Reliability Assessment of Systems with Dependent Degradation Processes Based on Piecewise-Deterministic Markov Process

Yan-Hui Lin, Yan-Fu Li and Enrico Zio

Abstract This chapter presents a reliability assessment framework for multi-component systems whose degradation processes are modeled by multi-state and physics-based models. The piecewise-deterministic Markov process modeling approach is employed to treat dependencies between the degradation processes within one component or/and among components. The proposed method can handle the dependencies between physics-based models, between multi-state models and between these two types of models. A Monte Carlo simulation algorithm is designed to compute the systems reliability. A case study on one subsystem of the residual heat removal system of a nuclear power plant is illustrated as exemplification of the proposed modeling framework.

Keywords Multi-state model • Physics-based model • Dependent degradation processes • Piecewise-deterministic markov process • Monte Carlo simulation method

Y.-H. Lin (✉)
School of Reliability and Systems Engineering, Beihang University, Beijing, China
e-mail: linyanhui@buaa.edu.cn

Y.-F. Li
Department of Industrial Engineering, Tsinghua University, Beijing, China
e-mail: liyanfu@tsinghua.edu.cn

E. Zio
Chair on Systems Science and the Energetic Challenge, Fondation EDF, at Centrale Supelec, Paris, France
e-mail: enrico.zio@ecp.fr; enrico.zio@polimi.it

E. Zio
Politecnico di Milano, Milan, Italy

A. Lisnianski et al. (eds.), *Recent Advances in Multi-state Systems Reliability*,
Springer Series in Reliability Engineering, DOI 10.1007/978-3-319-63423-4_11

1 Introduction

Safety-critical plants, like the nuclear power plants (NPPs), are designed not to fail, i.e. with very high reliability, because of the potentially catastrophic consequences of their failures. Traditional data-based reliability analysis, based on failure data, is, then, unsuitable. On the other hand, most failure mechanisms can be traced to underlying degradation processes (e.g. wear, stress corrosion, shocks, cracking, fatigue, etc.) [30], for which models exist.

In general, the reliability of a system decreases as the degradation processes develop, eventually leading to failure [31]. In reliability engineering, degradation processes have been widely studied and different degradation models have been developed. The existing degradation models can mainly be classified into the following categories:

- statistical models of time to failure, based on degradation data (e.g. Bernstein distribution [9], Weibull distribution [21]).
- stochastic process models (e.g. Gamma processes [14], inverse Gaussian process [2]) describing the evolution of one or more degradation parameters by gradual degradation increments over time, and the failure occurs when the degradation parameter values reach predefined thresholds.
- physics-based models (PBMs), based on the knowledge of the physics of degradation, which is translated into equations to give a quantitative description (e.g. the physics functions based on critical environmental stresses, e.g. amplitude and frequency of mechanical loads, used to model the pitting and corrosion-fatigue degradation mechanisms [3]).
- multi-state models (MSMs) describing the underlying degradation process by finite degradation states (e.g. semi-Markov models for the deterioration of infrastructure systems [1]).

Among these categories, PBMs [6, 13, 27] and MSMs [18, 19, 23] can be used to describe the evolution of degradation in structures, systems and components, for which statistical degradation/failure data are insufficient, e.g. the highly reliable devices in the nuclear and aerospace industries.

In reality, components and systems are often subject to multiple competing degradation processes and any of them may cause failure [29]. The dependencies among these processes within one component (e.g. the wear of rubbing surfaces influenced by the environmental stress shock within a micro-engine [12]), or/and among different components (e.g. the degradation of the pre-filtrations stations leading to a lower performance level of the sand filter in a water treatment plant [26]) need to be considered, under certain circumstances. This renders challenging the analysis and prediction of the components and systems reliability [25]. Wang and Pham [29] applied time-varying copulas for describing the dependencies between the degradation processes modeled by statistical distributions. Straub [28] used a dynamic Bayesian network to represent the dependencies between degradation processes modeled by multi-state models.

