Chapter 1
Introduction

1.1 Applications for Fluid-structure Interactions

Many application problems involve the coupling of fluid-dynamics and solid-dynamics. If a large aircraft is flying, the aerodynamical forces cause a substantial bending of the wings. This deformation significantly alters the geometry of the aircraft and affects its aerodynamical properties. Airborne, the tip of the wing will be deflected upwards by up to 10 m compared to the runway position. A simulation of the dynamics of flying planes must take this coupling into account.

If we consider the hemodynamical flow of blood in large vessels like the aorta or the blood flow in the heart, the forces on the vessel walls will lead to a widening and stretching of the tissue that again changes the overall geometry. Then, on the other hand, the new geometry will call for a new flow-field.

The lubricant flow in ball-bearings takes place in a very narrow channel between the balls and the surrounding ring. This gap can be as small as 100 nm. Enormous pressures and forces act and will cause a deformation of the surrounding steel parts. Steel of course is very stiff and deformation is small, but even a widening of the channel of only 50 nm can completely change the flow channel. Under the extreme conditions in the lubrication film, the oil reacts nonlinear on the forces, such that non-Newtonian material laws must be considered.

These problems have different characteristics, leading to different dynamics of the coupled system with different demands to the describing models and to the computational approaches. First of all, aerodynamical problems are in the regime of high Reynolds numbers with turbulent flow patterns and strictly three-dimensional character. The two other problems take place in the laminar regime. Furthermore, blood can be modeled as incompressible fluids, whereas air—at high velocity—or oil—at extreme pressures—must be treated as compressible. Numerical approaches for laminar and turbulent flows strictly differ. Here and in this context, we will deal with laminar problems only.

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Another striking difference between these problems is the type of material in the two phases, fluid and solid. The density of air is about $\rho_{\text{air}} \approx 1\,\text{kg}\cdot\text{m}^{-3}$, the density of water, blood or oil approximately $\rho_{\text{water}} \approx 1000\,\text{kg}\cdot\text{m}^{-3}$. Aluminum, as an important material in aircraft design has a density of $2700\,\text{kg}\cdot\text{m}^{-3}$, steel a density of about $7850\,\text{kg}\cdot\text{m}^{-3}$ and biological tissue about $\rho_{\text{tissue}} \approx 1000\,\text{kg}\cdot\text{m}^{-3}$. We will see that the ratio of the masses plays an important role in the dynamics of the coupling between an elastic solid and an incompressible fluid. Approaches that may work very efficiently for coupling the flow of air interacting with steel $\rho_{\text{air}}/\rho_{\text{steal}} \approx 0.00013$, may cease work, when used to describe a coupled configuration of blood and biological tissue with $\rho_{\text{blood}}/\rho_{tissue} \approx 1$. This effect of instability is called *added-mass effect*.

1.2 Dynamics of Fluid-structure Interactions

The dynamics of fluid-structure interactions are caused by the interplay of the two different phases, fluid and structure. Physically, the coupling is usually realized by three different principles. First, as a *geometric condition*, the common domain of fluid and solid—we always call it Ω—is divided into the fluid part $\mathcal{F}$ and the solid part $\mathcal{S}$. These two domains do not overlap $\mathcal{F} \cap \mathcal{S} = \emptyset$ and they are divided by the interface $\mathcal{I} = \partial\mathcal{F} \cap \partial\mathcal{S}$, such that $\Omega = \mathcal{F} \cup \mathcal{I} \cup \mathcal{S}$. The domains can change over time $\Omega \mapsto \Omega(t)$, $\mathcal{F} \mapsto \mathcal{F}(t)$ and $\mathcal{S} \mapsto \mathcal{S}(t)$, but the *geometric condition* will always hold, e.g. $\mathcal{S}(t) \cap \mathcal{F}(t) = \emptyset$ and $\Omega(t) = \mathcal{F}(t) \cup \mathcal{I}(t) \cup \mathcal{S}(t)$, where $\mathcal{I}(t) = \partial\mathcal{F}(t) \cap \partial\mathcal{S}(t)$. Usually, no holes will appear between fluid and structure. There are of course models, e.g. cavitation, where exactly such a thing has to be considered. The second coupling condition is the *kinematic condition*: the velocity of the fluid at the interface $\mathcal{I}(t)$ is the same, as the velocity of the solid at the interface. This means that the fluid will stick to the boundary, which is the moving interface. This model is similar to the no-slip condition in viscous fluid-dynamics. The fluid will simply stick to the wall which now might move. Alternative conditions are possible. In the inviscid regime, e.g. when considering problems of aeroelasticity, the no-slip condition can be relaxed to a non-penetration condition that only prescribes the motion in normal direction: the fluid will not enter the solid (and it will not move apart to open holes). Finally, the third *dynamic condition* prescribes a balance of normal stresses at the boundary in terms of *actio et reactio*.

