

Field-Based Analysis and Optimal Shape Synthesis of Switched Reluctance Motors

Paolo Di Barba, Maria Evelina Mognaschi, Marek Przybylski,
Najmeh Rezaei, Barbara Slusarek and Slawomir Wiak

Abstract This study focuses on a class of a three-phase switched reluctance motor. The aim of the paper is to optimize torque and iron loss as a function of the geometry. To enhance the efficiency of the motor, a procedure of automated optimal design is adopted. The analysis model of the motor is based on 2D finite element method simulation, while the design optimization is based on evolutionary computing.

Keywords Finite elements methods · Genetic algorithm · Multi-objective optimization · Switched reluctance motor

1 Introduction

Switched reluctance motors (SRM) have inherent advantages such as simple structure with non-winding construction in rotor side, fail-safe because of high tolerances, robustness, low cost with no permanent magnet in the structure, and also

P. Di Barba (✉) · M.E. Mognaschi · N. Rezaei
Department of Electrical, Computer and Biomedical Engineering,
University of Pavia, Pavia, Italy
e-mail: paolo.dibarba@unipv.it

M.E. Mognaschi
e-mail: eve.mognaschi@unipv.it

N. Rezaei
e-mail: najmeh.rezaei01@ateneopv.it

M. Przybylski · B. Slusarek
Tele and Radio Research Institute, Warsaw, Poland
e-mail: marek.przybylski@itr.org.pl

B. Slusarek
e-mail: barbara.slusarek@itr.org.pl

S. Wiak
Institute of Mechatronics and Information Systems,
Lodz University of Technology, Lodz, Poland
e-mail: swiak@wp.pl

possible operation at high temperatures or under strong temperature variations [1]. With these advantages, the SRM recently are increasingly used in a broad range of applications. The fundamental theory, design procedure, modeling, and analysis of SRM have been presented in the literature [2–4]. In recent years, machine designers have focused greatly on evolutionary computation based design optimization techniques to fulfill the desired performance requirements under various constraints such as converter rating, winding configuration, and outline dimensions [5]. Automated optimal design of motors has been applied since the 90s [6] and nowadays it is still being used successfully [7]. Multi-objective optimization methodology has been used in various optimization problems in different areas of engineering considering the aspect multi-physics of the electrical devices and MEMS actuator like those proposed in [8, 9]. From the literature [10–12], it is also evident that computational intelligence techniques like genetic algorithm (GA) has been effectively implemented for design optimization of SRM.

As the design of SRM for a particular application is a compromise between various performance criteria, improvement of a performance parameter may result in the degradation of other important features. Consequently, the designer has to search for solutions that are feasible with respect to all performance parameters. To deal with this trade-off and achieve efficient design, multi-objective optimization based design techniques seem to be the most suitable approach. Hence, there is growing interest towards the application of multi-objective optimization techniques for solving a wide variety of SRM design optimization problems [13].

Shape design of electromagnetic devices usually demands that multiple criteria be fulfilled concurrently. The most general solution is represented by the front of nondominated solutions [14]. The aim of this study is to optimize geometrical parameters of a 12/8 SRM to improve the maximum torque and also to reduce the total iron losses. The machine then is analyzed through finite element model (FEM) due to its accuracy in modeling complex geometry and considering physical phenomena like saturation. Genetic algorithm (GA) optimization code was carried out under MATLAB software coupled to FEM.

2 Motor Model and Field Analysis

The proposed structure of the 12/8 SRM, characterized by nominal power 450 W, 14,000 rpm and supply voltage of whole drive of 230 V, 50 Hz is shown in Fig. 1. It can be used as the drive of house appliances like washing machines [15]. The values of physical sizes of motor are reported in Table 1.

The stator and the rotor are assumed to be made of “M27: USS Motor - 26 Gage (M330-50-A5 according to IEC 60404-8-4 standard)” 0.5 mm thick electrical steel which the B–H curve is shown in Fig. 2 and loss curve is shown in Fig. 3. An advanced design procedure is needed in view of the design optimization in terms of geometry properties.

