On Possibilistic Dependencies: A Short Survey of Recent Developments

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Abstract Carlsson and Fullér introduced the notions of possibilistic mean value and variance of fuzzy numbers. Fullér and Majlender introduced a measure of possibilistic covariance between marginal distributions of a joint possibility distribution as the average value of the interactivity relation between the level sets of its marginal distributions. Fullér et al. introduced the possibilistic correlation ratio, the possibilistic correlation coefficient and the possibilistic informational coefficient of correlation. In this paper we give a short survey of some later works which extend and develop these notions.

1 Introduction

In probability theory the notion of mean value of functions of random variables plays a fundamental role in defining the basic characteristic measures of probability distributions: the measure of covariance, variance and correlation of random variables can all be computed as probabilistic means of their appropriately chosen real-valued functions. Similarly, in possibility theory we can use the principle of *average value* of appropriately chosen real-valued functions to define mean value, variance, covariance and correlation of possibility distributions. Marginal probability distributions are determined from the joint one by the principle of 'falling integrals' and marginal possibility distributions are determined from the joint possibility distribution by the principle of 'falling shadows'. Probability distributions can be interpreted as carriers of *incomplete information* [43], and possibility distributions can be interpreted as carriers of *imprecise information*. A function $f : [0, 1] \to \mathbb{R}$ is said to be a weighting

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function if f is non-negative, monotone increasing and satisfies the following normalization condition $\int_0^1 f(\gamma)\mathrm{d}\gamma = 1$. Different weighting functions can give different (case-dependent) importances to γ-levels sets of fuzzy numbers. It is motivated in part by the desire to give less importance to the lower levels of fuzzy sets [34] (it is why f should be monotone increasing).

We can define the mean value (variance) of a possibility distribution as the f-weighted average of the probabilistic mean values (variances) of the respective uniform distributions defined on the γ-level sets of that possibility distribution. A measure of possibilistic covariance (correlation) between marginal possibility distributions of a joint possibility distribution can be defined as the f-weighted average of probabilistic covariances (correlations) between marginal probability distributions whose joint probability distribution is defined to be uniform on the γ-level sets of their joint possibility distribution [29]. We should note here that the choice of uniform probability distribution on the level sets of possibility distributions is not without reason. Namely, these possibility distributions are used to represent imprecise human judgments and they carry non-statistical uncertainties. Therefore we will suppose that each point of a given level set is equally possible. Then we apply Laplace's principle of Insufficient Reason: if elementary events are equally possible, they should be equally probable (for more details and generalization of principle of Insufficient Reason see [26], page 59). The main new idea here is to equip the level sets of joint possibility distributions with uniform probability distributions and to derive possibilistic mean value, variance, covariance and correlation of possibility distributions, in such a way that they would be consistent with the extension principle. The idea of equipping the level sets of fuzzy numbers with a uniform probability refers to early ideas of simulation of fuzzy sets by Yager [60], and possibility/probability transforms by Dubois et al. [25] as well as the pignistic transform of Smets [55].

2 Possibilistic Mean Value, Variance, Covariance, Correlation Coefficient and Correlation Ratio

In this section we will recall the possibilistic mean value, variance, covariance and correlation of fuzzy numbers, which are consistent with the extension principle and with the well-known definitions of expectation and variance in probability theory. A *fuzzy number A* is a fuzzy set $\mathbb{R}$ with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers is denoted by $\mathscr{F}$. Fuzzy numbers can be considered as possibility distributions [24, 63]. A fuzzy set C in $\mathbb{R}^2$ is said to be a joint possibility distribution of fuzzy numbers $A, B \in \mathscr{F}$, if it satisfies the relationships $\max\{x \mid C(x,y)\} = B(y)$ and $\max\{y \mid C(x,y)\} = A(x)$ for all $x, y \in \mathbb{R}$. Furthermore, A and B are called the marginal possibility distributions of C.