These three conditions together mainly determine the dynamics of the coupled problem. One of the big challenges connected to fluid-structure interactions is the motion of the underlying domains. We have to deal with equations of fluid- and solid-dynamics on moving domains. In structure mechanics, this is typical, as the motion of the solid is exactly the unknown solution. In fluid-dynamics one usually deals with fixed domains. Moving domains only play a role in certain applications, like large scale ocean or atmospheric dynamics, where the rotation of the earth, the underlying domain, has to be taken into account. Here, in addition to dealing with moving domains, we need to face the fact that the motion is not pre-described,

but unknown part of the coupled solution. Section 5.1.1 and Chap. 11 will focus on an analysis of the nonlinear and non-stationary dynamics of coupled fluid-structure interaction problems.

1.3 Mathematical Challenges

The mathematical challenges in the analysis of fluid-structure interactions are manifold. Even for the governing equations of the two sub-systems, the Navier-Stokes equations in fluid-dynamics and conservation equations for nonlinear hyper-elastic materials, many theoretical questions are still not answered. We can show existence and uniqueness of solutions only for regular data and only locally in time. Without a full understanding of these subproblems, it is obvious that we cannot give complete results for the coupled system of equations. It will show that the main mathematical problem of coupled fluid-structure interactions will come from the motion of the domains and from the realization of the coupling conditions. While the (incompressible) Navier-Stokes equations are of parabolic type, the structural equations are of hyperbolic type. We will see that the notation of the kinematic coupling condition that glues velocities of fluid and solid together, is not well-posed. Furthermore, by the motion of the fluid domain (which follows the deflection of the structural domain) it is easily possible to lose regularity and smoothness of the interface, where all the coupling takes place.

It is well beyond the scope of this book to give new results concerning the mathematical theory of existence and uniqueness of the underlying equations. Instead, we will provide the reader with an overview of topical results and research that is currently under investigation.

1.4 Partitioned Approaches

Numerical simulations of fluid-structure interaction problems have a long history. Early approaches are based on the independent experience with problems of fluid-dynamics and structure-dynamics, which are well-understood. Existing simulation tools for these two types of problems are well developed and very efficient. In industrial applications they are established and used on a daily basis. It is not astonishing that this long experience is also adapted to the coupled configuration. This leads to the concept of *partitioned approaches* for fluid-structure interactions. Existing methods for handling fluid- and structure-dynamical problems are coupled by means of an outer control. This outer coupling is to be taken literally and extends to the coupling of two completely different software-frameworks used for the two subproblems: efficient finite volume schemes for fluid-dynamics and state of the art finite element schemes in solid dynamics are connected by means of boundary

condition and traction forces. Such an ad hoc approach will guarantee quick success and allows to treat complex application problems.

The basic approach for coupling is build on temporal discretization of the two problems by simple time stepping schemes. For approaching a new time step, one has to solve both subproblems for fluid and solid and one has to take into account the coupling conditions. These three tasks are fully coupled and each one has an effect on the others: motion of the solid changes the geometry and therefore the fluid-domain (geometric condition). Motion of the interface prescribes a motion of the attached fluid (kinematic condition). Modification of the fluid-domain will alter the flow field and hence generate new forces on the interface (dynamic condition).

