto front and their closest neighbour from the true Pareto front [3]. It is determined according to the following formula

$$\mathrm{GD} = \frac{\sqrt{\sum_{i=1}^{k} d_i^2}}{m} \tag{1}$$

where d_i represents the Euclidean phenotypic distance between i-th solution of the obtained Pareto front and the closest member of true Pareto front, and m denotes the number of obtained Pareto-optimal solutions. The lower the value GD, the better approximation of the true Pareto front.

Spacing, SP, describes the spread of the solutions of the obtained Pareto front that is calculates as [3]

$$\mathrm{SP} = \sqrt{\frac{1}{m-1}\sum_{j=1}^{m}(d_{avg} - d_j)^2}, \quad d_j = min_k \left\{ \sum_{l=1}^{n} |f_l^j(x) - f_l^k(x)| \right\} \tag{2}$$

where d_{avg} represents the mean of all d_j, n is the number of objective functions, and m denotes the number of the obtained Pareto-optimal solutions. For SP $\cong 0$, all solutions in the obtained Pareto front are evenly distributed.

Considering, for instance, the measure HV, one has to keep in mind that the objective space needs to be convex, which is a hard condition to fulfil in practice: If it is not convex, the results may be misleading, and, moreover, a true Pareto front has to be known [1]. The index GD measures the mean distance between the computed P-front, and the true one. When seeking for optimal parameters in continuous search domains, the Pareto front, or the Pareto optimal solution set, is infinite, and often difficult to be found for practical optimization tasks.

The EMOO indices HV and GD need an external (archive) population consisting of non-dominated individuals. Similarly, to determine the rate of MS the GAs must store the non-dominated solutions found during evolutionary cycles in an archive population (a known Pareto front). The GGA, however, does not utilize any external population (we use only one set of individuals evolving through the generations).

In contrast to the various other approaches and measures, the GGA algorithm returns 'good' representatives of the P-front of non-dominated solutions in a natural way of inheritance, and additionally can be extra-assessed by other tools, as the proposed GOL index.

Computational applications of EMOO algorithms (also those presented in this article), illustrate that determination of the true Pareto front would be computationally expensive: (a) due to the form of the criteria, requiring simulation of the optimized system, or (b) due to complex numerical computations necessary for getting the value of the vector criterion. What is more, a final assessment of solutions can always be a concern for further research studies.

Most of known benchmarks for multi-objective optimization concern only a few objectives, and they are not interesting patterns in the context of GGA/VGA, which are meant for truly multi-dimensional problems. Still, even the sample of results presented below appears to be sufficient to illustrate both the nature and power of the gender approach, that evidently is revealed in highly dimensional objective spaces [19].

2.1 Approximate Quality Criteria

We suggest the following approximate qualitative criteria: Hyper Cube (HC), Hyper Radius (HR), Approximate Hyper Volume (AHV), Approximate Spacing (ASP), Approximate Global Generational Distance (AGGD), and Approximate Directional Generational Distance (ADGD).

Hyper Cube (HC) Δ^n is defined as follows

$$\Delta^n = \prod_{i=1}^{n} \Delta_i, \quad \Delta_i = \overline{f_i} - \underline{f_i} \tag{3}$$

where $\overline{f_i}$ denotes the maximal value of an i-th objective function (f_i) of a solution in the Pareto front, $\underline{f_i} = 0$ is the minimal value of the i-th function among all Pareto solutions (assumed here to be zero), while n represents the number of objective functions.

Hyper Radius (HR) ρ_n is calculated as

$$\rho_n = \sqrt[n]{\Delta^n} \tag{4}$$

where Δ^n denotes the Hyper Cube of the given Pareto front.

Approximated Hyper Volume (AHV) ζ_n is determined as

$$\zeta_n = \frac{a_n}{2^n}\rho_n, \quad a_n = \begin{cases} \frac{\pi^n}{n!} & \text{if } n \text{ is even} \\ \frac{2^n \pi^n}{n!!} & \text{if } n \text{ is odd} \end{cases} \tag{5}$$

where the symbol $n!!$ means double factorial.