The possibilistic mean (or expected value), variance, covariance and correlation were originally defined from the measure of possibilistic interactivity (as shown in [10, 29]) but for simplicity, we will present the concept of possibilistic mean value, variance, covariance and possibilistic correlation in a probabilistic setting and point out the fundamental difference between the standard probabilistic approach and the possibilistic one. Let $A \in \mathcal{F}$ be fuzzy number with $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$ and let U_γ denote a uniform probability distribution on $[A]^\gamma$, $\gamma \in [0, 1]$. Recall that the probabilistic mean value of U_γ is equal to

$$M(U_\gamma) = \frac{a_1(\gamma) + a_2(\gamma)}{2},$$

and its probabilistic variance is computed by

$$\mathrm{var}(U_\gamma) = \frac{(a_2(\gamma) - a_1(\gamma))^2}{12}.$$

In 1987 Dubois and Prade [23] defined an interval-valued expectation of fuzzy numbers, viewing them as consonant random sets. They also showed that this expectation remains additive in the sense of addition of fuzzy numbers. In 2003 Fullér and Majlender [28] introduced the f-weighted *possibilistic mean value* of $A \in \mathcal{F}$ as

$$E_f(A) = \int_0^1 M(U_\gamma) f(\gamma) \mathrm{d}\gamma = \int_0^1 \frac{a_1(\gamma) + a_2(\gamma)}{2} f(\gamma) \mathrm{d}\gamma,$$

where U_γ is a uniform probability distribution on $[A]^\gamma$ for all $\gamma \in [0, 1]$. If $f(\gamma) = 1$ for all $\gamma \in [0, 1]$ then we get

$$E(A) = \int_0^1 M(U_\gamma) f(\gamma) \mathrm{d}\gamma = \int_0^1 \frac{a_1(\gamma) + a_2(\gamma)}{2} \mathrm{d}\gamma,$$

which the possibilistic mean value of A originally introduced in 2001 by Carlsson and Fullér [3]. In 2003 Fullér and Majlender [28] introduced the f-weighted *possibilistic variance* of $A \in \mathcal{F}$ as

$$\mathrm{Var}_f(A) = \int_0^1 \mathrm{var}(U_\gamma) f(\gamma) \mathrm{d}\gamma = \int_0^1 \frac{(a_2(\gamma) - a_1(\gamma))^2}{12} f(\gamma) \mathrm{d}\gamma.$$

In 2004 Fullér and Majlender [29] introduced a measure of possibilistic covariance between marginal distributions of a joint possibility distribution C as the expected value of the interactivity relation between the γ-level sets of its marginal distributions. In 2005 Carlsson et al. [10] showed that the possibilistic covariance between fuzzy numbers A and B can be written as the weighted average of the probabilistic covariances between random variables with uniform joint distribution on the level sets of their joint possibility distribution C. The *f-weighted measure of possibilistic covariance* between $A, B \in \mathscr{F}$ (with respect to their joint distribution C) [29] can be written as,

$$\mathrm{Cov}_f(A, B) = \int_0^1 \mathrm{cov}(X_\gamma, Y_\gamma) f(\gamma) \mathrm{d}\gamma,$$

and the f-weighted *possibilistic correlation coefficient* of $A, B \in \mathscr{F}$ (with respect to their joint distribution C) is defined by [31],

$$\rho_f(A, B) = \int_0^1 \rho(X_\gamma, Y_\gamma) f(\gamma) \mathrm{d}\gamma$$

where

$$\rho(X_\gamma, Y_\gamma) = \frac{\mathrm{cov}(X_\gamma, Y_\gamma)}{\sqrt{\mathrm{var}(X_\gamma)}\sqrt{\mathrm{var}(Y_\gamma)}}$$

and X_γ and Y_γ are random variables whose joint distribution is uniform on $[C]^\gamma$, $\mathrm{cov}(X_\gamma, Y_\gamma)$ denotes their covariance, for all $\gamma \in [0, 1]$.