Fig. 1 Geometry, design variables, flux lines

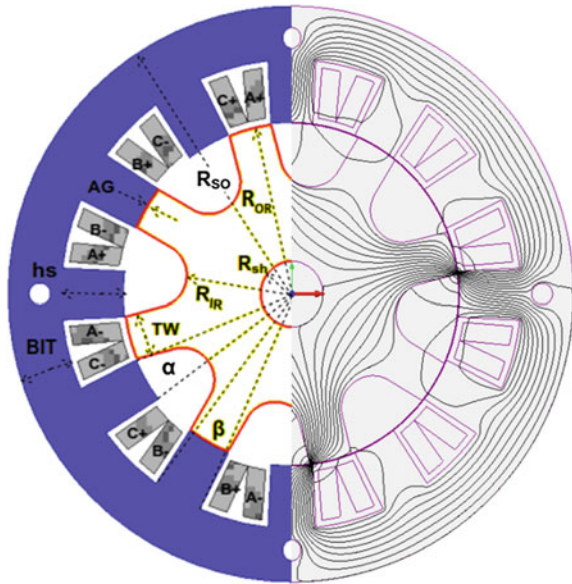


Table 1 Size of switched reluctance motor

Parameters	Value	Symbol
Stator outer radius	70 mm	R_{SO}
Stator inner radius	41.75 mm	R_{SI}
Shaft radius	8 mm	R_{Sh}
Air gap length	0.25 mm	AG
Rotor outer radius	41.5 mm	R_{OR}
Rotor pole width	11 mm	TW
Rotor inner radius	26.5 mm	R_{IR}
Stator pole height	15.25 mm	h_s
Back iron thickness	13 mm	BIT
Stator tooth outer span	15.14 ($^{\circ}$)	α
Stator tooth inner span	12.58 ($^{\circ}$)	β
Number turns of a coil winding	140	N
Axial length of a magnetic core	46.6 mm	

Since the motor exhibits a four-pole magnetic field which is shown in Fig. 1; currents in the motor windings have been set to simulate one-phase control mode ($i_a \neq 0, i_b = 0, i_c = 0$). Each phase incorporates four coils; phase “a” is driven by unipolar current with a constant value of 1 A. The two-dimensional finite element model of the motor is implemented using MagNet code by Infolytica [16]. The mesh with maximum element size of 0.5 mm with a detail of which is shown in Fig. 4 is considered. In simulation, the axial stack length has been set equal to 1 m.