In this chapter, we present a reliability assessment framework for multi-component systems whose degradation processes are modeled by MSMs and PBMs, capturing dependencies among the components and among multiple degradation processes within one component. The piecewise-deterministic Markov process (PDMP) modeling approach is employed. The PDMP, firstly introduced by Davis in [7, 8], and further studied by Jacobsen [11] and Cocozza-Thivent [4], is well-suited to describe degradation dependence. The remainder of this chapter is organized as follows. Section 2 presents the proposed degradation model for systems with degradation dependence. Monte Carlo simulation procedures to solve the model are presented in Sect. 3. Section 4 presents a case study on one subsystem of a residual heat removal system (RHRS) of a NPP. Section 5 concludes the work.

2 Dynamic Reliability Models for Systems with Degradation Dependence

For highly reliable systems, such as nuclear safety systems, it is relatively difficult to model their degradation and failure behaviors due to the limited amount of data available. In these cases, PBMs and MSMs are two modeling frameworks that can be used to model degradation. Systems are often subject to multiple competing degradation processes and any of them may cause failure. The dependences among these processes need to be considered under certain circumstances. In this chapter, a PDMP modeling framework is developed to treat degradation dependence in a system whose degradation processes are modeled by PBMs and MSMs.

2.1 Degradation Models

We consider a multi-component system made of Q components denoted by $O = \{O_1, O_2, \ldots, O_Q\}$. Each component may be affected by multiple degradation mechanisms or processes, possibly dependent. The degradation processes can be separated into two groups: (1) $L = \{L_1, L_2, \ldots, L_M\}$ modeled by M PBMs; (2) $K = \{K_1, K_2, \ldots, K_N\}$ modeled by N MSMs, where $L_m, m = 1, 2, \ldots, M$ and $K_n, n = 1, 2, \ldots, N$ are the indexes of the degradation processes.

2.1.1 PBMs

The following assumptions on PBMs are made:

- A degradation process $X_{L_m}(t), L_m \in L$ in the first group, has d_{L_m} time-dependent continuous variables $X_{L_m}(t) = \left(x^1_{L_m}(t), x^2_{L_m}(t), \ldots, x^{d_{Lm}}_{L_m}(t)\right) \in \mathbb{R}^{d_{Lm}}$. A system of first-order differential equations (i.e. physics equations) $\dot{X}_{L_m}(t) = f_{L_m}(X_{L_m}(t), t|\boldsymbol{\theta}_{L_m})$, are used to characterize its evolution, where $\boldsymbol{\theta}_{L_m}$ are the environmental factors influential to L_m (e.g. temperature and pressure) and the parameters used in f_{L_m}. This assumption is made in [20] and widely used in practice [5, 6]. Note that higher-order differential equations can be converted into a system of a large number of first-order differential equations by introducing extra variables [33].
- $X_{L_m}(t)$ can be divided into two groups of variables $X_{L_m}(t) = \left(X^D_{L_m}(t), X^P_{L_m}(t)\right)$: (1) $X^D_{L_m}(t)$ are the non-decreasing degradation variables describing the degradation process (e.g. leak area of the piston of the valve [6]), where D is the set of degradation variables indices; (2) $X^P_{L_m}(t)$ are the physical variables influencing $X^D_{L_m}(t)$ (e.g. velocity and force [5]), where P is the set of physical variable indices. For example, the friction-induced wear of the bearings is considered as one degradation process in [5]. It is represented by the increase in friction coefficients. The two friction coefficients associated with sliding and rolling friction are considered as the degradation variables. The rotational velocity of the pump is considered as the physical variable, since it influences the increase in the coefficients of friction. The evolution of physical variables can be characterized by physics equations. If the variables can be modeled by physics equations and influence certain degradation variables, then, they are considered as physical variables. As long as one $x^i_{L_m}(t) \in X^D_{L_m}(t)$ reaches or exceeds its corresponding failure threshold $x_{(L_m)}i^*$ the generic degradation process L_m fails. Let $\mathcal{F}_{L_m}$ denote the failure state set of L_m and $x^*_{L_m}$ denote the set of all the failure thresholds of $X^D_{L_m}(t)$. An example of L_1 is shown in Fig. 1.