In statistics, the correlation ratio is a measure of the relationship between the statistical dispersion within individual categories and the dispersion across the whole population or sample. The correlation ratio was originally introduced by Pearson [52] and it was extended to random variables by Kolmogorov [44] as,

$$\eta^2(X|Y) = \frac{\mathrm{var}[M(X|Y)]}{\mathrm{var}(X)},$$

where X and Y are random variables. If X and Y have a joint probability density function, denoted by $g(x, y)$, then we can compute $\eta^2(X|Y)$ using the following formulas

$$M(X|Y = y) = \int_{-\infty}^{\infty} x g(x|y) \mathrm{d}x$$

and

$$\mathrm{var}[M(X|Y)] = M(M(X|y) - M(X))^2,$$

and where,

$$g(x|y) = \frac{g(x, y)}{g(y)}.$$

In 2010 Fullér et al. [30] introduced the *f-weighted possibilistic correlation ratio* of marginal possibility distribution A with respect to marginal possibility distribution B as

$$\eta_f^2(A|B) = \int_0^1 \eta^2(X_\gamma|Y_\gamma) f(\gamma) \mathrm{d}\gamma$$

where X_γ and Y_γ are random variables whose joint distribution is uniform on $[C]^\gamma$, $\mathrm{cov}(X_\gamma, Y_\gamma)$ denotes their covariance and $\eta(X_\gamma|Y_\gamma)$ denotes their probabilistic correlation ratio [44], for all $\gamma \in [0, 1]$.

In 2012 Fullér et al. [32] introduced the *f-weighted possibilistic informational coefficient of correlation*. For any two continuous random variables X and Y (admitting a joint probability density), their mutual information is given by,

$$I(X, Y) = \iint g(x, y) \ln \frac{g(x, y)}{g_1(x)g_2(y)} \mathrm{d}x \, \mathrm{d}y$$

where $g(x, y)$ is the joint probability density function of X and Y, and $g_1(x)$ and $g_2(y)$ are the marginal density functions of X and Y, respectively. The informational coefficient of correlation of X and Y is defined by

$$L(X, Y) = \sqrt{1 - e^{-2I(X,Y)}}$$

Then the *f-weighted possibilistic informational coefficient of correlation* of marginal possibility distributions A and B is defined by

$$L(A, B) = \int_0^1 L(X_\gamma, Y_\gamma) f(\gamma) \mathrm{d}\gamma$$

where X_γ and Y_γ are random variables whose joint distribution is uniform on $[C]^\gamma$, for all $\gamma \in [0, 1]$.

Note 1 There exist several other ways to define correlation coefficient for fuzzy numbers, e.g. Liu and Kao [47] used fuzzy measures to define a fuzzy correlation coefficient of fuzzy numbers and they formulated a pair of nonlinear programs to find the α-cut of this fuzzy correlation coefficient, then, in a special case, Hong [39] showed an exact calculation formula for this fuzzy correlation coefficient. Vaidyanathan [57] introduced a new measure for the correlation coefficient between triangular fuzzy variables called credibilistic correlation coefficient.

Fullér and colleagues have extensively used the possibilistic mean value, variance, covariance and correlation in their later works for real option valuation [4, 8, 14], portfolio selection problems [6, 7, 11, 12] and strategic planning [9, 13]. For example, in 2007 Carlsson et al. [12] developed a methodology for valuing options on R&D projects, when future cash flows are estimated by trapezoidal fuzzy numbers. In particular, they presented the following fuzzy mixed integer programming model for the R&D optimal portfolio selection problem,

$$\text{maximize} \sum_{i=1}^{N} u_i \mathscr{F}_i$$
$$\text{subject to} \sum_{i=1}^{N} u_i X_i + \sum_{i=1}^{N} (1 - u_i) c_i \leq B$$
$$u_i \in \{0, 1\},\ i = 1, \ldots N,$$

where N is the number of R&D projects, B is the whole investment budget, u_i is the decision variable that takes value one if the i-th project should start now (at time zero) or takes the value zero if it should be postponed and started at a later time, c_i denotes the cost of the postponment (i.e. keep the option alive), X_i is the investment cost, and $\mathscr{F}_i$ denotes the possibilistic deferral flexibility of the i-th project for $i = 1, \ldots, N$. Furthermore, they discussed how their methodology can be used to build decision support tools for optimal R&D project selection in a corporate environment. They also claimed that the imprecision we encounter when judging or estimating future cash flows is not stochastic in nature, and the use of probability theory gives us a misleading level of precision and a notion that consequences somehow are repetitive. This is not the case, the uncertainty is genuine, i.e. we simply do not know the exact levels of future cash flows. Without introducing fuzzy real option models it would not be possible to formulate this genuine uncertainty.