Approximated Spacing (ASP) η_n is defined as follows

$$\eta_n = \sqrt[n]{\frac{\zeta_n}{m}} \tag{6}$$

where m represents the number of Pareto-optimal solutions.

Approximated Global Generational Distance (AGGD) γ_n is calculated in the following way:

$$\gamma_n = \frac{1}{m}\sqrt{\sum_{j=1}^{m}(\rho_n - r_j)^2}, \quad r_j = \sqrt{\sum_{i=1}^{n} f_i^2} \tag{7}$$

Approximated Directional Generational Distance (ADGD) $\widetilde{\gamma_n}$ is assessed as

$$\widetilde{\gamma_n} = \frac{1}{m}\sqrt{\sum_{j=1}^{m} p_j}, \quad p_j = \frac{1}{n}\sum_{i=1}^{n}(\Delta_i - f_i)^2 \tag{8}$$

It should be noted that in all the above definitions of the proposed indicators (of low computational complexity), one does not require the knowledge of the actual Pareto front, which is simply identified with the surface of a hyper-sphere.

3 Illustrative Benchmark Examples

This section presents the results of optimization for exemplary multi-objective problems [19]. Two groups of evolutionary algorithms have been selected for consideration in our comparative study: (a) four non-gender representatives: SMS-EMOA, MOEA/D-DE, GDE3, NSGA2, and three gender-based/multi-sexual algorithms: our gender propositions, i.e. GGA, Genetic Gender Approach, and VGA, Virtual Gender Algorithm with HPR, and MSGA, most similar to GGA. All can be run in attended mode, using elitism. The implementation parameters of the considered algorithms have been set as follows: the type of arithmetic: floating point; population size: 120; crossover probability: 0.8; mutation probability: 0.2; and maximal number of generations: 200; number of repeated runs: 30 (for statistical averaging by calculating the median). All algorithms start execution from the same initial population.

3.1 Optimization Results

All the considered algorithms, both gendered and not-gendered, have been compared using several optimization tasks designated as: UF1-UF10, DTLZ1-DTLZ7 [3,24,37]. In this study, the relatively difficult task UF7 is exercised.

The number of criteria of solutions' matching represents not only the overall complexity of a given problem, but it is also important for the 'structure' of gender mechanism used in the gender algorithms xGA. The UF7 describes a two-objective optimization test (with 30 decision variables/parameters):

$$f_1(\boldsymbol{x}) = \sqrt[5]{x_1} + \frac{2}{14} \sum_{i \in 3,5,\dots,30} \left(x_i - sin\left(6\pi x_1 + \frac{i\pi}{30}\right)\right)^2 \tag{9}$$

$$f_2(\boldsymbol{x}) = 1 - \sqrt[5]{x_1} + \frac{2}{15} \sum_{i \in 2,4,\dots,30} \left(x_i - sin\left(6\pi x_1 + \frac{i\pi}{30}\right)\right)^2 \tag{10}$$

where $0 \leq x_1 \leq 1$ and $-1 \leq x_j \leq 1 \quad (j = 1, 2, ..., 30)$.

Clearly, such a simple set of criterion functions leads to the natural division into two one-dimensional attributes (and two genetic-gender sets), where the first gender set is characterized by the first objective and the other determined by the second function $f_2(x)$. In programming the virtual-gender approach (VGA) the simplest (optional) III-level fitness hierarchy in the genetic gender distribution has been applied [13,14,16–19].

As can be seen in Figs. 1 and 2, the results of simple, approximate functions (Fig. 1), and the original indicators (Fig. 2), are very similar to each other.

Figure 3 presents the average computational time of the approximate quality indices and original quality indices. The abbreviation MCAHV applied in Fig. 3b and d means the Monte Carlo Approximation method of calculation of the Hyper Volume indicator [34]. Figures 3a and b refer to the 2-objective UF7 problem, while Figs. 3c and d concern the 3-objective UF10 problem. As shown in Fig. 3, the average computation times of the approximate quality indices are at least two orders lower as compared to the indices calculated by means of the accurate methods: GD and SP according to the definition of Eqs. (1) and (2), and HV by means of the Lebesgue measure algorithm [7,34]. In the case of the problems with the three criteria, the calculation time of the Hyper Volume indicator is enormous about 7.37 s (see the high bar, which does not fit to the applied scale).

