Reliability Analysis of a Modified IEEE 6BUS RBTS Multi-state System

Thomas Markopoulos and Agapios N. Platis

Abstract In this chapter, we attempt to develop a stochastic model based on a modification of a standard energy system. Aiming to achieve a high level of reliability in the system, it is necessary to implement specific modifications that are necessary to improve the structure of the system, in order to meet the demanded requirements. This improvement is actually a restructuring of an IEEE 6 BUS RBTS system by using an alternative combination of its generation units that presents the lowest possible failure rates using the same kind of generators and maintaining the level of output specifications according to the minimum reliability requirements. Using Multi-state systems and Semi-Markov modeling, the final result is a modified system that presents more flexibility and operates in less uncertainty environment, leading to a better level of reliability.

Keywords Markov chains · Semi-Markov chains · Multi-state system · Power system

1 Introduction

Reliability is a timeless problem closely related with human activity since the 18th century and has passed through stages of evolution in the course of time [35]. The systematic study of reliability has taken place at the end of the 20th century due to dramatic increase of complexity of electric and electronic systems and the cost reduction of them. In case of power generating systems, the operational parameters are the frequency of interruptions and the expected time to repair the failure [11]. The reliability as a concept is incident to two states of a system, "operation" and

T. Markopoulos · A.N. Platis (✉)
School of Science and Technology, Hellenic Open University, Patra, Greece
e-mail: platis@aegean.gr

A.N. Platis
Department of Financial and Management Engineering,
University of the Aegean, Chios, Greece

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“failure”. This approach works for relative simple systems. However, a question about its effectiveness arises when we need to apply it in more complex systems, leading to the development of multi state systems analysis in the middle of 1980’s [22]. This topic is considered one of the most pioneering in the research of reliability theory [18]. Although the basic operational states of a system are two, normal operation and failure, each state of operation could consist of more than one “sub-states” (e.g. 80% or 60%, etc.) [35]. According to the literature, the term “*multi state system*” refers to a system that can be operating in a finite number of states [7]. Each complex system, consisting of a number of simple “two-state” sub-systems that have a cumulative effect on its performance can be considered a multi state system. The final performance of the whole system depends on the availability of the sub-systems and it is proportional with those that are operating [16].

In this study, we will consider failure of the system the non-acceptable level of operation due to specific requirements concerning the output level of the system, whereas some of the generators work normally and others fail. Since the failures are occurring events, and are related with independent systems (such as generators), we assume that they follow the Poisson distribution with parameter λ, whereas the time needed to repair a failed system follows the exponential distribution with a mean $1/\mu$. Referring to the IEEE 6 BUS RBTS, these parameters are depicted in Table 4. There is significant number of studies on multi state systems (MSS), because of their applicability especially in power generating and communication systems is broad [17]. Multi State Systems provide the advantage of flexibility and their representation is more accurate compared to the simple two state systems. On the other hand, their complexity holds the understanding and their performance evaluation back [33], e.g. there are systems that have hundreds or thousands of possible states. The development and the handling of such models results to depletion of the conventional Markov methods, emphasizing the need to apply innovative methods such as the EUGF [18]. The systems that are affected by the ageing of the materials with their maintenance effectiveness within limits confirm the need to combine the complexity and the flexible analysis capability [16]. Another advantage of multi-state systems is that they focus to the acceptable or non-acceptable level of operation on a specific time, instead of the “time to failure” of the simple systems, contributing to the analysis of applications closer to the real world [19], providing more accurate assessments [17] and a significant cut of the time required to develop an acceptable solution [4]. Concerning the power generating systems, all the above characteristics refer to the level of power available over a minimum acceptable level, in accordance with the requirements. An additional factor related to the reliability of power generating systems is the modeling of shocks that affect a system during its operation, perceiving these shocks in three major categories, such as the cumulative [10], the extreme [28] and the mixed [20]. All kinds of shocks mentioned above refer to a complete failure of the system. Actually, they could be an early variation of multi state systems, where the shock pushes the system from one critical state to another one, resulting to a partial failure of the system [7]. Due to their inherent complexity and the probable interaction among the existing subsystems, multi state systems present a dynamic behavior, when a sub system fails; the result on the whole system is much more severe [35]. Depending on the research requirements, the number of the states can be increase, leading to extreme complexity. This happens because it is possible to consider them as multi state systems and breaking them down in subsystems of lower level, making the need for further research intensified [18]. The mathematical form of a multi state system depicts the set of the possible states such as:

$$G_j = \{g_{j1}, g_{j2}, \ldots, g_{ji}, \ldots, g_{jk}\} \tag{1}$$

Where g_{ji} is the performance level of the subsystem j and $i \in \{1, 2, \ldots, k\}$ is the set of the possible states of the subsystems. Since we consider time of operation of the whole system, its state over time is a random variable included in a stochastic process [16]. Due to the stochastic nature of the states, there is a mean, a variance and a distribution of them as a random variable and this is precisely the importance to calculate the limits of operation of the system, in order to achieve a minimum level of reliability. One step further, the reliability function of a multi state system is:

$$R(t, w) = P\{G(t) \geq w\} \tag{2}$$

where $G(t)$ is the state of operation, at time t and w is the minimum required level of operation. The above equation leads to the separation of all states of the system in two groups. The first one is the set of acceptable states $\{d(w), d(w) + 1, \ldots, M\}$ and the second one $\{0, 1, \ldots, d(w) - 1\}$ is the set of non acceptable states [29]. The reliability function concerning the required level of operation describes the sum of the probabilities of all those acceptable states that are independent and is expressed by:

$$R(t, w) = P\{\Phi(t) \geq d(w)\} = \sum_{j=d(w)}^{M} p_j(t) \tag{3}$$

or with another expression:

$$R_{MSS}(t, W^*) = P\left[W(t) \geq W^*\right] \tag{4}$$

[14] where $W(t)$ is the level of output of the system at time t and W^* is the minimum required performance. Assuming that the states of non-acceptable operation level are equivalent to the state of failure of a simple system, the function of "state of failure" is:

$$F(t, w) = 1 - R(t, w) = \sum_{j=1}^{d(w)-1} p_j(t) \tag{5}$$

The Eq. (5) confirms the argument that there are more states of operation except for the normal operation and the complete failure [33]. Another parameter on the performance of the multi state systems is also the time the system spends in a state of operation. Assuming a transition of the system in $M + 1$ different states, where $M \in \{1, 2, 3, \ldots\}$ and $M \geq 1$. We denote that state *0* corresponds to complete failure of the system and state M reflects normal operation. Concerning the lifetime of the system in each state, we can consider the time the system lies in a state j or higher level of performance ($T \geq j$ for $j, j+1, \ldots, M$) [32]. Since $R_k(t)$ is the reliability function of the MSS (with discrete states) it is simply the probability of the system to operate in a state level higher than w at time t, this function is:

$$R_k(t) = \sum_{\phi = k}^{M} P\{\Phi(t) = \phi\}, k \in \{1, 2, 3, \ldots, M\} \tag{6}$$

with $0 \leq k \leq M$ [19]. Depending on the circumstances and because of their dynamic behavior, they proceed to states of partial operation until the state of total failure. Especially when the number of states is high, the transition is not always among consecutive states, but it is possible to omit intermediary states. The evaluation of a system is based on the assessment of reliability parameters, such as the rate the system downgrades. This parameter is related with all subsystems of the system and can be grouped in a matrix form such as

$$\Lambda = \begin{bmatrix} \lambda_{M,M-1} & \lambda_{M,M-2} & \cdots & \lambda_{M,0} \\ 0 & \lambda_{M-1,M-2} & \cdots & \lambda_{M-1,0} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \lambda_{0,0} \end{bmatrix} \tag{7}$$

With the probability the system to operate normally to be

$$P[\Phi(t) = M] = e^{[-(\lambda_{M,M-1} + \lambda_{M,M-2} + \ldots + \lambda_{M,0})t]} \tag{8}$$

the matrix of Eq. (7) can facilitate the understanding of the complexity that the researcher is possible to face, by understanding the system and the evaluation of its performance [33]. However, this complexity is useful because in many cases the two state systems often lead to erroneous and disappointing results [4]. One way to deal with the problem of the complexity is to break down the system in smaller subsystems and then to analyze them as multi state systems [15]. After all, multi state systems are a useful tool and a challenge to researchers, in order to solve different kinds of problems and research questions. It is important to understand that multi state systems are an approach in the field of probabilities modeling. Of course, there are other methodologies such as research on the optimal maintenance policy [1], or the Universal Generation function [14]. Also, Monte Carlo Simulation is a useful approach for complex systems, especially when restrictions of time exist

[23], and the recursive method, when the researcher evaluates the reliability of k out of n multi state systems [34].

Attempting to analyze and deal with the problems of power systems management, researchers are necessary to develop quantitative methods as useful tools in the decision-making. Depending on the specific needs of the problem, the system we examine, during its operation passes through certain states that could be values of random variables and the set of those states is called *state space* [16]. Using the continuous time methodology, we can study phenomena occurring in any time. The lack of memory in Markov chains implies a relationship between them and the exponential distribution which is the only one presenting this property. This property in Semi Markov models presents certain limitations concerning the time distribution that should be exponential, in case of continuous time and geometric, in case of discrete time. Especially in real world applications, these limitations could lead to erroneous conclusions [2, 13]. This means that if the system remains for certain time T in a state i, the probability to remain in this state for additional time is independent of the time T. This property is useful in the case of Semi Markov processes, where except for transient probabilities we consider the mean sojourn time for each state. This characteristic provides significant flexibility and the complexity of the calculations remains at relative low level. Semi Markov models are a generalized approach of Markov models providing an additional advantage of flexibility, concerning the distribution of the sojourn time in each state. Another characteristic of Semi Markov models is that the property of the lack of memory applies also in past states and the time the system was in those states as well. The difference comparing Semi Markov with Markov models is that time is a random variable and presumably the transition concerns only *different* states, because the probability remaining at the same state i is zero, since the system remains in this state for variable time (sojourn time). A general form of a Semi Markov Model can be represented mathematically as follows [2, 24]:

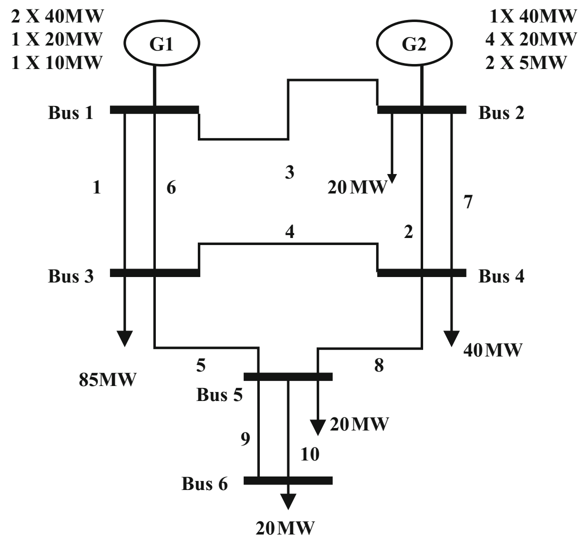
$$\begin{aligned} &P(J_{n+1}=j, S_{n+1}-S_n=k|J_0, \ldots, J_n; S_0, \ldots, S_n) \\ &\quad = P(J_{n+1}=j, S_{n+1}-S_n=k|J_n) \end{aligned} \tag{9}$$

where J_n is the system state at the nth jump time and S_n is nth jump time. The embedded Markov chain associated to the jumps from one state to another of the previous Semi-Markov model is defined by its transition matrix, i.e. $p_{ij} = P(J_{n+1} = j \mid J_n = i)$.

Let $H_i(t)$ be the sojourn time distribution in state i. If $H_i(t)$ is assumed to have the exponential distribution for all i, then the previous Semi-Markov Model is simply an alternate description of a homogeneous CTMC [30].

In order now, to compute the steady state solution of the Semi-Markov Process, we first need to compute the steady state probability distribution of the previously defined embedded Markov chain by solving the following equation: $\boldsymbol{v} = \boldsymbol{v}\,\boldsymbol{P}$, with $\sum_i v_i = 1$, where v_i is the steady state probability for state i and $\boldsymbol{P}$ the transition probability matrix of the embedded Markov chain. Let additionally, define,

Fig. 1 IEEE 6 BUS RBTS system. *Source* Setreus [26]



the mean sojourn time in state i by: $h_i = \int_0^\infty [1 - H_i(t)]dt$, then the steady state probability π_i for the semi-Markov Process is given by [6]:

$$\pi_i = \frac{v_i h_i}{\sum_j v_j h_j} \tag{10}$$

The above formulation gives us a general framework to model repairs with different distributions, other than the classical exponential, given that the erroneous use of those distributions will lead to also erroneous conclusions [5]. Semi Markov models can also be applied with other distributions such as the uniform distribution [21], or combination of exponential and uniform [30], where the calculations are getting more complex, or other more "general" and flexible such as Weibul distribution [9]. When due to specific reasons of the model there is combination of two or more distributions, the sojourn time in each state is the minimum value in that state of all those distributions and/or their combinations. Of course, attempting to develop a Semi Markov model, one should consider the dramatic increase of the volume (and the complexity) of the calculations in order to balance the requirements and the available computing power. The bottom line is that the Semi Markov methodology is a useful tool especially in the analysis of multi state systems, leading to valuable inferences concerning the maintenance policy of complex systems (Fig. 1).

2 System Analysis—Original System

The IEEE 6BUS RBTS is a power generation and transmission system [3]. The main characteristics of the system are the small size, facilitating the study and the solution of different problems. Additionally, its detailed description permits the examination of new methods concerning their adequacy. The main field of the system in research is the power transmission systems, although, there are recent studies [26, 27] where it is used as a basic tool for solving problems and developing methods, concerning the power generation and power transmission as well.

The development of the IEEE 6 BUS RBTS System aims to the study of reliability, regarding the power generation and transmission; therefore, it could be a sufficient initial point of a research project. As displayed in Table 1, the capacity of the system is 240 MW, with a peak load of 185 MW and an AC Nominal voltage of 230 kV. This output is achieved by using eleven generators in two groups, #1 and #2 which consist of four and seven generators respectively. Their output power is shown in Table 2. Considering the operational conditions of each one of the generators, the system could be in different states of power output and depending on the required level of load, the possible states could be "acceptable" or "not acceptable". This fact provides an inherent uncertainty which may not have crucial extent; however, it might cause undesired consequences.

The reliability of the standard system has been studied [31] using a Markov Chain model. The interesting findings of this approach imply that the probability of the system to be in an acceptable state is inversely proportional to time. It is obvious that the probability of the system to be in a "non-acceptable" state is proportional to time. Consequently, there are inherent vulnerabilities for which there have been efforts to overcome [12]. Table 3 contains probabilities of each state.

Table 1 Basic parts of IEEE 6 BUS RBTS

Number of buses	6
Number of generators	11
Number of load points	5
Number of transmission lines	9
Number of generation buses	2
Installed generation (MW)	240
System peak load (MW)	185
AC nominal voltage (kV)	230

[27]

Table 2 Output power of the generators

Generator group	Generators—output power
#1	G1-G2 = 40 MW, G3 = 20 MW, G4 = 10 MW
#2	G5 = 40 MW, G6-G7-G8-G9 = 20 MW, G10-G11 = 5 MW

Table 3 States and operational probabilities of IEEE 6BUS RBTS

G_A	G_B	Output (MW)	State	Probability
1	1	240	P_1	0.999897
1	0	110	P_2	9.9851e−5
0	1	130	P_3	2.9998e−6
0	0	0	P_4	2.9956e−10

Table 4 Rates of failure and repair for IEEE 6 BUS RBTS

Group	Generator	MTTF (hrs)	Failure rate (Annual)	MTTR (hrs)
#1	G1	1460	6	45
	G2	1460	6	45
	G3	2190	4	45
	G4	1752	5	45
#2	G5	4380	2	45
	G6	4380	2	45
	G7	2920	3	60
	G8	3650	2.4	55
	G9	3650	2.4	55
	G10	3650	2.4	55
	G11	3650	2.4	55

Considering the reliability requirements of a system, which is able to respond at all times, the results of the study show that a system modification could cope with certain restrictions. The main purpose of this study is not to develop a new different system, but to enhance the existing one. Considering the initial layout, the system presents a relative high level of reliability. According to the literature [31] the expected probability of the complete operation of the system, is 0.99989. Table 4 shows the failure (repair) rates, and the mean time to failure (repair).