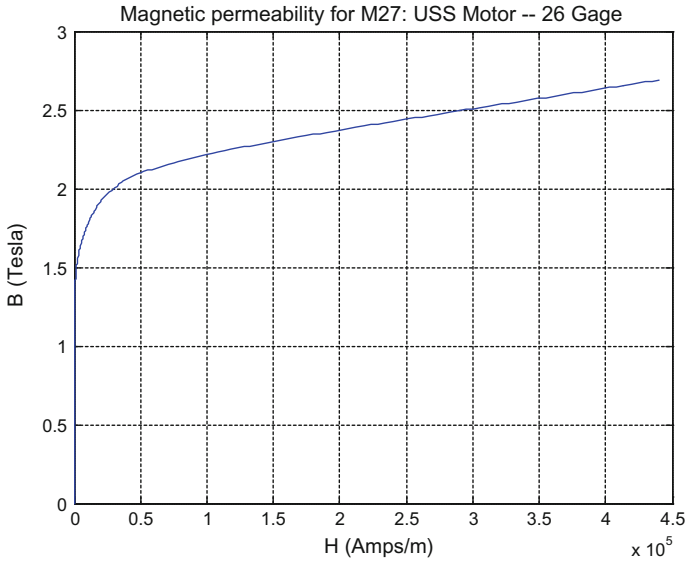


Fig. 2 Specific B-H curve for M27 material

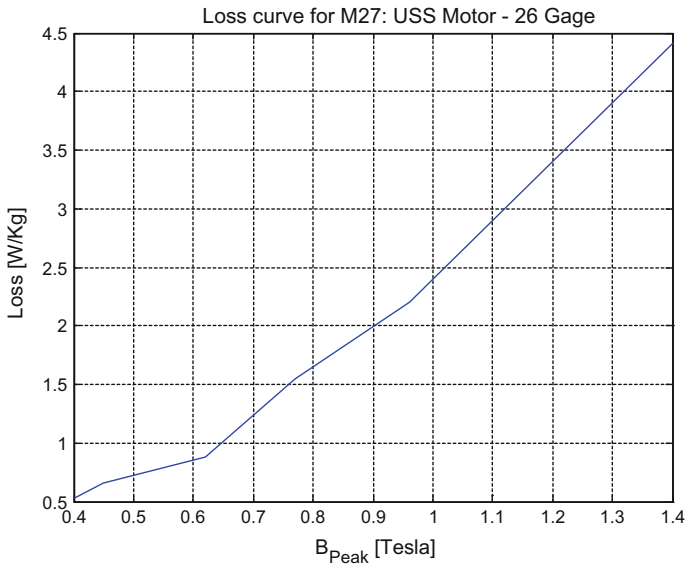
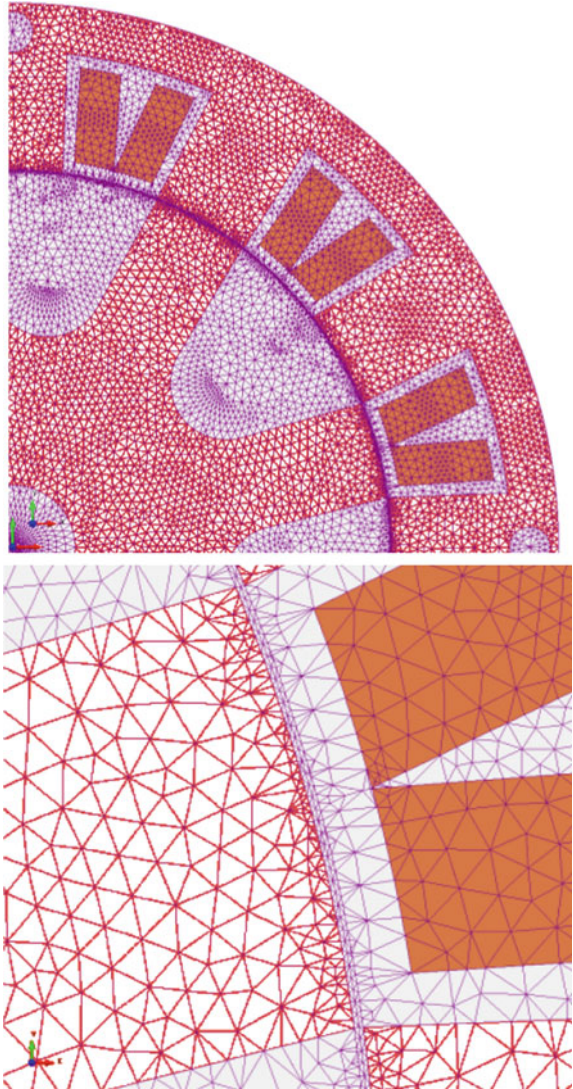


Fig. 3 Loss curve for M27 material at a frequency = 60 Hz

Fig. 4 Detail of the mesh

A typical solution for the field analysis of prototype model is shown in Fig. 1. The torque versus rotor position over 45° is shown in Fig. 5. Moreover, total losses were calculated by assuming supplying phase A with 1 A current with 60 Hz frequency and is also shown in Fig. 5. Additionally, magnetic induction along the air gap midline is shown in Figs. 6 and 7.