2.1.2 MSMs

The following assumptions on MSMs are made:

- A degradation process, $Y_{K_n}(t), K_n \in K$ in the second group, takes values from a finite state set denoted by $S_{K_n} = \{0, 1, \ldots, d_{K_n}\}$, where '$d_{K_n}$' is the perfect functioning state and '0' is the complete failure state. The transition rates $\lambda_i(j|\boldsymbol{\theta}_{K_n}), \forall i, j \in S_{K_n}, i > j$ characterize the degradation transition probabilities from state i to state j, where $\boldsymbol{\theta}_{K_n}$ is the set of the environmental factors to K_n and

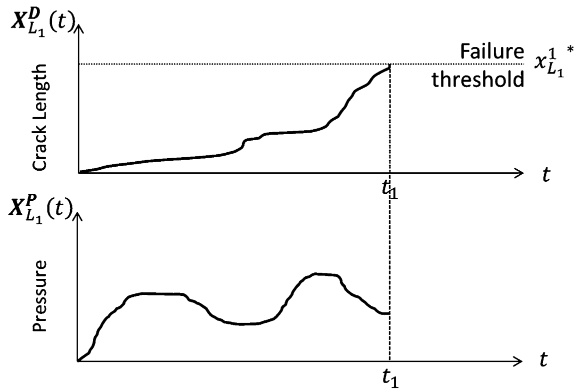

Fig. 1 An illustration of L_1

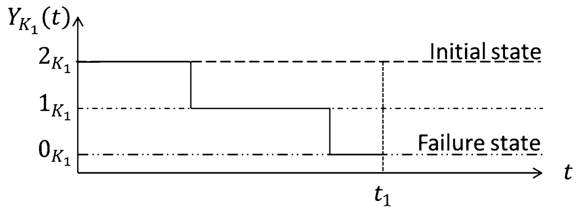

Fig. 2 An illustration of K_1

the related parameters used in λ_i. We follow the assumption of Markov property which is widely used in practice to describe components degradation processes [10]. The transition rates between different degradation states are estimated from the degradation and/or failure data from historical field collection. Let $\mathcal{F}_{K_n} = \{0\}$ denote the failure state set of K_n. An example of K_1 is shown in Fig. 2.

2.2 Degradation Model of the System Considering Dependence

The dependencies between degradation mechanisms or processes may exist within each group and between the two groups. The evolution trajectories of the continuous variables in the first group may be influenced by the degradation states of the second group. The transition times and transition directions of the degradation processes of the second group may depend on the degradation levels of the components in the first group [17]. PDMPs [4], which are a family of Markov processes involving deterministic evolution punctuated by random jumps, can be employed to model this type of dependence (the detailed formulations are shown in Eqs. (2) and (3)). Let $X(t) = \begin{pmatrix} X_{L_1}(t) \\ \vdots \\ X_{L_M}(t) \end{pmatrix}$ denote the degradation processes of the first group and $Y(t) = \begin{pmatrix} Y_{K_1}(t) \\ \vdots \\ Y_{K_N}(t) \end{pmatrix}$ denote the degradation processes of the second group. The overall degradation process of the system is presented as

$$Z(\mathrm{t}) = \begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} \in E = \mathbb{R}^{d_L} \times S \tag{1}$$

where E is a space combining $\mathbb{R}^{d_L}$ $(d_L = \sum_{m=1}^{M} d_{L_m})$ and $S = \{0, 1, \ldots, d_S\}$ denotes the state set of process $Y(t)$. The evolution of $Z(t)$ has two parts: (1) the stochastic behavior of $Y(t)$ and (2) the deterministic behavior of $X(t)$ between two consecutive jumps of $Y(t)$, given $Y(t)$. The former is governed by the transition rates of $Y(t)$, which depend on the states of the degradation processes in $X(t)$ and also in $Y(t)$, as follows:

$$\begin{aligned} &\lim_{\Delta t \to 0} P\big(Y(t+\Delta t) = j | X(t), Y(t) = i, \boldsymbol{\theta}_K = \cup_{n=1}^{N} \boldsymbol{\theta}_{K_n}\big) / \Delta t \\ &= \lambda_i(j | X(t), \boldsymbol{\theta}_K), \forall t \geq 0, i, j \in S, i \neq j \end{aligned} \tag{2}$$

The latter is described by the deterministic physics, which depends on the states of the degradation processes in $Y(t)$ and also in $X(t)$, as follows:

$$\begin{aligned} \dot{X}(t) = \begin{pmatrix} \dot{X}_{L_1}(t) \\ \vdots \\ \dot{X}_{L_M}(t) \end{pmatrix} &= \begin{pmatrix} f_{L_1}^{Y(t)}(X(t), t | \boldsymbol{\theta}_{L_1}) \\ \vdots \\ f_{L_M}^{Y(t)}(X(t), t | \boldsymbol{\theta}_{L_M}) \end{pmatrix} \\ &= f_L^{Y(t)}\Big(X(t), t | \boldsymbol{\theta}_L = \bigcup_{m=1}^{M} \boldsymbol{\theta}_{L_m}\Big) \end{aligned} \tag{3}$$

Let $\mathcal{F}$ denote the system failure state set, which depends on the structure of the system: then, the system reliability at mission time T_{miss} can be obtained as follows:

$$R(T_{miss}) = P[Z(s) \notin \mathcal{F}, \forall s \leq T_{miss}] \tag{4}$$

The system failure state set is dependent on system structure. To determine this set, reliability analysis tools such as fault tree [15] can be used to identify the combination of primary failure events leading to system failure.

3 System Reliability Estimation Method

Analytically solving the PDMP is a difficult task due to the complex behavior of the system [22], which contains stochastic properties in the components modeled by MSMs and the time-dependent evolutions of the components modeled by PBMs. On the other hand, MC simulation methods are suited for the reliability estimation of the system.

Refer to the system presented in Sect. 2.2. Let $Z_k = Z(T_k) = \begin{pmatrix} X(T_k) \\ Y(T_k) \end{pmatrix} \in E, k \in \mathbb{N}$, where T_k denotes the time of the k-th transition of $Y(t)$ from the beginning. Then, $\{Z_k, T_k\}_{k \geq 0}$ is a Markov renewal process defined on the space $E \times \mathbb{R}^+$ [4], which is characterized as follows:

$$P[Z_{k+1} \in B, T_{k+1} \in [T_k, T_k + \Delta t] | Z_k = i, \boldsymbol{\theta} = \boldsymbol{\theta}_K \cup \boldsymbol{\theta}_L] \\ = \iint\limits_{B*[0,\,\Delta t]} N(i, dz, ds|\boldsymbol{\theta}), \forall k \geq 0, \Delta t \geq 0, i \in E, B \in \varepsilon \tag{5}$$

where ε is a σ-algebra of E and $N(i, dz, ds|\boldsymbol{\theta})$ is a semi-Markov kernel on E, which verifies that $\int\limits_{E*[0,\,\Delta t]} N(i, dz, ds|\boldsymbol{\theta}) \leq 1, \forall \Delta t \geq 0, i \in E$. It can be further developed as:

$$N(i, dz, ds|\boldsymbol{\theta}) = dF_i(s|\boldsymbol{\theta})\beta(i, dz|s, \boldsymbol{\theta}) \tag{6}$$

where

$$dF_i(s|\boldsymbol{\theta}) \tag{7}$$

is the probability density function of $T_{k+1} - T_k$ given $Z_k = i$ and

$$\beta(i, dz|s, \boldsymbol{\theta}) \tag{8}$$

is the conditional probability distribution of state Z_{k+1} starting from $Z_k = i$ given $T_{k+1} - T_k = s$.