3 Modified System—Application of the Model

Indeed, the level of operability is high, but some questions arise. Is an enhancement possible? What is the effect of a change to the synthesis of the power generating system (IEEE 6 BUS RBTS)? Is an assessment concerning the performance of the system possible? Could Semi Markov modeling contribute to assess this performance? Especially, in case of a large project, where the level of operation is crucial, any kind of improvement is necessary and always welcome. Actually, there is an inherent need for further research and improvement of the original system.

More specifically, the observation of the relationship between the probability of functionality and the time of operation manifests the necessity of a further change for the better [31].

Attempting to improve the system, the authors suggest the layout of the system in a minimal way, aiming to minimize the probability of a poor state to the power transfer. The main idea is to apply a different combination of the power generators that satisfies the following criteria:

- Lower failure rates
- The same level of power per group of generators.

One of the fundamental assumptions of the authors is that not only both groups, but all generators operate independently in parallel layout. Considering the modified system, there are certain advantages by using this layout, such as that there is independence among the generators. Therefore, since a single generator fails, all the other generators continue to operate normally, until another failure to another generator takes place and/or a repair follows the existing failure (Fig. 2).

Aiming to improve the reliability characteristics of the power generation units, the change of the layout would be a sufficient start for the study. As it is shown in Fig. 2, there are two groups of generators, but with five generators in group 1 (four generators before the change) and six generators in group 2 (seven generators before the change). More analytically, and considering Table 4, the characteristics of both groups are shown in Table 5. Actually, the main idea for this change is to replace some of the generators with those that present lower failure rate.

Of course, concerning the performance of the generators, a question about the time to repair could arise. Concerning the Group #1, the replacement of the G1 and G2, by the G5 and G6 respectively, improves the performance, since there is a

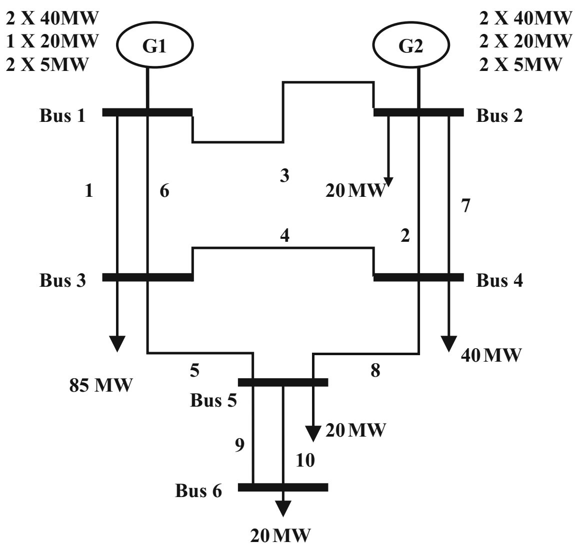


Fig. 2 Modified IEEE 6 BUS RBTS suggested by authors

Table 5 Change of layout of group #1 and #2

Group of generators #1							
Original				Modified			
Generator	Failure rate	Power (MW)	QTY	Generator	Failure rate	Power (MW)	QTY
G1	6	40	1	G5	2	40	2
G2	6	40	1	G6	2	20	1
G3	4	20	1	G10	2.4	5	1
G4	5	10	1	G11	2.4	5	1
Group of generators #2							
Generator	Failure rate	Power (MW)	QTY	Generator	Failure rate	Power (MW)	QTY
G5	2	40	1	G3	4	20	1
G6	2	20	1	G5	4	40	2
G7	3	20	1	G6	2	20	1
G8	2.4	20	1	G10	2.4	5	2
G9	2.4	20	1				
G10	2.4	5	1				
G11	2.4	5	1				

Table 6 Brief representation of states

State	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1	0	1
4	1	1	1	1	1	1	1	1	1	0	0
…	…	…	…	…	…	…	…	…	…	…	…
2047	0	0	0	0	0	0	0	0	0	0	1
2048	0	0	0	0	0	0	0	0	0	0	0

lowering of failure rate with the same time to repair. The other major change refers to the replacement of the G3 and G4 with the G10 and G11 respectively. Table 6 presents briefly the states of the system, where “1” is the normal operation of a generator and “0” is the status of failure. Concerning the reliability of the modified system, the states could be from full operative (240 MW—100%) to complete failure (0 MW—0%).