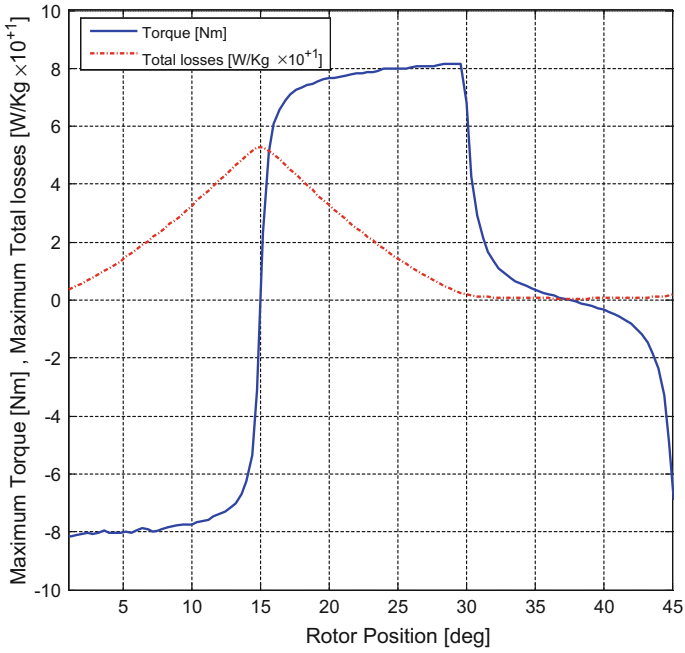


Fig. 5 Motor torque and iron losses versus rotor position for prototype

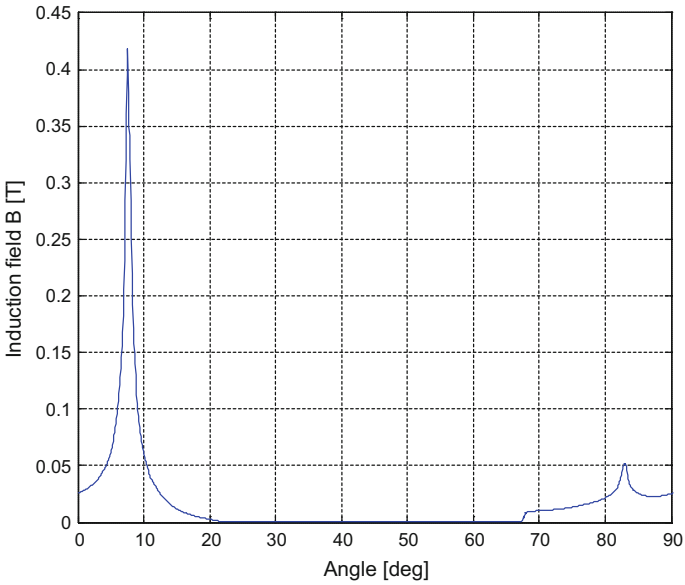


Fig. 6 Magnetic induction along the air gap midline at rotor initial position (see Fig. 1)

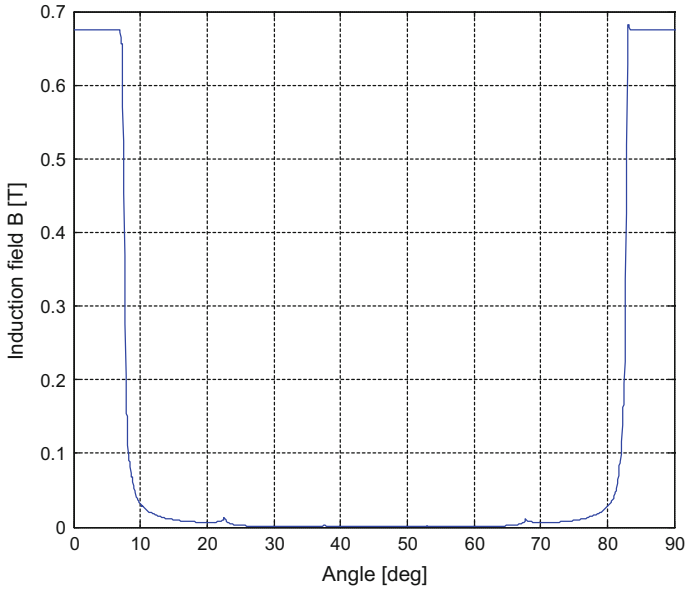


Fig. 7 Magnetic induction along the air gap midline with rotor aligned with stator poles

3 Inverse Problem: Optimal Shape Synthesis

Geometric sizes of machine are considered as unknown parameters for the optimization procedure. Specifically, the rotor outer radius, rotor inner radius, and back iron thickness are considered as design variables while the remaining parameters are considered as fixed.

- X_1 = rotor outer radius (R_{OR})
- X_2 = rotor inner radius (R_{IR})
- X_3 = back iron thickness (BIT)

The following inverse problem is considered:

given the material properties (i.e., B–H magnetization curve, P–B loss curve) and the power supply (one phase on, equal to 1 A), find the optimal values of geometric variables such that the maximum torque is maximized and the iron loss is minimized, subject to the problem constraints.

3.1 Design the Problem

A vector $X = \{X_i\}$, $i = 1,2,3$ of design variables is shown in Fig. 8. To satisfy variable bound as one of constraints, the overall diameter of the switched reluctance

Fig. 8 Prototype geometry, design variables

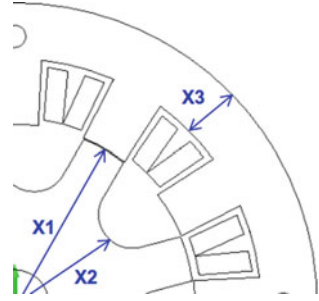


Table 2 Variation range of the design variables

Design variables X (mm)	X_1	X_2	X_3
Min	30.5	17	6
Max	46.6 mm	30.97	24

motor must not exceed 70 mm; and the air gap width is kept constant equal to 0.25 mm. The range of continuous-valued variables is reported in Table 2.