The simulation procedure consists of sampling the transition time from Eq. (7) and the arrival state from Eq. (8) for $Y(t)$, then, calculating $X(t)$ within the transition times, by using the physics equation (3) until the time of system evolution reaches a certain mission time T_{miss} or the system enters the failure space $\mathcal{F}$.

To calculate the system reliability, the procedure of the MC simulation is presented as follows:

Set N_{max} (the maximum number of replications) and $k = 0$ (index of replication)
Set $k' = 0$ (number of trials that end in the failure state)
While $k < N_{max}$
Initialize the system by setting $\boldsymbol{Z}' = \begin{pmatrix} \boldsymbol{X}(0) \\ \boldsymbol{Y}(0) \end{pmatrix}$ (initial state) and the time $T = 0$ (initial system time)
Set $t' = 0$ (state holding time)
While $T < T_{miss}$
Sample a t' by using the probability density function (11.7)
Sample an arrival state $\boldsymbol{Y}'$ for stochastic process $\boldsymbol{Y}(t)$ from all the possible states by using the conditional probability distribution (11.8)
Set $T = T + t'$
Calculate $\boldsymbol{X}(T)$ by using the physics eq. (11.3)
Set $\boldsymbol{Z}' = \begin{pmatrix} \boldsymbol{X}(T) \\ \boldsymbol{Y}' \end{pmatrix}$
If $T \leq T_{miss}$
If $\boldsymbol{Z}' \in \boldsymbol{\mathcal{F}}$
Set $k' = k' + 1$
Break
End if
Else (when $T > T_{miss}$)
Calculate $\boldsymbol{Z}(T_{miss})$
If $\boldsymbol{Z}(T_{miss}) \in \boldsymbol{\mathcal{F}}$
Set $k' = k' + 1$
Break
End if
End if
End While
Set $k = k + 1$
End While

The estimated probability of occurrence of one path at time T_{miss} can be obtained by

$$\widehat{R}(T_{miss}) = 1 - k^{'}/N_{max} \tag{9}$$

with the sample variance [16] as follows:

$$var_{\widehat{P(T_{miss})}} = \widehat{R}(T_{miss})\left(1 - \widehat{R}(T_{miss})\right)/(N_{max} - 1) \tag{10}$$

4 Case Study

The case study refers to one subsystem of the RHRS of a NPP. The system consists of a centrifugal pump and a pneumatic valve in series. Given the series configuration, the failure of anyone of the two components can lead the subsystem to failure. Dependence in the degradation processes of the two components has been indicated by the experts: the pump vibrates due to degradation [32] which, in turn, leads the valve to vibrate, aggravating its own degradation processes [24].

The pump is modeled by a MSM, modified from the one originally supplied by EDF upon discussion with the experts. It is a continuous-time homogeneous Markov chain as shown in Fig. 3.

$S_p = \{0, 1, 2, 3\}$ denotes its degradation states set, where 3 is the perfect functioning state and 0 is the complete failure state. The parameters λ_{32}, λ_{21} and λ_{10} are the transition rates between the degradation states. Due to degradation, the pump vibrates when it reaches the degradation states 2 and 1. The intensity of the vibration of the pump on states 2 and 1 is evaluated as by the experts 'smooth' and 'rough', respectively. We assume that $\lambda_{32} = \lambda_{21} = \lambda_{10} = 3e - 3/s$.

The simplified scheme of the pneumatic valve is shown in Fig. 4. It is a normally closed, gas-actuated valve with a linear cylinder actuator.