The probability of the system to be in an intermediate state between normal operation and complete failure proofs that it is a multi state system. After the formulation of the modified system, it is necessary to assess the new parameters and its expected behavior. After the modification, there is a lowering of the failure rate by a percentage of 40%, with an increase to the time to repair by a percentage of 22.2% (see Table 5). Consequently, this change contributes to an improvement of

Table 7 States and power output of the modified system

State #	Power output (MW)	# of states	State #	Power output (MW)	# of states	State #	Power output (MW)	# of states	State #	Power output (MW)	# of states
1	240	1	**14**	175	52	**26**	115	88	**38**	55	28
2	235	4	**15**	170	78	**27**	110	132	**39**	50	42
3	230	6	**16**	165	52	**28**	105	88	**40**	45	28
4	225	4	**17**	160	31	**29**	100	40	**41**	40	10
5	220	4	**18**	155	72	**30**	95	72	**42**	35	12
6	215	12	**19**	150	108	**31**	90	108	**43**	30	18
7	210	18	**20**	145	72	**32**	85	72	**44**	25	12
8	205	12	**21**	140	40	**33**	80	31	**45**	20	4
9	200	10	**22**	135	88	**34**	75	52	**46**	15	4
10	195	28	**23**	130	132	**35**	70	78	**47**	10	6
11	190	42	**24**	125	88	**36**	65	52	**48**	5	4
12	185	28	**25**	120	44	**37**	60	20	**49**	0	1
13	180	20									

the system as well. Considering the Group #2, there is a pure improvement of the system's performance, because of the similarity of the generators (see Table 5).

The possible states of the system are described in Table 6. There are five generators for the Group #1 and six generators for the Group #2, eleven totally. Placing both groups in parallel layout the number of states increases dramatically.

Thus, the actual number is $2^5 \cdot 2^6 = 2^{11} = 2048$. The calculation of all states aims finally to the estimation of the probability of the transition of the system to each state. These states are finally 49 *unique* states from 0 to 240 MW with an incremental step of 5 MW, due to the structure of the system (see Table 7).

Thus, the objective of the lowering of the failure rates is achieved, with only a partial increase of the time to repair. In the same time, the level of power output remains the same, achieving the same level of service with a better level of the output requirements with the original system, avoiding any major changes related with power transferring etc. Additionally, the independence of the generators mentioned above provides the flexibility that the minimum load capability is achieved with more combinations and there are forty nine output levels (as mentioned previously) compared with the four of the original system. The increase of the number of states provides more flexibility to the assessments that are necessary to manage the system and facilitate the decision-making concerning the parameters of the system. For representation purposes, due to the large size of the matrices, and in order to facilitate the understanding of the process, the authors selected to use three generators of the system in order to present the basic idea about the model. Therefore, in case of presenting systems of equations, there are eight equations (presenting a system of eight states instead of 2048). In the case of presenting matrices, there are abbreviated matrices for 2048 states. There is also another aspect concerning the issue of the reliability of the modified system. Trying to understand the meaning of reliability, it is necessary to determine the time of operation and the respective probability of an upcoming failure. First of all, in order to develop the Semi Markov Model, we should solve the system of equations in steady state conditions. The main assumption in this case is that the sojourn time in one state is exponentially distributed. Additionally, this transition is analyzed in two parts. The first one refers to the time spent in the particular state and the other one refers to the probability the system to be in another state. The jump of the system from one state to another is a result of the combination of failures and repairs of different generators. This occurs because the system consists of eleven generators and a probability to fail and/or repair one at a time or some of them simultaneously always exists. This transition will take place when the first combination of failure and/or repair comes. The transition matrix based on the failure rates and/or repairs will have the form

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,2048} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,2048} \\ \cdots & \cdots & \cdots & \cdots \\ p_{2048,1} & p_{2048,2} & \cdots & p_{2048,2048} \end{bmatrix} \tag{11}$$

Obviously, the number and the difficulty of the calculations increase dramatically with the increase of generators, confirming the findings of researchers about the complexity and the difficulty of the models. The failure and repair rates are expressed in failures and repairs per hour. These numbers are extracted by dividing the annual failure rates and repairs by 8,760 h per year. According to this analysis, the failure and repair rates are shown in Table 8. Continuing with the calculation of the steady state probabilities, we should solve the following matrix equation [30]:

$$v = vP \tag{12}$$

Table 8 Failure and repair rates

GEN	MTTF (hrs)	Failure rate (per year)	MTTR (hrs)	Failure rate (per hour)	Repair rate (per hour)
G1	1460	6	45	0.00000	0.00000
G2	1460	6	45	0.00000	0.00000
G3	2190	4	45	0.00000	0.00000
G4	1752	5	45	0.00000	0.00000
G5	4380	2	45	0.00046	0.01027
G6	4380	2	45	0.00023	0.00514
G7	2920	3	60	0.00000	0.00000
G8	3650	2.4	55	0.00000	0.00000
G9	3650	2.4	55	0.00000	0.00000
G10	3650	2.4	55	0.00027	0.00628
G11	3650	2.4	55	0.00027	0.00628

Where P is the transition probabilities Matrix and v is the vector of the discrete time Markov Chain.

$$v = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & \ldots & \ldots & v_{2047} & v_{2048} \end{bmatrix} \tag{13}$$

Of course, a unique solution of the Eq. (17) is possible only under the restriction [32].

$$\sum_{i=1}^{2048} v_i = 1 \tag{14}$$

The mean sojourn time for each state is given by the formula [30].

$$h_i = \int_0^{\infty} [1 - H_i(t)]dt \tag{15}$$

Solving the formula above, we find that the mean sojourn time has the form

$$h_i = \frac{1}{\lambda_i + \mu_i} \tag{16}$$

Once again, this expression is only indicative for representation reasons only and it is not applicable for all states.

$$V \cdot P_{semi} = U \Leftrightarrow V = U \cdot P_{semi}^{-1} \tag{17}$$

Vector U is

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & \ldots & 0_{(2047)} & 0_{(2048)} \end{bmatrix} \tag{18}$$

and V the matrix that will be combined with the mean sojourn times to calculate the final steady state probabilities. Solving the matrix Eq. (17), we have the results in Table 9. At this point, we notice that this table contains probabilities of **all** states of the system. Thus, assessing the probability of the failure related to the time of operation, these reliability parameters are shown in Table 10. An initial interest point of these results is the probability of the system to provide a level of power output.