Overall, the constraints define the feasible design space of $\Omega \in \mathbb{R}^3$. The problem of determining optimal value for these parameters is formulated to provide trade-off solutions between torque density and power loss in the iron core. The objective functions are defined as:

$$f_1(x) = \text{Minimization of total iron losses}$$

$$f_2(x) = \text{Maximization of the maximum value of torque.}$$

The above criteria are defined by:

$$f_1(x) = \int_{S(x)} P[B(x)]^2 ds, x \in \Omega$$

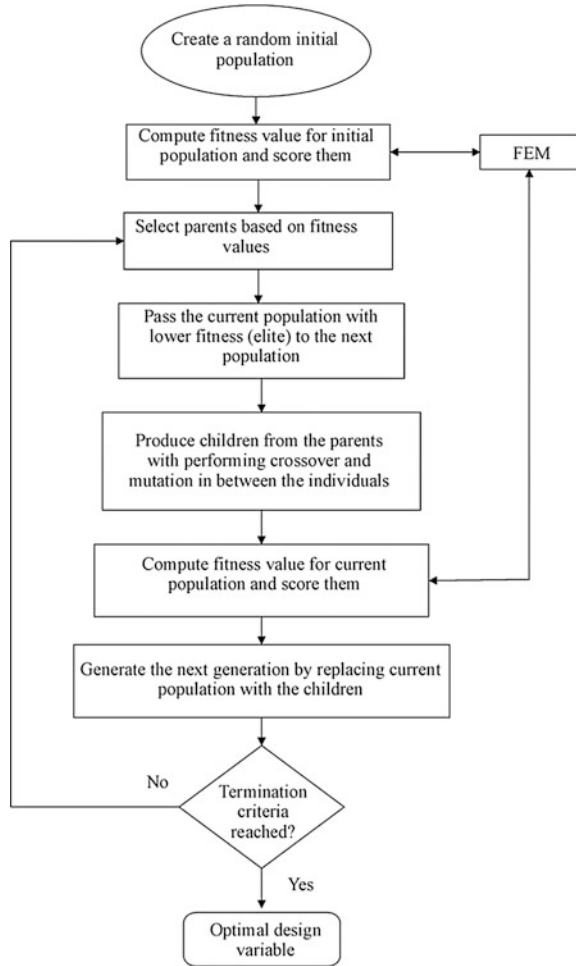
$$f_2(x) = \int r_0 \times f dv,$$

$$f = \int_{\Omega} \nabla \cdot \overline{\overline{T}} d\Omega = \int_{\Gamma} \overline{\overline{T}} \cdot \overline{\overline{n}} d\Gamma$$

where P is the specific power loss in the ferromagnetic material subject to induction B , S is iron volume, r_0 is the vector going from the origin to an element of volume in the body, V is the volume of the body, f is the force on body calculated by $\overline{\overline{T}}$ the Maxwell's stress Tensor, Γ is a closed surface enclosing the body and $\overline{\overline{n}}$ is the outward normal versor, respectively. Due to this fact that the induction, iron volume, and force values are dependent on x (the design variables), thus the inverse problem should be investigated for finding optimal values of geometric parameters for a given material to satisfy problem criteria.

In this study, a multi-objective genetic algorithm optimization method based on Pareto-optimal solutions is implemented for solving the problem. Figure 9 shows the flowchart diagram that is used for multi-objective optimal design of an SR motor in MATLAB [17].

Fig. 9 Optimal design of SR motor by GA in MATLAB



3.2 Optimization Process and Problem Formulation

Genetic algorithm (GA) is one kind of direct search algorithms, based on the developing mechanism from genetic evolution and natural selection. It begins by randomly creating its population. Each individual of the population represents a search point in the space of potential solutions of the given optimization problem. Candidate solutions are combined by a crossover operator to produce offspring, which expands the current population of solutions. Thus, the individuals in the population are evaluated via the fitness function. Meanwhile, a mutation operator is performed at a certain probability level to increase variation in the search space. By favoring the mating of the more fit individuals, the more promising areas of the search space are explored. The process of evaluation, selection, crossover and

mutation is repeated until a predetermined number of generations are reached or a satisfied solution has been found. The following sections describe each of the components of our GA method [18].

Multi-objective optimization involves minimizing or maximizing multiple objective functions subject to a set of constraints that are often contradictory, as the minimization of an objective leads to an increase of another goal, so the solution we seek is always a compromise between these objectives. The general multi-objective optimization problem (MOOP) can be stated as finding the n-dimensional vector, x , which

$$\text{Min or Max } f(x) = f_1(x), f_2(x), f_3(x), \dots, f_n(x) \quad (1)$$

$$x \in F$$

where $x \in R^n, f_i : R^n \rightarrow R$ and F is the feasible set of problem including inequality, equality and/or variable bounds to be satisfied such as:

$$F = \{x \in R^n : g_i(x) \leq 0 \ \& \ h_i(x) = 0 \ \& \ a x_{lb} \leq x_i \leq x_{ub} \quad i = 1, 2, \dots, n \quad (2)$$

The vector $f(x)$ includes several objective functions. An ideal solution of (1) introduced as Pareto-optimal solution (nondominated set) would be a point $x^* \in F$ such that (Table 3):

$$f_i(x^*) \leq f_i(x), \forall x \in F, \forall i \in \{1, 2, \dots, k\}$$

Population size is the number of individuals in each generation.