Most basic partitioned methods for fluid-structure interactions can be regarded as semi-implicit time stepping schemes. Instead of performing a coupled solution step for fluid, structure and interface conditions, one first solves the one problem while neglecting the other and then takes a step of the second problem. Let us assume that f^n and s^n are the states for fluid and solid at time t_n and that Ω^n, $\mathcal{F}^n$ and $\mathcal{S}^n$ is the domain partitioning. Then, a simple partitioned scheme could consist of performing a step of the solid problem, driven by the fluid's normal stresses at time t_n

$$S(f^n, s^n, \Omega^n) \mapsto s^{n+1}. \tag{1.1}$$

This new solid state s^{n+1} includes a prediction for the deformation, i.e. the shape of the domain Ω^{n+1}, at time t_{n+1}. Furthermore, by s^{n+1} and s^n we can estimate the velocity of the interface. Using this interface velocity as boundary condition for the fluid's velocity, we can perform a step of the flow problem on the predicted domain Ω^{n+1}

$$F(f^n, s^n, s^{n+1}, \Omega^{n+1}) \mapsto f^{n+1}. \tag{1.2}$$

Such weakly coupled partitioned schemes are very easy to implement, as they only require the successive solution of the two different subproblems and the possibility to modify the fluid-domain during the simulation. In particular in aeroelasticity, partitioned schemes are widely used.

These simple schemes cannot guarantee a solution of the fully coupled problem. In each of the two sub-steps, one focuses on one problem only and one takes only parts of the coupling conditions into account. There will be a splitting error in the interface condition which is at least as large as the time step size. For many applications, this simple approach is furthermore not stable and may require vanishing time step sizes to show convergence at all.

A first step towards an accurate and controllable solution of the coupled problem is to iteration the two solution steps (1.1) and (1.2) until convergence is reached. Denoting the solution of (1.1) $s^{n+1,(0)}$ and the solution of (1.2) $f^{n+1,(0)}$ we iterate for $i = 1, 2, \ldots$

$$\begin{aligned} S(f^{n+1,(i)}, s^{n+1,(i)}, \Omega^{n+1,(i)}) &\mapsto s^{n+1,(i+1)}, \Omega^{n+1,(i+1)}, \\ F(f^{n+1,(i)}, s^{n+1,(i+1)}, \Omega^{n+1,(i+1)}) &\mapsto f^{n+1,(i+1)}, \end{aligned} \qquad i = 1, 2, \ldots$$

This iteration is repeated, until convergence is reached. Strongly coupled partitioned schemes solve the fully coupled fluid-structure interaction problem. For some applications however—in particular if the *added mass effect* acts—many sub-iterations of very small time steps can be required.

There exists vast literature on the development of acceleration schemes for strongly coupled partitioned iterations and their analysis. Most real-world applications nowadays use partitioned solvers. This is partially due to the effect that partitioned schemes allow for the use of highly tuned and efficiently implemented software frameworks for the two subproblems. Partially the success of this approach must be attributed to the necessity of reusing existing tools.

1.5 Monolithic Models and Finite Element Discretizations

From a mathematical viewpoint, partitioned approaches are not satisfying. First and most important: there exists no complete description of the coupled problem that includes fluid-problem, solid-problem and the interface conditions. Instead, the coupled problem is replaced by an algorithm. Without a fully coupled formulation of the complete problem it will never be possible to design discretization schemes that are fully implicit. For reasons of stability and to be allowed to use large time steps, this however is highly desirable. Having a fully coupled—we call it *monolithic*—model for the whole problem, we can furthermore use strongly coupled solution schemes like Newton linearization, multigrid or Krylov subspace methods for the complete problem without the need to sub-iterate between fluid and solid.

The derivation of monolithic models will be the first important task of this book. The main difficulty will be the realization of the coupling conditions and in particular mastering the moving domains. Once we have derived monolithic variational formulations of the coupled problem, we will be able to use all the powerful techniques of Galerkin methods, like simple spatial and temporal discretization, a priori and a posteriori error analysis and strongly coupled solution methods for the discrete variational problems.