So, using the probabilities for each state (Table 9) and adding all probabilities with the same output, we have the final probabilities based on the level of output (see Table 10), giving the opportunity to the decision maker to shape the big picture concerning the expected level of the power output. According to Table 10, the expected level of power output which is E[X] = **237.267 MW**[1] and its variance found to be Var[X] = **81.83 $\mathbf{MW^2}$** and finally its standard deviation is at the level

[1]Calculation of expected value E[X].

Table 9 Probabilities of each state

State	Group #1					Group #2						Power (MW)	Prob
	G5	G5	G6	G10	G11	G3	G5	G5	G6	G10	G11		
1	1	1	1	1	1	1	1	1	1	1	1	240	8.681E−01
2	1	1	1	1	1	1	1	1	1	1	0	235	1.308E−02
3	1	1	1	1	1	1	1	1	1	0	1	235	1.308E−02
4	1	1	1	1	1	1	1	1	1	0	0	230	1.971E−04
5	1	1	1	1	1	1	1	1	0	1	1	220	8.918E−03
…	…	…	…	…	…	…	…	…	…	…	…	…	…
2047	0	0	0	0	0	0	0	0	0	0	1	5	1.532E−19
2048	0	0	0	0	0	0	0	0	0	0	0	0	2.541E−16

of **9.05 MW**.[2] These values assure that the system fulfills the criterion of a minimum output or peak load of **185 MW** with a probability of an almost total certain operation. Comparing the final results, the original system presents a probability for "normal operation >130 MW" equal to **0.999897**. The respective probability of the modified is **0.999995**. the probability of ">110 MW" is **0.999999** and for total failure of the original is **2.9956e − 10** and for the modified is **2.54109E − 16**. But, the most important finding is that the minimum output limit of 185 MW is achieved in the modified system with a probability of **0.997809**. Since the peak load is 185 MW there is a difference of 5.76σ (standard deviations). Considering Chebyshev's Inequality [25] since the power output is a random variable with mean **237.267 MW** and variance **81.83 MW²** then for any value k > 0,

$$P\{|X-\mu|\geq k\}\leq\frac{\sigma^2}{k^2} \tag{19}$$

Therefore, if $k=\frac{237.267-185}{9.05}\Leftrightarrow k=5.76$ standard deviations, then the formula (19) gives $P\{|X-237.267|\geq 52.267\}\leq\frac{81.83}{52.267^2}\Leftrightarrow P\{|X-237.267|\geq 52.267\}\leq 0.02995$. And finally, the probability of the power output to be higher than 185 MW in any case is $P\{|X-237.267|\leq 52.267\}\geq 1-0.02995\Leftrightarrow P\{|X-237.267|\leq 52.267\}\geq 0.97$. This result agrees and confirms the findings of the analysis.

Concerning the expected time of operation in each state it is shown in Table 11. The main finding is that in annual basis, the system operates in more than 185 MW for 8,740 h, 48 min and 36 s out of 8,760 h totally. The output from 130 MW to 184 MW is 19 h, 9 min and 4 s. The operation between 110 and 130 MW is 2 min and 16 s. Finally, the operation in level lower than 110 MW is only 4 s in annual basis. All the above findings show that the suggested modification upgrades the system and its reliability and could be a starting point for further improvement.

[2]Calculation of Variance by var[X] and its standard deviation.

Table 10 Probabilities and cumulative probabilities of each output level

State (MW)	Exp hours p.a.	Prob	Cummul prob	State (MW)	Exp hours p.a.	Prob	Cummul prob
240	7,604.214	8.68060E-01	8.68060E-01	115	0.002	2.44451E-07	9.99999E-01
235	458.336	5.23214E-02	9.20382E-01	**110**	**0.000**	**5.52527E-09**	**9.99999E-01**
230	10.360	1.18260E-03	9.21565E-01	105	0.000	5.55064E-11	9.99999E-01
225	0.104	1.18800E-05	9.21576E-01	100	0.001	1.55940E-07	9.99999E-01
220	312.502	3.56737E-02	9.57250E-01	95	0.000	9.39912E-09	9.99999E-01
215	18.836	2.15019E-03	9.59400E-01	90	0.000	2.12444E-10	9.99999E-01
210	0.426	4.86003E-05	9.59449E-01	85	0.000	2.13583E-12	9.99999E-01
205	0.004	4.88222E-07	9.59449E-01	80	0.000	1.16590E-08	9.99999E-01
200	316.515	3.61318E-02	9.95581E-01	75	0.000	7.02737E-10	9.99999E-01
195	19.078	2.17781E-03	9.97759E-01	70	0.000	1.58837E-11	9.99999E-01
190	0.431	4.92245E-05	9.97808E-01	65	0.000	1.60973E-13	9.99999E-01
185	**0.004**	**4.94492E-07**	**9.97809E-01**	60	0.000	4.05634E-10	1.00000E + 00
180	12.859	1.46792E-03	9.99277E-01	55	0.000	2.44497E-11	1.00000E + 00
175	0.775	8.84777E-05	9.99365E-01	50	0.000	5.52498E-13	1.00000E + 00
170	0.018	1.99984E-06	9.99367E-01	45	0.000	5.99760E-15	1.00000E + 00
165	0.000	2.00897E-08	9.99367E-01	40	0.000	5.10464E-12	1.00000E + 00
160	4.981	5.68594E-04	9.99936E-01	35	0.000	3.07528E-13	1.00000E + 00
155	0.300	3.42714E-05	9.99970E-01	30	0.000	7.16627E-15	1.00000E + 00
150	0.007	7.74628E-07	9.99971E-01	25	0.000	1.57224E-16	1.00000E + 00
145	0.000	7.78165E-09	9.99971E-01	20	0.000	2.07729E-14	1.00000E + 00
140	0.199	2.26705E-05	9.99994E-01	15	0.000	1.22322E-15	1.00000E + 00
135	0.012	1.36644E-06	9.99995E-01	10	0.000	1.33671E-16	1.00000E + 00
130	**0.000**	**3.08853E-08**	**9.99995E-01**	5	0.000	1.96904E-17	1.00000E + 00
125	0.000	3.10264E-10	9.99995E-01	**0**	**0.000**	**2.54108E-16**	**1.00000E + 00**
120	0.036	4.05567E-06	9.99999E-01	**Expected Power (MW) E[X]=**			**237.26**