By regulating the pressure of the pneumatic ports to fill or evacuate the top and bottom chambers, the position of the piston can be controlled. A return spring is linked with the piston to ensure the closure of the valve, when pressure is lost. The

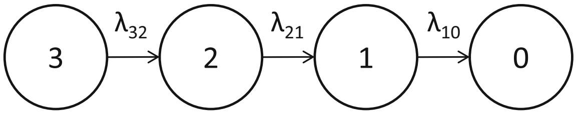

Fig. 3 Degradation process of the pump

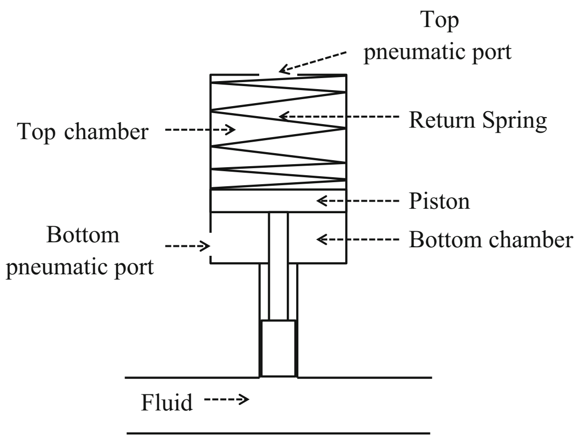

Fig. 4 Simplified scheme of the pneumatic valve [9]

external leak at the actuator connections to the bottom pneumatic port due to corrosion and other environmental factors is chosen as the degradation mechanism of the valve, which is much more significant than the other degradation mechanisms according to the results shown in [6].

Let $D_b(t)$ denote the area of the leak hole at the bottom pneumatic port at time t, the development of the leak size is described by:

$$\dot{D}_b(t) = \omega_b\left(1 + \beta_{Y_p(t)}\right) \tag{11}$$

where $\omega_b = 1e - 8m^2/s$ is the original wear coefficient and where $\beta_{Y_p(t)}$ is the relative increment of the developing rate of the external leak at the bottom pneumatic port, caused by the vibration of the pump at degradation state '2' or '1'. We assume that $\beta_2 = 10\%$ and $\beta_1 = 20\%$.

The leak will lead the valve to be more difficult to open but easier to close than in case without leak. The threshold of the area of the leak hole $D_b^* = 1.06\mathrm{e} - 5m^2$ is defined as the value above which $(D_b(t) > D_b^*)$ the valve cannot reach the fully open position from the fully closed position, within the 15 s time limit, after an opening command is executed.

The degradation processes affecting the system are modeled by PDMP as follows:

$$Z(t) = \begin{pmatrix} D_b(t) \\ Y_p(t) \end{pmatrix} \in \mathbb{R}^+ \times S_p \tag{12}$$

where $Y_p(t)$ denotes the degradation state of the pump at time t and $D_b(t)$ denotes the area of the leak hole at the bottom pneumatic port of the valve at time t. The space of the failure states of $Z(t)$ is $\mathcal{F} = [0, +\infty) \times \{'0'\} \cup \left[D_b^*, +\infty\right) \times \{1, 2, 3\}$.

The initial state of the system is assumed as follows:

$$Z_0 = \begin{pmatrix} D_b(0) \\ Y_p(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{13}$$

which means that the two components are both in their perfect states. The initial probability distribution of the processes $\left(D_b(t), Y_p(t)\right)_{t \geq 0}$, $p_0(dz|\boldsymbol{\theta})$, hence, equals to $\delta_{Z_0}(dz)$, where δ is the Dirac delta function.

We perform MC simulation for the estimation of the system reliability over a time horizon of $T_{miss} = 1000$ s. The results of 10^6 trials are shown in Fig. 5. We can see from the Figure that the system reliability decreases more rapidly after around 885 s, because at that time the valve could fail, corresponding to the situation when the pump jumps to the state '1' very quickly and stays there until the valve fails.