Selection function defines the selection method of parents for the next generation. “Tournament” selection chooses each parent by choosing tournament size players at random and then choosing the best individual out of that set to be a parent.

Crossover function is a genetic operator that combines two individuals, or parents to produce a new child for the next generation. The “Intermediate” crossover function creates children by taking a weighted average of the parents. Intermediate crossover (IC) is controlled by a single parameter ratio which can be a scalar or a row vector of length number of variables.

Table 3 Genetic optimization parameters

Parameters	Value/type
Population size	20
Selection function	Tournament
Crossover function	Intermediate
Mutation function	Adaptive feasible
Elite count	1
Crossover fraction	0.8
Pareto fraction	1
Stopping criterion	50 generations

$$\text{child} = \text{parent1} + \text{rand} * \text{Ratio} * (\text{parent2} - \text{parent1})$$

The ratio parameter in this study is set to 1.

Mutation function is used by genetic algorithm to make small random changes in the individuals in the population in order to create mutation children that provides genetic diversity. Thus, mutation enables the genetic algorithm to search a broader space. “Adaptive Feasible” mutation function is used so that mutation satisfied constraints and bounds.

Elite count specifies the number of individuals that are guaranteed to survive to the next generation.

Crossover fraction sets the fraction of the next generation, other than elite children, that are produced by crossover.

Pareto fraction defines the fraction of individuals to keep on the first Pareto front while the solver selects individuals from higher fronts.

Stopping criterion determines what causes the algorithm to terminate [17].

4 Results

The results of the optimization (solution of a Pareto-optimal set in objective space) and the objective function space of the synthesis problem are shown in Fig. 10. The solutions (marked by circles) are examples of best compromise solution between conflicting design criteria, i.e., torque and losses [19].

To illustrate the specific improvements obtained from optimization, two solutions are selected and their comparison with initial design (prototype) is presented in Table 4. The comparison of the results shows that solution B has best maximum torque but higher iron loss, whereas solution A has low iron loss and poor torque. The comparison of the results verified that each objective gets improved at the cost of the other and there is a clear trade-off between maximum torque and total losses. Notice that there is no clear best design, therefore the design can be selected based on the preferences of the designer, for a given specific application.

Flux and field distributions of the two optimal designs in the nominal condition are carried out by Magnet. The geometry and field results are shown in Figs. 11 and 12.

With respect to the optimal solutions obtained on Pareto front (circles in Fig. 9), the trend of varying design variables along Pareto front is shown in Fig. 12. By analyzing Figs. 9 and 12, the contribution of each design variables to performance of the model is defined. It shows that as rotor outer radius is increased, the maximum torque and consequently maximum iron losses are increased too. In contrary, as back iron thickness is increased, the maximum torque and maximum losses are decreased. It is worth highlighting that rotor inner radius makes nondominant contribution to quality characteristics. Additionally, it is also interesting to note that the prototype is one of the solutions on the front (Fig. 13).

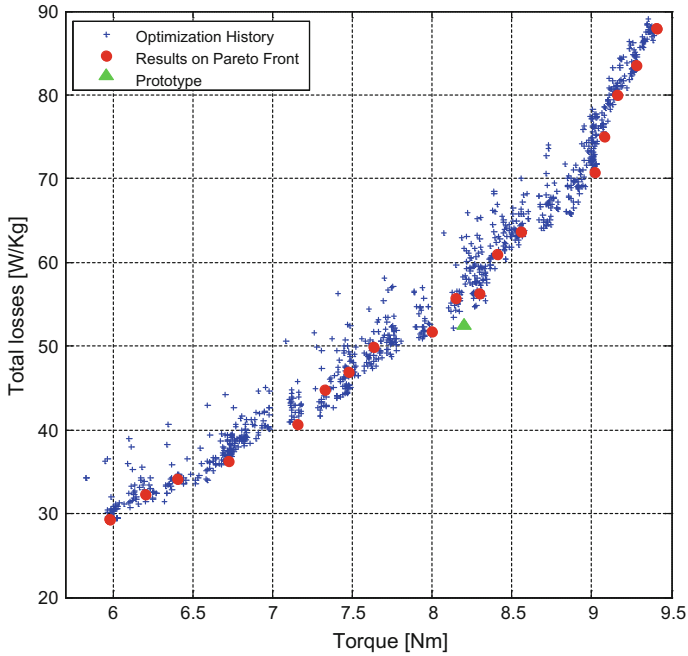


Fig. 10 Objective space, prototype (*triangle*), nondominated solutions (*circle*)