Power levels marked in bold refer to power levels (states) of initial IEEE 6BUS RBTS

Table 11 Reliability based on hrs of operation

Time (Hrs)	Prob group #1	Prob group #2	Total	Time (Hrs)	Prob group #1	Prob group #2	Total
0	1.0000E+00	1.0000E+00	1.0000E+00	4500	8.6712E−01	8.8414E-01	9.8460E-01
250	1.0000E+00	1.0000E+00	1.0000E+00	4750	8.4636E−01	8.6392E-01	9.7909E-01
500	9.9998E−01	1.0000E+00	1.0000E+00	5000	8.2456E−01	8.4245E−01	9.7236E−01
750	9.9987E−01	9.9996E−01	1.0000E+00	5250	8.0185E−01	8.1988E−01	9.6431E−01
1000	9.9951E−01	9.9982E−01	1.0000E+00	5500	7.7841E−01	7.9640E−01	9.5488E−01
1250	9.9871E−01	9.9944E−01	1.0000E+00	5750	7.5439E−01	7.7217E−01	9.4404E−01
1500	9.9723E−01	9.9863E−01	1.0000E+00	6000	7.2994E−01	7.4738E−01	9.3178E−01
1750	9.9482E−01	9.9715E−01	9.9999E−01	6750	6.5539E−01	6.7120E−01	8.8669E−01
2000	9.9123E−01	9.9475E−01	9.9995E−01	7000	6.3056E−01	6.4567E−01	8.6910E−01
2250	9.8627E−01	9.9119E−01	9.9988E−01	7250	6.0591E−01	6.2030E−01	8.5036E−01
2500	9.7977E−01	9.8623E−01	9.9972E−01	7500	5.8155E−01	5.9517E−01	8.3060E−01
2750	9.7161E−01	9.7970E−01	9.9942E−01	7750	5.5754E−01	5.7039E−01	8.0991E−01
3000	9.6171E−01	9.7144E−01	9.9891E−01	8000	5.3396E−01	5.4604E−01	7.8844E−01
3250	9.5004E−01	9.6137E−01	9.9807E−01	8250	5.1088E−01	5.2218E−01	7.6629E−01
3500	9.3664E−01	9.4945E−01	9.9680E−01	8500	4.8833E−01	4.9889E−01	7.4360E−01
3750	9.2154E−01	9.3570E−01	9.9496E−01	8750	4.6637E−01	4.7619E−01	7.2048E−01
4000	9.0484E−01	9.2016E−01	9.9240E−01	8800	4.6206E−01	4.7173E−01	7.1582E−01
4250	8.8666E−01	9.0294E−01	9.8900E−01				

4 Conclusions

In this study, we have evaluated the performance of a modified IEEE 6BUS RBTS and have shown that the recommended modification contributes to the improvement of the system performance by increasing the probability of operation within the required limits. The modification of the system aims to the reduction of failure rates maintaining the power output specifications. This objective is achieved through the restructuring of the group of generators whereas all generators are supposed to be in parallel operation. This technique led to approach the problem through the multi state systems theory. As described in previous paragraphs, the increase of the parts of the system resulted to a dramatic increase of the states of the system and consequently increased the complexity of the calculations. This problem showed in practice that probable restrictions of the computing power are possible to be overcome through programming algorithms and/or advanced software. In the case of even more complex systems, a breakdown of the system in smaller parts is a suggested complementary alternative. As multi state systems theory suggests, this model presents more flexibility than that of the original IEEE 6BUS RBTS. More specifically, there are more levels of power output and their respective probabilities. This characteristic contributes to the lowering of the uncertainty of the system, assisting decisively in the decision-making and the management of the system. Comparing the models, there is a significant improvement after the suggested modification. The reduction of the probability of "non-acceptable" output level and the increased probability of operation over the minimum required level contributes to the effective management of the system. The managerial aspect of the system's modification is that it contributes to the simplicity of the system. Since there are fewer types of generators, it simplifies the management of the system concerning the schedule of supplies and its maintenance as well. All these findings could be a starting point for further study and expansion of the methodology in other research topics. In addition, using advanced programming algorithms, the researchers could apply analytical methods in an effort to overcome the existing barriers of the computing power. This strategy can be combined with other methods such as Monte Carlo Simulation, in order to verify the effectiveness of each other method and to reduce the uncertainty of the models.