For deriving variational monolithic models, we must overcome the difficulty of different coordinate frames usually used to describe the two subproblems. In this book, we follow the concept of domain mappings to bring the two different systems together. The *Arbitrary Lagrangian Eulerian* coordinate framework has

been introduced in the seventies of the last century to work with flow problems on moving domains [120, 192, 202]. It consists of defining a reference configuration $\hat{\mathcal{F}}$ and a mapping $T_f(t) : \hat{\mathcal{F}} \to \mathcal{F}(t)$ that captures the motion of the fluid-domain. The reference domain $\hat{\mathcal{F}}$ is called *arbitrary*, as it does no necessarily (it usually does not) have any physical meaning. $T_f(t)$ does not describe the mapping between the Lagrangian system and the Eulerian current system. Instead, $\hat{\mathcal{F}}$ is a fixed domain that is somewhere in between the Eulerian and the Lagrangian system. As an example, one can consider the observation of a car that is driving next to the own car at same speed. Viewing through the side window, the neighboring car appears fixed. This viewpoint is neither the Eulerian, where the car would move, or the Lagrangian, where we would follow the air-particles. If such a reference domain $\hat{\mathcal{F}}$ fits to the Lagrangian reference system of the solid $\mathcal{S}$ (as it does for the car example) and if the mapping $T_f(t)$ is such that it follows the motion of the interface, the *Arbitrary Lagrangian Eulerian* (ALE) framework is a simple way to transform the coupled fluid-structure interaction problem into a system that is fixed in space and time. While $\mathcal{S}$ is—at all times—the Lagrangian framework, $\hat{\mathcal{F}}$ is without physical meaning.

By the transformation onto the reference system, all problems can be formulated on fixed domains. The interface between solid $\mathcal{S}$ and fluid $\hat{\mathcal{F}}$ in fixed reference state will not move any more. This comes at the price of a transformation of the Navier-Stokes equations. As the motion of the domain cannot simply disappear, it enters as additional nonlinearity and as additional transport terms, comparable to the Coriolis effect in large scale flows of the ocean on the rotating earth. In ocean simulations, the computational domain is fixed and the rotation of the earth enters the equation as a Coriolis term.

The ALE framework is widely used for fluid-structure interactions. Not only to derive monolithic formulations, but also as a simple way to realize the coupling conditions in partitioned approaches. The success of the ALE method strongly depends on the quality, i.e. the regularity, of the mapping $T_f(t)$. If this mapping loses its regularity, the problem of the transformed Navier-Stokes equations will not longer be valid. ALE methods often fail, if the motion of the domains (in particular of the fluid domain) gets very large or if it is very fast. This can easily be explained using an extreme yet simple case: we want to describe the interaction of an elastic (or rigid that does not matter for this discussion) ball with a surrounding fluid. The ball is supposed to be more dense, such that it falls down. Realizing this problem in ALE coordinates requires a reference system $\hat{\mathcal{F}}$ and the ALE map $T_f(t) : \hat{\mathcal{F}} \to \mathcal{F}(t)$ mapping onto the current configuration. At some point, the ball will get close to the bottom boundary of the fluid-domain and may even touch it. Then, the topology of $\hat{\mathcal{F}}$ and $\mathcal{F}(t)$ differs. While $\hat{\mathcal{F}}$ has a hole (the ball), the domain $\mathcal{F}(t)$ at contact is simply connected, as the ball is now longer a hole in the domain, if it touches the exterior boundary. Such a mapping T_f between domains of different topology cannot be differentiable, not even continuous and the ALE formulation will fail.

Hence, we will introduce a second monolithic model for fluid-structure interaction that goes the other way around: the fluid-problem will be untouched and

reside in the Eulerian current system on the moving domain $\mathcal{F}(t)$. Instead, we will introduce a mapping $T_s(t)$ that maps the Lagrangian solid reference system to the Eulerian current system $T_s(t) : \mathcal{S} \rightarrow \mathcal{S}(t)$. The conceptual difference between the ALE map T_f and the Lagrangian-Eulerian map T_s of the solid problem is its physical background: T_f maps an artificial reference domain onto the current configuration. T_s maps between the Lagrangian and the Eulerian configuration. Both of them are physical and the elastic solid problem is well-posed in both configurations. The Lagrangian framework is usually considered, as it is more convenient for carrying out computations. The real world however is Eulerian. Regardless of the deformation or motion of the solid, both frameworks will be equivalent and can be used for describing the coupled problem. Coupling of fluid and solid will now take place on the moving interface $\mathcal{I}(t) = \partial\mathcal{F}(t) \cap \partial\mathcal{S}(t)$. This concept of *Fully Eulerian Coordinates* for fluid-structure interaction sounds strikingly simple and convincing. However, there are reasons for its very late introduction by Dunne [126] in 2005, as it allows for a simple mathematical description, but brings along plenty of difficulties. Most important, it requires to deal with moving domains and with a moving interface. While the interface location is fixed in the ALE method, its location is not even a priori known—but it has to be *captured*—during the simulation. Furthermore, convergence of all approximation techniques like finite elements or finite differences worsens, if interfaces or boundaries are not correctly approximated. The possibility to describe problems with arbitrarily large motions and deformations in a variationally coupled monolithic formulation comes at a high price.