In 2009 Collan et al. [18] presented a new method (fuzzy pay-off method) for real option valuation using fuzzy numbers that is based on findings from earlier real option valuation methods and from fuzzy real option valuation. They also presented the use of number of different types of fuzzy numbers with the method and an application of the new method in an industry setting. In 2010 Carlsson et al. [13] used fuzzy real option models for the problem of closing/not closing a production plant in the forest products industry sector. In 2013 Carlsson and Fullér [15] implemented a hybrid probabilistic and possibilistic model to assess the success of computing tasks in a Grid. Using the predictive probabilistic approach they developed a framework for resource management in grid computing, and by introducing an upper limit for the number of possible failures, they approximated the probability that a particular computing task can be executed. Coroianu and Fullér [19] studied he problem of additivity property of the weighted possibilistic mean operator for interactive fuzzy numbers. They showed that the weighted possibilistic mean operator is additive on the set of symmetric fuzzy numbers if their joint possibility distribution is defined by a triangular norm. They also showed some results for general joint-possibility-distribution-based additions of fuzzy numbers of symmetrical opposite sides.

3 Recent Developments

The notions of possibilistic mean value, variance, covariance and correlation are used in many different research areas and by many different authors (Google Scholar finds over 2,000 citations to papers [3, 8, 10, 12, 28, 29]).

In 2005 Yoshida et al. [61] evaluated the randomness and fuzziness in fuzzy stochastic processes by the probabilistic expectation and the mean values defined by fuzzy measures and λ-weighting functions. The mean values are demonstrated particularly in three kinds of important fuzzy measures: possibility measure, necessity measure and credibility measure. Furthermore, by introducing fuzziness to stochastic processes in optimization/decision-making, they considered a new model with uncertainty of both randomness and fuzziness, which is a reasonable and natural extension of the original stochastic process.

In 2006 Fang et al. [27] proposed a portfolio rebalancing model with transaction costs based on fuzzy decision theory and illustrated the behaviour of their proposed model using real data from the Shanghai Stock Exchange. Huang [41] selected the optimal portfolio with fuzzy returns by criteria of chance represented by credibility measure. He introduced two types of credibility-based portfolio selection models: (i) by one chance criterion, the objective is to maximize the investor's return at a given threshold confidence level; (ii) by another chance criterion, the objective is to maximize the credibility of achieving a specified return level subject to the constraints. To solve the resulting problems he designed a hybrid intelligent algorithm integrating fuzzy simulation and genetic algorithm.

In 2007 Zhang et al. [64] proposed two kinds of portfolio selection models based on lower and upper possibilistic means and possibilistic variances, respectively, and introduced the notions of lower and upper possibilistic efficient portfolios. They also presented an algorithm which can derive the explicit expression of the possibilistic efficient frontier for the possibilistic mean-variance portfolio selection problem dealing with lower bounds on asset holdings. Zhang and Wang [65] investigated the relationship between several crisp possibilistic variances and covariances of fuzzy numbers. Silva et al. [54] presented and developed an original and novel fuzzy sets based method that solves a class of quadratic programming problems with vagueness in the set of constraints. The method uses two phases to solve fuzzy quadratic programming problems, which eventually can be considered in the portfolio context. In the first phase they parametrize the fuzzy problem in several classical alpha-problems with different cutting levels. In the second phase each of these alpha-problems is solved by using conventional solving techniques.

In 2008 by introducing the concept of semivariance of fuzzy variable Huang [42] proposed two fuzzy mean-semivariance models for portfolio selection problems in fuzzy environment. To solve the new models in general cases, he presented a fuzzy simulation based genetic algorithm. By morphing mean-variance optimization portfolio model into semi-absolute deviation model, Gupta et al. [36] applied multi criteria decision making via fuzzy mathematical programming to develop comprehensive models of asset portfolio optimization for the investors' pursuing either of the aggressive or conservative strategies.