References

1. Aven T, Jensen U (1999) Stochastic models in reliability. Springer, New York
2. Barbu VS, Limnios N (2008) Semi Markov chains and hidden semi Markov models towards applications. Springer Business Media, New York
3. Billinton R, Li Y (1994) Reliability assessment of electric power systems using Monte Carlo methods. Springer Science Business Media, New York
4. Billinton R, Li Y (2007) Incorporating multi-state unit models in composite system adequacy assessment. Eur Trans Electr Power 17:375–386

5. Ciardo G, Marie RA, Sericola B, Trivedi K (1990) Performability analysis using semi-Markov reward processes 39(10):1251–1264
6. Cinlar E (1975) Introduction to stochastic processes. Prentice-Hall, Englewood Cliffs, N.J
7. Eryilmaz S (2015a) Assessment of a multi-state system under a shock model. Appl Math Comput 269:1–8
8. Eryilmaz S (2015b) Dynamic assessment of multi state systems using phase type modeling. Reliab Eng Syst Saf 140:71–77
9. Foucher Y, Mathieu E, Sait-Pierre P, Durand J-F, Daures J-P (2005) A semi markov model based on generalized Weibull distribution with an illustration for HIV disease. Biometrical J 47:1–9
10. Gut A (1990) Cumulative shock models. Adv Appl Probab 22:504–507
11. Koval et al (2007) IEEE recommended practice for the design of reliable industrial and commercial power systems. IEEE Gold Book, New York
12. Kumar S, Sankar V (2013) Enhancement of reliability analysis for a 6-bus composite power system using combination of TCSC & UPFC. International conference on recent trends in power, control & instrumentation engineering PCIE-2013. Elsevier
13. Limnios N, Oprisan G (2001) Semi-Markov processes and reliability. Springer Science Business Media, New York
14. Lisnianski A, Levitin G, Ben Heim H, Elmakias D (1999) Power system structure optimization subject to reliability constraints. Electr Power Syst Res 39:145–152
15. Lisnianski A (2007) Extended block diagram method for a multi-state system reliability assessment. Reliab Eng Syst Saf 92(12):1601–1607
16. Lisnianski A, Frenkel I, Ding Y (2010) Multi-state system reliability analysis and optimization for engineers and industrial managers. Springer, London
17. Lisnianski A, Elmakias D, Laredo D, Haim HB (2012) A multi-state Markov model for a short-term reliability analysis of a power generating unit. Reliab Eng Syst Saf 98:1–6
18. Lisnianski A (2016) Application of extended universal generating function technique to dynamic reliability analysis of a multi state system. In: The Second International Symposium on Stochastic Models in Reliability Engineering, Life Science and Operations Management
19. Liu YW, Kapur KC (2006) Reliability measures for dynamic multistate nonrepairable systems and their applications to system performance evaluation. IIE Trans 38(6):511–520
20. Mallor F, Omey E, Santos J (2006) Asymptotic results for a run and cumulative mixed shock model. J Math Sci 138:5410–5414
21. Nabli H (2004) Performability: asymptotic distribution and moment computation. Comput Math Appl 48:1–8
22. Natvig B (2007) Multi-state reliability theory, department of Mathematics. University of Oslo Statistical Research Report No 1, ISSN 0806–3842
23. Ramirez-Marquez J, Coit D (2005) A Monte-Carlo simulation approach for approximating multi-state two-terminal reliability. Reliab Eng Syst Saf 87:253–264
24. Ross S (1995) Stochastic processes (2). Wiley, Canada
25. Ross S (2003) Introduction to probability models (8). Academic Press, Burlington
26. Setreus J (2009) On reliability methods quantifying risks to transfer capability in electric power transmission systems. Licentiate Thesis in Electrical Systems Stockholm, Sweden
27. Setreus J (2011) Identifying critical components for system reliability in power transmission systems. Doctoral Thesis in Electrical Systems, Stockholm, Sweden
28. Shanthikumar G, Sumita U (1983) General shock models associated with correlated renewal sequences. J Appl Probab 20:600–614
29. Shey-H Chang C, Chen Y, Zhang ZG (2015) Optimal preventive maintenance and repair policies for multi-state systems. Reliab Eng Syst Saf 140:78–87
30. Trivedi KS (2002) Probability and statistics with reliability, queuing and computer science applications. Wiley, New York
31. Vijayalaxmi D, Karjagi B (2015) Modeling and analysis of RBTS IEEE-6 BUS system based on Markov Chain. Int J Eng Res Gen Sci 3(2):686–696

32. Violentis J, Koutras V, Platis A, Gravvanis G (2008) Asymptotic availability of an electrical substation via a semi-markov process computed by generalized approximate inverse preconditioning. HERCMA Conference Proceedings.
33. Yingkui G, Jing L (2012) Multi-state system reliability: a new and systematic review. Procedia Eng 29:531–536
34. Zhao X, Cui L (2012) Reliability evaluation of generalised multi-state k out-of-n systems based on FMCI approach. Int J Syst Sci 41:1437–1443
35. Zio E (2009) Reliability engineering: old problems and new challenges. Reliab Eng Syst Saf 94:125–141