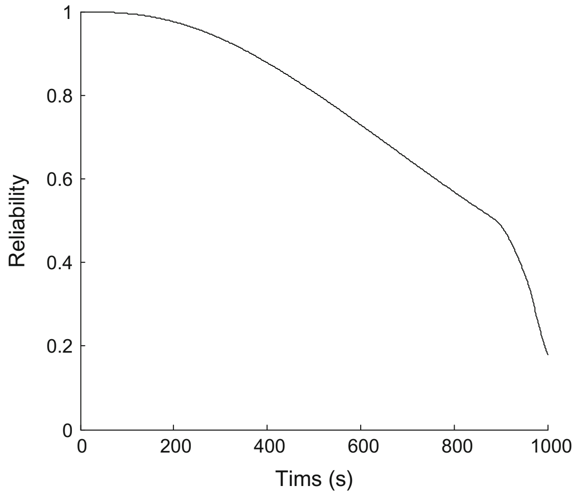

Fig. 5 Estimated system reliability

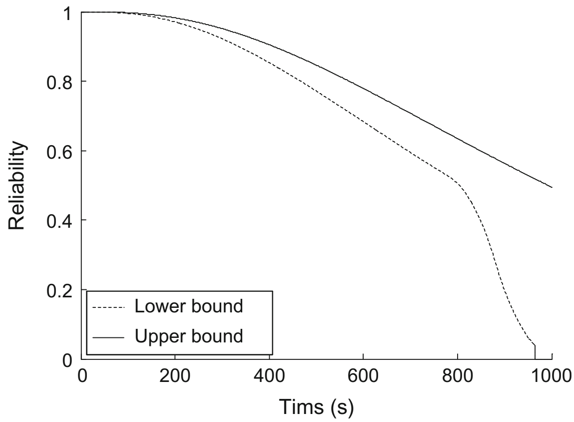

Fig. 6 Estimated system reliability in consideration of uncertainty

We further consider a relative uncertainty of ±10% of the original parameters values. In this case study, higher parameters values lead to rapider degradation development and lower system reliability. The results of 10^6 trials are shown in Fig. 6. The lower bound of the system reliability decreases more sharply after around 790 s. It is seen that the system fails after around 964 s, because at that time the valve is completely failed. The upper bound of the system reliability does not experience a rapid decrease because the valve is mostly functioning over the time horizon.

5 Conclusion

We have illustrated a PDMP modeling approach for modeling multiple, dependent, competing degradation processes. The significance of the proposed method lies in its capability to describe the degradation dependence. A MC simulation algorithm for the system reliability assessment has been designed and an example from a real industrial system has been used to illustrate the capabilities of the modeling and simulation framework.

Limitation of the MC simulation lies in the computational burden. As future work, we plan to study acceleration techniques to improve computation efficiency, thus, enabling to extend the applications to systems of larger sizes.

References

1. Black M, Brint A, Brailsford J (2005) A semi-Markov approach for modelling asset deterioration. J Oper Res Soc 56(11):1241–1249
2. Chen N, Ye ZS, Xiang Y, Zhang L (2015) Condition-based maintenance using the inverse Gaussian degradation model. Eur J Oper Res 243(1):190–199
3. Chookah M, Nuhi M, Modarres M (2011) A probabilistic physics-of-failure model for prognostic health management of structures subject to pitting and corrosion-fatigue. Reliab Eng Syst Saf 96(12):1601–1610
4. Cocozza-Thivent C (2011) Processus de renouvellement markovien, Processus de Markov déterministes par morceaux. In: Online book. http://perso-math.univ-mlv.fr/users/cocozza.christiane/recherche-pageperso/PresentationRMetPDMP.html
5. Daigle M, Goebel K (2011) Multiple damage progression paths in model-based prognostics. In: Aerospace conference, 2011. IEEE, pp 1–10
6. Daigle M, Goebel K (2011) A model-based prognostics approach applied to pneumatic valves. Int J Prognostics Health Manage 2:84–99
7. Davis M (1984) Piecewise-deterministic Markov processes: a general class of non-diffusion stochastic models. J R Stat Soc Ser B (Methodol) 46(3):353–388
8. Davis M (1993) Markov models & optimization vol 49. CRC Press
9. Gebraeel N, Elwany A, Pan J (2009) Residual life predictions in the absence of prior degradation knowledge. IEEE Trans Reliab 58(1):106–117
10. Giorgio M, Guida M, Pulcini G (2011) An age-and state-dependent Markov model for degradation processes. IIE Trans 43(9):621–632
11. Jacobsen M (2006) Point process theory and applications: marked point and piecewise deterministic processes. Birkhäuser, Boston
12. Jiang L, Feng Q, Coit D (2012) Reliability and maintenance modeling for dependent competing failure processes with shifting failure thresholds. IEEE Trans Reliab 61(4):932–948
13. Keedy E, Feng Q (2012) A physics-of-failure based reliability and maintenance modeling framework for stent deployment and operation. Reliab Eng Syst Saf 103:94–101
14. Lawless J, Crowder M (2001) 4) Covariates and random effects in a gamma process model with application to degradation and failure. Lifetime Data Anal 10(3):213–227
15. Lee WS, Grosh D, Tillman F, Lie C (1985) Fault tree analysis, methods, and applications: a review. IEEE Trans Reliab 34(3):194–203
16. Lewis E, Böhm F (1984) Monte Carlo simulation of Markov unreliability models. Nucl Eng Des 77(1):49–62