In 2009 Chen et al. [16] considered a possibilistic mean-variance (FMVC) portfolio selection model and proposed a cutting plane algorithm to solve it. Xu et al. [59] presented a fuzzy normal jump-diffusion model for European option pricing, with uncertainty of both randomness and fuzziness in the jumps, which is a reasonable and a natural extension of the Merton [50] normal jump-diffusion model. Based on the crisp weighted possibilistic mean values of the fuzzy variables in fuzzy normal jump-diffusion model, they also obtained the crisp weighted possibilistic mean normal jump-diffusion model. Yoshida [62] discussed value-at-risk portfolio model under uncertainty. In his proposed model the means, the variances and the measurements of imprecision for fuzzy numbers/fuzzy random variables are evaluated in the possibility case and the necessity case, and the rate of return in portfolio is estimated regarding the both random factors and imprecise factors. Zhang et al. [66] proposed a new portfolio selection model with the maximum utility based on the interval-valued possibilistic mean and possibilistic variance, which is a two-parameter quadratic programming problem. They also presented a sequential minimal optimization (SMO) algorithm to obtain the optimal portfolio. The remarkable feature of their algorithm is that it is extremely easy to implement, and it can be extended to any size of portfolio selection problems for finding an exact optimal solution.

In 2010 Zhang et al. [67] proposed a possibilistic portfolio adjusting model with transaction costs and bounded constraints on holdings of assets, which can be transformed into a linear programming problem. Both the lower bounds on holdings and the total investment constraints influence the optimal portfolio adjusting strategies. Gładysz and Kasperski [33] discussed the problem of computing the mean absolute deviation in a set of uncertain variables. The uncertainty is modelled by closed intervals and fuzzy intervals. Some polynomial algorithms for determining the lower and upper bounds for the mean absolute deviation under interval uncertainty are proposed. Possibility theory is then applied to generalize the interval uncertainty representation to the fuzzy one.

In 2011 Ho and Liao [38] proposed a fuzzy binomial approach for investment project valuation in uncertain environments from the aspect of real options. Their approach also reveals the value of flexibilities embedded in the project. Duan and Stahlecker [22] considered static portfolio selection problem, in which future returns of securities are given as fuzzy sets. In contrast to traditional analysis, they assume that investment decisions are not based on statistical expectation values, but rather on maximal and minimal potential returns resulting from the α-cuts of these fuzzy sets. By aggregating over all α-cuts and assigning weights for both best and worst possible cases they get a new objective function to derive an optimal portfolio. Lee and Lee [45] examined the strategic characteristic of RFID (Radio Frequency Identification) investment and proposed a fuzzy real options technique that can consider various situations of expected cash flow or investment costs as a plan to support investment decisions.

In 2012 Deng and Li [20] proposed a portfolio selection model with borrowing constraint by means of possibilistic mean, possibilistic variance, and possibilistic covariance under the assumption that the returns of assets are fuzzy numbers. They presented a quadratic programming model with inequality constraints when the returns of assets are trapezoid fuzzy numbers and utilized the Lemke algorithm to solve the problem. Zhang et al. [68] dealt with a multi-period portfolio selection problem with fuzzy returns and presented a possibilistic mean-semivariance-entropy model for multi-period portfolio selection by taking into account four criteria: return, risk, transaction cost and diversification degree of portfolio. In their proposed model, the return level is quantified by the possibilistic mean value of return, the risk level is characterized by the lower possibilistic semivariance of return, and the diversification degree of portfolio is measured by the originally presented possibilistic entropy. Based on the possibilistic mean and the possibilistic variance/covariance of fuzzy numbers, Chrysafis [17] proposed a method to reduce some problems arising from the Capital Asset Pricing Model (CAPM) assumptions.