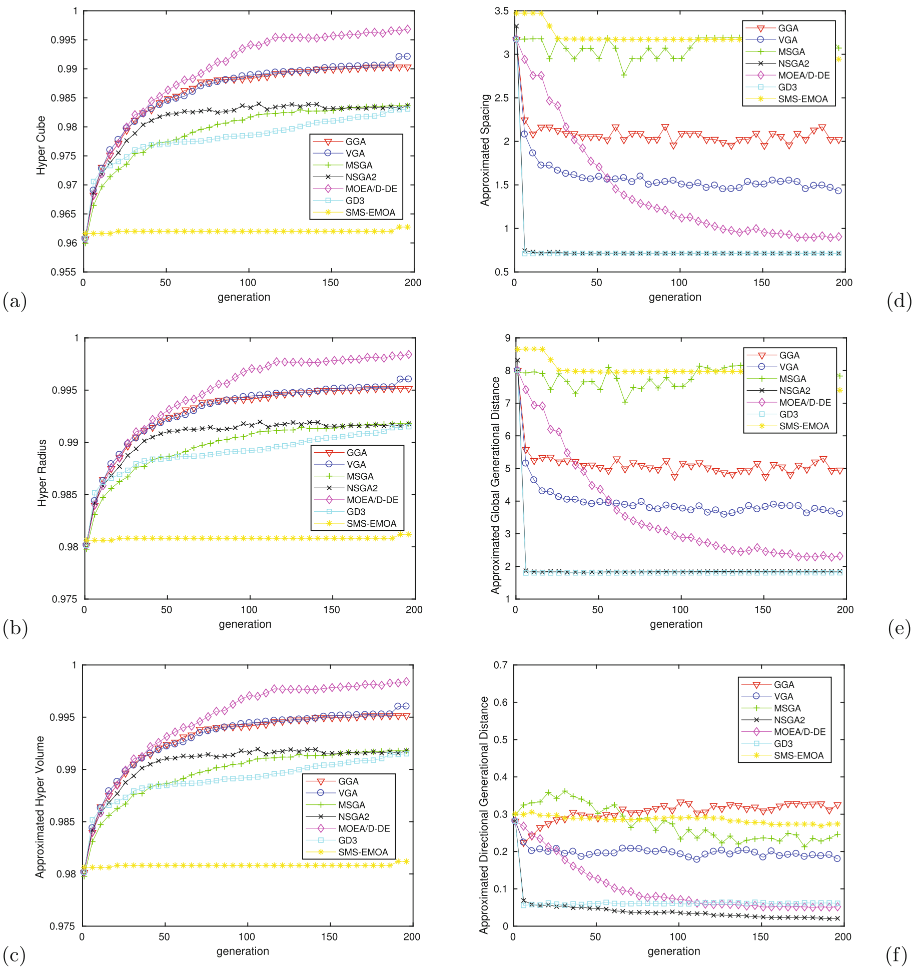

Fig. 1. Indicators of the seven unattended (without elitism) EMOO algorithms in their averaged simulation runs: (a) Hyper Cube; (b) Hyper Radius SP; (c) Approximated Hyper Volume; (d) Approximated Spacing, (e) Approximated Global Generational Distance, (f) Approximated Directional Generational Distance.