17. Lin YH, Li Y, Zio E (2014) Dynamic reliability models for multiple dependent competing degradation processes. In: Proceedings of ESREL 2014
18. Li W, Pham H (2005) Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks. IEEE Trans Reliab 54(2):297–303
19. Lisnianski A, Frenkel I, Ding Y (2010) Multi-state system reliability analysis and optimization for engineers and industrial managers. Springer, London
20. Lorton A, Fouladirad M, Grall A (2013) A methodology for probabilistic model-based prognosis. Eur J Oper Res 225:443–454
21. Lu C, Meeker W (1993) Using degradation measures to estimate a time-to-failure distribution. Technometrics 35:161–174
22. Marseguerra M, Zio E (1996) Monte Carlo approach to PSA for dynamic process systems. Reliab Eng Syst Saf 52:227–241
23. Moghaddass R, Zuo M (2014) Multistate degradation and supervised estimation methods for a condition-monitored device. IIE Trans 46:131–148
24. Moussou P, Cambier S, Lachene D, Longarini S, Paulhiac L, Villouvier V (2001) Vibration investigation of a French PWR power plant piping system caused by cavitating butterfly valves. ASME Publications-PVP 420:99–106
25. Peng H, Feng Q, Coit D (2010) Reliability and maintenance modeling for systems subject to multiple dependent competing failure processes. IIE Trans 43:12–22
26. Rasmekomen N, Parlikad A (2013) Maintenance optimization for asset systems with dependent performance degradation. IEEE Trans Reliab 62:362–367
27. Reggiani S, Poli S, Denison M, Gnani E, Gnudi A, Baccarani G, Pendharkar S, Wise R (2011) Physics-based analytical model for HCS degradation in STI-LDMOS transistors. IEEE Trans Electron Dev 58:3072–3080
28. Straub D (2009) Stochastic modeling of deterioration processes through dynamic Bayesian networks. J Eng Mech 135:1089–1099
29. Wang Y, Pham H (2012) Modeling the dependent competing risks with multiple degradation processes and random shock using time-varying copulas. IEEE Trans Reliab 61:13–22
30. Yang G (2002) Environmental-stress-screening using degradation measurements. IEEE Trans Reliab 51:288–293
31. Ye ZS, Xie M (2014) Stochastic modelling and analysis of degradation for highly reliable products. Appl Stoch Models Bus Ind 31:16–32
32. Zhang S, Hodkiewicz M, Ma L, Mathew J (2006) Machinery condition prognosis using multivariate analysis. In: Engineering Asset Management. Springer, pp 847–854
33. Zwillinger D (1998) Handbook of differential equations. Academic Press