Table 4 Compared solutions obtained for objective functions

Performance parameters	Rotor outer radius (mm)	Rotor inner radius (mm)	Back iron thickness (mm)	Torque (Nm)	Total losses (W/kg)
Solution A	48.37	17.54	6.13	9.41	87.90
Solution B	30.50	18.87	24.00	5.98	29.25
Prototype	41.50	26.50	13.00	8.20	52.46

Fig. 11 Geometry and flux lines for solution A

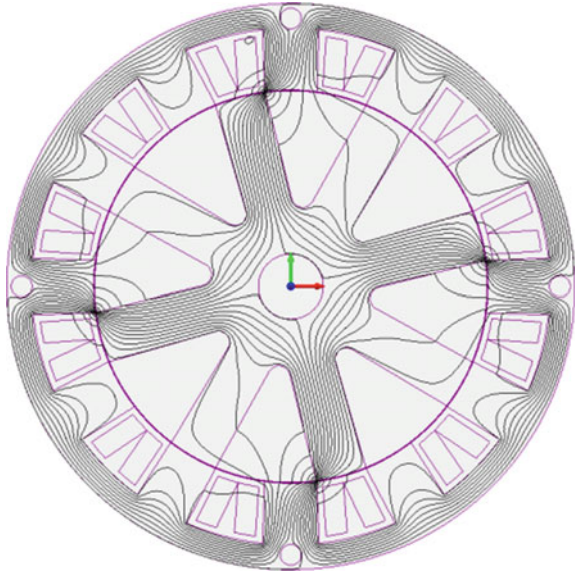
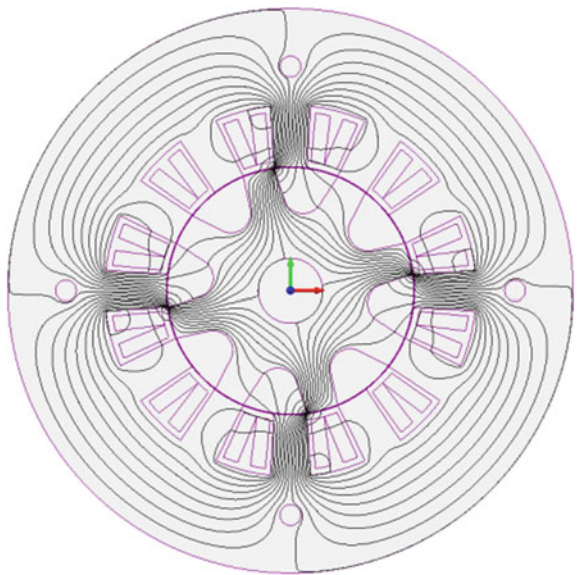