3.2 Discussion

In the above, the algorithms have been compared in terms of the dispersion or standard spacing metric (SP), which measures the spread or dispersion of the solutions contained in the derived Pareto front [3]. The third quality index has been hypervolume (HV). Due to the computational complexity in determining the hypervolume indicator, an off-line Monte Carlo method [34] has been used, which is necessary especially for the demanding multi-objective problems

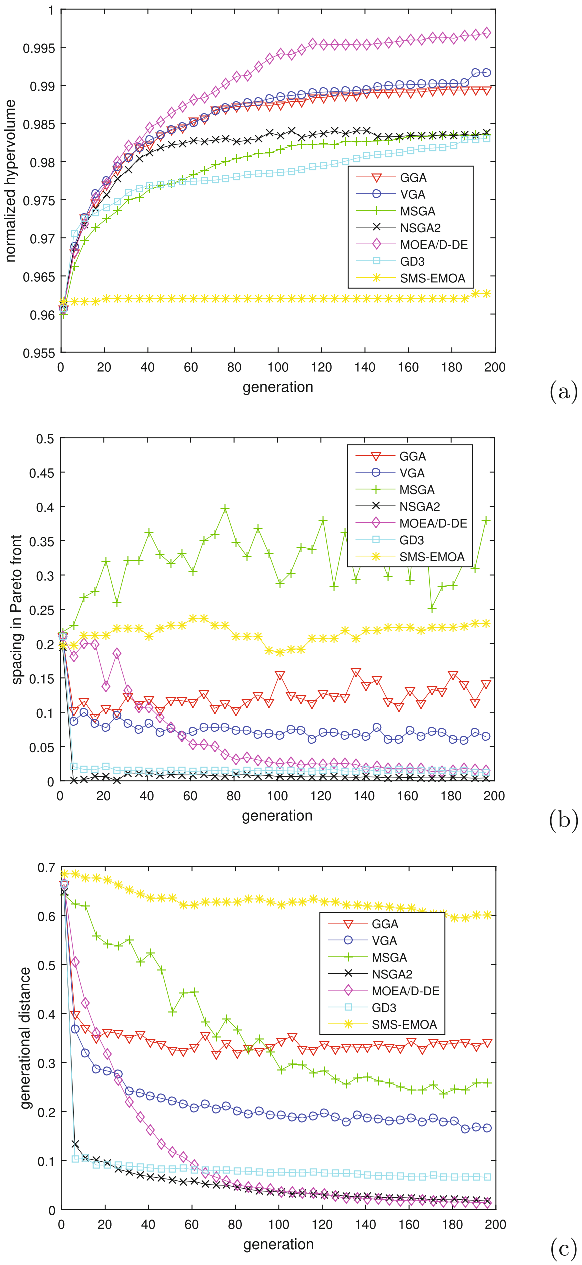

Fig. 2. Original indicators for the seven unattended EMOO algorithms in their averaged simulation runs [19]: (a) Hyper Volume; (b) Spacing, (c) Generational Distance.