Even if the focus of this book is on monolithic models, as they allow for more rigorous analysis of discretization and solution techniques, we will frequently discuss partitioned methods. An efficient numerical solver for a monolithic model will often make use of partitioned techniques.

1.6 Outline

The book is divided into three parts. The first one, *Fundamentals* deals with the basic models of fluid- and solid-mechanics, with the theory of coupling and with basic discretization techniques. The second part, *Realizations* details two monolithic models for fluid-structure interactions, the ALE approach and the Fully Eulerian Formulation. Further, we specify discretization and solution techniques for these two formulations. In the third part *Applications* we cover advanced topics as contact problems, interaction to chemistry or optimization problems.

Chapter 2 gives an overview of the models for fluid- and structure-dynamics that are involved in coupled fluid-structure interactions. We highlight the concept of continuum mechanics in different coordinate frameworks and configurations. Where it helps the understanding, we provide proofs for the fundamental theorems. Readers familiar with these basics can directly jump to the following Chap. 3. This overview

is too short for a comprehensive analysis, we however mention the main results and note plenty of literature for further reading.

In Chap. 3 the coupling of the two subproblems is detailed. We describe the different coupling conditions and their theoretical and practical implications. Here, we also derive variational formulations for coupled fluid-structure interactions in the two different coordinate frameworks: Arbitrary Lagrangian Eulerian and Fully Eulerian.

We continue in Chap. 4 with introducing the finite element method and techniques for discretization in time. After some basics on the discretization of elliptic and parabolic problems, followed by a discussion of saddle-point problems like the incompressible Navier-Stokes equations or conservation laws of incompressible solids, we quickly proceed to the special efforts of fluid-structure interactions. Most of this chapter is generic and does not only apply to fluid-structure interactions.

In Chaps. 5 and 6, we focus on the realization of the two monolithic models considered in this book, fluid-structure interactions in Arbitrary Lagrangian Eulerian and in Fully Eulerian coordinates. For both formulations we discuss details of the numerical realization, discretization in space and time, questions of linearization and discuss the solution of the resulting problems.

Chapter 7 is devoted to techniques for the solution of the very large linear systems arising from discretization and linearization of the coupled fluid-structure interaction problems. These large and complex problems lack any desirable structure like symmetry or positivity and are very ill-conditioned. To derive efficient numerical solvers, it will be necessary to abandon the strict monolithic character and to adapt ideas from partitioned approaches.

Advanced techniques for local mesh refinement, as a mechanism for reducing the dimension of the discretized problems are described in Chap. 8. In particular three dimensional problems quickly get huge. By introducing a posteriori error estimators for construction of efficient finite element meshes, we can strongly reduce the dimension of the discrete problem thus the time to solution.

The third part comprises four chapters with different advanced techniques. We call this part *Applications*, although we do not present industrial applications but discuss relevant configurations, that go beyond the scope of standard approaches. In Chap. 9 we start with optimization problems that involve fluid-structure interactions as constraint. We describe gradient based methods that will require the assembly of sensitivity information. Next, in Chap. 10 we tackle coupled mechano-chemical fluid-structure interaction problems. Chemistry will enter as a third field. In a prototypical problem we model the growth of plaques in blood vessels. In Chap. 11 we study the nonlinear dynamics of a fluid-structure interaction problem and will discuss the special properties that arise from the coupling of the non-stationary Navier-Stokes equations with the elastic solids. Finally, Chap. 12 will provide a method for handling fluid-structure interaction problems that include contact. Here, we make full use of the Fully Eulerian formulation. This last chapter is a guest article of Stefan Frei [151], who contributed the essential analysis and derived efficient numerical techniques for an accurate discretization of fluid-structure interactions in the Fully Eulerian formulation.