In 2013 Thavaneswaran et al. [56] used fuzzy set theory to price binary options. Namely, they studied binary options by fuzzifying the maturity value of the stock price using trapezoidal, parabolic and adaptive fuzzy numbers. Hsieh and Tsaur [40] proposed a simplified fuzzy regression equation based on possibilistic mean and variance method and used it for modeling the constraints and objective function of a fuzzy regression model without determining the membership function of extrapolative values. Liu and Zhang [48] discussed a multi-objective portfolio optimization problem for practical portfolio selection in fuzzy environment, in which the return rates and the turnover rates are characterized by fuzzy variables. Based on the possibility theory, they quantified fuzzy return and liquidity by possibilistic mean, and market risk and liquidity risk are measured by lower possibilistic semivariance. They proposed a fuzzy multi-objective programming technique to transform their proposed models into corresponding single-objective models and then designed a genetic algorithm for their solution.

In 2014 Wang et al. [58] employed the weighted possibilistic mean (WPM), weighted interval valued possibilistic mean (WIVPM) of fuzzy number as a sort of representative values for the fuzzy attribute data, and establish new fuzzy control charts with WPM and WIVPM. They compared the performance of the charts to the existing fuzzy charts with a fuzzy *c*-chart example via newly defined average number of inspection for variation of control state. Based on possibility theory and the assumption that the returns of assets are triangular fuzzy numbers, Deng and Li [21] proposed a bi-objective nonlinear portfolio selection model. They show that their nonlinear bi-objective model is equivalent to the linear bi-objective minimizing programming model on the basis of possibilistic mean and possibilistic variance.

In 2015 Nguyen et al. [51] initiated the fuzzy Sharpe ratio in the fuzzy modeling context. In addition to the introduction of the new risk measure, they also put forward the reward-to-uncertainty ratio to assess the portfolio performance in fuzzy modeling. Zhang [69] considered a multi-period portfolio selection problem in a fuzzy investment environment, in which the return and risk of assets are characterized by possibilistic mean value and possibilistic semivariance, respectively. Based on the theories of possibility, he proposed a new multi-period possibilistic portfolio selection model, which contains risk control, transaction costs, borrowing constraints, threshold constraints and cardinality constraints. By redefining the concepts of mean and variance for fuzzy numbers, Li et al. [46] formulated a fuzzy mean-variance-skewness portfolio selection model.

In 2016 Mashayekhi and Omrani [49] proposed a novel multi objective model for portfolio selection, where the asset returns are considered as trapezoidal fuzzy numbers. Their model incorporates the DEA cross-efficiency into Markowitz mean-variance model and considers return, risk and efficiency of the portfolio. Rubio et al.

[53] proposed the weighted fuzzy time series method to forecast the future performance of returns on portfolios. They modelled the uncertain parameters of the fuzzy portfolio selection models using a possibilistic interval-valued mean approach, and approximate the uncertain future return on a given portfolio by means of a trapezoidal fuzzy number. Guo et al. [35] considered a fuzzy multi-period portfolio selection problem with V-Shaped transaction cost. Compared with the traditional studies assuming that assets have the same investment horizon, they handled the practical but complicated situation in which assets have different investment horizons. Within the framework of credibility theory, they formulate a mean-variance model with the objective of maximizing the terminal return under the total risk constraint over the whole investment.

In 2017 Babazadeh et al. [1] presented a multi-objective possibilistic programming model to design a second-generation biodiesel supply chain network under risk. Their model minimizes the total costs of biodiesel supply chain from feedstock supply centers to customer centers besides minimizing the environmental impact of all involved processes under a well-to-wheel perspective. Brunelli and Mezei [2] presented an analysis of approximate operations on fuzzy numbers. By focusing on the ranking and defuzzification procedures as essential tools in fuzzy decision making problems, they studied the errors produced by the application of approximate operations.

4 Concluding Remarks

Possibility theory is mathematically the simplest uncertainty theory for dealing with incomplete information [26]. This may be the reason why possibilistic dependencies are used in many different research areas like information sciences, geosciences, social sciences, economics, mathematical and computer modelling, financial engineering, systems engineering, military engineering, and robotics. We have shown several applications of possibilistic dependencies ranging from multi-period portfolio selection problem with fuzzy returns to designing a second-generation biodiesel supply chain network. However, it is still an open problem to construct joint possibility distribution for correlated variables in applications [37].

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