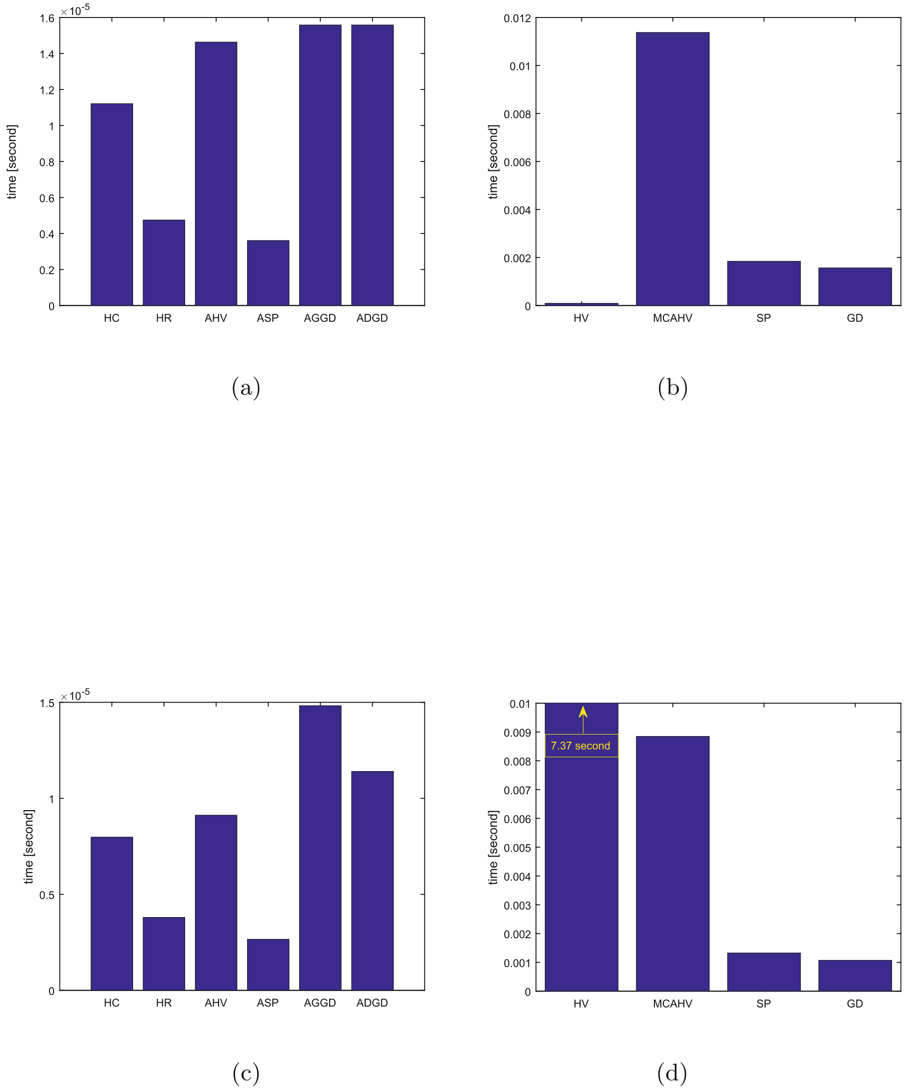

Fig. 3. Average computational time of: (a) approximate quality indices for UF7; (b) quality indices for UF7, (c) approximate quality indices for UF10, (d) quality indices for UF10.

spanned by 3 to 10 objectives. When using SMS-EMOA the hypervolume contribution must always be determined on-line.

In the simplest case of UF7, it has been quite easy to compute the GD index, representing the generational distance [3], and referring the analysed solutions to a known true Pareto front. When dealing with other complex problems (like UF10, DTLZ4, DTLZ5, DTLZ6, DTLZ7), however, this index appears to be too computationally complex, and therefore impractical (mainly due to the need to determine the true Pareto front). Therefore, in the other more complex cases, in place of the generational distance GD, a median version of the Global Level Optimization (median GOL) has been applied as the fourth indicator.

Besides the easy-to-calculate index GOL, all other quality indicators have been estimated off-line. For statistical analysis of the obtained results, the median approach has been primarily used. Consequently, the running statistical results of multi-objective unattended optimization for UF7 will be shown below in terms of the indicators: maximal GOL, spacing SP, normalized hypervolume (HV), and generational distance GD, whereas for the complex MOO tasks (UF10, DTLZ4 $\div$ 7) we will use: maximal GOL, SP, HV, and median GOL.

In the case of the hypervolume index, the exact algorithm is based on the Lebesgue measure, and the profit made from the exact criterion is disproportionate to the time it takes for its calculation. Also, the accurate determination of the distance GD requires knowledge of the true Pareto front. In practical problems of optimization, it may be impossible or very difficult. In the case of the approximated GD, this knowledge is not needed, which is a huge advantage of the proposed approach.

4 Conclusion

The basic advantage and the success of the application of the proposed approximate functions should be attributed to the fact that they are computationally much cheaper than those of the original ones (Fig. 3), and as such can be easily and efficiently calculated.

The presented results obtained for the chosen simple benchmarks, for which you can determine the Pareto front, show the high compatibility of the approximated indicators AHP, ASP, AGGD and ADGD (Fig. 1) with the original functions HV, SP, GD (Fig. 2).

It is also extremely important that the proposed approximate functions (AGGD, ADGA) do not require knowledge of the true Pareto front. This gives them a huge advantage over the original indicators in solving complex multi-objective optimization problems in engineering design.

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