Fig. 12 Geometry and flux lines for solution B



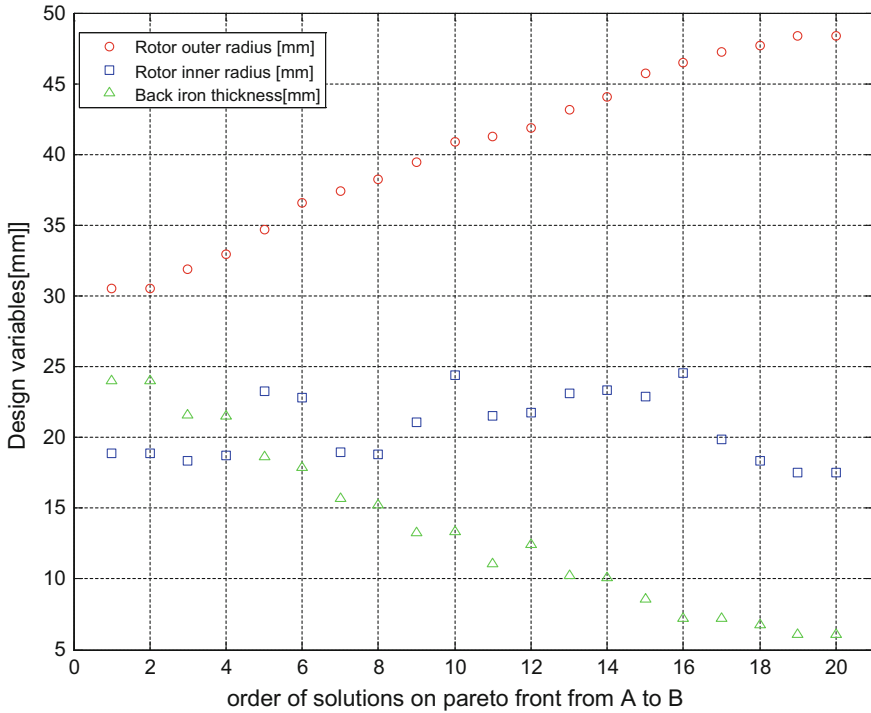


Fig. 13 Pareto front solutions comparison as a function of design variables

5 Conclusion

The optimization approach used in this work has proved that the optimal shape design problem is well posed because it has achieved its objectives, namely improving the performance of a 12/8 SRM prototype through the optimization of geometrical parameters under constraints. According to the results, it is possible to identify solutions which improved both torque and total iron losses.

References

1. Ahn, J.W.: Switched Reluctance Motor, Torque Control. InTech, Croatia (2011)
2. Stephenson, J.M., Blenkinsop, P.T., Corda, J., Fulton, N.N.: Variable-speed switched reluctance motors. *IEE Proc. Electr. Power Appl.* **127**, 253–265 (1980)
3. Krishnan, R.: Switched Reluctance Motor Drives: Modeling, Simulation, Analysis, Design, and Applications. CRC Press, Boca Raton (2001)
4. Miller, T.J.E.: Optimal design of switched reluctance motors. *IEEE Trans. Ind. Electron.* **49**, 15–27 (2002)

5. Uler, G.F., Mohammed, O.A., Koh, C.S.: Utilizing genetic algorithms for the optimal design of electromagnetic devices. *IEEE Trans. Magn.* **30**, 4296–4298 (1994)
6. Palka, R.: Synthesis of magnetic fields by optimization of the shape of areas and source distributions. *Archiv für Elektrotechnik* **75**, 1–7 (1991)
7. Di Barba, P., Mognaschi, M.E., Palka, R., Paplicki, P., Szkolny, S.: Design optimization of a permanent-magnet excited synchronous machine for electrical automobiles. *Int. J. Appl. Electromagn. Mech.* **39**, 889–895 (2012)
8. Di Barba, P., Dolezel, I., Mognaschi, M.E., Savini, A., Karban, P.: Non-linear multi-physics analysis and multi-objective optimization in electroheating applications. *IEEE Trans. Magn.* **50**, 673–676 (2014)
9. Di Barba, P., Dughiero, F., Mognaschi, M.E., Savini, A., Wiak, S.: Biogeography-inspired multiobjective optimization and MEMS design. *IEEE Trans. Magn.* **52**, 1–4 (2016)
10. Mirzaeian, B., Moallem, M.: Multiobjective optimization method based on a genetic algorithm for switched reluctance motor design. *IEEE Trans. Magn.* **38**, 1524–1527 (2002)
11. Raminosa, T., Blunier, B., Fodorean, D., Miraoui, A.: Design and optimization of a switched reluctance motor driving a compressor for a PEM fuel-cell system for automotive applications. *IEEE Trans. Ind. Electron.* **57**, 2988–2997 (2010)
12. Goldberg, D.E.: *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison Wesley Longman Publishing, Boston (1989)
13. Balaji, M., Kamaraj, V.: Evolutionary computation based multi-objective pole shape optimization of switched reluctance machine. *Int. J. Electr. Power Energy Syst.* **43**, 63–69 (2012)
14. Di Barba, P., Mognaschi, M.E.: Recent experiences of multiobjective optimization in electromagnetic: a comparison of methods. *Int. J. Comput. Math. Electr. Electron. Eng.* **24**, 921–930 (2005)
15. Jankowski, B., Kapelski, D., Karbowski, M., Przybylski, M., Ślusarek, B.: Analysis of static characteristics of a switched reluctance motor. In: Gołębiowski, L., Mazur, D. (eds.) *LNEE*, vol. 324, pp. 289–304. Springer, Heidelberg (2015)
16. MagNet, Infolytica Corporation. <http://www.infolytica.com>
17. *MATLAB Genetic Algorithm*, The MathWorks, Inc., Natick, USA (2013)
18. Xu, J.X., Panda, S.K., Zheng, Q.: Multiobjective optimization of current waveforms for switched reluctance motors by genetic algorithm. *Int. J. Model. Simul.* **24**, 1860–1865 (2002)
19. Di Barba, P., Mognaschi, M.E.: Sorting Pareto solutions: a principle of optimal design for electrical machines. *Int. J. Comput. Math. Electr. Electron. Eng.* **28**, 1227–1235 (2009)